Hybrid simulation with multiple actuators: a state-of-the-art review

Amirali Najafi[a], Gaston A. Fermandois[b,*], Shirley J. Dyke[c], and Billie F. Spencer, Jr.[d]

[a] *Center for Advanced Infrastructure and Transportation, Rutgers University, Piscataway, NJ, USA*

[b] *Departamento de Obras Civiles, Universidad Técnica Federico Santa María, Santiago, Chile*

[c] *School of Mechanical Engineering, Purdue University, West Lafayette, IN, USA*

[d] *Department of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, Urbana, IL, USA*

[*] *Correspondence: gaston.fermandois@usm.cl*

Abstract

This paper reviews the conceptual and technical advances in multi-actuator dynamic loading in modern structural testing. In particular, a focus is given to the developments and challenges in multi-axial hybrid simulation (maHS) and multi-axial real-time hybrid simulation (maRTHS), where a specimen is subjected to multi-directional dynamic loading by interacting with a numerical simulation of its surrounding structural subsystems and components. This review introduces the general framework for maHS and maRTHS, describing substructuring techniques, loading equipment, and nonlinear kinematics. In particular, the process of dynamic compensation for multi-actuator loading assemblies in maRTHS is explored. Different compensation architectures in the task (Cartesian) and joint (actuator) spaces are covered, and each alternative is assessed on its own merits for dynamic synchronization of multi-actuator loading platforms. Finally, current challenges in maHS and maRTHS testing are identified, with recommendations for future research endeavors for the scientific community.

Keywords: Hybrid simulation; structural testing; substructuring; multiple actuators; nonlinear kinematics; specimen-actuator interaction; dynamic compensation.

Contents

Terminology

Boundary conditions physical interfaces between experimental and numerical substructures enforced by actuators.

Actuator compensation algorithm intended to minimize the synchronization error between target and measurement signals from an actuator.

Coupled compensation every actuator's control signal is determined by feedback from all other actuators. Also known as centralized or multi-input multi-output (MIMO) compensation.

Decoupled compensation every actuator's control signal is determined by feedback from itself. Also known as decentralized or single-input single-output (SISO) compensation.

Parallel manipulator actuated system where the ends of all actuators are connected to a rigid platform, creating a kinematic loop.

Substructuring process of simulating the dynamics of a mechanical system by means of analyzing the sum of its constituents.

1 Introduction

Experimental testing is a fundamental step in the development of innovative, sustainable, and reliable materials and structural systems. The predominant structural test methods employed have been: (1) quasi-static testing, where a cyclic trajectory often with increasing amplitude is imposed at slow (i.e., near static) speeds on a physical specimen to identify the nonlinear hysteretic behaviors under load reversals; (2) shake table testing, for identifying the behavior of a complete structure through the application of base motion; and (3) hybrid simulation (also called pseudo-dynamic testing, dynamic virtualization, and hardware-in-the-loop testing), where the behavior of a complete structure is simulated via the interaction of numerical modeling and experimental testing [1, 2].

The response of a structural component is a function of the loading history it has experienced and the boundary conditions with the greater structural system. Thus, hybrid simulation (HS) was proposed as an alternative to quasi-static testing, which is capable of incorporating system-level interactions with realistic excitations [3, 4, 5, 6]. HS is a versatile methodology that addresses many of the limitations with other test methods. For example, quasi-static testing often employs simplistic cyclic trajectories which are not entirely representative of the behaviors experienced by a structural element under environmental loading. Also, shake table testing is limited by the equipment available to test an entire floor plan with a base excitation. Shake tables have size and payload limitations, and often the consequence is testing of scaled structures. With HS, only the structural elements of interest are experimentally tested and the excitation can be applied with more flexibility through different actuator configurations. Although size and capacity limitations continue to exist with HS, a wider range of experiments are possible in a wider range of labs.

Real-time hybrid simulation (RTHS) is a variation of hybrid simulation, where the simulation has real-time constraints, thus enabling the study of physical specimens with rate-dependent behaviors [7]. Whether or not real-time testing is possible depends on the availability of dynamically-rated actuators and real-time computational resources. Servo-hydraulic actuators saw vast growth due to the need to simulate realistic flight conditions with the onset of the space age in the 1950s and 1960s [8]. At that time, dynamic structural testing became possible due to improvements in servo-valve technology, higher flow capacity, resonant load stabilization, and static compensation for structural compliance [9]. The exponential growth in computational capabilities combined with the diminishing costs also played a critical role in realizing the first RTHS tests in the 90s and various more sophisticated implementations since [10].

The choice between slow speed and real-time tests also depends on the rate-dependence of the materials and structures under consideration, the natural frequency of the structure, and the characteristics of the structural loading. For instance, a stiff structure (i.e, having large natural frequencies) and an excitation with a high frequency content may experience strain-rate induced increases in capacity. Some studies have explored the dependence of common building materials (e.g., steel and concrete) to the rate of loading [11, 12, 13]. Many studies have reported negligible rate-dependent findings in common structural materials [14, 15, 16]. The discussion on the need for real-time testing of common building materials is not settled. Nonetheless, dampers, isolation systems, and many modern materials are rate-dependent [17, 18].

Other external factors may influence the consideration between HS and RTHS. In the hybrid fire test conducted by [19], strain rate is not high but the rate of temperature increase is quite rapid. Therefore, the experiment had to be conducted in real-time to ensure the temperature gradient in the physical specimen is realistic.

Hybrid simulation researchers have considered many extensions to the original technique, including the use of multiple actuators in conjunction for higher loading capacity and to prescribe displacements over a physical specimen at more than one degree-of-freedom (DOF). The authors have identified several literature reviews pertaining to these expansions which discuss the general framework of slow speed and real-time methodologies and the variants of the dynamic substructuring concept [20, 21, 22, 23, 24, 25]. However, a review of the developments with multiple actuators coupled through a continuum body was not identified.

This review article provides an updated perspective on the various contributions in HS and

RTHS with multi-actuator loading. In Section 2, a general framework for multi-actuator hybrid simulation is described including developments made in actuator compensation and kinematic transformations. In Section 3, noteworthy classes of multi-actuator devices for structural testing are listed, including shake tables, boundary condition devices, and shell element testers. Section 4 and 5 are devoted to developments in multi-axial hybrid simulation (maHS) and multi-axial real-time hybrid simulation (maRTHS), respectively. Section 5 describes multi-actuator RTHS developments which operate in single-axis configurations. Lastly, Section 6 highlights many of the current challenges with multi-actuator loading and suggests research avenues for the maHS/maRTHS community to explore.

2 General framework for multi-actuator loading

In this section, the general framework and technical prerequisites for multi-actuator HS and RTHS are discussed, and the two variations are distinguished from one another. The procedure for maHS and maRTHS can be simplified into four tasks: (1) simulation of the numerical substructure subject to external loading (e.g., ground motion); (2) imposition of displacements and forces at the boundary interface between the numerical and experimental substructures through a multi-actuator loading assembly; (3) direct measurement of experimental substructure response; and (4) feedback of measured experimental responses to the numerical substructure to close the hybrid simulation loop. The framework discussed herein is the foundation upon which many of the developments in multi-actuator hybrid simulation rest, and will help in explaining many of the references discussed in this review.

2.1 Substructuring for hybrid testing

Consider a system of second-order differential equations (i.e., equation of motion, EOM) used to represent the dynamics of a reference structure in a domain Ω:

$$\Omega : \quad \mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{R}(\mathbf{x}, \dot{\mathbf{x}}) = \mathbf{p}(t) \tag{1}$$

where the vectors $\mathbf{x}(t) \in \mathbb{R}^n$, $\dot{\mathbf{x}}(t) \in \mathbb{R}^n$, and $\ddot{\mathbf{x}}(t) \in \mathbb{R}^n$ represent the displacement, velocity, and acceleration vectors relative to the ground floor, respectively. $\mathbf{M} \in \mathbb{R}^{n\times n}$ and $\mathbf{C} \in \mathbb{R}^{n\times n}$ are the mass and damping matrices, respectively. The damping matrix is representative of the various frictional and dissipative mechanisms that exist in structures. Because damping is a

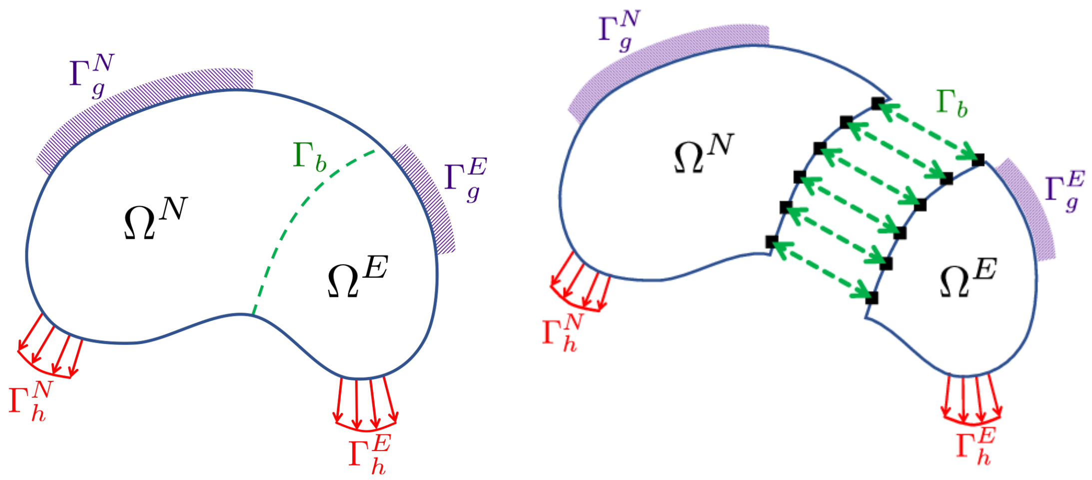


Figure 1: Substructuring of dynamical system

difficult phenomenon to model, it is customary to assume the damping matrix as proportional to the mass and stiffness matrices [26]. $\mathbf{R}(\mathbf{x}, \dot{\mathbf{x}}) \in \mathbb{R}^n$ is the vector of restoring forces, which is as a function of states $\{\mathbf{x}, \dot{\mathbf{x}}\}$. Finally, $\mathbf{p}(t) \in \mathbb{R}^n$ is the total load vector. Note that the time t is the load time, while the hybrid testing process may actually occur on an extended time scale.

Instead of solving the equations pertaining to the entire reference structure, a process known as *substructuring* is performed to subdivide it into smaller substructures, shown in Fig. 1. These equations can be solved independently, provided that the coupling between components is enforced by means of compatibility and equilibrium conditions at their boundary conditions [20, 27]. Then, a reference structure can be defined as the union of the two smaller substructures, $\Omega = \Omega^N \cup \Omega^E$, where Ω^N and Ω^E are the domains of numerical and experimental substructures, respectively. Each substructure has its own DOFs and boundaries. Let the displacement vector of the associated numerical and experimental substructures be defined as:

$$\mathbf{x}^N = \begin{Bmatrix} \mathbf{x}_i^N \\ \mathbf{x}_b^N \end{Bmatrix}, \quad \mathbf{x}^E = \begin{Bmatrix} \mathbf{x}_i^E \\ \mathbf{x}_b^E \end{Bmatrix} \tag{2}$$

where the superscripts N and E refer to the numerical and experimental substructures, respectively; and subscripts i and b refer to the interior and boundary DOFs, respectively, as shown in Fig. 2.

Then, the coupled EOM for both numerical and experimental substructures are expressed as follows:

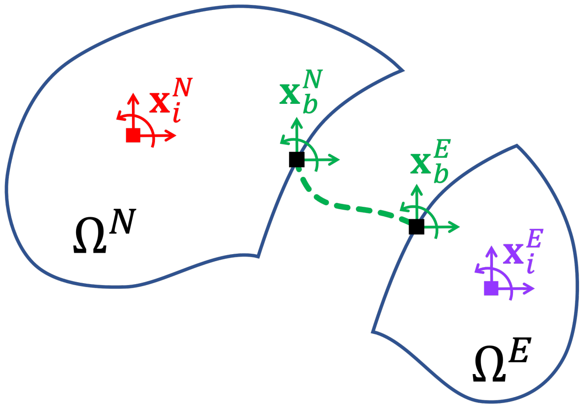


Figure 2: Degrees-of-freedom (DOF) of numerical (Ω^N) and experimental (Ω^E) substructures

$$\Omega^N: \qquad \mathbf{M}^N\ddot{\mathbf{x}}^N + \mathbf{C}^N\dot{\mathbf{x}}^N + \mathbf{R}^N(\mathbf{x}^N, \dot{\mathbf{x}}^N) = \mathbf{p}^N + \mathbf{g}^N \tag{3}$$

$$\Omega^E: \qquad \mathbf{M}^E\ddot{\mathbf{x}}^E + \mathbf{C}^E\dot{\mathbf{x}}^E + \mathbf{R}^E(\mathbf{x}^E, \dot{\mathbf{x}}^E) = \mathbf{p}^E + \mathbf{g}^E \tag{4}$$

and the coupling force vector applied over each substructure is defined by:

$$\mathbf{g}^N = \begin{Bmatrix} \mathbf{g}_i^N \\ \mathbf{g}_b^N \end{Bmatrix}, \quad \mathbf{g}^E = \begin{Bmatrix} \mathbf{g}_i^E \\ \mathbf{g}_b^E \end{Bmatrix} \tag{5}$$

The main assumption in this formulation is that the substructures are only coupled through the boundary Γ_b. Therefore, the coupling forces at interior DOFs for each substructure should be equal to zero:

$$\mathbf{g}_i^N = \mathbf{0}_i^N, \qquad \mathbf{g}_i^E = \mathbf{0}_i^E \tag{6}$$

To solve this coupled problem, both displacement compatibility and force equilibrium conditions must be satisfied:

$$\text{(Displacement compatibility):} \qquad \mathbf{x}_b^N = \mathbf{x}_b^E \tag{7}$$

$$\text{(Force equilibrium):} \qquad \mathbf{g}_b^N + \mathbf{g}_b^E = \mathbf{0}_b \tag{8}$$

Therefore, by substituting (8) and (5) into (3), the following "coupled" numerical substructure EOM is obtained:

$$\mathbf{M}^N\ddot{\mathbf{x}}^N + \mathbf{C}^N\dot{\mathbf{x}}^N + \mathbf{R}^N = \mathbf{p}^N + \begin{Bmatrix} \mathbf{0}_i^N \\ -\mathbf{g}_b^E \end{Bmatrix} \tag{9}$$

where $\mathbf{g}_b^E$ is the coupling force vector from the experimental component, which includes all the effects associated with nonlinear restoring forces, nonlinear damping, and inertial forces, along with any external excitation that can be induced directly to the experimental substructure. In slow speed experiments, rate-dependent damping and inertial forces are ignored. The result is:

$$\mathbf{g}^E = \begin{Bmatrix} \mathbf{0}_i^E \\ \mathbf{g}_b^E \end{Bmatrix} = \mathbf{M}^E\ddot{\mathbf{x}}^E + \mathbf{C}^E\dot{\mathbf{x}}^E + \mathbf{R}^E - \mathbf{p}^E \tag{10}$$

while noting that the coupling vector $\mathbf{g}_b^E$ is a function of displacement vector $\mathbf{x}_b^N$ to satisfy (7):

$$\mathbf{x}^E = \begin{Bmatrix} \mathbf{x}_i^E \\ \mathbf{x}_b^E = \mathbf{x}_b^N \end{Bmatrix} \tag{11}$$

To obtain an admissible solution, compatibility and equilibrium must be satisfied for all boundary DOFs at all times. Therefore, an algorithm should be considered to prescribe displacements and/or forces at the boundary Γ_b for the solution of the dynamical system. Three different classes of algorithms are found in the literature:

Displacement-based After solving the EOM (9) of the numerical substructure Ω^N through a time-stepping integration algorithm, the output $\mathbf{x}_b^N$ is commanded to the experimental substructure Ω^E for execution by actuator(s) to satisfy displacement compatibility at the boundary Γ_b. Displacement transducer(s) ensure that the command is achieved. Then, the coupling force $\mathbf{g}_b^E$ is measured directly from the test specimen after displacement-controlled loading, using load cell sensors in a laboratory facility, and this measured output is inserted back into the numerical substructure Ω^N, to satisfy the equilibrium condition at the boundary Γ_b. This "hybrid loop" procedure is repeated until the simulation reaches the final simulation time.

Force-based Similar to displacement-based, the EOM of the numerical substructure Ω^N is solved, but now the coupling force $\mathbf{g}_b^N$ is calculated and commanded to the experimental substructure Ω^E for execution by actuator(s). Load cell(s) ensure that the desired coupling force is achieved. Then, the displacement $\mathbf{x}_b^E$ is measured directly from the test specimen after force-controlled loading, and is fed back into the numerical substructure Ω^N, to satisfy displacement compatibility at the boundary condition Γ_b.

Mixed-mode Also called displacement-force control, this approach consists of calculating a set of displacements $\mathbf{x}_b^N$ and coupling forces $\mathbf{g}_b^N$ from the numerical substructure to be enforced on the experimental substructure simultaneously, satisfying compatibility and equilibrium over the boundary Γ_b.

In the context of hybrid simulation, a structural component of interest is usually selected from the reference structure to become the experimental substructure (i.e., physical specimen), as illustrated in Fig. 3. The choice for the experimental substructure can vary based on the research problem under consideration. But generally, the experimental substructure is comprised of elements with large uncertainty, or are expected to show a nonlinear response, for which appropriate models are not available, or for designs and materials that are perhaps new technologies and require further study.

As an illustrative example, consider a typical n-story shear building subjected to arbitrary excitation in the form of external forces $\mathbf{F}(t)$, and ground excitation $\ddot{x}_g(t)$. This reference structure may have any number of DOFs for added complexity and realism, but for the sake of establishing the abstract concepts for substructuring of an EOM, only the lateral DOFs are shown. In this case, the total load vector $\mathbf{p}(t)$ is defined as:

$$\mathbf{p}(t) = -\mathbf{M}\iota\ddot{x}_g(t) + \mathbf{F}(t) \tag{12}$$

where $\iota \in R^n$ is an inertial influence vector.

Here, the numerical substructure is assumed to behave elastically for simplicity, with a stiffness matrix $\mathbf{K}^N$, and restoring force $\mathbf{R}^N(\mathbf{x}^N) = \mathbf{K}^N\mathbf{x}^N$. The boundary point between the numerical and experimental substructure is indicated at the locations of the mass m_1. Following a displacement-based algorithm, the boundary condition between the numerical and experimental substructures is indicated with the DOFs $x_1^N(t)$ and $x_1^E(t)$, respectively. In an ideal world, the boundary condition calculated through integration of the numerical substructure would be perfectly executed via actuators located at the boundary with the experimental substructure with $x_1^N(t) = x_1^E(t)$ (displacement compatibility). Actuation of the physical specimen in the experimental substructure results in the generation of forces, measured by load cells. These coupling forces are returned to the numerical substructure as feedback forces, as illustrated in

Figure 3.

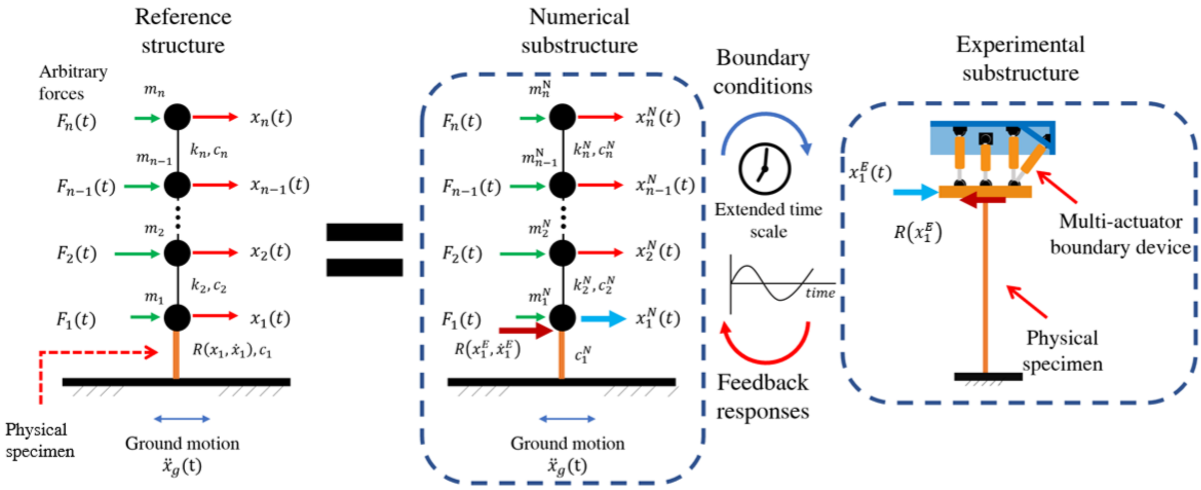


Figure 3: Conventional (slow-speed) hybrid simulation

In conventional (i.e., slow speed) hybrid simulation (HS) applications, the boundary conditions are imposed on the physical specimen over an extended time scale. As a result, velocity- and acceleration-dependent forces (i.e., damping and inertia) of the physical specimen are not acquired experimentally and must instead be modeled numerically. The feedback forces measured in conventional HS, $R(x_1^E)$, are therefore only comprised of experimental restoring forces. Meanwhile, in RTHS applications, dynamic effects are included because the boundary conditions are imposed on the physical specimen at the real time according to the input excitation. Therefore, specimen inertial and damping forces are automatically incorporated into the feedback forces. The inertial component of the experimental specimen must be removed from the numerical structure, as shown in Fig. 4.

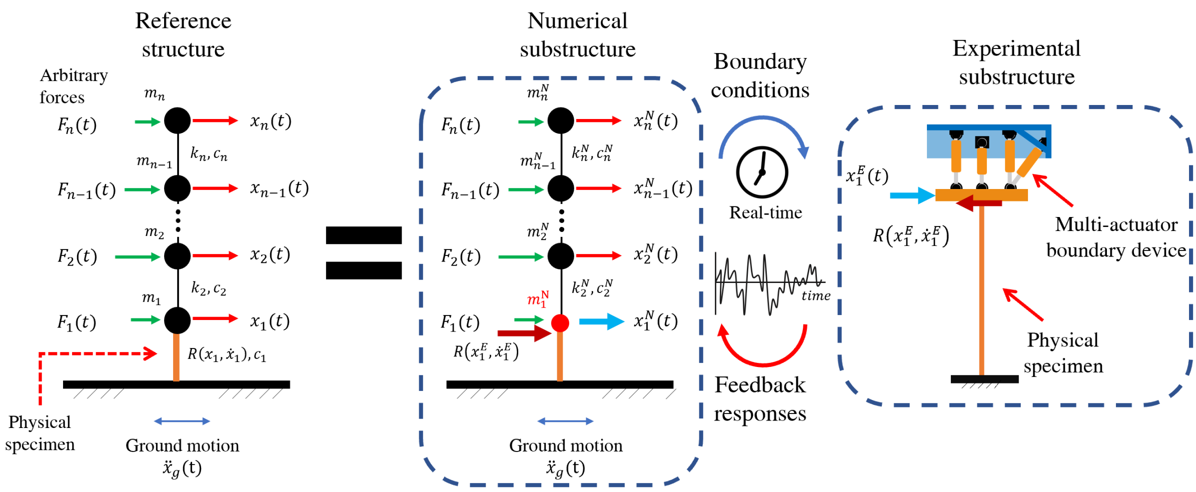


Figure 4: Real-time hybrid simulation

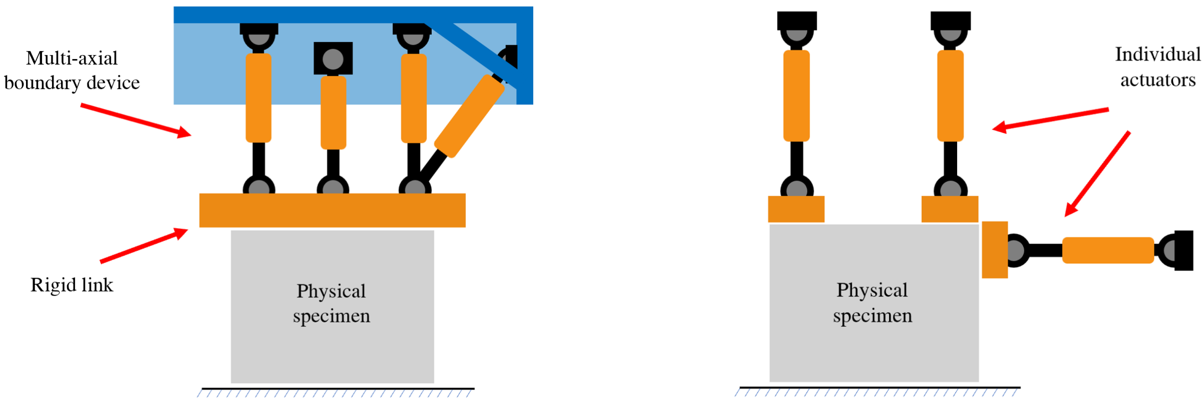
It is worth mentioning that other substructuring techniques have been used for hybrid simulation testing, such as overlapping methods [28], where the substructures are overlapping by more than the boundary nodes and can also share redundant elements. This overlapping technique is conceived for the main purpose of alleviating the requirements on the number of actuators at the boundary of experimental subassemblies. Similarly, [29] developed a weakly-coupled HS where two DOFs of the physical specimen are measured experimental and one DOF is obtained numerically.Also, switch control [30, 31] has been developed to command forces in addition to displacements over the experimental substructure. The UT-SIM [32] is a generalized distributed data exchange and communication protocol framework that integrates numerous numerical analysis software and experimental testing equipment.

Certain multi-actuator devices have been developed for properly imposing the boundary conditions to perform more complex hybrid simulations. These devices can be classified as (i) multi-axial boundary devices, and (ii) individually attached actuators to a common physical specimen, as illustrated in Fig. 5. In nearly all actuator setups, actuator dynamics will affect trajectory tracking and stability of the hybrid simulation. In addition, actuators linked through a stiff continuum (e.g., test specimen and/or loading fixtures) tend to be mechanically coupled with forces in one actuator resulting forces in all other actuators. Control algorithms are typically required in multi-actuator HS to satisfy synchronization between substructures, and in multi-actuator RTHS to stabilize and tackle the dynamics inherent to actuators, as well as the mechanical coupling between the actuators [33]. Successful operation of multi-actuator devices also requires a mathematical understanding of the geometry of the motions, also commonly known as the *kinematics*. Kinematic transformation algorithms capture the mapping needed between each actuator and a Cartesian frames of reference.

2.2 Tracking algorithms in multi-actuator hybrid simulation

Tracking algorithms are mathematical formulations that help an actuator execute a command displacement in a stable and timely manner. Studies have indicated addition of delays to the closed-loop system in experiments where stiffness is dominant [34, 35], and addition of leads where inertial forces are dominant [36]. The dynamics of actuators are considerably different between slow speed and real-time hybrid tests, and so are the tasks of compensating.

In (conventional) HS, actuator displacements are typically applied in a repeated pattern of slow ramp loading and pausing. A target displacement is first calculated by the numerical



(a) Multi-axial boundary device

(b) Individually attached actuators

Figure 5: Multi-actuator configurations

substructure. Next, a controller generates a ramp-shaped command signal for the actuators to execute the target. The actuator is paused once the target trajectory is achieved and restoring forces are measured. This process is repeated for all other time steps [37, 38, 39, 40, 41]. Ramp-hold algorithms may be insufficient however for compensating strong actuator coupling in multi-actuator platforms.

Displacement- and *force-control* are two types of commonly employed compensators in multi-actuator HS. Displacement-control is more appropriate for DOFs requiring a large actuator stroke and small specimen stiffness [42, 43, 44, 45]. DOFs with high stiffness (i.e., large force and small displacement variations) are best compensated with force-control [46]. *Mixed-control* is the combined use of both control methodologies. In mixed-control, the translations and rotations for lateral DOFs are compensated using displacement-control, and highly stiff axial DOFs are compensated using force-control [47, 30, 48, 49, 50, 51].

For an RTHS, response inaccuracies and instabilities will result, unless appropriate steps are taken toward compensating for the coupling between the actuators and dynamics of the multi-actuator device. The open-loop behavior of a servo-hydraulic actuator is inherently unstable. For the purpose of stabilization, the dynamics of a servo-hydraulic actuator is typically first stabilized using a proportional-integral-derivative (PID) compensator, known as an *inner-loop* [52]. Additional compensation techniques then take the form of outer-loops, which aim to achieve accurate target tracking. In the context of the RTHS example in Fig. 4, accurate tracking means $x_1^E(t) \rightarrow x_1^N(t)$ in a finite time, where $x_1^N(t)$ is the target boundary condition calculated from the numerical substructure, and $x_1^E(t)$ is the experimental realization (i.e., measurements) of the boundary condition.

The majority of compensation algorithms used in RTHS today are based on displacement-control. *Decoupled* and *coupled* control are the two types of real-time control used for multi-actuator devices. Decoupled control refers to the case when individual actuators are treated as single-input, single-output (SISO) systems, and are compensated for independently. Decoupled controllers are easier to design, optimize, and have been widely used throughout the literature [53, 54, 55, 56, 57, 58, 59, 60]. Such controllers may have limitations in experiments where the coupling between the actuators is large, possibly due to presence of a very stiff physical specimen. Coupled controllers treat actuators as multi-input, multi-output (MIMO) systems, and compensate for the system-wide actuator dynamics [61, 62, 63]. These controllers are challenging to optimize for ideal stability and tracking behavior, due to the large number of parameters that require tuning. Other forms of RTHS compensation are summarized as: mixed-control [17], and acceleration-control [64, 65, 66]. The stiffness of the physical continuum that connects the actuators largely determines the extent of the mechanical coupling in multi-actuator devices. The literature listed have tackled application-specific coupling challenges. However, a generalizable solution for realistic RTHS performance and stability does not exist to-date. [67] developed a predictive indicator focused on assessing the stability of MDOF RTHS to use as a design tool. [68] provided a sufficient condition for stability of RTHS with multiple actuators, by employing the small gain theorem [69]. [70] found that analytical stability indicators are not accurate for discrete systems.[71] investigates the critical time delay in multi-DOF RTHS systems using the root locus technique. Usefulness of analysis models for predicting stability and performance in experimental RTHS are highlighted.

2.3 Kinematics of multi-actuator loading assemblies

Kinematics refers to the mathematical operations that describe the geometry of motion and forces in robotic assemblies with respect to time. The kinematic transformation of multi-actuator systems must be well-understood for successful use of these devices in hybrid simulation. There are two types of kinematic transforms: *Forward* and *Inverse*. Forward kinematic transform considers the strokes in each individual actuator and sensing device (e.g., displacement transducer) for deriving the position and orientation of the Cartesian boundary conditions and forces. Inverse kinematic transformation uses the available information about the desired positions of the boundary conditions to calculate what the strokes of individual actuators must be.

Kinematic relationships are mathematically expressible via nonlinear equations. Solutions to the kinematic equations can be approximated for a finite range of motion, solved iteratively, or solved directly. [42] introduces a kinematic transformation matrix for performing a bi-directional HS. [43], [47], and [72] extend the prior development for handling of the geometric nonlinearities of a planer actuator setup. [44] presents two non-iterative kinematic transformation algorithms: linearized transformation for approximations, and nonlinear transformation which yield exact results. These approaches can be applied to real-time problems. [73] introduces an online iterative kinematic scheme for ensuring multi-actuator systems achieve a desired Cartesian motion. For real-time tests, iterative solutions however are not applicable. [63] and [60] present real-time kinematic transform methods based on direct and linearized approximations of the kinematic equations of motion.

A brief mathematical summary of kinematic transformations are provided next for the convenience of the reader. A typical multi-actuator boundary device is comprised of several servo-hydraulic prismatic actuators moving a single highly stiff platform. These devices are known as *parallel manipulators*, and possess large load-carrying capacities due to the load sharing ability of the actuators. This quality is attractive in structural testing applications due to the large forces desired. A schematic of a generalized parallel manipulator is presented in Fig. 6.

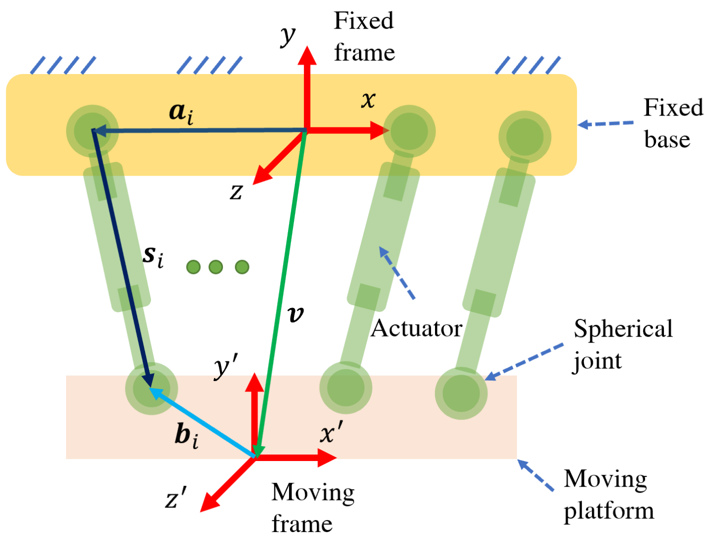


Figure 6: n-DOF parallel manipulator

A kinematic analysis of a parallel manipulator requires evaluation of the actuator environment as a vector space. A Cartesian fixed frame of reference is typically selected in an arbitrary position, and a moving frame of reference is selected on the moving platform. In many experiments, the location of choice for the moving frame can be the centroid of the attachment with the physical specimen. The linear strokes of the prismatic limbs (e.g., actuators) result in displacements and rotations of the moving platform.

With the frames of reference and parallel manipulator components visualized, the focus will next shift to kinematic transforms. Following the simulation of numerical substructures, to obtain the target Cartesian motion, an inverse kinematic transform calculates the stroke of each actuator. The vectors $\boldsymbol{a}_i$, $\boldsymbol{b}_i$, $\boldsymbol{s}_i$ and $\boldsymbol{v} \in \mathbb{R}^3$, describe the position of the fixed and moving frames of reference, vector stroke of the actuators, and total translational vector, respectively. [52] introduces a sensitivity-based calibration method for multi-actuator devices based on external measurements. Incorporating a rotational matrix $\boldsymbol{A} \in \mathbb{R}^{3\times3}$, the three-dimensional forward kinematics of the actuated assemblies are:

$$\boldsymbol{s}_i = \boldsymbol{v} + \boldsymbol{A}\boldsymbol{b}_i - \boldsymbol{a}_i \tag{13}$$

$$|\boldsymbol{s}_i| = f(\boldsymbol{\omega}) \tag{14}$$

where $|\boldsymbol{s}_i|$ is the absolute length of actuator i for some target Cartesian motion of a moving frame of reference $\boldsymbol{\omega} = (x, y, z, \theta_x, \theta_y, \theta_z)^T$. The above derivations are based on an assumption that the load transfer elements (e.g., fixed and moving platforms) are rigid.

Actuator and sensor coordinate measurements are converted to Cartesian coordinates via the forward kinematic transform. In parallel manipulators, the forward transform is a challenging task that involves solving several implicit nonlinear equations per Eq. 15. Solutions to these equations can be achieved iteratively for HS or approximated using Taylor expansion around an stationary operation point for RTHS.

$$\boldsymbol{\omega} = f^{-1}(|\boldsymbol{s}|) \tag{15}$$

Regardless, compliance and slippage of multi-actuator connectors can induce wrong estimates of true Cartesian coordinates of the moving platform. Thus, some studies have proposed solutions to forward kinematics including these undesired effects. [74] proposed an online correction method that adjusts Cartesian displacement commands by minimizing Cartesian displacement errors through optimization methods. [75] proposes a model-based adaptive kinematic method where the elastic deformations of connectors are included in a system model which is employed to compensate for estimation errors of Cartesian coordinates from actuator coordinate measurements.

Finally, the kinematic transformation procedures described herein are generalizable for multi-actuator setups with individually attached actuators per Fig. 5(b).

3 Structural testing with multi-actuator devices

This section explores many of the multi-actuator devices around the world dedicated to structural testing and hybrid simulation. There are three main reasons to consider the use of a multi-actuator layouts in structural testing: (i) to increase loading capacity over a stiff and high capacity physical specimen, (ii) to apply realistic 3D loading over specimens with multi-axial boundary conditions, and (iii) for applications involving loading at multiple boundary devices. Some of the commonly used multi-actuator devices include shake tables, boundary condition devices, and shell element testers. The objective here is to introduce the available capabilities and functionalities of these testing systems to the reader.

3.1 Shake tables

Shake tables are a class of actuated assemblies, where a moving platform is used to excite an onboard structure. These devices are used to acquire the global nonlinear dynamical behaviors of complete structures. Many large shake tables have been built around to world to test large and full-scale structures. The *E-Defense* (6-DOF full-scale earthquake testing facility) is the world's largest shake table, with a dimension of 20 m $\times$15 m, a payload capacity of 12 MN, horizontal motion of 1 m at > 9 m/s^2, and vertical motion of 0.5 m at > 15 m/s^2 [76, 77]. Tianjin University is currently constructing an even larger shake table with underwater capabilities [78]. The NHERI@UCSD shake table, shown in Fig. 7(a) has a 7.6 m $\times$12.2 m platform, a payload capacity of 20 MN [79], and was recently upgraded with additional actuators and servo-hydraulics to have 6-DOF capabilities, with X-direction motion of 0.89 m at 5.9 g, Y-direction motion of 0.38 m at 4.6 g, and Z-direction (vertical) motion of 0.127 m at 4.7 g [80]. In other shake table facilities, size has been traded for flexible physically distributed testing capabilities. Shake table arrays like those at the University of Nevada – Reno, and Tongji University, can test long-span structures with multiple independent base excitations [81, 82]. Underwater shake tables tests allow for experimentally attained hydrodynamic pressures for studying structural behavior under seismic maritime environments [83, 84, 85]. Examples of shake table use in HS and RTHS are provided in Sections 4.2 and 5.1.

3.2 Boundary condition devices

Boundary condition devices are mechanical manipulators made from several prismatic servo-hydraulic actuators connected by swivel joints. The number of servo-hydraulic actuators is typically equal to or higher than the number of DOFs expected from the boundary condition device [47]. For instance, a 6-DOF boundary condition device has six actuators or more. Actuators are pinned to a fixed based at one end, and a (stiff) moving platform at the other end. The fixed base is typically attached to a rigid reaction wall, and the moving platform is attached to the experimental substructure. Multi-axial boundary condition devices overcome the payload limitations in shake tables and are advantageous in their flexibility for testing structures with various configurations and experimental costs.

The Load and Boundary Condition Boxes (LBCBs) at the University of Illinois at Urbana-Champaign are 6-DOF devices with force and position control capabilities, and X-direction stroke and force capacities of 0.254 m and $2,402$ kN, respectively. These capacities are 0.127 m and $1,201$ kN in the Y-direction, and $0.127m$ and $3,603$ kN in the Z-direction. The LBCBs, shown in Fig. 7(b), are attached to highly stiff strong-wall and strong-floor reaction frames, which allow flexibility in testing configurations [86]. Similar LBCB devices are also available in smaller scales at the University of Illinois, the University of Southern California [87], and the Institute of Engineering Mechanics, China Earthquake Administration in Beijing.

The Multi-Axial Subassemblage Testing (MAST) system is another type of boundary condition device, first built at the University of Minnesota. The MAST is comprised of a highly stiff moving platform (i.e., crosshead) that imposes boundary conditions and forces to the top of the experimental substructure. The MAST has horizontal stroke and load capacities of 0.4 m and $3,910$ kN, and vertical capacity of 0.25 m and $5,870$ kN, respectively [88]. Two new MAST facilities were built recently at the Swinburne University of Technology and at the ETH Zürich [89, 90].

The multi-use structural testing (HNU-MUST) system at MOE Key Laboratory of Building Safety and Energy Efficiency at the Hunan University (HNU) is similar in design to the MAST, with two horizontal and four vertical actuators with loading capacities of $4,000$ kN and $20,000$ kN, respectively. The stroke capabilities are $0.35m$ in the horizontal direction and 0.5 m in the vertical direction [91]. The multi-directional hybrid testing system at the Structural Engineering

Laboratory of Polytechnique Montreal is a similar design to the MAST and has four horizontal and four vertical actuators [92]. The Taiwan National Centre for Research on Earthquake Engineering (NCREE) multi-axial testing system (MATS) is a self-reacting loading frame with more than 15 actuators, which combine to enable 6-DOF boundary condition operation. MATS was designed such that experimental substructure would be fixed from the top and excited from the base [93].

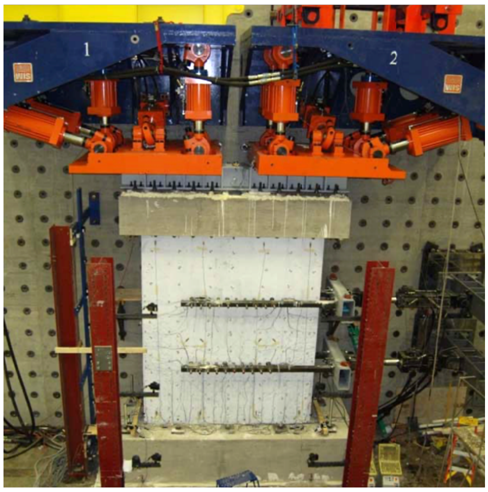
3.3 Shell element testers

Shell element testers are experimental assemblies, composed of a large number of actuators, that impose load combinations on four sides of shell elements. These devices are largely used for testing of reinforced concrete (RC) shell elements and have led to important breakthroughs in the field of mechanics of RC, including Compression Field Theory [94]. At the University of Toronto, the Shear Panel Tester was developed in 1979 for in-plane tests, and the Shell Element Tester was developed in 1984 for in-plane and out-of-plane tests and was upgraded to have 60 servo-hydraulic actuators. The UT10 Simulator is an augmentation to the Shell Element Tester that allows hybrid testing of up to 10 elements simultaneously [95, 96]. Other shell element testers are the Universal Element Tester at the University of Houston with 60 actuators [97], and the Large Universal Shell Element Tester (LUSET) at the ETH Zürich with 100 actuators illustrated in Fig. 7(c) [98]. HS and RTHS have not been implemented using shell element testers to-date.

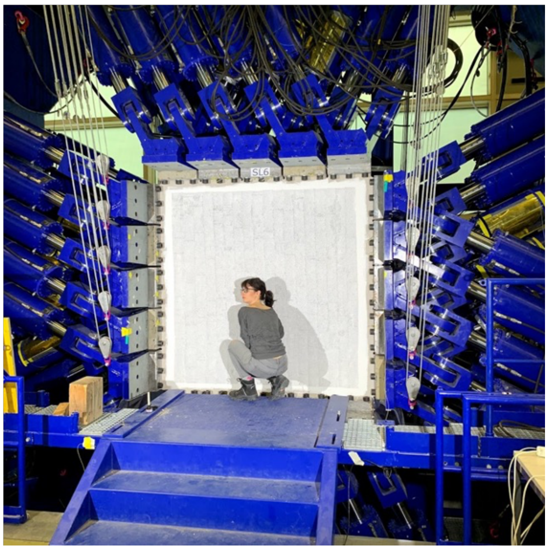
3.4 Individually attached actuators

As discussed in Section 2.1, individual actuators can be combined to realize a customized multi-actuator boundary condition. Testing of frame structures with translational DOFs for instance requires individually attached actuators to each story. The damped braced frame setup at the Lehigh University real-time multi-directional (RTMD) large-scale testing facility, illustrated in Fig. 7(d), is an example. In the testbed shown here, two stiff braced frames are attached to the faces of the test frame to prevent out-of-plane motions [59, 99]. In other developments, actuators were used for simulating both translational and rotational behaviors in the frame structures [100, 53, 54, 101]. Many applications using individually attached actuators are discussed in sections 5.1 and 6.1.

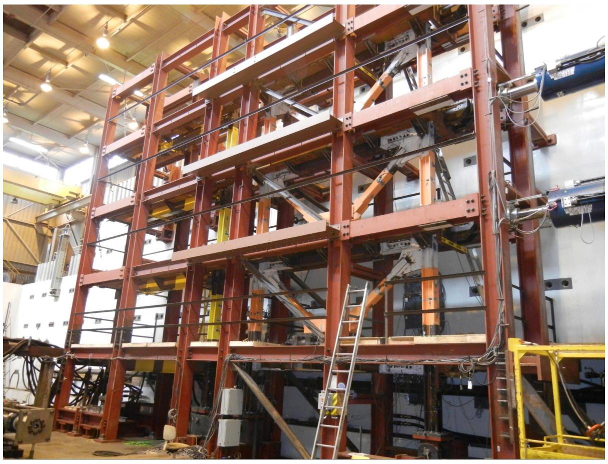
(a) Shake table, NHERI@UCSD



(b) LBCB, UIUC



(c) LUSET, ETH Zurich



(d) Damped Braced Frame, Lehigh RTMD

Figure 7: Types of multi-actuator devices

4 Multi-axial hybrid simulation (maHS)

The widespread use of (conventional) HS since the 1980s can be attributed to major advances in control and measurement techniques, substructuring formulations, test-pausing ability, and reliability in reproducing dynamic behaviors. This section provides a chronological summary of various illustrative maHS applications with LBCBs, MASTs, and other multi-actuator setups.

4.1 Multi-actuator applications

From the earliest days, developments in multi-actuator HS were driven by the need to simulate realistic seismic performances of structures in laboratory environments. The first multi-actuator

SSHS studies were on multi-story frame structures subject to uni-axial ground motions, where actuators were attached to the each story to simulate floor translations [102, 103, 104]. Frame structures with 6 and 8 actuator configurations were studied in [105] and [106], respectively. [107] presents a a bi-directional two-actuator test setup for evaluating a bi-axial ground motion on a frame.

4.2 Multi-axial applications

Only planer hybrid simulation were considered up until the mid-1990s, as the dynamics of planar structures are more easily verifiable on a shake table. The limitations of the planar frameworks are that multi-component excitations and 3D strength envelopes of structures are not incorporated into the hybrid test. Extensions of the HS method into the multi-axial setting requires tackling of the geometric nonlinearities resulting from the actuator kinematics, and compensation for the mechanical coupling introduced when a stiff physical specimen is shared among multiple actuators. The work in the multi-axial domain began with planar (i.e., 3-DOF) developments [42, 43, 72, 100, 44].

In the early 2000s, great emphasis was placed on earthquake engineering and hybrid simulation research through the Network for Earthquake Engineering Simulation (NEES) program [22]. The MUST-SIM facility at the University of Illinois was influential in advancing multi-axial HS methods to enable research. [86] outlines the use of full-scale and $1/5^{\text{th}}$-scale LBCBs at MUST-SIM for vertical accelerations assessment on shear capacity and demand of RC bridge piers in a geographically distributed HS between University of Illinois at Urbana-Champaign and Lehigh University. [108] presents a mixed-mode control strategy for HS of a skewed RC bridge. [109] concludes from a vulnerability study of RC bridge piers that without consideration of vertical accelerations, the severity of earthquake-induced damages can be widely underestimated. [110] conducts a multi-axial test on RC slender walls using two LBCBs connected over a rigid link, to achieve the necessary vertical loads and overturning moments. Multiple LBCBs were used in a four-span bridge test, where the piers were physical and the deck was numerically evaluated [111, 112]. The results from this study were compared to analytical simulations for verification. [45] studies moment frames where two LBCBs test a full-scale beam-column connection, while the rest of the assembly is modeled numerically. [113] performs multi-axial simulation on an RC frame structure under pulse-type ground motion and evaluates shear failures in pre-1970 structures. Despite immediate failure of the columns, the 10-story building structure did not under go collapse.

The MAST facilities at the University of Minnesota and the Swinburne University of Science and Technology enable a broad scope of SSHS experiments. [88] and [114] discuss the possibilities for hybrid simulation using the MAST systems. [115] uses the MAST system to generate collapse fragility curves of an RC column using quasi-static testing and SSHS. The probability of collapse is discovered to be less when column is SSHS tested and the realistic boundary conditions are imposed. Building on this research, [116] also evaluates the effectiveness of carbon fiber reinforcement polymer repairs in restoring earthquake-damaged columns to their original performances. [117] implements a 6-DOF seismic hybrid simulation of RC bridge piers including excitation in the vertical direction.

A vast body of literature is designated to multi-axial and multi-actuator HS frameworks. Because of the iterative algorithms embedded in these frameworks, they are, however, unable to produce real-time results. The next section will explore the various developments in RTHS.

5 Multi-axial real-time hybrid simulation (maRTHS)

Although RTHS and SSHS are similar in architecture, successful implementation of each method requires solving different challenges. Sensors, computers, and actuators used in real-time tests must acquire, process, and execute at higher speeds. A complete RTHS loop: numerical integration, kinematic transformation, actuator compensation, physical execution, and measurement of feedback forces, must be completed in a very small time increment. In addition, the fast loading speeds required to perform RTHS create frequency-dependent actuator dynamics and often reveal nonlinearities. Fast operation also impacts the interactions between an actuator setup and the physical structure, in what is known as *control-structure interaction* [33]. This section presents literature in RTHS and is divided in two subsections on multi-actuator and multi-axial testing, respectively.

5.1 Multi-actuator applications

The earliest multi-actuator RTHS targeted a simple portal frame [53, 54], where a column with a 2-DOF boundary condition at the top is experimentally substructured. Two-actuator setup is connected to a highly stiff loading bracket, which deforms the experimental column through translation and rotation. The oscillating instabilities that resulted from actuator coupling were solved using a delay compensation algorithm.

The next set of multi-actuator RTHS experiments focused on dynamic substructuring of mass-dashpot-spring (MDS) systems arranged in a series configuration. [55] explores a triple MDS system, while [56] studies two systems of four and five MDSs, respectively. The boundary condition between the substructures are realized by connecting actuators to the experimental springs on either side of the middle mass. MDS systems allow for dynamic coupling studies by varying the stiffness of the spring elements. These systems are typically limited to only two actuators.

Shake tables are useful for testing of multi-story structures, and have been used in RTHS studies. [118] introduces a shake table RTHS for a two-story structure, with an experimental first story, and a numerical second story. A shake table and an actuator excite the base and mass of the first story, respectively. [119] studied a three-story structure where the second story was experimentally tested. Applied accelerations for the base and mass are first converted to displacement commands. [120] reviews various RTHS tests with shake tables. Most of the tests discussed are however limited to 1-DOF actuation. [121] studied full-scale rolling bearings used as floor isolation systems using multi-axial RTHS shake table tests in the Natural Hazards Engineering Research Infrastructure (NHERI) Experimental Facility at Lehigh University.

Several RTHS studies are dedicated to multi-story frame structures. [61] conducts a numerical RTHS of a three-story steel frame structure with a magnetorheological (MR) damper installed at the first story. The author proposes a coupled model-based controller for the three-actuators exciting the frame in simulation. [62] evaluates a two-story steel moment resisting frame (MRF) with a first-story MR damper in three different RTHS configurations: fully numerical, experimental MR damper with a numerical frame, and fully experimental frame with numerical mass. [59], [99], and [122] explore the passive and semi-active use of MR dampers for vibration mitigation in a three-story MRF. Three actuators excite a damped brace frame containing three MR dampers as part of the experimental substructure, and the MRF is modeled numerically. [123] studies the behavior of a two-story steel frame structure with an experimental first story column. A setup of three actuators (i.e., two vertical and one lateral) drive the boundary conditions for the column. A nonlinear finite element analysis program for hybrid testing is also discussed which shortens the computational time.

5.2 Multi-axial applications

Multi-axial RTHS is challenging due to the need for fast experimental hardware, high levels of actuator coupling, and accuracy of the kinematic calculations. The boundary condition devices used for multi-axial RTHS require different algorithms for kinematic transformations and actuator compensations than those listed in section 4. Iterative solutions developed for slow speed testing must be replaced with rapid solutions, to generate stable and accurate trajectories in one or few discrete time steps.

Two classes multi-axial real-time hybrid simulation (maRTHS) frameworks have been proposed in the recent years for boundary condition devices such as the LBCB and the MAST. The difference between these approaches is in how actuator compensation is conducted. [63] proposes a coupled compensation, while [60] and [124] propose a decoupled compensation.

The general architecture of both frameworks involves directing target displacements and rotations obtained from the numerical substructure through an outer-loop controller, to compute control signals for boundary condition device execution. Model-based outer-loop controllers are proposed in these frameworks, for addressing the dynamic coupling that exists between the actuators in the boundary condition devices. Individual hydraulic actuators are identified with a transfer function model $G_i(s)$, where i is the actuator index. Next, the kinematic relationships for the boundary condition devices are acquired, including Jacobian matrices from the linearization of Eq.(15). In [63], the Jacobian and the diagonal transfer function matrix of the actuators are combined to create a MIMO transfer system representing the nominal boundary condition device dynamics in Cartesian coordinates:

$$\hat{\boldsymbol{G}}(s) = \begin{bmatrix} \hat{G}_{11}(s) & \hat{G}_{12}(s) & \dots \\ \vdots & \ddots & \\ \hat{G}_{n1}(s) & & \hat{G}_{nn}(s) \end{bmatrix} = \boldsymbol{J}^{-1} \begin{bmatrix} G_1(s) & & \\ & \ddots & \\ & & G_n(s) \end{bmatrix} \boldsymbol{J} \tag{16}$$

where n is the total number of hydraulic actuators used in the boundary condition device. Next, feedforward and feedback controllers are designed as coupled systems according to the model-based architecture proposed in [61]. Lastly, feedback forces from the experiment coordinate transformed and returned to the controller responsible for carrying out the computation for the next time step, thus closing the maRTHS loop.

Studies on the rotational DOFs were found to cause oscillating instabilities, due to the lack of sufficient control authority provided by the coupled controller in this study. Tuning and optimization are a challenging task, due to the numerous parameters in these compensators.

To minimize the role that dynamic coupling plays in the compensation task, [60] proposed an maRTHS framework with the compensation taking place in the actuator frame of reference as opposed to Cartesian coordinates where coupling is substantial. Cartesian signals (e.g., target and measured) are first converted to actuator signals via kinematic transformation. SISO controllers are designed for each individual actuator following the system identification and acquisition of the diagonal transfer matrix $\boldsymbol{G}(s)$. The decoupled maRTHS framework has also been extended for studies where use of more than one boundary condition device is desired [124].

5.3 Multi-axial real-time testing in other engineering disciplines

Due to the listed challenges, applications of maRTHS are rare and few in the field of Civil Engineering. Methodologies similar to RTHS have been implemented in other engineering disciplines and are commonly referred to as *Hardware-in-the-loop* (HIL) tests. The HIL uses of serial and parallel robotics common in Aerospace and Mechanical applications involving aircraft, automobiles, and spacecrafts. The first uses of multi-axial robots for vibration based applications were by V. E. Gough for tire testing, K. L. Cappel and D. Stewart for flight simulators [125, 126]. These replicas of aircraft cockpits installed on parallel manipulators are simulating the flight environment for pilot training and other in-flight studies. In the automotive industry HIL is used for rapid prototyping. HIL constitutes a synergy between various physical components (e.g., powertrain, axles, and chassis) and virtual models (e.g., environment conditions, driver commands, and road profile) [127, 128]. [129] and [130] introduced an HIL test rig for mechatronic vehicle axles. A hexapod manipulator with six hydraulic actuators imposes multi-axial forces and torques. In the space industry, spacecrafts are often multi-axial vibration tested for system identification, verification of mathematical models, and simulation of in-flight shocks and vibrations. The Mechanical Vibrations Facility (MVF) at the Glenn Research Center contains a vibration table with 16 vertical and 4 horizontal actuators, used for modal testing [131]. Other commercial products, such as the MTS Multi-axial Simulation Tables [132], have been used for various multi-axial dynamic tests.

6 Current challenges and opportunities

Hybrid simulation with multiple actuators is an active topic of research and has proven to be an effective tool for various investigations and applications. However, a great deal more research is needed to establish generalizable theories and build the capacity needed to truly exploit hybrid simulation, and especially RTHS, to study complex structural engineering problems. This section sheds light on some of the remaining challenges and unanswered questions in this domain. In doing so, the aim is to share research insights, and to direct the attention of the research community to existing research gaps and future research directions.

6.1 Robustness of multi-actuator closed-loop systems

Design of an RTHS test determines the quality of the tracking behavior achieved at the boundary conditions and the closed-loop robustness to uncertainty. When stability and performance are critical in validation studies, emphasis is typically placed on compensation design, and when easier compensation design is desired, emphasis is on the choice for the RTHS partition [67]. For multi-actuator setups, the operational challenge is increased due to the dynamic coupling that exists between actuators. Coupled compensation may be a more rational approach for developing multivariate models. For instance, multivariate robust control approaches take into account all the coupling effects as uncertainties. Decoupled compensation is easier to program, and allows for adaptive control developments in actuator space. However, the effects of stiff specimen on Cartesian performance need to be studied. Use of adaptive and robust compensation methods should be explored for improving the robustness of RTHS experiments.

Most applications in the maRTHS domain have used displacement-based compensation algorithms. Displacement compensation is not suitable for testing highly stiff physical specimen, as the smallest actuator motion result in sharp increases in reaction forces from the physical specimen [46]. Another challenge with testing a stiff physical specimen is in the accurate measurement of small multi-DOF deformations. Force compensation allows for a more stable and accurate control of highly stiff DOFs. Displacement compensation also does not allow for maintaining a prescribed force level (e.g., gravity forces) over a physical specimen. Gravity forces result in application of axial forces which can alter with the failure mode and capacity of physical specimen [133]. The MTS control software is a generic kinematic transformation tool that provides a layer of force control loop is included for over constrained systems when the number of actuators exceeds the number of controlled DOFs [134]. Therefore, incorporation of force and displacement compensations into maRTHS can combine the benefits of both approaches and improve the realism of this experimental framework.

A key aspect of hybrid simulation is the black-box and reference-free nature of the experimental substructures. Quantifying the uncertainty associated with the experimental substructuring, and exploiting this information for the design of a suitable controller and updating of the numerical substructure are critical to the accuracy and confident generalization of this method [135, 136]. [137] presents foundational ideas for uncertainty quantification via a modular framework that divides RTHS into smaller units on multi-rate coordination, actuator control, state estimation and model updating, stability and performance indicators, and real-time decision making. Multivariate representation of uncertainty for a multi-actuator system remains a major challenge.

6.2 Mechanical design of multi-actuator loading assemblies

The choice for the closed-kinematic chain architecture of multi-actuator loading assemblies will dictate the success of the kinematic planning. Consider the linearization of Eq.(15), which generates a Jacobian matrix $\boldsymbol{J}$. Certain arrangements of the actuators over the test specimen or loading platform may result in formulation of a singular $\boldsymbol{J}$ matrix [138]. When designing multi-actuator assemblies, the presence of singularities must be explored over a given trajectory workspace. In addition, loading platform must be designed to have optimal coupling, stiffness, and capacity. This challenge exists for individually attached non-modular multi-actuator configurations too.

The next challenge is with multi-actuator hardware requirements. RTHS with multiple actuators require large hydraulic capabilities (e.g., accumulators for flow demands, manifolds with additional accumulators for fast transient response) [139]. For instance, [140] provide details of the hydraulic power system for the NHERI@UCSD outdoor shake table, with a model of the hydraulic accumulators.

Other mechanical design challenges include interactions among the various fixtures including actuator friction, bearing slippage, moving platform inertia, and flexibility of the loading and support assemblies. These phenomena create erroneous load cell and displacement transducer measurements, which result in inaccurate simulations [92]. Therefore, the contribution of these phenomena must be minimized through measurement and compensation. For example, [73] provide an online positioning correction algorithm for multi-actuator loading platforms with flexible fixtures (i.e., reaction wall and floor). Also, adaptive kinematic transformations were proposed by [75] to compensate the errors associated to fixture compliance in multi-axial hybrid simulation at slow speeds. Future developments should focus on strategies to circumvent fixture properties on the accuracy of multi-axial real-time testing.

6.3 Time constraints and computational efforts

Real-time solutions to the numerical substructure, model updating, actuator compensation, and coordinate transformation constitute most of the computational efforts in RTHS. With more actuators and increased degree of actuator coupling, the computational efforts grow further. Most computational platforms however cannot execute real-time simulations at the rapid rates typically used in RTHS testing (e.g., 2048 Hz or higher). Parallel computing is an affordable way of overcoming computational constraints while meeting the increased computational demands for real-time multi-actuator applications. [141] introduces a platform for parallel computing of complex numerical substructures on standard off-the-shelf multi-core computers. Field programmable gate arrays (FPGAs) are also affordable means to speed up the computational capabilities [142].

Efficient computational programs (e.g., codes) for the numerical substructures are rarely available. OpenSees finds it challenging to meet the efficiency requirements for real-time testing. Instead, researchers often resort to the Bouc-Wen simulation code [143]. [144] developed an efficient Timoshenko hysteretic beam model with nonlinear behavior. [145] developed a state-space formulation for structural analysis with plastic and geometric nonlinearities.

6.4 Validation of multi-actuator RTHS

Another challenge with multi-actuator HS and RTHS applications is validation. In many applications, RTHS results are validated against numerical analysis results. This approach is however difficult when the nonlinearity and physics of the experimental substructure are unknown. The need for validation grows stronger as RTHS methods grow more complicated.

New benchmark problems are needed for advancing new technologies in maHS and maRTHS. [146] presented an RTHS benchmark control problem which offered challenges of unmodeled dynamics and uncertainty. A nonlinear RTHS benchmark control problem is currently in the works.

6.5 Other applications regarding multi-axial testing

Other hybrid testing applications pertain to the multi-physics problems, which refers to simultaneous presence and coupling of physical phenomena in a single system or simulation. Multi-physics problems involving fluid-solid interaction have been studied in recent years. Some examples of hybrid simulation with wind-structure behavior as the focus of the study are: offshore monopile wind turbines [147], semi-submersible floating wind turbines [148], aeroelastic base-pivoting building model [36], and wind-tunnel model for flexible bridges [149]. For multi-physics seismic research, underwater shake tables are useful for realizing hydrodynamic pressures [83, 84, 85]. Coupling of structural and thermal loads have also been explored [150, 151, 19]. Multi-physics cyber-physical testing to examine complex systems involving thermal and structural coupling is also being developed [152, 153]. Traditional multi-actuator approaches discussed in this review paper may not be suitable for multi-physics problems. Hence, new generalizable approaches should be explored.

7 Concluding remarks

Multi-actuator hybrid simulation is the process of emulating the dynamical behavior of a structural system through closed-loop simulation of its constituent substructures (i.e., numerical and experimental) via multiple actuators. This methodology serves as a middle ground between pure numerical simulation, which offers rapidness, and full experimental testing, which offers realism and means for validation. In addition, multi-actuator hybrid simulation offers flexibility which addresses a broad range of experimental configurations.

This review paper highlights the historical roots, evolution and key enablers of HS and RTHS methods. A greater emphasis has been placed on multi-actuator and multi-axial test setups, as single-actuator HS and RTHS have already been discussed in other review literature. The general framework for the methodology are outlined, starting with the concepts of substructuring, compensation, and kinematic transformation, and the fundamental differences between HS and RTHS are highlighted. Capabilities of several experimental facilities around the world are presented and their significance to HS and RTHS are discussed. A thorough review of multi-actuator and multi-axial HS and RTHS are next provided. Finally, several insights are provided on current challenges and future research directions.

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References

[1] M. S. Williams and A. Blakeborough. Laboratory testing of structures under dynamic loads: An introductory review. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 359(1786):1651–1669, 2001. ISSN 1364503X. doi: 10.1098/rsta.2001.0860.

[2] Narutoshi Nakata, Shirley J. Dyke, Jian Zhang, Gilberto Mosqueda, Xiaoyun Shao, Hussam Mahmoud, Monique H. Head, Michael Erwin Bletzinger, Gemez A. Marshall, Ge Ou, and Cheng Song. Hybrid simulation primer and dictionary, 2014. URL `https://mechs.designsafe-ci.org/media/cms_page_media/965/Primer.pdf`.

[3] Motohiko Hakuno, Masatoshi Shidawara, and Tsukasa Hara. Dynamic destructive test of a cantilever beam, controlled by an analog-computer. *Proceedings of the Japan Society of Civil Engineers*, 1969(171):1–9, 1969. ISSN 1884-4936. doi: 10.2208/jscej1969.1969.171{_}1. URL `http://ci.nii.ac.jp/naid/130003978834/http://joi.jlc.jst.go.jp/JST.Journalarchive/jscej1969/1969.171_1?from=CrossRef`.

[4] Koichi Takanashi, Kuniaki Udagawa, Matsutaro Seki, Tsuneo Okada, and Hisashi Tanaka. Nonlinear Earthquake Response Analysis of Structures by a Computer-Actuator On-Line System. *Bulletin of Earthquake Resistant Structure Research Center*, (8):1–17, 1975. doi: 10.3130/aijsaxx.229.0.

[5] Robert D Hanson and N Harris McClamroch. Pseudo dynamic test method for inelastic building response. In *Proceedings of the 8th World Conference on Earthquake Engineering*, pages 123–134, San Francisco, 1984.

[6] Stephen A Mahin and Pui-shum B Shing. Pseudodynamic method for seismic testing. *Journal of Structural Engineering*, 111(7):1482–1503, 1985.

[7] Masayoshi Nakashima, Hiroto Kato, and Eiji Takaoka. Development of real-time pseudo dynamic testing. *Earthquake Engineering and Structural Dynamics*, 21(1):79–92, 1992.

[8] Andrew Plummer. Electrohydraulic servovalves - past, present, and future. In *10th International Fluid Power Conference*, pages 405–424, Dresden, 2016.

[9] R. H. Maskrey and W. J Thayer. A brief history of electrohydraulic servomechanisms. *Journal of Dynamic Systems Measurement and Control*, 1978. URL http://www.mylesgroupcompanies.com/moog_pdfs/MoogBriefHistoryofServovalves.pdf.

[10] Hans Moravec. *Robot: Mere Machine to Transcendent Mind*. Oxford University Press, 2000. ISBN 0195136306.

[11] L. Javier Malvar and C. Allen Ross. Review of Strain Rate Effects for Concrete in Tension. *ACI Materials Journal*, 95:735–739, 1998.

[12] Matthew P. Murray, Stephen P. Rowell, and Trace A. Thornton. Effects of High Strain Rates on ASTM A992 and A572 Grade 50 Steel. Technical report, 2014.

[13] Yunbyeong Chae, Minseok Park, Chul-young Kim, and Young Suk. Experimental study on the rate-dependency of reinforced concrete structures using slow and real-time hybrid simulations. *Engineering Structures*, 132:648–658, 2017. ISSN 0141-0296. doi: 10.1016/j.engstruct.2016.11.065. URL http://dx.doi.org/10.1016/j.engstruct.2016.11.065.

[14] W. Ghannoum, V. Saouma, G. Haussmann, K. Polkinghorne, M. Eck, and D.-H Kang. Experimental Investigations of Loading Rate Effects in Reinforced Concrete Columns. *Journal of Structural Engineering*, 138(8):1032–1041, 2012. doi: 10.1061/(ASCE)ST.1943-541X.0000540.

[15] Min Li and Hongnan Li. Effects of Loading Rate on Reinforced Concrete Beams. In *17th World Conference on Earthquake Engineering*, Lisbon, 2012.

[16] Guoxi Fan, Yupu Song, and Licheng Wang. Experimental study on the seismic behavior of reinforced concrete beam-column joints under various strain rates. *Journal of Reinforced Plastics & Composites*, 33(7):601–618, 2014. doi: 10.1177/0731684413512706.

[17] Narutoshi Nakata, Richard Erb, and Matthew Stehman. Mixed force and displacement control for testing base-isolated bearings in real-time hybrid simulation. *Journal of Earthquake Engineering*, 23(6):1055–1071, 2019. doi: 10.1080/13632469.2017.1342296.

[18] Sarah Tell, Andreas Andersson, Amirali Najafi, Bill F. Spencer Jr., and Raid Karoumi. Real-time hybrid testing for efficiency assessment of magnetorheological dampers to mitigate train-induced vibrations in bridges. *International Journal of Rail Transportation*, 0 (0):1–20, 2021. doi: 10.1080/23248378.2021.1954560.

[19] Xuguang Wang, Robin E. Kim, Oh-Sung Kwon, In-Hwan Yeo, and Jae-Kwon Ahn. Continuous real-time hybrid simulation method for structures subject to fire. *Journal of Structural Engineering*, 145(12):04019152, 2019. doi: 10.1061/(ASCE)ST.1943-541X.0002436.

[20] D de Klerk, D J Rixen, and S N Voormeeren. General Framework for Dynamic Substructuring: History, Review and Classification of Techniques. *AIAA Journal*, 46(5): 1169–1181, 5 2008. ISSN 0001-1452. doi: 10.2514/1.33274. URL https://arc.aiaa.org/doi/pdfplus/10.2514/1.33274http://arc.aiaa.org/doi/10.2514/1.33274.

[21] Masayoshi Nakashima, J. McCormick, and T. Wang. Hybrid Simulation: A historical perspective. In Victor Saouma and Mettupalayam Sivaselvan, editors, *Hybrid Simulation: Theory, Implementation, and Applications*, chapter 1, page 3–12. Taylor & Francis, 2008. URL https://books.google.com/books?hl=en&lr=&id=fJzmEkGuzWQC&oi=fnd&pg=PA3&ots=QOessMEFmL&sig=lr3JCp9svu8NnDJiXK9ygUJgw2Q.

[22] Daniel Gomez, Shirley J. Dyke, and Amin Maghareh. Enabling role of hybrid simulation across NEES in advancing earthquake engineering. *Smart Structures and Systems*, 15(3): 913–929, 2015. ISSN 17381991. doi: 10.12989/sss.2015.15.3.913.

[23] D P McCrum and M S Williams. An overview of seismic hybrid testing of engineering structures. *Engineering Structures*, 118:240–261, 2016. ISSN 01410296. doi: 10.1016/j.engstruct.2016.03.039.

[24] Masayoshi Nakashima. Hybrid simulation: An early history. *Earthquake Engineering & Structural Dynamics*, (March):1–14, 4 2020. ISSN 00988847. doi: 10.1002/eqe.3274. URL http://doi.wiley.com/10.1002/eqe.3274.

[25] Stathis N Bousias. Seismic Hybrid Simulation of Stiff Structures: Overview and Current Advances. *Journal of Structures*, 2014:1–8, 2014. ISSN 2356-766X. doi: 10.1155/2014/825692. URL http://www.hindawi.com/archive/2014/825692/.

[26] Anil K Chopra. *Dynamics of Structures: Theory and Applications to Earthquake Engineering*. Prentice Hall, fourth edi edition, 2012. ISBN 978-0-13-285803-8.

[27] Xu Huang and Oh-Sung Kwon. An integrated simulation method for coupled dynamic systems. *Computer-Aided Civil and Infrastructure Engineering*, 35(10):1115–1131, 2020. doi: https://doi.org/10.1111/mice.12556. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/mice.12556.

[28] Javad Hashemi and Gilberto Mosqueda. Innovative substructuring technique for hybrid simulation of multistory buildings through collapse. *Earthquake Engineering & Structural Dynamics*, 43:2059–2074, 2014.

[29] Georgios Giotis, Oh-Sung Kwon, and Shamim A. Sheikh. Weakly coupled hybrid simulation method for structural testing: Theoretical framework and numerical verification. *Journal of Structural Engineering*, 146(2):04019196, 2020. doi: 10.1061/(ASCE)ST.1943-541X.0002492.

[30] Peng Pan, Masayoshi Nakashima, and Hiroshi Tomofuji. Online test using displacement-force mixed control. *Earthquake Engineering and Structural Dynamics*, 34(8):869–888, 2005. ISSN 00988847. doi: 10.1002/eqe.457.

[31] T. Y. Yang, Dorian P. Tung, Yuanjie Li, Jian Yuan Lin, Kang Li, and Wei Guo. Theory and implementation of switch-based hybrid simulation technology for earthquake engineering applications. *Earthquake Engineering and Structural Dynamics*, 46(14):2603–2617, 2017. ISSN 10969845. doi: 10.1002/eqe.2920.

[32] Xu Huang and Oh-Sung Kwon. A Generalized Numerical/Experimental Distributed Simulation Framework. *Journal of Earthquake Engineering*, 24(4):682–703, 2020. doi: 10.1080/13632469.2018.1423585.

[33] S. J. Dyke, Billie F. Spencer, Jr., P. Quast, and M. K. Sain. Role of Control-Structure Interaction in Protective System Design. *Journal of Engineering Mechanics*, 121(2):322–338, 1995. ISSN 0733-9399. doi: 10.1061/(ASCE)0733-9399(1995)121:2(322). URL http://ascelibrary.org/doi/10.1061/%28ASCE%290733-9399%281995%29121%3A2%28322%29.

[34] Pui-shum B Shing and Stephen A Mahin. Cumulative experimental errors in pseudo-

dynamic tests. *Earthquake Engineering and Structural Dynamics*, 15(4):409–424, 1987. ISSN 10969845. doi: 10.1002/eqe.4290150402.

[35] Pui-shum B. Shing and Stephen A. Mahin. Experimental Error Effects in Pseudodynamic Testing. *Journal of Engineering Mechanics*, 116(4):805–821, 1990. ISSN 0733-9399. doi: 10.1061/(asce)0733-9399(1990)116:4(805).

[36] Moniruzzaman Moni, Youchan Hwang, Oh-Sung Kwon, Ho-Kyung Kim, and Un Yong Jeong. Real-time aeroelastic hybrid simulation of a base-pivoting building model in a wind tunnel. *Frontiers in Built Environment*, 6, 2020. ISSN 2297-3362. doi: 10.3389/fbuil.2020.560672. URL https://www.frontiersin.org/article/10.3389/fbuil.2020.560672.

[37] Koichi Takanashi and K. Ohi. Earthquake response analysis of steel structures by rapid computer-actuator on-line system. *Bulletin of Earthquake Engineering*, 16:103–109, 1983.

[38] R. Peek and W. H. Yi. Error analysis for pseudodynamic test method: 1. Analysis. *Journal of Engineering Mechanics*, 116:1618–1637, 1990.

[39] Georges Magonette. Development and application of large-scale continuous pseudo-dynamic testing techniques. *Philosophical Transactions of the Royal Society A Mathematical, Physical and Engineering Sciences*, 359(1786):1771–1799, 2001.

[40] Gilberto Mosqueda, Bozidar Stojadinovic, and Stephan A. Mahin. Geographically distributed continuous hybrid simulation. In *13th World Conference on Earthquake Engineering*, number 0959, Vancouver, 2004.

[41] Oh-Sung Kwon. Multi-platform Hybrid (Experiment-Analysis) Simulations. In *Dynamic Response of Infrastructure to Environmentally Induced Loads*, pages 37–63. Springer, 2017. doi: https://doi.org/10.1007/978-3-319-56136-3{_}3.

[42] Christopher R. Thewalt and Stephen A. Mahin. Non-planar pseudodynamic testing. *Earthquake Engineering & Structural Dynamics*, 24(5):733–746, 1995. ISSN 10969845. doi: 10.1002/eqe.4290240509.

[43] F. Javier Molina, G. Verzeletti, G. Magonette, Ph Buchet, and M. Géradin. Bi-directional pseudodynamic test of a full-size three-storey building. *Earthquake Engineering and Structural Dynamics*, 28(12):1541–1566, 1999. ISSN 00988847. doi: 10.1002/(SICI)1096-9845(199912)28:12⟨1541::AID-EQE880⟩3.0.CO;2-R.

[44] Oya Mercan, James M. Ricles, Richard Sause, and Thomas Marullo. Kinematic transformations for planar multi-directional pseudodynamic testing. *Earthquake Engineering & Structural Dynamics*, 38:1093–1119, 2009. doi: 10.1002/eqe.886.

[45] Hussam N Mahmoud, Amr S Elnashai, Billie F Spencer, Jr., Oh-Sung Kwon, and David J Bennier. Hybrid Simulation for Earthquake Response of Semi-rigid Partial-Strength Steel Frames. *Journal of Structural Engineering*, 139(7): 1134–1148, 7 2013. ISSN 0733-9445. doi: 10.1061/(ASCE)ST.1943-541X.0000721. URL http://ascelibrary.org/doi/10.1061/(ASCE)ST.1943-541X.0000721http://ascelibrary.org/doi/10.1061/%28ASCE%29ST.1943-541X.0000721.

[46] Narutoshi Nakata, Erin Krug, and Aaron King. Experimental implementation and verification of mDOF effective force testing. *Earthquake Engineering & Structural Dynamics*, 43:413–428, 2014.

[47] Lung-Wen Tsai. *Robot Analysis: The Mechanics of Serial and Parallel Manipulators*. Wiley-Interscience, 1999.

[48] Tao Wang, Chun Cheng, and Xun Guo. Model-based predicting and correcting algorithms for substructure online hybrid tests. *Earthquake Engineering & Structural Dynamics*, 41: 2331–2349, 2012. doi: 10.1002/eqe.2190.

[49] Peng Pan, Gang Zhao, Xinzheng Lu, and Kailai Deng. Force–displacement mixed control for collapse tests of multistory buildings using quasi-static loading systems. *Earthquake Engineering & Structural Dynamics*, 43:287–300, 2014.

[50] Guoshan Xu, Zhen Wang, Bin Wu, Oreste S. Bursi, Xiaojing Tan, Qingbo Yang, and Long Wen. Seismic performance of precast shear wall with sleeves connection based on experimental and numerical studies. *Engineering Structures*, 150:346–358, 2017. ISSN 18737323. doi: 10.1016/j.engstruct.2017.06.026. URL http://dx.doi.org/10.1016/j.engstruct.2017.06.026.

[51] Huimeng Zhou, Tao Wang, Chunbo Du, and Cheng Chen. Multi-degree-of-freedom force-displacement mixed control strategy for structural testing. *Earthquake Engineering & Structural Dynamics*, (July):eqe.3334, 8 2020. ISSN 0098-8847. doi: 10.1002/eqe.3334. URL https://onlinelibrary.wiley.com/doi/abs/10.1002/eqe.3334.

[52] Narutoshi Nakata, Billie F Spencer, Jr., and Amr S Elnashai. Sensitivity-Based External Calibration of Multiaxial Loading System. *Journal of Engineering Mechanics*, 136(2): 189–198, 2010. ISSN 0733-9399. doi: 10.1061/(ASCE)0733-9399(2010)136:2(189). URL http://dx.doi.org/10.1061/(ASCE)0733-9399(2010)136:2(189)http://ascelibrary.org.proxy2.library.illinois.edu/doi/abs/10.1061/(ASCE)0733-9399(2010)136:2(189)http://ascelibrary.org.proxy2.library.illinois.edu/doi/pdf/10.1061/(ASCE)0733-9399(2010)136:2(189).

[53] A Blakeborough, M S Williams, A P Darby, and D M Williams. The development of real–time substructure testing. *Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, 359(1786): 1869–1891, 9 2001. ISSN 1364-503X, 1471-2962. doi: 10.1098/rsta.2001.0877. URL http://rsta.royalsocietypublishing.org/content/359/1786/1869http://rsta.royalsocietypublishing.org/content/roypta/359/1786/1869.full.pdf.

[54] A P Darby, M S Williams, and A Blakeborough. Stability and delay compensation for real-time substructure testing. *Journal of Engineering Mechanics*, 128(12):1276–1284, 2002. ISSN 0733-9399. doi: 10.1061/(ASCE)0733-9399(2002)128:12(1276). URL http://opus.bath.ac.uk/742/.

[55] M I Wallace, D J Wagg, and S A Neild. An adaptive polynomial based forward prediction algorithm for multi-actuator real-time dynamic substructuring. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 461(2064):3807–3826, 12 2005. ISSN 1364-5021. doi: 10.1098/rspa.2005.1532. URL http://rspa.royalsocietypublishing.org/content/461/2064/3807http://rspa.royalsocietypublishing.org/content/461/2064/3807.full.pdfhttp://rspa.royalsocietypublishing.org/cgi/doi/10.1098/rspa.2005.1532.

[56] Paul A Bonnet, C N Lim, M S Williams, A Blakeborough, S A Neild, D P Stoten, and C A Taylor. Real-time hybrid experiments with Newmark integration, MCSmd outer-loop control and multi-tasking strategies. *Earthquake Engineering & Structural Dynamics*, 36(1):119–141, 1 2007. ISSN 00988847. doi: 10.1002/eqe.628. URL http://onlinelibrary.wiley.com/doi/10.1002/eqe.628/abstracthttp://doi.wiley.com/10.1002/eqe.628.

[57] Rae-young Jung, P Benson Shing, Eric Stauffer, and Bradford Thoen. Performance of a real-time pseudodynamic test system considering nonlinear structural response. *Earthquake Engineering and Structural Dynamics*, 36(12):1785–1809, 2007. doi: 10.1002/eqe.

[58] Cheng Chen, James M Ricles, Ian C Hodgson, and Richard Sause. Real-Time Multi-Directional Hybrid Simulation of Building Piping Systems. In *The 14th World Conference on Earthquake Engineering*, pages Beijing, China, 2008.

[59] Yunbyeong Chae, James M Ricles, and Richard Sause. Large-scale real-time hybrid simulation of a three-story steel frame building with magneto-rheological dampers. *Earthquake Engineering & Structural Dynamics*, 139 (7):1215–1226, 4 2014. ISSN 1096-9845. doi: 10.1002/eqe.2429. URL http://onlinelibrary.wiley.com/doi/10.1002/eqe.2429/abstracthttp://onlinelibrary.wiley.com/doi/10.1002/eqe.2429/abstract?campaign=woletoc.

[60] Amirali Najafi, Gaston A Fermandois, and Billie F Spencer. Decoupled model-based real-time hybrid simulation with multi-axial load and boundary condition boxes. *Engineering Structures*, 219(October 2019):110868, 9 2020. ISSN 01410296. doi: 10.1016/j.engstruct.2020.110868. URL https://linkinghub.elsevier.com/retrieve/pii/S0141029619344219.

[61] Brian M Phillips and Billie F Spencer, Jr. Model-Based Multiactuator Control for Real-Time Hybrid Simulation. *Journal of Engineering Mechanics*, 139(2):219–228, 2013. ISSN 0733-9399. doi: 10.1061/(ASCE)EM.1943-7889.0000493. URL http://dx.doi.org/10.1061/(ASCE)EM.1943-7889.0000493http://ascelibrary.org/doi/abs/10.1061/(ASCE)EM.1943-7889.0000493http://ascelibrary.org/doi/pdf/10.1061/(ASCE)EM.1943-7889.0000493.

[62] Xiuyu Gao, Nestor Castaneda, and Shirley J Dyke. Experimental Validation of a Generalized Procedure for MDOF Real-Time Hybrid Simulation. *Journal of Engineering Mechanics*, 140(4):4013006, 4 2014. ISSN 0733-9399. doi: 10.1061/(ASCE)EM.1943-7889.0000696. URL http://ascelibrary.org/doi/10.1061/%28ASCE%29EM.1943-7889.0000696.

[63] Gaston A Fermandois and Billie F Spencer. Model-based framework for multi-axial real-time hybrid simulation testing. *Earthquake Engineering and Engineering Vibration*, 16

(4):671–691, 10 2017. ISSN 1671-3664. doi: 10.1007/s11803-017-0407-8. URL http://link.springer.com/10.1007/s11803-017-0407-8.

[64] Narutoshi Nakata. Acceleration trajectory tracking control for earthquake simulators. *Engineering Structures*, 32(8):2229–2236, 8 2010. ISSN 01410296. doi: 10.1016/j.engstruct.2010.03.025. URL http://www.sciencedirect.com/science/article/pii/S0141029610001203https://linkinghub.elsevier.com/retrieve/pii/S0141029610001203.

[65] Ruiyang Zhang, Brian M Phillips, Shun Taniguchi, Masahiro Ikenaga, and Kohju Ikago. Shake table real-time hybrid simulation techniques for the performance evaluation of buildings with inter-story isolation. *Structural Control and Health Monitoring*, 24(10):1–19, 2017. ISSN 15452263. doi: 10.1002/stc.1971.

[66] Amirali Najafi and Billie F Spencer, Jr. Validation of Model-Based Real-Time Hybrid Simulation for Lightly-Damped and Highly-Nonlinear Structural System. *Journal of Applied and Computational Mechanics*, 2020.

[67] Amin Maghareh, Shirley Dyke, Siamak Rabieniaharatbar, and Arun Prakash. Predictive stability indicator: a novel approach to configuring a real-time hybrid simulation. *Earthquake Engineering & Structural Dynamics*, 46(1):95–116, 2017. doi: https://doi.org/10.1002/eqe.2775. URL https://onlinelibrary.wiley.com/doi/abs/10.1002/eqe.2775.

[68] Rui M. Botelho, Xiuyu Gao, Muammer Avci, and Richard Christenson. A robust stability and performance analysis method for multi-actuator real-time hybrid simulation. *Structural Control and Health Monitoring*, page e3017, 2022. doi: https://doi.org/10.1002/stc.3017. URL https://onlinelibrary.wiley.com/doi/abs/10.1002/stc.3017.

[69] Geir E. Dullerud and Fernando Paganini. *A Course in Robust Control Theory - A Convex Approach*. Springer, 2000. ISBN 9781475732900.

[70] Ning Li, Zihao Zhou, and Zhong-Xian Li. Stability prediction for real-time hybrid simulation with different physical and numerical substructure discretization using asynchronous multirate simulation. *Journal of Engineering Mechanics*, 147(11), 2021. doi: 10.1061/(ASCE)EM.1943-7889.0001992.

[71] L Liqiao, W Jinting, D Hao, and Z Fei. Theoretical and experimental studies on critical time delay of multi-DOF real-time hybrid simulation. *Earthquake Engineering and Engineering Vibration*, 21:117–134, 2022. doi: 10.1007/s11803-021-2073-0.

[72] G.Y. Liu and S.Y. Chang. Bi-axial pseudodynamic testing. *Proceedings, 12th World Conference on Earthquake Engineering, New Zealand*, (1):1–8, 2000. URL http://www.iitk.ac.in/nicee/wcee/article/0151.pdf.

[73] Chia Ming Chang, Thomas M. Frankie, Billie F. Spencer, and Daniel A. Kuchma. Multiple degrees of freedom positioning correction for hybrid simulation. *Journal of Earthquake Engineering*, 19(2):277–296, 2014. ISSN 13632469. doi: 10.1080/13632469.2014.962670. URL http://dx.doi.org/10.1080/13632469.2014.962670.

[74] Chia-Ming Chang, Thomas M. Frankie, Billie F. Spencer, and Daniel A. Kuchma. Multiple Degrees of Freedom Positioning Correction for Hybrid Simulation. *Journal of Earthquake Engineering*, 19(2):277–296, feb 2015. ISSN 1363-2469. doi: 10.1080/13632469.2014.962670. URL http://www.tandfonline.com/doi/full/10.1080/13632469.2014.962670.

[75] Jamin Park, Raymond Ma, and Oh-Sung Kwon. Model-based adaptive kinematic transformation method for accurate control of multi-DOF boundary conditions in conventional tests and hybrid simulations. *Earthquake Engineering & Structural Dynamics*, 51(5):1076–1095, apr 2022. ISSN 0098-8847. doi: 10.1002/eqe.3605. URL https://onlinelibrary.wiley.com/doi/10.1002/eqe.3605.

[76] Nobuyuki Ogawa, Keiichi Ohtani, Tsuneo Katayama, and Heki Shibata. Construction of a three-dimensional , large-scale shaking table and development of core technology. *Philosophical Transactions of the Royal Society A Mathematical, Physical and Engineering Sciences*, 359:1725–1751, 2001.

[77] Keiichi Ohtani, Nobuyuki Ogawa, Tsuneo Katayama, and Heki Shibata. Construction of E-Defense (3-D Full-scale Earthquake Testing Facility). In *13th World Conference on Earthquake Engineering*, page Paper No. 1989, Vancouver, 2004.

[78] Liao Shouqin. ianjin university to take lead in building world's largest earthquake engineering simulation research facility. http://www.tju.edu.cn/english/info/1012/1122.htm, 2022. Accessed: 2020-05-19.

[79] J E Luco, O Ozcelik, and J P Conte. Acceleration Tracking Performance of the UCSD-NEES. *Journal of Structural Engineering*, (May):481–490, 2010.

[80] Lelli Van Den Einde, Joel P Conte, José I Restrepo, Ricardo Bustamante, Marty Halvorson, Tara C Hutchinson, Chin Ta Lai, Koorosh Lotfizadeh, J Enrique Luco, Machel L Morrison, Gilberto Mosqueda, Mike Nemeth, Ozgur Ozcelik, Sebastian Restrepo, Andrés Rodriguez, P Benson Shing, Brad Thoen, and Georgios Tsampras. NHERI@UC San Diego 6-DOF Large High-Performance Outdoor Shake Table Facility. *Frontiers in Built Environment*, 6(January):1–21, 2021. ISSN 22973362. doi: 10.3389/fbuil.2020.580333.

[81] Siavash Soroushian, E Maragakis, Arash E Zaghi, Esmaeel Rahmanishamsi, Ahmad M Itani, and Gokhan Pekcan. Response of a 2-story test-bed structure for the seismic evaluation of nonstructural systems. *Earthquake Engineering and Engineering Vibration*, 15 (1):19–29, 2016. ISSN 1993503X. doi: 10.1007/s11803-016-0302-8.

[82] Xiao Yan, Juyun Yuan, Haitao Yu, Antonio Bobet, and Yong Yuan. Multi-point shaking table test design for long tunnels under non-uniform seismic loading. *Tunnelling and Underground Space Technology*, 59:114–126, 2016. ISSN 08867798. doi: 10.1016/j.tust.2016.07.002. URL http://dx.doi.org/10.1016/j.tust.2016.07.002.

[83] Bo Song, Fei Zheng, and Yue Li. Study on a simplified calculation method for hydrodynamic pressure to slender structures under earthquakes. *Journal of Earthquake Engineering*, 17(5):720–735, 2013. ISSN 13632469. doi: 10.1080/13632469.2013.771592.

[84] Yang Ding, Rui Ma, Yun Dong Shi, and Zhong Xian Li. Underwater shaking table tests on bridge pier under combined earthquake and wave-current action. *Marine Structures*, 58(September 2017):301–320, 2018. ISSN 09518339. doi: 10.1016/j.marstruc.2017.12.004. URL https://doi.org/10.1016/j.marstruc.2017.12.004.

[85] Liao Shouqin. Tianjin University, 2016. URL http://www.tju.edu.cn/english/info/1009/1463.htm.

[86] Amr S Elnashai, Billie F Spencer, Jr., Daniel A Kuchma, Guangqiang Yang, Juan E Carrion, Quan Gan, and Sung Jig Kim. The Multi-Axial Full-scale Sub-structured Testing and Simulation (MUST-SIM) Facility at the University of Illinois at Urbana-Champaign. In *Advances in Earthquake Engineering for Urban Risk Reduction*, pages

245–260. Kluwer Academic Publishers, 2006. doi: 10.1007/1-4020-4571-9{_}16. URL http://link.springer.com/10.1007/1-4020-4571-9_16.

[87] Bora Gencturk and Farshid Hosseini. Evaluation of reinforced concrete and reinforced engineered cementitious composite (ECC) members and structures using small-scale testing. *Canadian Journal of Civil Engineering*, 42(3):164–177, 2015. ISSN 12086029. doi: 10.1139/cjce-2013-0445.

[88] Catherine W French, Arturo E Schultz, Jerome F Hajjar, Carol K Shield, Douglas W Ernie, Robert J Dexter, David H.-C. Du, Steven A Olson, Drew J Daugherty, and Chen P Wan. Multi-axial subassemblage testing (MAST) system: description and capabilities. In *13th World Conference on Earthquake Engineering*, number August, Vancouver, 2004.

[89] M J Hashemi, J Wilson, and G Burnett. Mixed-Mode Hybrid Simulation of Large-Scale Structures through Multi-Axis Substructure Testing (MAST) System. In *Proceedings of the Tenth Pacific Conference on Earthquake Engineering*, number November, Sydney, 2015.

[90] Giuseppe Abbiati, Catherine A Whyte, Vasilis Dertimanis, and Bozidar Stojadinovic. Hybrid simulation of large-scale structures at ETH Zurich: the new 8-actuator multi-axial subassemblage testing (MAST) setup. In *16th World Conference on Earthquake Engineering*, number January, page 712, Santiago, 2017. ISBN 1462715524.

[91] Y. Xiao. Experimental Methods for Seismic Simulation of Structural Columns: State-of-the-Art Review and Introduction of New Multiuse Structural Testing System. *Journal of Structural Engineering*, 145(3):1–11, 2019. doi: 10.1061/(ASCE)ST.1943-541X.0002269.

[92] Ali Imanpour, Robert Tremblay, Martin Leclerc, Romain Siguier, Guillaume Toutant, Yasaman Balazadeh Minouei, and Shawn You. Development and application of multi-axis hybrid simulation for seismic stability of steel braced frames. *Engineering Structures*, 252 (September 2021):113646, feb 2022. ISSN 01410296. doi: 10.1016/j.engstruct.2021.113646. URL https://linkinghub.elsevier.com/retrieve/pii/S0141029621017351.

[93] Kung-Juin Wang, Ming-Chieh Chuang, Keh-Chyuan Tsai, Chao-Hsien Li, Pu-Yuan Chin, and Shen-Yuo Chueh. Hybrid testing with model updating on steel panel damper substructures using a multi-axial testing system. *Earthquake Engineering and Structural Dynamics*, 48:347–365, 2019. doi: 10.1002/eqe.3139.

[94] Evan C Bentz, Frank J Vecchio, and Michael P Collins. Simplified modified compression field theory for calculating shear strength of reinforced concrete elements. *ACI Structural Journal*, 103(4):614–624, 2006. ISSN 08893241. doi: 10.14359/16438.

[95] Saeid Mojiri, Oh Sung Kwon, and Constantin Christopoulos. Development of a ten-element hybrid simulation platform and an adjustable yielding brace for performance evaluation of multi-story braced frames subjected to earthquakes. *Earthquake Engineering and Structural Dynamics*, 48(7):749–771, 2019. ISSN 10969845. doi: 10.1002/eqe.3155.

[96] Saeid Mojiri, Pedram Mortazavi, Oh-Sung Kwon, and Constantin Christopoulos. Seismic response evaluation of a five-story buckling-restrained braced frame using multi-element pseudo-dynamic hybrid simulations. *Earthquake Engineering & Structural Dynamics*, 50(12):3243–3265, 2021. doi: https://doi.org/10.1002/eqe.3508. URL https://onlinelibrary.wiley.com/doi/abs/10.1002/eqe.3508.

[97] Thomas T C Hsu, Abdeldjelil Belarbi, and Xiaobo Pang. A Universal Panel Tester. *Journal of Testing and Evaluation*, 23(1):41–49, 1995. ISSN 00903973. doi: 10.1520/jte10397j.

[98] Walter Kaufmann, Alexander Beck, Demis Karagiannis, and Dominik Werne. The Large Universal Shell Element Test LUSET. Technical report, 2009. URL https://doi.org/10.3929/ethz-a-010025751.

[99] Anthony J Friedman, Shirley J Dyke, Brian M Phillips, Ryan Ahn, Baiping Dong, Yunbyeong Chae, Nestor Castaneda, Zhaoshuo Jiang, Jianqiu Zhang, Young-Jin Cha, Ali Irmak Ozdagli, Billie F Spencer, James Ricles, Richard E Christenson, Anil Agrawal, and Richard Sause. Large-Scale Real-Time Hybrid Simulation for Evaluation of Advanced Damping System Performance. *Journal of Structural Engineering*, 141(6):4014150, 6 2015. ISSN 0733-9445. doi: 10.1061/(ASCE)ST.1943-541X.0001093. URL http://ascelibrary.org/doi/10.1061/%28ASCE%29ST.1943-541X.0001093.

[100] Keh-Chyuan Tsai, Hong-Yuan Wang, Chih-Hong Chen, Gee-Yu Liu, and Kung-Juin Wang. Substructure Pseudo Dynamic Performance of Hybrid Steel Shear Panels. *Steel structures*, 1:95–103, 2001.

[101] Jamin Park, Elias Strepelias, Nikos Stathas, Oh-Sung Kwon, and Stathis Bousias. Application of hybrid simulation method for seismic performance evaluation of rc coupling

beams subjected to realistic boundary condition. *Earthquake Engineering & Structural Dynamics*, 50(2):375–393, 2021. doi: https://doi.org/10.1002/eqe.3335. URL https://onlinelibrary.wiley.com/doi/abs/10.1002/eqe.3335.

[102] M. Seki, M. Teshigawara, and T. Okada. Simulation of Earthquake Response of Reinforced Concrete Building Frame by Computer-Actuator On-Line System. In G. A. Keramidas and C. A. Brebbia, editors, *Proceedings of the International Conference*, pages 317–328, Washington D.C., 1982. Springer, Berlin, Heidelberg. doi: https://doi.org/10.1007/978-3-662-11353-0.

[103] Pui-shum B Shing and Stephen A. Mahin. Experimental error propagation in pseudodynamic testing. Technical Report UCB/EERC-83/12, UC Berkeley, Berkeley, 1983.

[104] S. N. Dermitzakis and Stephen A. Mahin. Development of substructuring techniques for on-line computer controlled seismic performance testing. Technical report, UC Berkeley, Berkeley, 1985.

[105] Douglas A. Foutch, Subhash C. Goel, and Charles W. Roeder. Preliminary report on seismic testing of a full-scale six-story steel building. Technical report, University of Illinois at Urbana-Champaign, 1987.

[106] A. Igarashi, F. Seible, and G. A. Hegemier. Testing of full scale shear wall structures under seismic load. In *10th World Conference on Earthquake Engineering*, pages 2653–2658, Rotterdom, 1992.

[107] Koichi Takanashi, Hidetake Taniguchi, and Hisashi Tanaka. Inelastic response of H shaped columns to two dimensional earthquake motions. *Bulletin of Earthquake Resistant Structure Research Center*, 13:15–28, 1980.

[108] Narutoshi Nakata, Billie F. Spencer, and Amr S. Elnashai. Multi-Dimensional Hybrid Simulation Using A Six-Actuator Self-Reaction Loading System. In *14th World Conference on Earthquake Engineering*, number January, Beijing, 2008.

[109] Sung Jig Kim, Curtis J Holub, and Amr S Elnashai. Experimental investigation of the behavior of RC bridge piers subjected to horizontal and vertical earthquake motion. *Engineering Structures*, 33(7):2221–2235, 7 2011. ISSN 01410296. doi: 10.1016/j.engstruct.2011.03.013. URL http://linkinghub.elsevier.com/retrieve/

pii/S0141029611001453https://www.sciencedirect.com/science/article/pii/S0141029611001453.

[110] Laura N Lowes, Dawn E Lehman, Anna C Birely, Daniel A Kuchma, Kenneth P Marley, and Christopher R Hart. Earthquake response of slender planar concrete walls with modern detailing. *Engineering Structures*, 43:31–47, 10 2012. ISSN 01410296. doi: 10.1016/j.engstruct.2012.04.040. URL http://dx.doi.org/10.1016/j.engstruct.2012.04.040https://linkinghub.elsevier.com/retrieve/pii/S0141029612002350.

[111] Thomas M Frankie, Adel E Abdelnaby, Pedro Silva, David Sanders, Amr S Elnashai, Billie F Spencer, Jr., Daniel Kuchma, and Chia-Ming Chang. Hybrid Simulation of Curved Four-Span Bridge: Comparison of Numerical and Hybrid Experimental/Analytical Results and Methods of Numerical Model Calibration. In *ASCE Structures Congress 2013*, pages 721–732, Reston, VA, 4 2013. ISBN 978-0-7844-1284-8. doi: 10.1061/9780784412848.064. URL http://ascelibrary.org/doi/abs/10.1061/9780784412848.064.

[112] Adel E Abdelnaby, Thomas M Frankie, Amr S Elnashai, Billie F Spencer, Daniel A Kuchma, Pedro Silva, and Chia-Ming Chang. Numerical and hybrid analysis of a curved bridge and methods of numerical model calibration. *Engineering Structures*, 70:234–245, 7 2014. ISSN 01410296. doi: 10.1016/j.engstruct.2014.04.009. URL http://www.sciencedirect.com/science/article/pii/S0141029614002223http://www.sciencedirect.com/science/article/pii/S0141029614002223/pdfft?md5=e2d71a4bfd092d7e6a4cff945d1fff35&pid=1-s2.0-S0141029614002223-main.pdfhttp://linkinghub.elsevier.com/retrieve/.

[113] Justin A Murray and Mehrdad Sasani. Near-collapse response of existing RC building under severe pulse-type ground motion using hybrid simulation. *Earthquake Engineering & Structural Dynamics*, 45(7):1109–1127, 2016. doi: 10.1002/eqe.

[114] Riadh Al-Mahaidi, M Javad Hashemi, Robin Kalfat, Graeme Burnett, and John Wilson. *Multi-axis Substructure Testing System for Hybrid Simulation*. SpringerBriefs in Applied Sciences and Technology. Springer Singapore, Singapore, 2018. ISBN 978-981-10-5866-0. doi: 10.1007/978-981-10-5867-7. URL

https://link.springer.com/content/pdf/10.1007/978-981-10-5867-7.pdfhttp://link.springer.com/10.1007/978-981-10-5867-7.

[115] M Javad Hashemi, Hing-Ho Tsang, Yassamin Al-Ogaidi, John L Wilson, and Riadh Al-Mahaidi. Collapse Assessment of Reinforced Concrete Building Columns through Multi-Axis Hybrid Simulation. *ACI Structural Journal*, 114(2):437–449, 3 2017. ISSN 0889-3241. doi: 10.14359/51689438. URL http://www.concrete.org/Publications/InternationalConcreteAbstractsPortal.aspx?m=details&i=51689438.

[116] M. Javad Hashemi, Yassamin Al-Ogaidi, Riadh Al-Mahaidi, Robin Kalfat, Hing-Ho Tsang, and John L. Wilson. Application of Hybrid Simulation for Collapse Assessment of Post-Earthquake CFRP-Repaired RC Columns. *Journal of Structural Engineering*, 143(1), 2017. doi: 10.1061/(ASCE)ST.1943-541X.0001629.

[117] Ali Y Al-Attraqchi, M Javad Hashemi, and Riadh Al-Mahaidi. Hybrid simulation of bridges constructed with concrete-filled steel tube columns subjected to horizontal and vertical ground motions. *Bulletin of Earthquake Engineering*, 18(9):4453–4480, 7 2020. ISSN 1570-761X. doi: 10.1007/s10518-020-00871-7. URL https://doi.org/10.1007/s10518-020-00871-7http://link.springer.com/10.1007/s10518-020-00871-7.

[118] Andrei M Reinhorn, Mettupalayam V Sivaselvan, and Z Liang. Large scale real time dynamic hybrid testing technique – shake tables substructure testing. In Yoshito Itoh and Tetsuhiko Aoki, editors, *The First International Conference on Advances in Experimental Structural Engineering*, Nagoya, Japan, 2005. URL http://civil.eng.buffalo.edu/$~$reinhorn/PUBLICATIONS/05-08-19-AESE-Hybridtesting.pdf.

[119] Xiaoyun Shao, Andrei M Reinhorn, and Mettupalayam V Sivaselvan. Real-Time Hybrid Simulation Using Shake Tables and Dynamic Actuators. *Journal of Structural Engineering*, 137(7):748–760, 7 2011. ISSN 0733-9445. doi: 10.1061/(ASCE)ST.1943-541X.0000314. URL http://ascelibrary.org/doi/10.1061/%28ASCE%29ST.1943-541X.0000314.

[120] Yingpeng Tian, Xiaoyun Shao, Huimeng Zhou, and Tao Wang. Advances in Real-Time Hybrid Testing Technology for Shaking Table Substructure Testing. *Frontiers in Built Environment*, 6(August), 2020. ISSN 22973362. doi: 10.3389/fbuil.2020.00123.

[121] Esteban Villalobos Vega, P S Harvey Jr, J M Ricles, L Cao, and Daleen M Torres Burgos. Multi-Directional Real-Time Hybrid Simulation Study of Rolling Pendulum Isolation Systems for Seismic Risk Mitigation of Critical Building Contents. In *Proceedings of the 2022 International Modal Analysis Conference XL*, 2022.

[122] Baiping Dong, Richard Sause, and James M Ricles. Accurate real-time hybrid earthquake simulations on large-scale MDOF steel structure with nonlinear viscous dampers. *Earthquake Engineering & Structural Dynamics*, 44(12):2035–2055, 9 2015. ISSN 00988847. doi: 10.1002/eqe.2572. URL http://doi.wiley.com/10.1002/eqe.2572.

[123] O Na, S Kim, and S Kim. Multi-Directional Structural Dynamic Test using Optimized Real-time Hybrid Control System. *Experimental Techniques*, pages 1–12, 2 2014. ISSN 07328818. doi: 10.1111/ext.12080. URL http://doi.wiley.com/10.1111/ext.12080.

[124] Amirali Najafi and Billie F. Spencer, Jr. Multi-Axial Real-Time Hybrid Simulation for Substructuring with Multiple Boundary Points. *In press*, 2021.

[125] J. P. Merlet. Parallel manipulators: state of the art and perspectives. *Advanced Robotics*, 8(6):589–596, 1993. ISSN 15685535. doi: 10.1163/156855394X00275.

[126] E. F. Fichter, D. R. Kerr, and J. Rees-Jones. The Gough-Stewart platform parallel manipulator: A retrospective appreciation. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 223(1):243–281, 2009. ISSN 09544062. doi: 10.1243/09544062JMES1137.

[127] Hosam K. Fathy, Zoran S. Filipi, Jonathan Hagena, and Jeffrey L. Stein. Review of hardware-in-the-loop simulation and its prospects in the automotive area. *Modeling and Simulation for Military Applications*, 6228E, 2006. ISSN 0277786X. doi: 10.1117/12.667794.

[128] R. Isermann, J. Schaffnit, and S. Sinsel. Hardware-in-the-loop simulation for the design and testing of engine-control systems. *Control Engineering Practice*, 7(5):643–653, 1999. ISSN 09670661. doi: 10.1016/S0967-0661(98)00205-6.

[129] S. Olma, A. Kohlstedt, P. Traphöner, K. P. Jäker, and A. Trächtler. Substructuring and Control Strategies for Hardware-in-the-Loop Simulations of Multiaxial Suspension

Test Rigs. *International Federation of Automatic Control*, 49(21):141–148, 2016. ISSN 24058963. doi: 10.1016/j.ifacol.2016.10.533.

[130] Andreas Kohlstedt, Phillip Traphöner, Simon Olma, Karl Peter Jäker, and Ansgar Trächtler. Fast hybrid position / force control of a parallel kinematic load simulator for 6-DOF Hardware-in-the-Loop axle tests. *IEEE/ASME International Conference on Advanced Intelligent Mechatronics, AIM*, pages 694–699, 2017. doi: 10.1109/AIM.2017.8014098.

[131] Kim D. Otten, Dzu K. Le, James C. Akers, and Vicente J. Suarez. Status and Design Features of the new NASA GRC Mechanical Vibration Facility (MVF), 2010. URL http://www.teamcorporation.com/images/technical_documents/Presentations/NASA_2010_SCLV_MVF.pdf.

[132] MTS Systems Corporation. MAST (Multi-axial Simulation Table) Systems, 2018. URL https://www.mts.com/cs/groups/public/documents/library/dev_002251.pdf.

[133] C J Holub. Interaction of variable axial load and shear effects in RC bridges. Technical report, 2005.

[134] B Theon. Generic kinematic transforms package. *MTS Systems Corporation*, 2013.

[135] C H Ligeikis. Exploring uncertainty in real-time hybrid substructuring. Technical report, University of Connecticut, 2019.

[136] N Tsokanas. Real-time and stochastic hybrid simulation. Technical report, ETH Zurich, 2021.

[137] Amin Maghareh, Yuguang Fu, Herta Montoya, Johnny Condori, Zixin Wang, Shirley J. Dyke, and Arturo Montoya. A reflective framework for performance management (reform) of real-time hybrid simulation. *Frontiers in Built Environment*, 6, 2020. doi: 10.3389/fbuil.2020.568742.

[138] Jean-Pierre Merlet and Clément Gosselin. Parallel Mechanisms and Robots. In Bruno Siciliano and Oussama Khatib, editors, *Springer Handbook of Robotics*, chapter 12, pages 269–285. Springer Berlin Heidelberg, Berlin, Heidelberg, 2008. ISBN 978-3-540-30301-5.

[139] A. R. Plummer. A Detailed Dynamic Model of a Six-Axis Shaking Table. *Journal of Earthquake Engineering*, 12(4):631–662, may 2008. ISSN 1363-2469. doi: 10.1080/13632460701457264. URL http://www.tandfonline.com/doi/full/10.1080/13632460701457264.

[140] Ozgur Ozcelik, Joel P. Conte, and J. Enrique Luco. Comprehensive mechanics-based virtual model of NHERI@UCSD shake table—Uniaxial configuration and bare table condition. *Earthquake Engineering & Structural Dynamics*, 50(12):3288–3310, oct 2021. ISSN 0098-8847. doi: 10.1002/eqe.3510. URL https://onlinelibrary.wiley.com/doi/10.1002/eqe.3510.

[141] J Condori, A Maghareh, and J Orr. Exploiting Parallel Computing to Control Uncertain Nonlinear Systems in Real-Time. *Experimental techniques*, 44:735–749, 2020. doi: 10.1007/s40799-020-00373-w.

[142] Y Duan, J Tao, H Zhang, Wang S, and C Yum. Real-time hybrid simulation based on vector form intrinsic finite element and field programmable gate array. *Structural Control and Health Monitoring*, 26, 2017. doi: 10.1002/stc.2277.

[143] Mohammed Islmail, Faycal Ikhouane, and Jose Rodellar. The hysteresis bouc-wen model, a survey. *Archives of Computational Methods in Engineering*, 16:161–188, 2009. doi: 10.1007/s11831-009-9031-8.

[144] M. Amir, K.G. Papakonstantinou, and G.P. Warn. A consistent timoshenko hysteretic beam finite element model. *International Journal of Non-Linear Mechanics*, 119:103218, 2020. ISSN 0020-7462. doi: https://doi.org/10.1016/j.ijnonlinmec.2019.07.003. URL https://www.sciencedirect.com/science/article/pii/S0020746219300836.

[145] M. Amir, K. G. Papakonstantinou, and G. P. Warn. State-space formulation for structural analysis with coupled degradation-plasticity and geometric nonlinearity. *Journal of Structural Engineering*, 148(4):04022016, 2022.

[146] Christian E. Silva, Daniel Gomez, Amin Maghareh, Shirley J. Dyke, and Billie F. Spencer. Benchmark control problem for real-time hybrid simulation. *Mechanical Systems and Signal Processing*, 135:106381, 2020. ISSN 0888-3270. doi: https://doi.org/10.1016/j.ymssp.2019.106381. URL https://www.sciencedirect.com/science/article/pii/S0888327019306028.

[147] W Song, C Sun, Y Zuo, V Jahangiri, Y Lu, and Q Han. Conceptual Study of a Real-Time Hybrid Simulation Framework for Monopile Offshore Wind Turbines Under Wind and Wave Loads. *Frontiers in Built Environment*, 6(129), 2020.

[148] M Thys, V Chabaud, T Sauder, L Eliassen, L O Saether, and O B Magnussen. Real-time hybrid model testing of a semi-submersible 10mw floating wind turbine and advances in the test method). In *Proceedings of the IOWTC 2018 1st International Offshore Wind Technical Conference*, page Paper No. 1081, San Francisco, 2018.

[149] Teng Wu, Shaopeng Li, and Mettupalayam Sivaselvan. Real-time aerodynamics hybrid simulation: A novel wind-tunnel model for flexible bridges. *Journal of Engineering Mechanics*, 145(9):04019061, 2019. doi: 10.1061/(ASCE)EM.1943-7889.0001649.

[150] Catherine A. Whyte, Kevin R. Mackie, and Bozidar Stojadinovic. Hybrid simulation of thermomechanical structural response. *Journal of Structural Engineering*, 142(2): 04015107, 2016. doi: 10.1061/(ASCE)ST.1943-541X.0001346.

[151] G Abbiati, O S Bursi, B Stojadinovic, N Tondini, and C Whyte. Hybrid simulation of heat transfer problems in structural applications). In *VI International Conference on Computational Methods for Coupled Problems in Science and Engineering*, pages 254–265, Sitges, 2015.

[152] S J Dyke, K Marais, I Bilionis, J Werfel, and R Malla. Strategies for the design and operation of resilient extraterrestrial habitats. Proc. SPIE Smart Structures + Nondestructive Evaluation Conference, 2021.

[153] A Maghareh, A Lenjani, M Krishnan, S J Dyke, and I Bilionis. Role of Cyber-Physical Testing in Developing Resilient Extraterrestrial Habitats. Proceedings of the ASCE Earth and Space Conference, 2021.