Predictive Resilience Modeling

Priscila Silva
Department of Electrical and Computer Engineering
University of Massachusetts Dartmouth
Dartmouth, Massachusetts
psilva4@umassd.edu

Mariana Hermosillo Hidalgo
Department of Industrial, Manufacturing and Systems Engineering
The University of Texas at El Paso
El Paso, Texas
mhermosillo3@miners.utep.edu

Igor Linkov
Engineer Research and Development Center
U.S. Army Corps of Engineers
Concord, Massachusetts
igor.linkov@usace.army.mil

Lance Fiondella
Department of Electrical and Computer Engineering
University of Massachusetts Dartmouth
Dartmouth, Massachusetts
lfiondella@umassd.edu

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***Abstract*—Resilience is the ability of a system to respond, absorb, adapt, and recover from a disruptive event. Dozens of metrics to quantify resilience have been proposed in the literature. However, fewer studies have proposed models to predict these metrics or the time at which a system will be restored to its nominal performance level after experiencing degradation. This paper presents two alternative approaches to model and predict performance and resilience metrics with techniques from reliability engineering, including (i) bathtub-shaped hazard functions and (ii) mixture distributions . Given their ease of accessibility, historical data sets on job losses during recessions in the United States are used to assess the predictive accuracy of these approaches. Goodness of fit measures and confidence interval are computed to assess how well the models perform on the data sets considered. The results suggest that both approaches can produce accurate predictions for data sets exhibiting V and U shaped curves, but that L and W shaped curves that respectively experience a sudden drop in performance or deviate from the assumption of a single decrease and subsequent increase cannot be characterized well by either class of model proposed, necessitating additional modeling efforts that can capture these more general scenarios.**

***Index Terms*—resilience metrics, bathtub-shaped hazard functions, mixture distributions**

I. Introduction

In reliability engineering [1], a system may be modeled as repairable or non-repairable. Models for non-repairable systems characterize the time between commissioning a system and its eventual failure, whereas models for repairable systems characterize failure and repair times to compute properties such as availability and optimal maintenance policies [2]. In either case, only two states are considered, the fully operational and failed state. In the failed state, the system performance is assumed to be zero. Resilience engineering [3], [4] may thus be regarded as a generalization of repairable systems from reliability engineering, in which the performance level is degraded due to aging or externals shocks but is proactively maintained to preserve nominal performance equivalent to the fully operational state in reliability modeling.

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Resilience engineering is a complex, hierarchal and multi disciplinary field [5], finding applications in a diverse spectrum of engineering and social science domains. As a result, researchers have proposed a variety of metrics [6] to quantify the resilience of these systems. However, application of these metrics is almost exclusively performed on data after recovering. This approach enables retrospective analysis to assess how well the system performed under stress and inform future design and operational decisions, yet this approach does not project when the system will recover to a specified level of performance or what actions to take in order to reach a target level of performance quickly and cost effectively. Without predictive models, emergency management teams tasked with making critical decisions at times of intense stress will struggle to optimally respond to disruptive scenarios that may impact the lives of thousands or millions of individuals.

Relevant research on quantitative resilience metrics includes the work of Biringer et al. [7] who presented seismic, probabilistic, and economic resilience indexes focusing in structural requirements for a system to behave resiliently under malevolent or natural hazard. Hosseini et al. [6] reviewed the definitions of resilience across different application domains, classifying over 50 articles according to the methods proposed, including qualitative and quantitative, further subdividing quantitative approaches into the categories of generic resilience metrics or structural modeling. Cheng et al. [8] performed a comprehensive survey of quantitative methods to assess system resilience, including interval-based, point-based, and probabilistic metrics as well as metrics that consider multiple system attributes. For example, taking interval-based metrics into consideration, Bruneau and Reinhorn [9] defined resilience as the area under the curve to quantify the normalized performance preserved relative to a baseline, whereas Ouyang and Dueñas-Osorio [10] defined resilience as the ratio of the area under the curve over the area under the baseline (average normalized performance preserved). Ouyang and

Dueñas-Osorio [11] subsequently extended to average performance preserved when the hazard of interest is characterized by a Poisson process. Yang and Frongopol [12] measured the loss of resilience due to a hazard (performance lost). Zhou et al. [13] defined resilience as the ratio of system performance lost and target performance during disruption and recovery period (average normalized performance lost). Zobel [14] defined resilience from the point at which performance is lowest (performance preserved from the minimum to recovery minus minimum performance). Reed et al. [15] proposed average performance preserved as a resilience metric, while Cimerallo [16] suggested a resilience metric that employs a user-specified weight to place emphasis on the area under the curve before and after the critical condition (minimum performance).

To overcome the limitations of past research, this paper considers two alternative approaches to model and predict resilience as well as various metrics with techniques from reliability engineering, including (i) bathtub-shaped hazard functions [17], and (ii) mixture distributions to characterize deterioration and recovery. We assess these alternative approaches in the context of widely accessible historical data on recessions in the United States. Goodness of fit measures and confidence interval are computed to quantify how well each model performs on the data sets considered. Our results indicate that the competing risks form of the bathtub-shaped function performed best with respect to the adjusted coefficient of determination. Moreover, data sets that contain a sudden drop in performance or deviate from the assumption of a single decrease and subsequent increase could not be fit to either class of proposed models. These results suggest that classical reliability modeling techniques are suitable for modeling and prediction of some resilience curves, but that explanatory factors and domain specific information may increase the predictive accuracy of the models.

The remainder of the paper is organized as follows: Section II presents two alternative approaches to model resilience. Section III describes model fitting techniques, validation, and statistical inference methods. Section IV reviews common interval-based resilience metrics. Section V provides illustrative examples of the alternative modeling and prediction approaches. Section VI offers conclusions and identifies future research.

II. Resilience Modeling Approaches

This section develops quantitative resilience models with alternative techniques from reliability engineering and statistics to characterize the performance of a system. The concept of performance is domain dependent, but may be defined generally as the level of goal achievement of a system or task. For example, in the context of cybersecurity [18], [19], performance may be measured in terms of the computational capacity or bandwidth preserved when some computers within a network are compromised due to diverse attacks that affect various functions and degrade the system. It is also important to distinguish between system and mission performance, since the available computational capacity or bandwidth may degrade services such as e-commerce, compromising business or economic activities.

Figure 1 provides a conceptual view of a resilience curve possessing a bathtub shape.

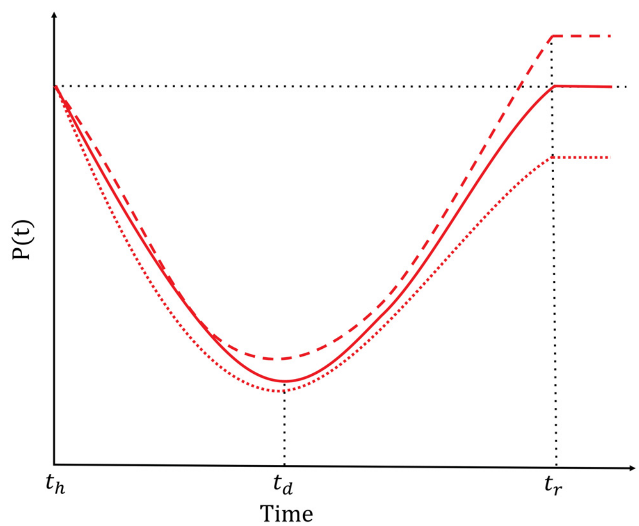

Fig. 1. Conceptual resilience curve.

The dotted horizontal line indicates the nominal system performance $P(t)$ at time t_h when the disruptive event occurs. Performance deteriorates until a minimum is experienced at time t_d. In some cases deterioration is instantaneous, in which case $t_d = t_h$. In other cases, the system is not resilient and performance deteriorates to $P(t_d) = 0$. In cases, where the system is resilient, system performance improves from minimum t_d until a new steady state performance is reached at at time t_r. Figure 1 indicates three possibilities, namely degraded performance (dotted), nominal (solid), and improved performance (dashed). Physical systems such as power generation may only exhibit recovery to nominal or degraded performance, whereas economic systems and computational systems such as machine learning are capable of achieving improved performance.

A. Bathtub Shaped Functions

The following piece wise function can be used to specify alternative resilience curves with bathtub shaped hazard functions $\lambda(t)$ from reliability engineering

$$P(t) = \begin{cases} P(t_h) & t \leq t_h \\ c \times \lambda(t) & t_h \leq t \leq t_r \\ P(t_r) & t_r \leq t \end{cases}$$

where the nominal performance before the hazard and after recovery are respectively $P(t_h)$ and $P(t_r)$. The normalizing constant c ensures continuity because $P(t_h) \neq \lambda(t_h)$.

The following subsections specify alternative bathtub-shaped curves to model resilience and derive expressions for the time at which the system recovers to a new steady state performance level $P(t_r)$ and area under the resilience curve $P(t)$ when analytical forms exist.

1) Quadratic Model: The form of the quadratic hazard function is

$$\lambda(t) = \alpha + \beta t + \gamma t^2 \tag{1}$$

is bathtub-shaped when $-2(\alpha\gamma)^{1/2} \leq \beta < 0$ and α, $\gamma \geq 0$.

The time at which the system recovers to performance level $P(t_r)$ is

$$t_r = \frac{-\beta - \sqrt{\beta^2 - 4\alpha\gamma + 4\alpha\gamma P(t_r)}}{2\gamma} \tag{2}$$

and area under the resilience curve characterized by the quadratic hazard function is

$$P(t) = \alpha t + \frac{\beta t^2}{2} + \frac{\gamma t^3}{3}\Big|_{t_h}^{t_r} \tag{3}$$

2) Competing Risks Model: The competing risks model [20] is capable of displaying increasing, decreasing, constant, and bathtub-shaped rates

$$\lambda(t) = \frac{\alpha}{1 + \beta t} + 2\gamma t \tag{4}$$

The time at which the system recovers to performance level $P(t_r)$ is

$$t_r = \frac{\beta P(t_r) - 2\gamma + \sqrt{\beta^2 P(t_r) + 4\beta\gamma P(t_r) - 8\alpha\beta\gamma + 4\gamma^2}}{4\beta\gamma} \tag{5}$$

and area under the resilience curve characterized by the competing risks model is

$$P(t) = \gamma t^2 + \frac{\alpha \log(1 + \beta t)}{\beta}\Big|_{t_h}^{t_r} \tag{6}$$

B. Mixture Distributions

The following general form can be used to specify alternative resilience curves as mixture distributions

$$P(t) = a_1(t)(1 - F_1(t)) + a_2(t)F_2(t) \tag{7}$$

where $a_1(t)$ is the transition from degradation, with $\lim_{t\to 0^+} a_1(t) = 1$ and $\lim_{t\to\infty} a_1(t) = 0$, and $a_2(t)$ is the transition to recovery. $F_1(t)$ and $F_2(t)$ are an arbitrary cumulative distribution functions (CDF). Thus, the terms $(1-F_1(t))$ and $F_2(t)$ respectively characterize degradation and recovery processes.

III. Model fitting, validation, and inference

This section describes methods to fit and validate models as well as make inferences based on these models.

A. Model fitting technique

Least squares estimation (LSE) is a common approach to estimate parameters of a model, which explains a resilience curve possessing deterministic variables. LSE [21] minimizes the disagreement between the empirical resilience data and a specific parametric model in order to identify numerical estimates of that model's parameters.

$$\min \sum_{i=1}^{n} (R(t_i) - P(t_i))^2 \tag{8}$$

where n is the number of measurements used for model fitting, $R(t_i)$ is the empirical resilience at time t_i, and $P(t_i)$ is the corresponding value of resilience according to the model.

B. Validation and statistical inference

This section describes methods to validate models and make inferences based on them, including statistical goodness of fit measures and establishing confidence intervals around model fits based on the sample size and corresponding uncertainty.

1) Goodness of fit measures: Goodness of fit measures assess how well a model performs on a given data set. In practice, no model performs best with respect to all measures. Goodness of fit measures provide a quantitative approach to compare alternative models. Models with lower error are preferred. However, model selection is ultimately a subjective choice that must be made by a decision-maker. A primary consideration is the tradeoff between model complexity and predictive accuracy.

Sum of squares error is calculated by fitting a model with n observations with least squares estimation as specified in Equation (8) and then computing the sum of squares difference between the observations and model predictions.

$$SSE = \sum_{i=1}^{n} (R(t_i) - P(t_i))^2 \tag{9}$$

where the difference $R(t_i) - P(t_i)$ is known as the error or residual.

Predictive mean square error fits a model with the first $n - \ell$ observations and then computes the sum of squares of the prediction residuals for the remaining ℓ observations not used to fit the model.

$$PMSE = \frac{1}{\ell} \sum_{i=(n-\ell+1)}^{n} (R(t_i) - P(t_i))^2 \tag{10}$$

Adjusted coefficient of determination is the proportion of the variation in the dependent variable that is explained by independent variables and is calculated according to

$$r_{adj}^2 = 1 - \left(1 - \frac{SSY - SSE}{SSY}\right)\left(\frac{n-1}{n-m-1}\right) \quad (11)$$

where

$$SSY = \sum_{i=1}^{n} \left(R(t_i) - \overline{R}(t)\right)^2$$

is the sum of squares error associated with the naive predictor $\overline{R}(t)$. Thus, the r_{adj}^2 value quantifies the degree of linear correlation between the changes in performance of the empirical resilience and those predicted by the model. A value of r_{adj}^2 closer to 1.0 indicates a strong relationship between the data and model.

2) Confidence intervals: A confidence interval (CI) [22] establishes a range for an estimated value or model parameter, according to a desired level of confidence specified by the user. To obtain an approximate confidence interval for the predictions of a regression model, the variance of SSE

$$\sigma^2 = \left(\frac{1}{n-2}\right) SSE \quad (12)$$

represents the dispersion between the set of predictions and their average value. Lower and upper confidence interval limits for the change in performance between interval $(i-1)$ and i is

$$CI = \Delta P(t_i) \pm z_{1-\alpha/2}\sqrt{\sigma^2} \quad (13)$$

where $z_{1-\alpha/2}$ is the critical value of the standard normal distribution and α the user-specified level of significance.
Empirical coverage (EC) is the percentage of observations contained by the confidence intervals, which is computed by dividing the number of observations within the interval by the total number of observations (n).

IV. Metrics

Categories of metrics [8] include interval-based, point-based, and probabilistic metrics as well as metrics that consider multiple system attributes. This paper focuses on interval-based resilience metrics, since they measure a system's performance over a period of time, quantifying resilience based on performance during the hazard and recovery periods.

The area under the curve [9] measures the performance preserved as

$$R(t_r) = \int_{t_h}^{t_r} P(t)dt \quad (14)$$

and it can also be expressed as the ratio of system actual and nominal performance (average normalized performance preserved) [10], [11].

$$R(t_r) = \frac{\int_{t_h}^{t_r} P(t)dt}{P(t_h)(t_r - t_h)} \quad (15)$$

Conversely, the performance lost is measured as area above the curve [12], and is simply the difference between actual and nominal performance.

$$R(t_r) = P(t_h)(t_r - t_h) - \int_{t_h}^{t_r} P(t)dt \quad (16)$$

It can also be expressed in its normalized form as the area above the curve over the nominal performance (average normalized performance lost) [13].

$$R(t_r) = \frac{\int_{t_h}^{t_r} (P(t_h) - P(t))dt}{P(t_h)(t_r - t_h)} \quad (17)$$

Performance from the minimum [14] has been expressed as the performance preserved from the minimum to recovery minus the rectangular region below the minimum performance.

$$R(t_r) = \int_{t_d}^{t_r} P(t)dt - P(t_d)(t_r - t_d) \quad (18)$$

The average performance preserved is expressed as the area under the curve from the hazard to recovery divided by the duration between hazard and recovery [15]

$$R(t_r) = \frac{\int_{t_h}^{t_r} P(t)dt}{t_r - t_h} \quad (19)$$

Similarly, the average performance lost [15] is

$$R(t_r) = \frac{P(t_h)(t_r - t_h) - \int_{t_h}^{t_r} P(t)dt}{t_r - t_h} \quad (20)$$

The weighted average of performance preserved before and after the minimum [16] provides an5 integrated measure of the degradation and recovery processes

$$R(t_r) = \alpha \frac{\int_{t_h}^{t_d} P(t)dt}{t_d - t_h} + (1-\alpha)\frac{\int_{t_d}^{t_r} P(t)dt}{t_r - t_d} \quad (21)$$

where α is a user specified weight factor in the interval $(0,1)$. Larger values of α place greater importance on the period from hazard to the minimum, while smaller values of α emphasize the period from the minimum to recovery.

To apply interval-based metrics in a predictive manner, t_h is replaced with the first time interval not used for model fitting, namely $t_{n-\ell+1}$. Similarly, t_r is set to the last time interval (t_n). Additional considerations include Equation (18) and Equation (21), which require knowledge of the minimum performance (t_d). In cases where the minimum is contained within the observed data, that value is used. Similarly, in cases, where the minimum has not yet been observed, the interval predicted by the fitted model to experience the minimum $(\hat{t}_d)$ is used for the purpose of calculating metrics. Moreover, t_h is set to t_0 in Equation (21), since this final metric utilizes the entire interval.

V. Illustrations

This section illustrates the proposed modeling approaches through a series of examples. The two alternative approaches to model resilience, including (i) bathtub-shaped hazard functions from reliability engineering, and (ii) mixture distributions to characterize deterioration and recovery. In each case, the model goodness of fit is assessed as well as the models' ability to predict performance and resilience metrics.

Although the modeling efforts are general and aspire to advance the general theory of predictive resilience engineering across domains such as infrastructure systems, including electric power and transportation networks, cybersecurity, and machine learning, data in these areas is not shared widely. Therefore, the experiments reported here are illustrated using seven U.S. recessions shown in Figure 2, which were documented by the Bureau of Labor Statistics' Current Employment Statistics Program [23], [24], including the most recent recession that began in March, 2020 at the start of the COVID-19 pandemic.

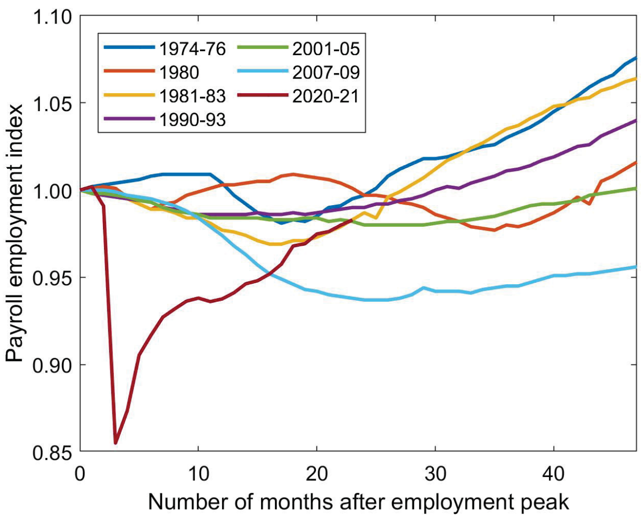

Fig. 2. Payroll change in U.S. recessions from peak employment

Each curve shows the normalized number of individuals employed with time step zero indicating peak employment prior to a period of job loss and recovery. Economists describe resilience curves [25] with various letters of the English alphabet, including V, U, W, L, J, K, and J. In a V-shaped recession, the economy suffers a sharp but brief degradation followed by a similarly strong recovery, while a U-shaped recession deteriorates and recovers more slowly. W-shaped recessions possess two successive periods of degradation and recovery in sequence. L-shaped recessions experience a sharp decline followed by a long period of under performance. J-shaped recessions are characterized by a slow recovery but eventually return to pre-recession growth trends. K-shaped recessions suffers a long sharp drop and divergent recovery paths that are difficult to describe.

Table I indicates the competing risks model produced lower PMSE on all data sets other than the 1974-76 and 2001-05 data. Moreover, the competing risks model was a close second on nearly all measures when the quadratic model performed best. Neither model performed well on the 1980 data, since it exhibits a W-shaped curve. Hence, both the quadratic model and competing risks model performed substantially poorer on the 1980 data, resulting in low or even negative r^2_{adj} in the case of the quadratic model. Both models also fit the 2020-21 data poorly because deterioration in performance occurred rapidly, which is a characteristic of L and K-shaped recessions. Thus, the competing risks model exhibited greater flexibility, but neither model could characterize the 1980 or 2020-2021 data satisfactorily.

TABLE I
VALIDATION OF PREDICTION USING TWO BATHTUB FUNCTIONS ON DATA FROM SEVEN U.S. RECESSIONS

U.S Recession	n	Measure	Quadratic	Competing Risks
1974-76	48	SSE	**0.00227675**	0.00255851
		PMSE	**0.00000037**	0.00000062
		r^2_{adj}	**0.91792100**	0.90776400
		EC	97.91%	95.83%
1980	48	SSE	0.00472714	**0.00430915**
		PMSE	0.00002572	**0.00002508**
		r^2_{adj}	-0.07161330	**0.02314130**
		EC	95.83%	95.83%
1981-83	48	SSE	0.00503712	**0.00183996**
		PMSE	0.00008464	**0.00000841**
		r^2_{adj}	0.84347000	**0.95378100**
		EC	93.75%	97.91%
1990-93	48	SSE	0.00005197	**0.00003800**
		PMSE	0.00000037	**0.00000015**
		r^2_{adj}	0.99511300	**0.99642600**
		EC	91.66%	97.91%
2001-05	48	SSE	**0.00008087**	0.00010226
		PMSE	**0.00000010**	0.00000012
		r^2_{adj}	**0.95919200**	0.94839300
		EC	95.83%	95.83%
2007-09	48	SSE	**0.00165841**	0.00186147
		PMSE	0.00000509	**0.00000008**
		r^2_{adj}	**0.92051800**	0.91078600
		EC	97.91%	95.83%
2020-21	24	SSE	0.02328560	**0.01771130**
		PMSE	0.00022676	**0.00002900**
		r^2_{adj}	0.11727200	**0.32858600**
		EC	90.47%	90.47%

Figure 3 shows the 2001-05 U.S. recession data, fitted quadratic model, and 95% confidence interval (grey region centered around the quadratic model fit) computed with Equation (13). The dashed vertical line at $t = 42$ indicates that the first 43 months were used for model fitting and the last five months use to compute predictive accuracy measures. Since all but two of the 48 observed data points are within the confidence interval, the EC is 95.83%, which is slightly conservative.

Figure 4 shows the 1990-93 U.S. recession data, fitted competing risks model, and 95% confidence interval. Since all but one of the 48 observed data points are within the confidence interval, the empirical coverage is 97.91%, which is also conservative.

In addition to curve fitting and traditional methods to assess the statistical validity of resilience models based on bathtub shaped distributions, it is also possible to make predictions for each of the interval-based metrics described in Section IV.

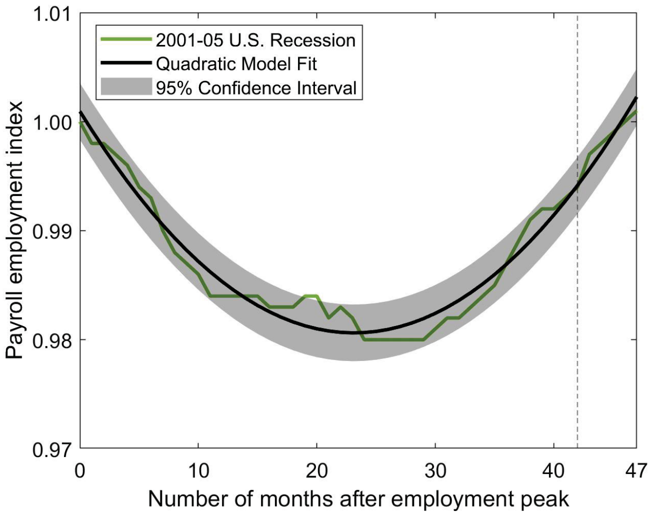

Fig. 3. Quadratic model fit to 2001-05 U.S recession data.

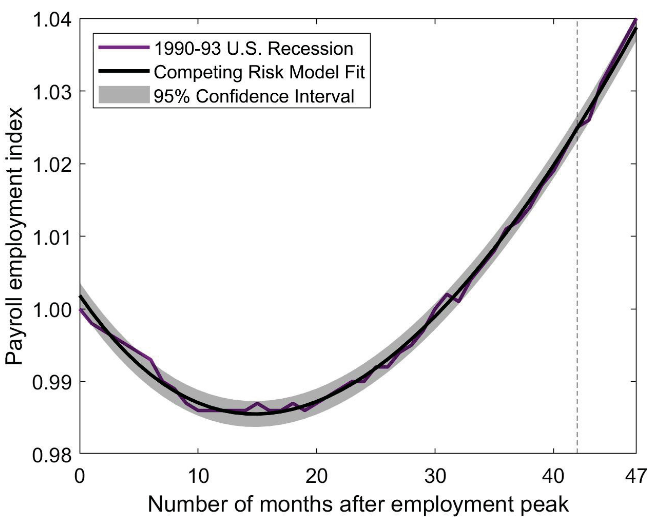

Fig. 4. Competing risks model fit to 2001-05 U.S recession data set.

Table II reports the predictive metrics for the quadratic and competing risks models fit to the 1990-93 data set, and their respective relative error according to

$$\delta = \left| \frac{R(t)_{actual} - R(t)_{predicted}}{R(t)_{actual}} \right| \tag{22}$$

and $\alpha = 0.5$ for weighted average of performance preserved before and after the minimum (Equation (21)). Table II indicates that the quadratic model exhibited lower relative error on five of eight metrics, but that both models achieve error of less than 0.01 on all metrics except for the normalized average performance loss, which was larger because of the normalization step. Negative values in the performance loss metrics can be interpreted as the system having recovered to a higher performance level than the time at which the disruption occurred, producing a negative performance loss in the predictive period.

TABLE II
INTERVAL-BASED RESILIENCE METRICS USING BATHTUB SHAPED FUNCTIONS AND 1990-93 U.S. RECESSIONS DATA

Metrics	Data	Quadratic	Competing Risks
Performance preserved	Actual	5.168000	5.168000
	Predicted	5.176470	5.165280
	δ	0.001638	**0.00052**
Performance lost	Actual	−1.064000	−1.064000
	Predicted	−1.061390	−1.055290
	δ	**0.00245**	0.008183
Normalized average performance preserved	Actual	1.259260	1.259260
	Predicted	1.257930	1.256760
	δ	**0.00105**	0.001982
Normalized average performance lost	Actual	0.001059	0.001059
	Predicted	−0.007925	−0.006763
	δ	**0.14403**	0.269552
Performance preserved from the minimum	Actual	1.603000	1.603000
	Predicted	1.598770	1.618710
	δ	**0.00264**	0.009798
Average performance preserved	Actual	1.292000	1.292000
	Predicted	1.294120	1.291320
	δ	0.001638	**0.00052**
Average performance lost	Actual	−0.266000	−0.266000
	Predicted	−0.265346	−0.263823
	δ	**0.00245**	0.008183
Average performance preserved before/after minimum	Actual	0.514662	0.514662
	Predicted	0.518942	0.518053
	δ	0.008316	**0.00658**

A. Example II: Mixture Distributions

In the second experiment, least squares estimation was applied according to Equation (8) in order to estimate the parameters of the resilience curve characterized by the mixture distribution model (Equation (7)), given 90% of each data set shown in Figure 2. Four pairwise combinations of $F_1(t)$ and $F_2(t)$ from reliability engineering [26] were considered to characterize degradation and recovery, including the Weibull (Wei) distribution, which possesses the form

$$F(t) = 1 - e^{-(\frac{t}{\lambda})^k} \tag{23}$$

and the simpler exponential (Exp) distribution, which is obtained by setting $k = 1$ in Equation (23). The trend describing the transition from degradation was held constant at $a_1(t) = 1$ for simplicity. Alternative forms of transition to recovery considered included

$$a_2(t) = \{\beta, \beta t, e^{\beta t}, \beta \ln(t)\}$$

each of which corresponds to an increasing trend characteristic of economic data. Predictions were then made for the last 10% of the data not used for model fitting and the SSE (Equation (9)), PMSE (Equation (10)), r^2_{adj} (Equation (11)), and EC computed, as shown in Table III for $a_2(t) = \beta \ln(t)$, which performed well for each data set shown in Figure 2. Table III indicates that the simplest mixture composed of the Exponential distributions, denoted (Exp-Exp), performed poorly with respect to all measures on all data sets. At least

TABLE III

VALIDATION OF PREDICTION USING MIXTURE DISTRIBUTIONS ON DATA FROM SEVEN U.S. RECESSIONS DATA

U.S Recession	Measures	Exp-Exp	Wei-Exp	Exp-Wei	Wei-Wei
1974-76	SSE	0.018194	**0.002471**	0.002473	0.005806
	PMSE	0.000029	**0.000009**	0.000009	0.000023
	r^2_{adj}	0.344729	**0.910974**	0.910926	0.790891
	EC	97.91%	100%	100%	100%
1980	SSE	0.007981	0.005087	0.007983	**0.005087**
	PMSE	0.000141	0.000055	0.000141	**0.000055**
	r^2_{adj}	−0.809364	−0.153224	−0.809915	**−0.153203**
	EC	91.66%	91.66%	93.75%	93.75%
1981-83	SSE	0.023730	0.003452	**0.003452**	0.004281
	PMSE	0.000115	0.000051	0.000051	**0.000021**
	r^2_{adj}	0.403901	0.913269	**0.913279**	0.892453
	EC	91.66%	89.58%	93.75%	100%
1990-93	SSE	0.021164	**0.000202**	0.000203	0.000570
	PMSE	0.000033	**0.000001**	0.000001	0.000002
	r^2_{adj}	−0.990459	**0.980913**	0.980908	0.946310
	EC	93.75%	97.91%	97.91%	100%
2001-05	SSE	0.019830	**0.000195**	0.000195	0.000486
	PMSE	0.000027	**0.000001**	0.000004	0.000003
	r^2_{adj}	0.006810	**0.901303**	0.901281	0.754430
	EC	95.83%	100%	100%	100%
2007-09	SSE	0.016917	0.016917	**0.002022**	0.005496
	PMSE	0.000067	0.000067	0.000025	**0.000015**
	r^2_{adj}	0.189229	0.189229	**0.903069**	0.736572
	EC	97.91%	97.91%	93.75%	97.91%
2020-21	SSE	0.017936	0.018654	0.017936	**0.015770**
	PMSE	0.000007	0.000216	0.000007	**0.000007**
	r^2_{adj}	0.320035	0.292833	0.320035	**0.402177**
	EC	85.71%	95.23%	90.47%	95.83%

one of the remaining three combinations for $F_1(t)$ and $F_2(t)$ (Wei-Exp, Exp-Wei, and Wei-Wei) achieved an r^2_{adj} greater than 0.9 on all data sets with the exception of the 1980 and 2020-21 data sets because of the W-shaped curve and sharp drop associated with the L and K-shaped curves respectively. In some cases, the bathtub shaped curves achieved a slightly higher r^2_{adj} than mixture distribution models partially because the number of parameters in the mixture models was not sufficient to substantially increase the model fit.

Figure 5 shows the 1990-93 U.S. recession data and the fitted (Wei-Exp) model, while Figure 6 shows the 1981-83 recession data and the fitted (Exp-Wei) and (Wei-Wei) models, since the former achieved better SSE and r^2_{adj}, but the latter attained a lower PMSE on that data set.

Figures 5 and 6 also show the corresponding 95% confidence intervals computed with Equation (13). Since all of the observed data points shown in Figure 5 are within the confidence interval, the empirical coverage was 100%. The Wei-Wei confidence interval (light grey) of Figure 6 also exhibited 100% empirical coverage, but the empirical coverage for the Exp-Wei confidence interval (dark grey) was 93.75%.

Predictions for each of the interval-based metrics described in Section IV are reported in Table IV for all four combinations of the mixture distributions fit to the 1990-93 recession data, and their respective relative error, where $\alpha = 0.5$ for the weighted average of performance preserved before and after the minimum (Equation (21)). Table IV indicates that Wei-Exp model achieved the lowest relative error on four of eight metrics predicted, the Exp-Exp two metrics, and the Wei-Wei two metrics. Considering Tables III and IV together, the combination characterizing the deterioration and recovery by the Weibull and exponential distributions respectively most frequently predicted performance as well as metrics for the data sets considered, while several of the other combinations also exhibited similar accuracy.

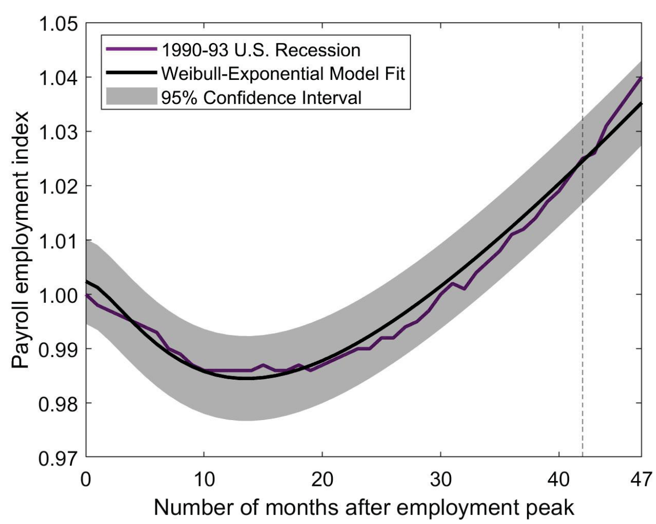

Fig. 5. Fit of Weibull-Exponential model fit to 1990-93 U.S recession data set.

TABLE IV
INTERVAL-BASED RESILIENCE METRICS USING MIXTURE DISTRIBUTIONS AND 1990-93 U.S. RECESSIONS DATA

Metrics	Data	Exp-Exp	Wei-Exp	Exp-Wei	Wei-Wei
Performance preserved	Actual	5.16800000	5.16800000	5.16800000	5.16800000
	Predicted	5.25294000	5.15458000	5.15457000	5.14434000
	δ	0.01643570	**0.0025963**	0.00259779	0.00457829
Performance lost	Actual	−1.06400000	−1.06400000	−1.06400000	−1.06400000
	Predicted	−1.08461000	−1.04811000	−1.04811000	−1.04552000
	δ	0.01937030	**0.0149350**	0.01493750	0.01736510
Normalized average performance preserved	Actual	1.25926000	1.25926000	1.25926000	1.25926000
	Predicted	1.26020000	1.25523000	1.25523000	1.25508000
	δ	**0.0007490**	0.00319698	0.00319726	0.00331930
Normalized average performance lost	Actual	0.00105905	0.00105905	0.00105905	0.00105905
	Predicted	−0.01020250	−0.00523343	−0.00523308	−0.00507940
	δ	**0.1018730**	0.43478900	0.43482700	0.45142500
Performance preserved from the minimum	Actual	1.60300000	1.60300000	1.60300000	1.60300000
	Predicted	4.90004000	1.67578000	1.67577000	1.65575000
	δ	2.05679000	0.04540480	0.04539900	**0.0329070**
Average performance preserved	Actual	1.29200000	1.29200000	1.29200000	1.29200000
	Predicted	1.31323000	1.28865000	1.28864000	1.28608000
	δ	0.01643570	**0.0025963**	0.00259779	0.00457829
Average performance lost	Actual	−0.26600000	−0.26600000	−0.26600000	−0.26600000
	Predicted	−0.27115200	−0.26202700	−0.26202700	−0.26138100
	δ	0.01937030	**0.0149350**	0.01493750	0.01736510
Average performance preserved before/after minimum	Actual	1.06091000	1.06091000	1.06091000	1.06091000
	Predicted	1.05576000	1.04862000	1.04862000	1.05871000
	δ	0.00485651	0.01158750	0.01158770	**0.0020730**

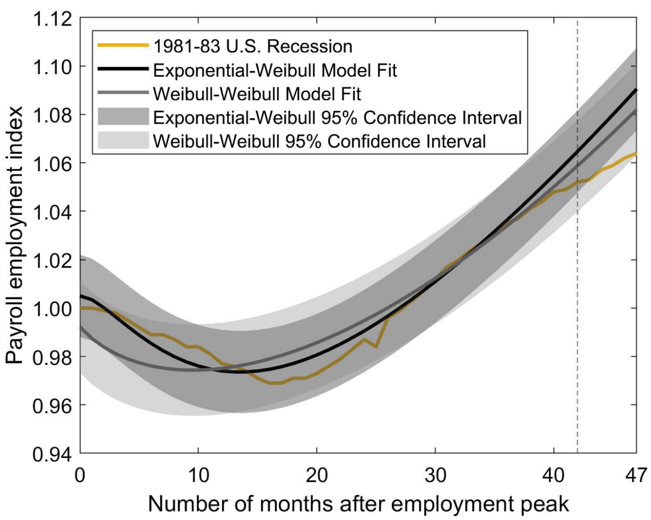

Fig. 6. Fit of Exponential-Weibull and Weibull-Weibull models fit to 1981-83 U.S recession data set.

VI. CONCLUSIONS AND FUTURE RESEARCH

This paper presented two alternative approaches to model and predict resilience as well as various metrics, including (i) bathtub-shaped hazard functions and (ii) mixture distributions to characterize deterioration and recovery. These alternative approaches were assessed in the context of historical data on jobs in the United States. Our results indicated that both bathtub-shaped hazard functions and mixture distributions fit performance and metrics well for several of the data sets exhibiting V and U, shaped curves, but that data sets possessing W, L, and K shaped curves could not be fit to either class of model proposed. Thus, the results suggested that classical reliability modeling techniques are suitable for resilience modeling and prediction, but that explanatory factors and domain specific information may increase the predictive accuracy of the models.

Future research will explore other alternative statistical approaches to predict the movement in performance as a function of disruptive events and activities to restore performance. These modeling extensions will be assessed in terms of their ability to accurately compute predictive metrics and the time at which performance can be restored to a specified level.

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