Combining Functional and Automata Synthesis to Discover Causal Reactive Programs

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We present a new algorithm that synthesizes functional reactive programs from observation data. The key novelty is to iterate between a *functional synthesis* step, which attempts to generate a transition function over observed states, and an *automata synthesis* step, which adds any additional latent state necessary to fully account for the observations. We develop a functional reactive DSL called Autumn that can express a rich variety of causal dynamics in time-varying, Atari-style grid worlds, and apply our method to synthesize Autumn programs from data. We evaluate our algorithm on a benchmark suite of 30 Autumn programs as well as a third-party corpus of grid-world-style video games. We find that our algorithm synthesizes 27 out of 30 programs in our benchmark suite and 21 out of 27 programs from the third-party corpus, including several programs describing complex latent state transformations, and from input traces containing hundreds of observations. We expect that our approach will provide a template for how to integrate functional and automata synthesis in other induction domains.

CCS Concepts: • **Software and its engineering → Automatic programming**; • **Computing methodologies → Knowledge representation and reasoning**.

Additional Key Words and Phrases: synthesis, automata, reactive, causal

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1 INTRODUCTION

In the last decade, the traditional view of program synthesis as a technique for automating programming tasks has broadened due to the following observation: Programs can compactly represent a wide variety of structured knowledge, making programming languages powerful *knowledge representations* in artificial intelligence systems [Ellis et al. 2021; Evans et al. 2021]. Program synthesis in this context is then not primarily concerned with improving programmer productivity, but instead captures a form of automated knowledge discovery, with a number of recent advances in areas as diverse as learning programs describing biological data [Köksal et al. 2013], learning computer-aided design programs from 3D meshes [Du et al. 2018], discovering phonological rules [Ellis et al. 2022; Zuidema et al. 2020], and animal behavior modeling [Tjandrasuwita et al. 2021].

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Much of this work at the intersection of program synthesis and AI can be framed as addressing the challenge of *theory induction*, an important knowledge discovery problem in both cognitive science and AI. At a high level, it asks, given some observations, what is the underlying theory or model that generates or explains those observations? We use *theory* to mean not just formal scientific theories, but also everyday cognitive explanations that humans intuit on the fly [Gopnik and Wellman 2012; Ullman and Tenenbaum 2020]. For example, a child who has figured out how a new toy works after a few minutes of play has come up with a *theory* of the toy's mechanism. In particular, the child has likely come up with a *causal theory* capturing precise causal relationships between observations. Causal theories are especially valuable as they can be used to predict how a system will react to future stimuli.

Unfortunately, existing methods of program synthesis are not yet suited to capture the rich space of theories that humans can learn from data. A major difficulty is that many real-world phenomena are *reactive*, time-varying systems—they change dynamically in reaction to occurring events. By itself, reactivity would not pose a challenge if the state in question was fully observable. In that case, we could synthesize a *transition function* mapping observed input states to observed output states at every point in time. However, current methods of inductive program synthesis—synthesizing programs from input-output examples—cannot cope with *time-varying latent state*—i.e. state that changes over time and cannot be directly observed but still affects the dynamics of the system.

There is a variety of prior work touching upon different parts of the problem of theory induction with latent state. Some work synthesizes functions with unseen inputs in the context of *unsupervised learning* [Ellis et al. 2015], but the reactive setting is more challenging because of the need to discover how that latent state *evolves* over time, rather than just its static value. More recently, the Apperception Engine [Evans et al. 2020] was developed in response to a very similar reactive problem setting as our own, but it largely contributes a new formalism for inducing theories as *logic programs* rather than a new synthesis algorithm.

Further, there is an extensive body of work on *reactive synthesis* that seeks to generate reactive programs with latent state, but much of this work focuses on synthesizing finite state programs from temporal logic specifications, rather than examples (e.g. [Bloem et al. 2012] or [Bansal et al. 2018]). Some work has been extended to infinite state systems [Beyene et al. 2014], and even to functional reactive programs [Finkbeiner et al. 2019]. Most recently, there have also been efforts to combine reactive synthesis with syntax-guided synthesis [Choi et al. 2022] given logical formulas to produce large programs with latent state, as well as to synthesize programs from a mix of logical specifications and examples [Newcomb and Bodik 2019]. However, even this recent work does not address the problem of inducing such programs purely from examples on which we focus, and the closer mixed specification approach tackles smaller benchmarks than our own.

To address this gap between current inductive program synthesis approaches and the reactive setting, we develop a novel program synthesis algorithm that unites two largely orthogonal traditions within programming languages and formal methods: *functional program synthesis* and *automata synthesis*. Specifically, we show that we can induce reactive programs by splitting synthesis into two procedures, a functional synthesis procedure and an automata synthesis procedure (Fig. 1). The functional synthesis step attempts to synthesize the parts of the program that do not depend on latent state. If functional synthesis fails to synthesize a program component explaining an observed output from observed inputs, our algorithm then uses automata synthesis to induce latent state in the form of a *finite state automaton*, where transitions in the automaton encode the dynamics of the latent state. At a high level, the automata synthesis procedure *enriches* the original program state with new latent structure that then allows a subsequent iteration of the functional synthesis step to succeed. Importantly, the algorithm is modular along two dimensions. First, it operates modularly over observed elements in the program, allowing the system to synthesize large programs with

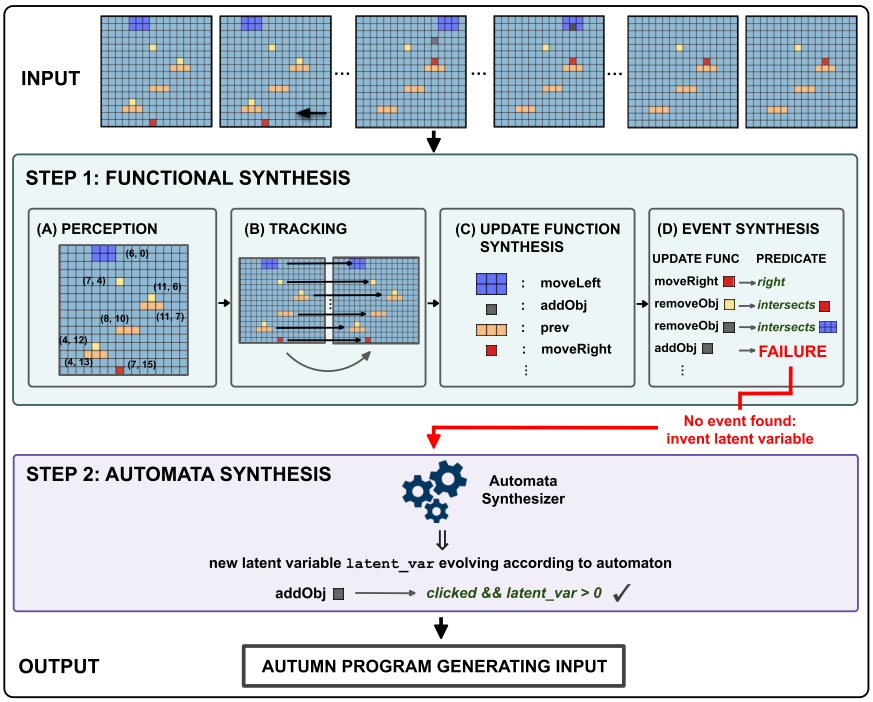


Fig. 1. Overview of the AutumnSynth algorithm. The input is a sequence of observed grid frames and associated user actions (clicks and arrow keypresses), and the output is a program in the Autumn language that generates the observed grid frames given the user actions. AutumnSynth first attempts to synthesize a fully functional program that generates the input sequence. This involves parsing a set of object variables in each frame (Perception); tracking how those objects change over time (Tracking); finding an Autumn expression called an *update function* describing how each object changes at each time (Update Function Synthesis); and synthesizing an *event*, or predicate over the observed state, that triggers each update function over time (Event Synthesis). If event synthesis fails, AutumnSynth augments the observed state by inventing an unobserved or *latent* variable via *automata synthesis*, which then allows the desired event to be expressed.

unbounded numbers of components, each with their own internal states. Second, it maintains clear boundaries between the functional and automata synthesis modules, allowing it to leverage existing algorithms for both procedures. In fact, our evaluation demonstrates the approach using both an off-the-shelf SAT-based synthesizer as well as a heuristic procedure for automata synthesis.

We anticipate that our two-layer approach to integrating functional and automata synthesis will be valuable to a wide breadth of synthesis domains. In this paper, we demonstrate its value by instantiating it in a particular domain of interactive 2D grid worlds. While much simpler than the real world, this domain still spans a wide range of dynamic theories of interest in artificial intelligence, cognitive science, and other scientific disciplines, including those from classic Atari-style video games and more recent physics-based games [Chollet 2019a]. Specifically, we have developed a functional reactive DSL called Autumn (from *automaton*) that is designed to concisely express the rich variety of causal dynamics within these grids (Fig. 2, Fig. 3, Fig. 4). The inductive synthesis problem addressed by our algorithm is, given a sequence of observed grid frames and corresponding user actions (clicks and keypresses), to synthesize the program in the Autumn language that generates the observations. The expressiveness of Autumn means that solving the

problem of causal theory induction in the context of Autumn programs will be an important step towards the goal of learning causal theories in cognitive science and AI.

In summary, our paper makes the following contributions:

(1) We present a new functional reactive domain-specific language (Autumn) suitable for expressing and synthesizing non-trivial grid world programs.
(2) We introduce a new algorithm, AutumnSynth, that induces Autumn programs from observation data. The fundamental novelty of the algorithm is its high-level, two-layer approach to combining functional and automata synthesis in an error-driven manner. The lower-level algorithms used for each step may be modified or swapped for other methods, which is useful when applying the approach to other theory induction domains.
(3) We introduce a benchmark suite of 30 Autumn programs which we call the *Causal Inductive Synthesis Corpus* (CISC), to spur the development of further algorithms in this space. The programs in this benchmark suite are designed to capture the diversity of time-varying causal models that may be manifested in 2D grids.
(4) We present an empirical evaluation of the scalability and expressiveness of AutumnSynth on both CISC and a third-party dataset of 27 grid-world-style games written in a Python video game framework [Tsividis et al. 2021]. We show that AutumnSynth can induce significantly more complex programs and from much longer inputs than previously known.

More broadly, we expect AutumnSynth will provide a template for how to integrate functional and reactive synthesis in the context of theory induction. In the rest of the paper, we provide a high-level overview of our work (Section 2), followed by an in-depth description of the algorithm (Sections 3 and 4), our evaluation (Section 5), and related and future work (Sections 6 and 7).

2 OVERVIEW

In this section, we briefly describe the Autumn language and AutumnSynth algorithm and walk through a concrete execution of the algorithm on a video-game-inspired example.

2.1 Running example

As a running example, we use a simple program we call Mario, which is inspired by the popular video game. In this program, there is an agent representing Mario, which is rendered as a single red pixel. Mario can move left or right and can jump onto platforms in response to user keyboard commands. Mario can also collect coins, which are a different object type rendered as gold pixels.

As an interesting added twist, when the player clicks on the grid, Mario shoots a bullet upwards, but each bullet costs one coin, so Mario can only shoot if it has collected at least one coin. If so, then shooting will decrement its coin count by one. Notably, the number of coins that Mario possesses is not displayed anywhere on the grid at any time; it is tracked by a scalar variable in the program. This creates a challenge for any synthesis algorithm trying to infer a program from a sequence of observations because the synthesizer has to infer the existence of this latent state to explain why sometimes clicking on the grid results in a bullet and other times it does not. At the top of the grid, an enemy object continuously moves between the left and right side of the frame and disappears if it is hit by one of Mario's bullets. Figure 2 illustrates a few steps of the game.

2.2 The Autumn Language

Autumn is a *functional reactive* (FR) language designed to concisely express a rich variety of causal mechanisms in interactive 2D worlds. These mechanisms range from distillations of real-world, everyday causal phenomena, such as water interacting with a sink or plants growing upon exposure to sunlight, to video game-inspired domains such as Atari's Space Invaders or our Mario running

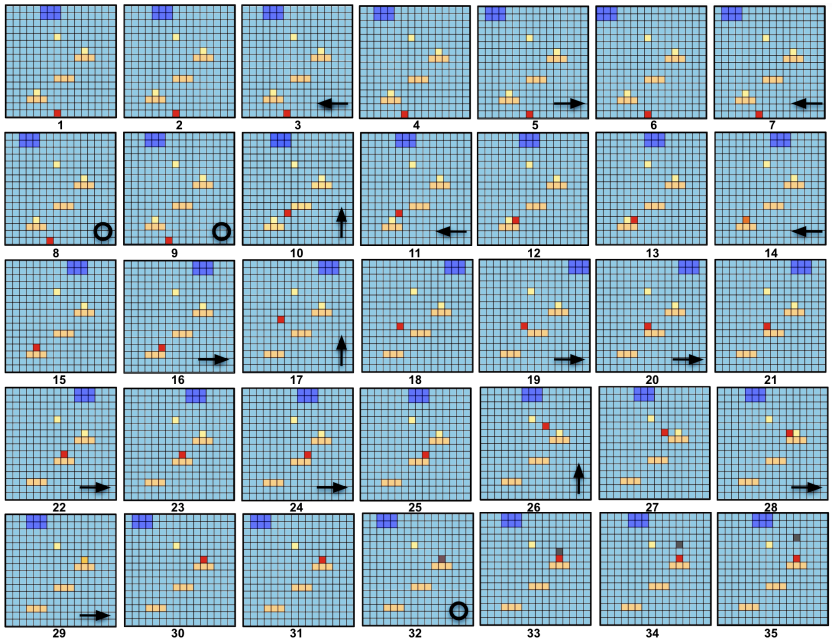


Fig. 2. An observation trace from the Mario program. Black arrows indicate user keypresses and circles indicate clicks. An agent (red) moves around with arrow key presses and can collect coins (yellow). If the agent has collected a positive number of coins, when the human player clicks, a bullet (gray) is released upwards from the agent's position, and the agent's coin count is decremented. Otherwise, clicking does nothing.

example (see Fig. 1 for more examples). The design of Autumn was inspired by prior FR languages such as Elm [Czaplicki and Chong 2013], but differs from those languages in some important respects to make the synthesis process more tractable as explained in the rest of this section.

Every Autumn program is composed of four parts: Environment setup, Type definitions, Stream definitions and Event handling. The *Environment setup* defines the grid dimensions and background color for a program. *Type definitions* define object types; each object type has a *shape* represented as a list of 2D positions relative to the object center and each associated with a color, as well as a set of *internal fields* which store additional information about the object (e.g. a Boolean `healthy` field may store an indicator of the object's health). The object type definitions for the Mario program are shown below. In this case, only the Enemy object type has additional state, in the form of a Boolean field indicating whether the enemy's current direction of motion is left or right. The definitions of the other types only include the list of colors and 2D positions that define how the object is rendered. Every instance of an object (e.g. every Mario or every Coin) also has a 2D position without it needing to be declared in the type definition.

```
object Mario ( [(0, 0, red)] )
object Coin ( [(0, 0, gold)] )
object Platform ( [(-1, 0, orange), (0, 0, orange), (1, 0, orange)] )
object Enemy (movingLeft : Bool) ( [(-1, -1, purple), (-1, 0, purple),
                                    (0, -1, purple), (0, 0, purple),
                                    (1, -1, purple), (1, 0, purple)] )
```

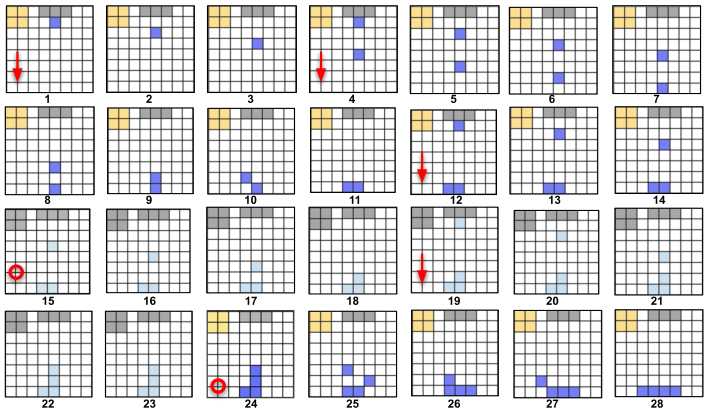


Fig. 3. An observation trace from the Ice program. At times 1 and 4, the user presses down (red arrow), releasing a blue water particle from the gray cloud. The water moves down to the lowest possible height, moving to the side (time 10) if necessary to reach this height. The user presses down again at time 12, and then clicks anywhere (red circle) at time 15. The click causes the sun to change color and the water to turn to ice, which *stacks* rather than tries to reach the lowest height. A down press at time 19 releases another ice particle from the cloud. Finally, a click at time 24 turns the sun yellow again and turns the ice back to water.

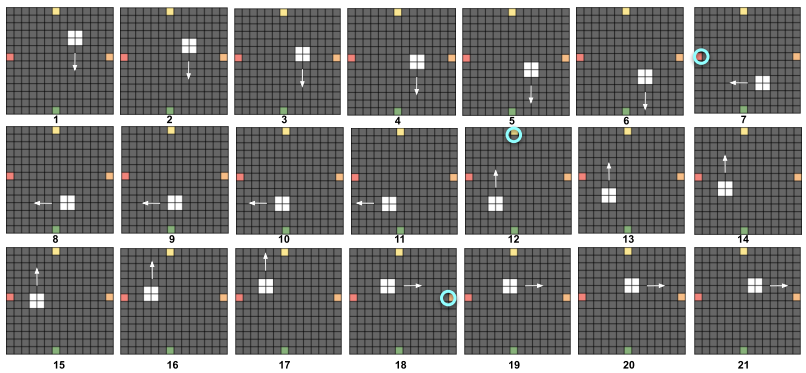


Fig. 4. An observation trace from the Gravity I program. A white 2×2 block initially moves down one cell per time, and clicking (indicated by the cyan circle) any of the four buttons on the grid borders causes the block to change direction. Specifically, clicking the left border button causes the block to move left (time 7), clicking the top button causes it to move up (time 12), clicking the right button causes it to move right (time 17), etc.

The next part of a program consists of *Stream definitions*, which define object instances and other auxiliary values and their evolution over time in the absence of external events. For example, in the Mario program, we have four stream variables: one for Mario, one for the coins, one for the platforms, and one for the number of coins—the invisible latent state that tracks how many coins have been gathered and not used. Each of these streams is defined using the primitive Autumn language construct called *initnext*, which defines a *stream* of values over time using the syntax `var = init expr1 next expr2`. The initial value of the variable (`expr1`) is set with **`init`**, and the value at later time steps is defined using **`next`**. The **`next`** expression (`expr2`) is re-evaluated at each subsequent time step to produce the new value of the variable at that time. Within the **`next`** section of the stream definition, it is possible to access the previous value of the stream using the primitive **`prev`**. For example, below are some of the stream definitions in the Mario program:

```
mario : Mario
mario = init (Mario (Pos 7 15)) next (moveDownNoCollision (prev mario))
coins : List Coin
coins = init (list (Coin (Pos 4 12)) (Coin (Pos 7 4)) (Coin (Pos 11 6)))
        next (prev coins)
bullets : List Bullet
bullets = init (list) next (prev bullets)
numCoins : Int
numCoins = init 0 next (prev numCoins)
```

For the `mario` stream, the **`init`** section initializes the agent, and the **`next`** definition uses a user-defined function `moveDownNoCollision` which specifies that later values of the agent should move down one unit from the previous value whenever that is possible without collision. The `coins` stream illustrates that streams can also be lists of objects, not just individual objects. In the absence of other events, the list of coins will stay the same throughout the game, but shortly we will see the code that will make the list shrink as coins are collected by Mario. Similarly, the initial empty `bullets` list will grow as Mario shoots bullets and shrink if a bullet hits the enemy and is hence removed. Finally, the `numCoins` stream represents the latent state that tracks the number of coins collected by Mario. In general, any value that is not an object will be latent state since it will not be directly observable through the interface. Streams corresponding to latent state can be of primitive types `int`, `string`, or `bool` as well as lists of such values.

The fourth segment of an Autumn program is *Event handling* and is expressed using a construct called *on-clauses* which are expressed via the high-level form

```
on event
   intervention
```

where `event` is a predicate and `intervention` is one or more assignment of the form `var = expr` that override the default next value in the stream defined by the **`next`** clause in the Stream definitions section. For example, the code below shows some of the on-clauses of the Mario game.

```
on intersects (prev mario) (prev coins)
   numCoins = (prev numCoins) + 1
   coins = removeObj coins (-> obj (intersects (prev mario) (prev obj)))
on clicked && ((prev numCoins) > 0)
   numCoins = (prev numCoins) - 1
   bullets = addObj bullets (Bullet ((prev mario).origin))
```

The first on-clause indicates that when Mario intersects with a coin in the list of coins, the coin is removed from the list, and the number of coins is incremented. The second one indicates that when the grid is clicked and the number of coins is positive, the number of coins is decremented and a bullet is added at the current position of Mario (the -> symbol denotes a lambda function). We note also that the coin removal syntax demonstrates that the **`prev`** function may be used not only to access the previous value of a stream but also the previous values of *individual objects* within a stream that is a list of objects (i.e. the use of **`prev`** `obj` instead of **`prev`** `coins`). Keeping track of object history in addition to stream history as such allows more fine-grained control over object dynamics. For example, it allows individual objects in a list to be modified or removed without resetting all other list objects to their previous values, as would happen if `removeObj coins` were replaced with `removeObj (prev coins)` above.

One important difference between Autumn and other FR languages like Elm is that on-clauses are evaluated sequentially, with the effect that later on-clauses may update a variable in a way that composes with updates from earlier on-clauses or completely overrides it. We found that for many programs, this led to significantly more concise programs, which made synthesis more efficient.

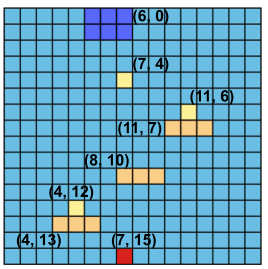
2.3 Synthesis Overview

The AutumnSynth algorithm, is an end-to-end synthesis algorithm that takes as input a trace of a program—corresponding to a sequence of grids for a range of time steps and the corresponding user inputs for those steps—together with a library of language components. From these, the algorithm synthesizes an Autumn program using the given components that, when given the observed sequence of user inputs, matches the behavior observed in the trace. The algorithm consists of four distinct steps, each producing a new representation of the input sequence. These steps are:

(1) **perception**: object types and instances are parsed from the observed grid frames;
(2) **object tracking**: links objects in consecutive time steps to distinguish between (1) objects that moved or changed from one time step to the next, (2) objects that were created or destroyed and therefore lack a matching object in the preceding or subsequent timestep respectively;
(3) **update function synthesis**: *update functions*—Autumn expressions describing each object-object mapping from Step 2—are synthesized from the given components; and
(4) **cause synthesis**: Autumn events (predicates) that *cause* each update function from Step 3 are sought, and new latent state in the form of automata is constructed upon event search failure.

We give details for these steps in Section 4, focusing primarily on cause synthesis since that procedure represents the most novel aspect of our work. First, we provide some intuition by briefly describing how these steps are used to synthesize the Mario program from the running example.

2.3.1 Perception. The object perception step first extracts the object types and object instances from the input sequence of grid frames. For the example, the perception phase will identify three different object types: (1) a general single-cell type with a color parameter corresponding to the (red) agent, (yellow) coin, and (gray) bullet objects; (2) a platform type that is a row of three orange cells; and (3) an enemy type that is a rectangle of six blue cells. A list of object instances is extracted from each grid frame in the input sequence, where an object instance describes the object's type, position, and any field values. For example, the object instances for the first grid frame in Fig. 2 (magnified on the right) are a red single-celled object (agent) at position (7, 15); three yellow single-celled objects (coins) at positions (4, 12), (7, 4), and (11, 6); three platform objects at positions (4, 13), (8, 10), and (11, 7); and an enemy object at position (6, 0). A few points to highlight are that, at this stage, the system does not know that the coins and the agent are different kinds of objects, as opposed to different colors for the same kind of object; also note that for multi-cell objects, the system assigns the object center to be the center of the pixel group. In addition, all Autumn objects are currently partially *transparent*, so occlusion is not a concern for the perception step.



2.3.2 Object Tracking. Next, the object tracking step determines how each object in each grid frame *changes* in the next grid frame. For example, it identifies that the agent object at position (7, 15) in the second grid frame corresponds to the agent object at position (6, 15) in the third grid frame (i.e. it moved left). Intuitively, this step *tracks* the changes undergone by every object across all grid frames. In our current implementation, both Perception and Tracking are heuristic-based.

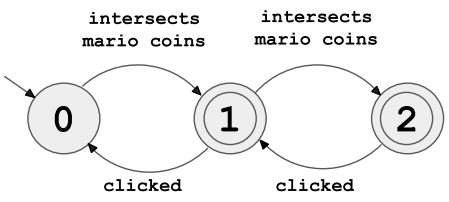
2.3.3 Update Function Synthesis. This step derives an expression for each object in a given grid frame that describes the change in the object's visible attributes (position and color) in that time step. The expressions are synthesized using the components given as input to the algorithm. In our example, this step identifies that the expression `agent = moveLeft (prev agent)` accurately describes the change undergone by the agent object between the second and third grid frames from Fig. 2. Often, there are multiple such expressions that match any given mapping. For example, the agent's left movement during the first time step might also be described by `agent = moveLeftNoCollision (prev agent)` or `agent = moveClosest (prev agent) Platform`, where the latter indicates movement one unit towards the nearest object of type `Platform`. The update function synthesis step will not try to disambiguate among these options. Instead, it will return a set of possibilities to the subsequent *Cause Synthesis* step, which will be responsible for identifying the correct update function.

2.3.4 Cause Synthesis. Finally, the cause synthesis step searches for an Autumn event that triggers each update function identified in the previous step. For now, we will assume that update function synthesis produced a unique update function for each object at each time step; in Section 3 we will elaborate on the general case where this is not true. With this assumption, the goal is now to explain *why* each function was triggered when it was triggered. Autumn will first attempt to explain the triggering of each update function based on observable events, and then it will synthesize latent state to explain the triggering of any remaining updates that cannot be explained by the observable events alone. To find an Autumn event that triggers a particular update function, the algorithm collects the set of times that the update function is used and enumerates through a space of Autumn events until it finds one that evaluates to true at exactly those times. For example, say that the Mario object undergoes the update function `agent = moveLeft (prev agent)` at times 1, 4, and 5. If the Autumn event `left`, which indicates that a left keypress has occurred, evaluates to true at only those three times, then the on-clause

```
on left
   agent = moveLeft (prev agent)
```

accurately describes that particular update function's occurrence. The search space of Autumn predicates is defined over the *program state*, which consists of the current object instances, latent variables, and user events. At the start of this step in the algorithm, there are not yet any latent variables in the program state, so the possible events use only the objects and user events (e.g. `clicked`, `clicked mario`, or `intersects bullet enemy`). Lastly, this event-finding process is simplified by the fact that on-clauses may override each other, so perfect alignment between the trigger event and observed update function is not always necessary. For example, even though `mario` does not undergo the update function `mario = moveDownNoCollision (prev mario)` at every time, the trigger event learned for this update function is simply `true`. This is because later on-clauses describing other behaviors like `moveRight` and `moveLeft` *override* the on-clause at appropriate times, so `mario` ends up undergoing `moveDownNoCollision` exactly when desired. Searching for a trigger event in the search space that exactly matches the times of `moveDownNoCollision`, in contrast, may be much more challenging or even impossible. This nuance will be explained in detail in Section 4.

The interesting case in the cause synthesis step is what happens when a matching Autumn event cannot be found for a particular update function. In the Mario example, this happens with the update function `bullets = addObj (prev bullets) (Bullet (mario.origin))`, which describes a bullet object being added to the list of objects named `bullets`. Bullet addition takes place at times 32, 41, and 57, but no event is found that evaluates to true at exactly those times. Since the existing program state does not give rise to any matching events, the algorithm must augment the program state by inventing a new latent variable that can be used to express the desired predicate.




```
% initialize start state
numCoins : Int
numCoins = init 0 next (prev numCoins)
% state transitions
on intersects mario coins && (numCoins == 0)
   numCoins = 1
on intersects mario coins && (numCoins == 1)
   numCoins = 2
on clicked && (numCoins == 2)
   numCoins = 1
on clicked && (numCoins == 1)
   numCoins = 0
```

Fig. 5. (a) Diagram of automaton representing the numCoins latent variable synthesized for the Mario program. The start value is zero, and the accept values (i.e. the values during which clicked causes a bullet to be added to the scene) are 1 and 2. (b) Description of the numCoins latent variable in the Autumn language. We note that the reason that the automaton states only go up to two collected coins instead of three is because the input sequence provided only demonstrates a maximum of two coins being collected at a time. We discuss this and give an example where the automaton has learned all three accept states instead in Section 5.

Specifically, the algorithm proceeds by finding the "closest" event in the event space that aligns with the update function. This is the event that *co-occurs* with every update function occurrence, but may also occur during *false positive times*: times when the event is true but the update function does not occur. For bullet addition, this event is `clicked`, as every bullet is added on a click, but some clicks do not add a bullet. Having identified this closest event, our goal is then to construct a latent variable that acts as a finite state automaton that *switches* states between the false positive times and true positive times (i.e. the times when `clicked` is true and the update function occurs). To be precise, the new variable takes one set of values during the false-positive times, and another set during the true positive times. Calling the values taken by the latent variable during true positive times *accept values*, and those during the false-positive times *non-accept values*, the event

```
clicked && (latentVar in [/* accept values */])
```

perfectly matches the observed update function times. This is because `clicked` is true during a set of false-positive times, and `latentVar` is in *non-accept* values at exactly those times, so bullet addition does not take place, as desired. The full Autumn definition of `latentVar`, including the *transition on-clauses* that change its value over time, is shown in Fig. 5. The variable name `numCoins` is substituted to note the equivalence to a *number of collected coins* tracker. We note that this automaton—and all automata synthesized by our algorithm—are *finite-state* automata, even though it is possible to write an Autumn program describing infinite-state automata (e.g. if the Mario program in Section 2 described coins being added to the program at regular intervals instead of a fixed set of three coins, its counter variable would be infinite-state). In these infinite cases, our method synthesizes a finite-state *approximation* to the ground-truth automaton that suffices at explaining the given finite input trace.

The challenge in constructing this latent variable is learning the transition on-clauses that update the value of the variable at the appropriate times. Note that these transition on-clauses represent *edges* in the *automaton* diagrammed in Fig. 5 (hence the use of the term *accept values* or *states*). We perform the transition learning step as part of a general automaton search procedure, implemented via a SAT solver as well as heuristically, to be discussed in Section 4.

3 PROBLEM FORMALIZATION

In this section, we formally specify the synthesis task solved by the AutumnSynth algorithm.

3.1 Modeling the Autumn language

We define the AutumnSynth algorithm in terms of a simplified model of the Autumn semantics. In this model, an Autumn program is represented as a tuple $\langle \mathcal{T}, \mathcal{X}, \mathcal{A}, \mathcal{O}, F, Z, X_0 \rangle$, defined below:

- **Object Types**: $\mathcal{T}$ is a set of *object types*, where each object type $t \in \mathcal{T}$ consists of a shape S and a list of additional data fields $data$, i.e. $t = \langle S, data \rangle$. A shape S is a set of cells, where each cell $c \in \mathbb{N} \times \mathbb{N}$ is a 2D position relative to center (0, 0). Each data field $f \in data$ is a symbol n.
- **States**: Each state $X \in \mathcal{X}$ is a tuple $\langle X_o, X_l \rangle$, where X_o is a set of *objects* and X_l is a set of *latent variables*. Each object $x_o \in X_o$ is a tuple $\langle t, p, d \rangle$ where $t \in \mathcal{T}$ is an object type, $p \in \mathbb{N} \times \mathbb{N}$ is the origin of the object in the 2D grid, and d is the set of data values associated with the data fields of t. Each latent variable $x_l \in X_l$ is a value with integer type. Note that in this simplified model of the language, the different kinds of stream variables (objects, lists of objects, scalars, etc.) have been flattened to a simpler representation where all the objects belong to a single set and all the scalars belong to another. After a program has been synthesized in this representation, the code generator will be responsible for organizing the state into individual streams.
- **Actions**: The action space $\mathcal{A}$ is a set of three types of elements a: (1) arrow key presses (left, right, up, and down); (2) clicks on the observed grid, where each click is associated with a 2D grid position $p \in \mathbb{N} \times \mathbb{N}$; or (3) no action.
- **Observations**: The observations $\mathcal{O}$ are 2D grids of colored cells, each of the same dimensions.
- **Transition Function**: Let $\mathcal{H} = \mathcal{X} \times \mathcal{A}$ be the space of *program histories*, where a program history is simply the most recent state and action at any given time. The transition function F is defined as a composition of a *next* function $next : \mathcal{H} \to \mathcal{X}$ corresponding to the next function in the stream definition and a set $\mathcal{C}$ of m *on-clause functions* $o_i : \mathcal{X} \times \mathcal{H} \to \mathcal{X}$. These functions are described in greater detail below:
  - *The next function.* The *next* function defines the "default" modification to the current set of state variables given history $H \in \mathcal{H}$. Note that in this simplified formalism, all the `next` clauses in the individual stream definitions are collapsed into a single *next* function applied to the entire state X.
  - *On-clause functions.* The on-clause functions are a set of m functions $o_1, \ldots, o_m$ where each o_i is a tuple $\langle \text{event}_i, \text{update}_i \rangle$. Each $\text{event}_i : \mathcal{X} \times \mathcal{H} \to \{0, 1\}$ is a Boolean predicate over the history and the new/intermediate state (described below). and each $\text{update}_i : \mathcal{X} \times \mathcal{H} \to \mathcal{X}$ is a function that modifies the current state X given the same input.
    From these event_i and update_i functions, each o_i is then

$$o_i(X, H) = \begin{cases} \text{update}_i(X, H) & \text{if event}_i(X, H), \\ X & \text{otherwise.} \end{cases}$$

  Given these components, we define the transition function $F : \mathcal{H} \to \mathcal{X}$ to be

$$F(H) = o_m(\ldots(o_2(o_1(next(H), H), H)\ldots), H). \tag{1}$$

  Note that each o_i has access to both the original state at the end of the previous timestep (which in the code can be accessed through `prev`), as well as to the new or intermediate state as computed by `next` or by any previous on-clauses. This models the overriding behavior of on-clauses that was defined in Section 2.2, in which later on-clauses override earlier ones.
- **Observation Function**: The deterministic partial observation function $Z : \mathcal{X} \to \mathcal{O}$ renders the shape of each object $x_o \in X_o$ at the object's position in the 2D grid. Specifically, for an object $x_o = \langle t, p, d \rangle$ with $t = \langle S, data \rangle$, Z translates the shape S by the position p to obtain the observed rendering of x_o.

- **Start State**: The start state X_0 gives the set of objects and latent variables present at the start of the simulation.

3.2 Inference Task

Given a sequence of observations $(O_1, \ldots, O_T)$ and corresponding sequence of actions $(A_1, \ldots, A_{T-1})$ from the decision process, our goal is to recover the object types $\mathcal{T}$, initial state X_0, and transition function F that correctly produces the observed data. These three components $(\mathcal{T}, X_0, F)$ specify an Autumn program, although there is still some additional work to do at that point to translate this program from this simplified formalism to the full Autumn syntax, for example by splitting the state and the *next* functions into individual stream definitions as explained in Section 4.6.

In general, the inductive synthesis problem is underdetermined, as there are many programs that will produce the correct observation sequence. Since it is challenging to identify whether a synthesized program is semantically equivalent to the ground-truth program from which the observation data was generated—especially since this generating program may be a black box—we define a *score function* to approximately measure closeness to the ground-truth.

4 SYNTHESIS ALGORITHM

We now give detailed descriptions of the steps of our algorithm introduced in Section 2. We focus on Steps 3 and 4—update function synthesis and cause synthesis—since the object perception and tracking steps use more standard techniques and are not a contribution of our work.

4.1 Step 1: Perception

For each observed grid frame $O_i \in \{O_1, \ldots, O_T\}$, the perception step produces a set $X_{o,i}$ of objects present in the frame, as well as a set of object types $\mathcal{T}$. Each object is a tuple $\langle t, p, d \rangle$ of an object type $t \in \mathcal{T}$, position p, and data values list d, where $t.data$ is either the empty set $\emptyset$ or the singleton set composed of the field $\langle$color, `string`$\rangle$. No other data fields beyond the observable color field are identified in this step. Latent data fields may be constructed in Step 4, upon which they are added as a modification to the existing type.

Our current Autumn implementation actually uses two different object parsing algorithms and runs the rest of the synthesis procedure on the result of each. The algorithm then returns the output program from the first parsing for which synthesis succeeds.

The simplest of the two is called *single-cell parsing*, which identifies each colored cell in a grid frame as an individual object. The set of object types is then the set of single-celled shapes each with a particular fixed color observed across the grid frames. The other algorithm is called *multi-cell parsing*, which identifies groups of adjacent cells with the same color as multi-celled objects. This is necessary when the observed grid-frames contain groups of pixels that move together as single objects, but when a frame contains multiple single-celled objects of the same color, this algorithm runs the risk of interpreting them as one object when they are adjacent to each other.

Neither of the two object parsing algorithms is especially sophisticated, but they are sufficient to demonstrate our approach on the benchmarks.

4.2 Step 2: Object Tracking

Object tracking pairs objects in one frame to corresponding objects in the next frame. A *correspondence* m_t is a binary relation over two sets of objects X_t and X_{t+1} from consecutive frames. Intuitively, $(x_1, x_2) \in m_t$ denotes that x_1 and x_2 are the *same* object; $x \in X_{t+1} \setminus \text{domain}(m)$ denotes that x was *added* in the transition; $x \in X_t \setminus \text{codomain}(m)$ denotes that x was *removed*.

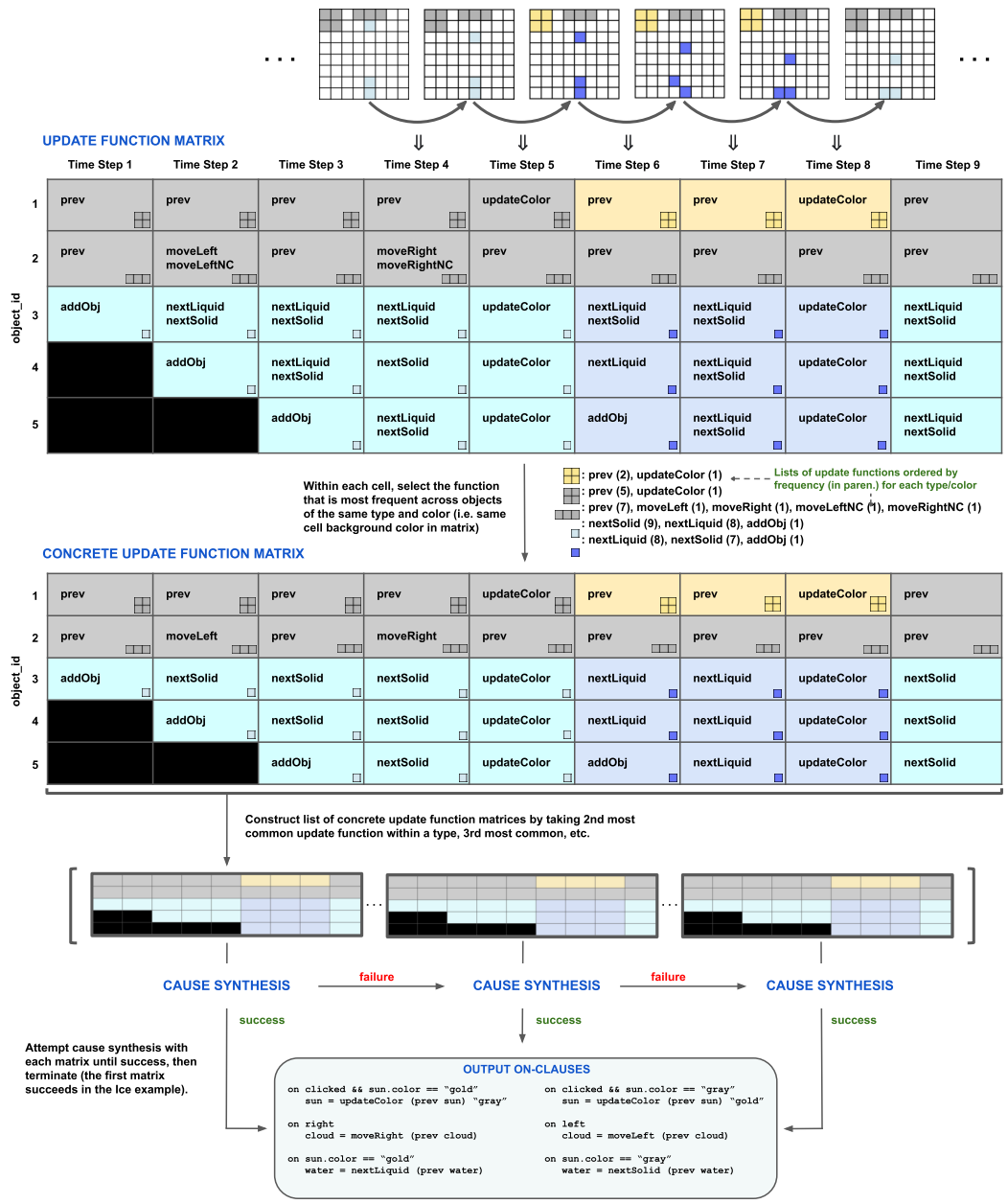


Fig. 6. Update Function Synthesis, demonstrated on the Ice benchmark program. In the ground-truth model, water particles are released from the cloud and move according to the `nextLiquid` update function when dark blue in color (i.e. melted) and according to the `nextSolid` update function when light blue in color (i.e. frozen). Clicking toggles the sun between gold to gray and the water particles between light and dark blue. Each cell of the update function matrix contains a set of update functions that each describes the change undergone by the object with object_id equal to the row index during a particular time step (column index). A list of concrete update function matrices, with one update function per cell, is extracted via frequency-based heuristic.

The algorithm that constructs the correspondence relations is a heuristic that is based on the assumption that objects are unlikely to move very far in a single time step. Hence, it attempts to maximally assign objects in frame t to their closest objects (by Manhattan distance) in frame $t+1$. Objects that remain unassigned by this proximity heuristic are deemed to have been added or removed. The algorithm also ensures that no two objects in time step t are mapped to the same object in time step $t+1$, and vice versa.

4.3 Step 3: Update Function Synthesis

Having tracked the objects in the program through time in Step 2, Step 3 synthesizes an Autumn expression, called an *update function*, that describes each object assignment, addition, or removal in each mapping relation m_i. These update functions are stored in a matrix that we call the *update function matrix*. We describe the construction of this matrix below.

From the sequence $\vec{X} = (X_1, \ldots, X_T)$ of objects produced by Perception, let $\mathcal{P}$ denote a set of unique object identifiers and $id(x) : X_i \rightarrow \mathcal{P}$ denote the mapping from objects to their unique identifiers that has two properties: (a) if $(x_a, x_b) \in m_t$ for some time t, then $id(x_a) = id(x_b)$, and (b) if $x_a \in X_t$ and $x_b \in X_t$ are two distinct objects in the same timestep t, then $id(x_a) \neq id(x_b)$.

Our goal is to construct an update matrix M, where $M_{i,t}$ is an Autumn expression defining how an object $x \in X_t$ with $id(x) = i$ transitions between time-steps t and $t+1$. We will use the shorthand $M_{x,t}$ to mean $M_{id(x),t}$. Specifically, M is a $|\mathcal{P}| \times (T-1)$ matrix, where each row contains the sequence of update functions undergone by the object with *id* corresponding to that row. To construct the update matrix, our method first constructs an *abstract update matrix* $\hat{M}$, where $\hat{M}_{x,t}$ is a set of candidate Autumn expressions such as `moveLeft` and `nextLiquid`, all of which denote object transformations. $\hat{M}$ is constructed such that each expression in $\hat{M}_{x,t}$ is consistent with $\vec{X}$. Formally, let $\mathcal{L}$ denote a set of Autumn expressions provided as input by the user to the algorithm, then $\hat{M}_{x,t} = \{f \mid f \in \mathcal{L}, (x, f(x)) \in m_t\} \cup Additions_{x,t}$. The final term of $Additions_{x,t}$ accounts for objects added or removed in a particular timestep.

$$Additions_{x,t} = \begin{cases} \{\texttt{addObj}\}, & x \text{ is added at time } t \\ \{\texttt{removeObj}\} & x \text{ is removed at time } t \end{cases}$$

4.3.1 Update Function Filtering. Our ultimate aim is to pair update functions with associated triggering events to generate on-clauses. Prior to this, the abstract update matrix $\hat{M}$, which describes many possible programs, must be concretized into a concrete update matrix. A concrete update function matrix M is a "filtering" of an abstract update matrix $\hat{M}$ in the sense that $M_{x,t} \in \hat{M}_{x,t}$.

There are combinatorially many concrete matrices corresponding to any given abstract update function matrix, so we follow a set of heuristics to filter and sort the possible matrices. The heuristics are somewhat involved, but in this section, we provide the reader a sense of the main ideas behind them. The heuristics fall into four categories: (1) local filtering, (2) temporal filtering, (3) type consistency, and (4) type update function frequency.

Local filtering corresponds to local heuristics that are applied independently to every cell in $\hat{M}$ to remove low-probability update functions. *Temporal filtering* analyzes individual objects over time and prioritizes update functions that are consistent with what has happened at other times. This is particularly relevant for objects that undergo update functions such as `moveNoCollision` since it is often ambiguous whether the object is actually stationary (i.e. undergoing the update function `prev obj`) or is attempting to move but is blocked by a collision (i.e. undergoing `moveNoCollision`). When an object is not moving, the fact that it was moving in other time steps makes it more likely that a `moveNoCollision` function was actually involved, compared to if there was not such a pattern.

The next two categories of heuristics, *type consistency* and *type update function frequency* both build on the intuition that the algorithm should prioritize update functions that occur *more frequently* in $\hat{M}$ across all rows of the abstract matrix with the same object type. The idea is that selecting more frequent update functions will allow the generated code to "maximally share" update functions, resulting in fewer on-clauses for the final program. These heuristics are both illustrated in Figure 6.

Before we define type consistency and type update function frequency filtering, we introduce some notation. First, all objects in the program are grouped by type, so $\Gamma(x)$ is the type of object x. In most cases, the type will correspond to the object type defined earlier, but for objects that change color over time, the synthesizer may introduce distinct types for their different forms. For each type, we define an ordering among the update functions corresponding to that type as follows. Let $a, b \in \hat{M}_{x,t}$ be two update functions; then $a \leq_\tau b$ if and only if $count_\tau(a) \leq count_\tau(b)$ where:

$$count_\tau(u) = \sum_{t \in T, x \in \mathcal{P} \wedge \Gamma(x) = \tau} \mathbf{1}_{\hat{M}_{x,t}}(u)$$

where $\mathbf{1}_{\hat{M}_{x,t}}(u)$ is an indicator function producing a value of 1 if $u \in \hat{M}_{x,t}$ and 0 otherwise.

Based on this type-based ordering, we now define the *type consistency* requirement that all our concrete update matrices should satisfy. A matrix M is type consistent if it satisfies the following:

$$\forall x, y \in \mathcal{P}, t_x, t_y \in T \text{ s.t. } \Gamma(x) = \Gamma(y) = \tau.\ a = M_{x,t_x} \wedge b = M_{y,t_y} \wedge a \neq b$$
$$\rightarrow (a = sup_{\leq_\tau}(\hat{M}_{x,t_x}) \wedge b \notin \hat{M}_{x,t_x}) \wedge (b = sup_{\leq_\tau}(\hat{M}_{y,t_y}) \wedge a \notin \hat{M}_{y,t_y})$$

In other words, type consistency enforces that all objects of the same type apply the same update function at every time step, with the exception that some object x may be different from the others if the update function that was chosen by others is not available for x at a given time step. In that case, x must chose the best update function available to it relative to the order defined earlier ($sup_{\leq_\tau}(\hat{M}_{x,t_x})$).

In the final step, *Type update function frequency* filtering, we first eliminate any matrix that unnecessarily uses any update function not among the top two based on the type update function ordering. More formally if $toptwo_\tau(\hat{M})$ returns the top two update functions based on the type update function order for a given type τ and a given matrix $\hat{M}$, then we filter away any matrix M that does not satisfy:

$$\forall x \in \mathcal{P}, t \in T, M_{x,t} \in toptwo_\tau(\hat{M}) \vee M_{x,t} = sup_{\leq_\tau}(\hat{M}_{x,t})$$

The matrices that pass this filtering are then sorted based on the partial order defined below. Let M^a and M^b be filterings of an abstract matrix $\hat{M}$. Then, $M^a \leq M^b$ iff

$$\forall x \in \mathcal{P}, t \in T. M^a_{x,t} \leq_\tau M^a_{x,t} \text{ where } \tau = \Gamma(x).$$

The end result of update function filtering is a list of update function matrices. All later steps of the algorithm are run independently on each of these candidate matrices until synthesis succeeds.

4.4 Step 4: Cause Synthesis

By this stage in the algorithm, the object types, the object instances, and the possible update functions undergone by each object at every time have been identified. Remaining to be synthesized are the *event predicates* associated with the update functions in on-clauses, and potentially *latent variables*. At a high level, this step enumerates through concrete update function matrices $M_1, M_2, \ldots, M_n$, and searches for events that could have triggered each update function, including trying to synthesize latent variables if necessary for these events to exist. If this procedure succeeds for a given concrete matrix, the overall algorithm terminates, returning the final program. If it fails

on the current concrete matrix, it is repeated on the next concrete matrix until success or until the end of the list is reached, which indicates overall synthesis failure.

To formalize the cause synthesis problem, we introduce the concepts of an *update function trajectory*, *event trajectory*, and a *match* between instances of the two. This step assumes that objects that belong to the same object type are controlled by the same set of on-clauses—if two objects of the same type both undergo `moveLeft` then a single event/on-clause was the cause. In contrast, if two objects undergo `moveLeft` and belong to different types, the method must synthesize a different event associated with each one. Thus, we synthesize events by enumerating through the object types and finding an event for each distinct update function that appears across objects of that type. Since this step of the algorithm is applied to each type independently, for convenience we shall assume that M contains objects of only a single type τ.

4.4.1 Event Space. The event space $\mathcal{E}$ is composed of conjunctions and disjunctions of a finite set of *atomic events*. There are two kinds of atomic events: *global* events and *object-specific* events.

Definition 4.1 (Global Event). A global event is a predicate over the program state, and may switch between occurring and not occurring as the program state evolves. Examples include user events such as `clicked`, `clicked obj1`, and `left` as well as object contact events like `intersects obj1 obj2` and `adjacent obj1 obj2`.

Definition 4.2 (Object-Specific Event). An object-specific event is a predicate on a single object. For example, `obj.color == "red"`. Object-specific events are used to apply update functions selectively to subsets of objects (those that make the predicate true) of a particular type.

The event trajectory for an event $e \in \mathcal{E}$ is its sequence of true/false values over time.

Definition 4.3 (Event Trajectory). For an event $e \in \mathcal{E}$, the event trajectory $E_{x,t}$ is constructed according to the following cases:

(1) Case 1: e is a global event. For all x, i.e., independent of x:

$$E_{x,t} = \begin{cases} 1 & e \text{ is true at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

(2) Case 2: e is an object-specific event.

$$E_{x,t} = \begin{cases} 1 & \text{if e is true for object } x \text{ at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

4.4.2 Update Function Trajectory. Informally, with respect to an update matrix M, the update function trajectory of an update function u is a matrix where each element is 0, 1 or $\frac{1}{2}$ to indicate whether u took place (1) or not (0) for object x at time t. The value $\frac{1}{2}$ represents uncertainty, indicating that u may have taken place but its effect could have been overridden by another update function. As described in Section 2.2, an update function can be overridden when there are other on-clauses that follow it in the Autumn program that are triggered at the same time. For synthesis, there is a choice of how to order on-clauses. With a similar rationale to the ordering of concrete matrices in the previous Section, our method orders update functions according to the ordering relation $\leq_\tau$, which favors update functions that occurred more frequently.

Definition 4.4 (Update Function Trajectory). Given a concrete update matrix M and an update function u, the update function trajectory $U_{x,t}$ is defined as:

$$U_{x,t} = \begin{cases} 1 & \text{if } M_{x,t} = u, \\ \frac{1}{2} & \text{if } M_{x,t} = u' \text{ where } u' \neq u \text{ and } u \leq_\tau u' \\ 0 & \text{otherwise.} \end{cases}$$

In other words, more frequent update functions appear earlier in the Autumn program than less frequent ones, and hence are overridden by those less frequent update functions when both are triggered at once.

4.4.3 Matching. Finally, an event is said to match an update function if their corresponding event and update function trajectories are the same over all objects at all times. If there is any ambiguity due to the overriding behavior, we tend towards being permissive in calling it a match. That is, the method counts any instance where $U_{x,t}$ is $\frac{1}{2}$ as a match, regardless of the event.

Definition 4.5. An event trajectory E *matches* an update function trajectory U if for all x, t, $E_{x,t} = U_{x,t}$, where for $a, b \in \{0, 1, \frac{1}{2}\}$, $a = b$ is true if a or b is $\frac{1}{2}$, and otherwise defined in the standard way.

If a matching event trajectory cannot be found for a particular update function trajectory, the algorithm moves on to the automata synthesis step, which attempts to augment the existing program state in such a way that a matching event may be written.

4.5 Step 4b: Automata Synthesis

Failure to find an event trajectory that matches an update function trajectory suggests that the domain of the event—the program state and known objects—may be missing something. That is, there may be some latent state, which if known, would allow our method to discover a matching event. The automata synthesis step discovers this latent state.

The input to the automata synthesis step is a set of update function trajectories, one for each unmatched update function from the previous step. The goal of the automata synthesis procedure is to construct the simplest latent state automaton that enables us to write matching latent-state-based event predicates. For ease of exposition, we will begin by describing the automata synthesis procedure for the scenario in which there is exactly one unmatched update function for which a latent-stated-based predicate must be constructed. We will then describe the extension to the more general scenario of multiple unmatched update functions.

4.5.1 Problem Formulation. To start, we formulate the problem of latent state synthesis within the classic formulation of automata synthesis given input-output examples, which aims to determine the minimum-state automaton that accepts a given set of accepted input strings (positive examples) and rejects a given set of rejected input strings (negative examples). In our scenario, we can construct positive and negative input "strings" from the sequence of program states.

To construct positive and negative examples, we consider the set of *prefixes*—sub-sequences of the program-state sequence, starting from the first position—that has, as their last element, a program state where the *optimal co-occurring event* is true. The optimal co-occurring event is the event that co-occurs with the update function in question and has the minimum number of false-positive times, i.e. times when the event is true but the update function does not occur. This event is selected from a user-defined *co-occurring event space*, which contains a subset of events more likely to be co-occurring triggers than the arbitrary events in the full event space. In the Mario example, the optimal co-occurring event is `clicked`. Our method partitions the set of program state

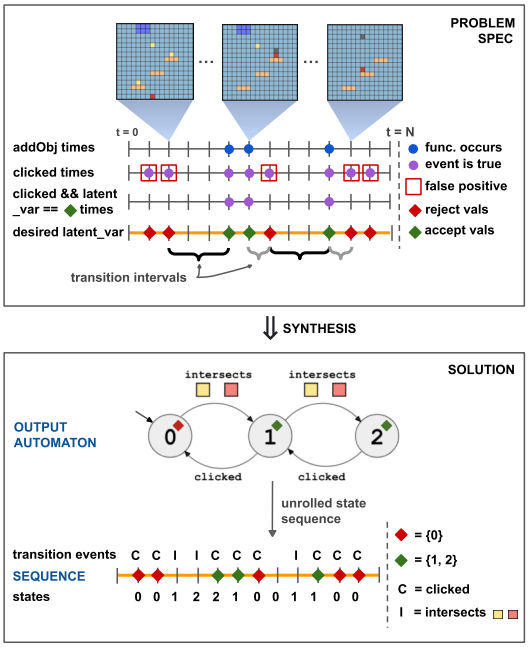


Fig. 7. Bird's-eye view of the automata synthesis problem, using the example of the Mario program. The bullet addition update function, indicated by `addObj`, does not have a matching trigger event. The closest event is `clicked`, which co-occurs with bullet addition but also is true at false positive times. We seek a latent variable that is true at one set of times (accept values) and false at another set of times (reject values), so that the conjunction of `clicked` and that latent variable perfectly matches `addObj`'s times. As shown in the solution, this latent variable initially has value zero, and changes to one then two on agent-coin intersection, and changes back down on clicks.

sequence prefixes into those that end with a program state in which the update function took place and those in which it did not. The former set is the set of positive examples and the latter is the set of negative examples for use in automata synthesis.

This construction of positive and negative input strings is motivated by the fact that, if there existed a latent state automaton that fit this specification, then the event

```
co_occurring_event && (latentVar in [/* accept values */])
```

would be a perfect match for the update function. This is because the co-occurring event is true during a set of false positive times with respect to the update function trajectory, and the latent automaton is in rejecting states at exactly those times (since those times correspond to the rejected program state prefixes). Thus, finding such an automaton would mean we would have an event that matches the update function under consideration.

4.5.2 Multiple unmatched update functions. The general setting has multiple unmatched update functions. In this scenario, each unmatched update function specifies its own inductive automata synthesis problem—a set of positive and negative input strings—that if solved will give rise to a matching latent-state-based predicate.

One solution to this "multi-automata" synthesis problem is to construct a distinct latent automaton (variable) that satisfies each update function. However, a smaller number of latent variables

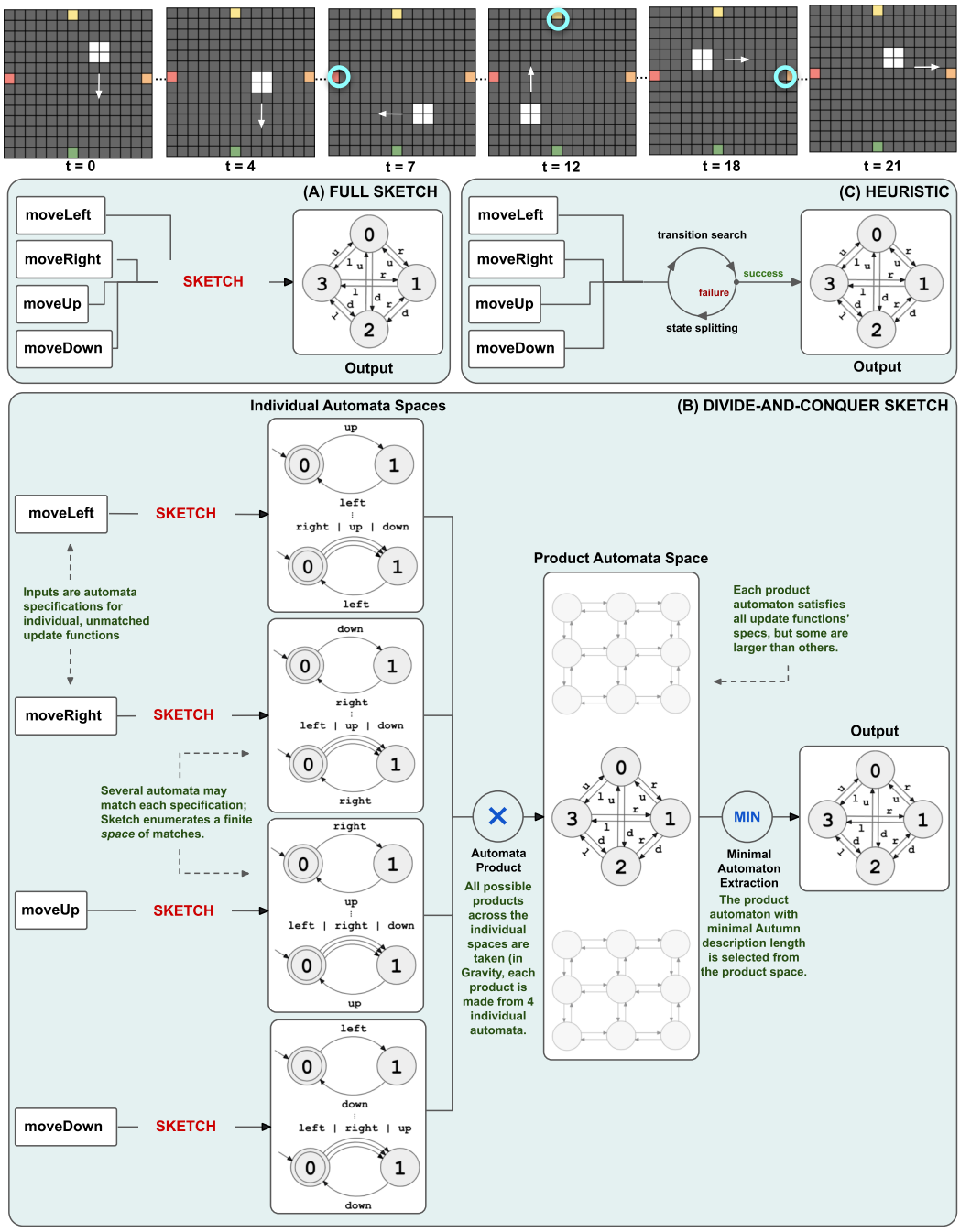


Fig. 8. Three variant methods for automata synthesis, shown for the Gravity I benchmark program. The white block move left, right, up, or down depending on the button last clicked. The transition label `left` abbreviates `(clicked leftButton)`, etc.

is often sufficient to explain all the update functions. In fact, the *product* of all the individual update function automata is a single automaton that satisfies all specifications, up to changing the accept states for each update function. However, taking the product of the simplest automata satisfying individual update functions—where we define *simplest* as that automaton with the fewest states—does not necessarily produce the simplest product automaton. This is because it is possible that larger automata for individual update functions will multiply to form the minimal product automaton. Thus, optimizing each automaton individually and multiplying is not sufficient.

We now discuss three distinct algorithms for solving this inductive automata synthesis problem: Full Sketch, Divide-and-Conquer Sketch, and Heuristic. Our current implementation synthesizes a single latent state automaton that satisfies all unmatched update functions within each object type, as opposed to a single automaton for the entire program (i.e. across all object types), which mirrors the latent state structure found in most real Autumn programs in practice.

4.5.3 Algorithm 1: Full Sketch. In the Full Sketch approach, the complete multi-automata synthesis problem (for each object type) is encoded as a Sketch [Solar-Lezama 2013] problem. In other words, Sketch is tasked with identifying the minimal automaton that accepts each update function's language, as specified by the observed examples, up to changing just the accept states. As an example, consider the Gravity I program shown in Fig. 4 and Fig. 8. The white block continuously moves left, right, up, or down depending on which of the four colored buttons was last pressed. A matching event cannot be found for any of the four update functions `moveLeft`, `moveRight`, `moveUp`, or `moveDown`, so their update function trajectories are fed to the Sketch solver to produce the 4-state automaton shown in Fig. 8a. This new latent variable then allows a matching predicate to be written for all four update functions: `true && latentVar == 1`, `true && latentVar == 2`, `true && latentVar == 3`, and `true && latentVar == 4`, where the optimal co-occurring event is `true`.

4.5.4 Algorithm 2: Divide-And-Conquer Sketch. Rather than attacking the full multi-automata synthesis problem, Divide-And-Conquer Sketch tasks Sketch with solving each update function's automata synthesis problem *individually*, and then combines those solutions together via product. The intuition behind this approach is that synthesizing an automaton matching *all* update functions at once may face scalability challenges, but finding an automaton matching a single update function, which is likely smaller, may be easier. As described previously, the smallest automaton satisfying a single update function may not give rise to the smallest product, so the Divide-and-Conquer algorithm identifies a small *set* of automata matching each update function instead. It then takes the product over all update functions' automata sets, and computes the minimal automaton from that product space. We illustrate this algorithm again with the Gravity I example (Fig. 8b). The algorithm first identifies a set of automata that solve the automata synthesis problems corresponding to the four unmatched update functions. Note that each of these automata have just two states instead of the full 4-state solution found in the Full SAT approach. Next, it computes all automata *products* over these four automata sets, and takes the minimal automaton from this product set, which is the 4-state solution seen previously.

4.5.5 Algorithm 3: Heuristic. Despite the simplicity of the Sketch-based formulations of automata synthesis, their scalability to problem settings with large automata is unclear, due to the scaling limitations of SAT solvers. As such, we also implemented a heuristic algorithm that synthesizes an automaton satisfying a set of update function trajectories via a series of greedy updates to an initial automaton (Fig. 8c). At a high level, this approach begins with an automaton with a small number of states, and repeatedly *splits* states into two based on a heuristic related to the search for transition events. More precisely, the algorithm begins by searching for transition events (edges) that result in an automaton that produces a particular initial state sequence that has few distinct states. If

the transition search fails, one of the original states is split into two, and the transition search is repeated. This process continues until a satisfying automaton is identified. Our heuristic algorithm bears some similarity to *counterexample-guided abstraction refinement*, in that it iteratively *refines* a specification in response to errors while trying to satisfy it; see Appendix A.2 for further detail.

4.5.6 Non-Determinism Handling in AutumnSynth. Having described the final cause synthesis step, we briefly comment on support for writing *nondeterministic* programs in Autumn. Autumn provides a library function called `uniformChoice`, which selects one element uniformly at random from a non-empty list. Using this function, many interesting causal probabilistic Autumn programs can be written. However, inferring the probability distribution described by a probabilistic Autumn program adds a new level of complexity, so our current synthesis algorithm is focused only on synthesizing *deterministic* Autumn programs.

We make one minimal, small-scoped exception to this, however: We allow use of `uniformChoice` at the *update function* level of an on-clause, but not at the *event* level. Explicitly, if no deterministic Autumn program is found by the synthesizer, which means cause synthesis fails on every concrete update function matrix M identified through the update function synthesis step, the algorithm will try to construct new concrete matrices using `uniformChoice`-based update functions. Currently, the algorithm only allows these random update functions to have the form `addObj (uniformChoice [ \* list of positions *\ ])`. For example, it is possible that the set of possible update functions in the unfiltered matrix for a certain object x at a certain time t is

$$M_{x,t} = \{\ \texttt{addObj (Bullet (Position 5 5))},\ \texttt{addObj (Bullet (uniformChoice (map (-> obj obj.origin) objects)))}\ \},$$

if (5, 5) is the location of an object in the list. Hence, a matching event might be found for a `uniformChoice`-based update function even if not found for deterministic update functions, so this limited form of nondeterminism in Autumn programs is supported by the synthesizer.

4.6 Code Generation

We implemented the Autumn language and AutumnSynth algorithm in Julia. The interpreter and library functions of the language are expressed in about 1,600 lines of Julia code, while the synthesizer is about 18,000 lines. To construct the correct Autumn syntax describing the object types $\mathcal{T}$, initial state X_0, and transition function F determined by AutumnSynth, we first express the stream definitions as follows. For each object type $t \in \mathcal{T}$, we define a list variable where the **`init`** value contains the values of all objects $x \in X_0$ with type t, and the **`next`** value is simply the default **`prev`** expression (i.e. synthesized Autumn programs do not use the **`next`** clause, keeping all updates in on-clauses instead). After these stream definitions, the on-clauses are expressed in order from most-frequent to least-frequent for each object type $t \in \mathcal{T}$. Precisely, for each on-clause o_i in $F(H) = o_m(\ldots(o_2(o_1(next(H), H), H)\ldots), H)$, the on-clause o_i is the ith on-clause to appear in the Autumn code when read from top to bottom. These stream definitions and on-clauses are inserted following the standard grid size definition, background definition (we currently change all background colors to white, for simplicity), and definitions of the object types $\mathcal{T}$ at the start of the program to form the complete synthesized output.

5 EVALUATION

To evaluate AutumnSynth, we constructed a suite of 30 Autumn programs, called the Causal Inductive Synthesis Corpus (CISC), and also evaluated against a preexisting corpus of grid-world video games written in Python. We evaluated the following questions:

(1) How expressive is the Autumn language for modeling grid world environments?
(2) Does the AutumnSynth algorithm scale to interesting programs and long input traces?

(3) Are the synthesized programs able to generalize to new scenarios as opposed to just memorizing the input trace?

5.1 Benchmarks

5.1.1 The Causal Inductive Synthesis Corpus. Descriptions of each of the 30 benchmark programs in the Causal Inductive Synthesis Corpus are given in Fig. 12 in the Appendix. Of these models, 24 possess latent state and hence require the automata synthesis step of our algorithm, whereas the remaining six models do not possess latent structure and thus only test the functional synthesis component. The largest latent automaton present across the benchmark models has 11 states and 20 edges (Count V in the figure), while the largest number of latent automata in a single benchmark is four (Water Plug), where there is one global variable and three object types that each contain one object-specific latent field.

5.1.2 The Exploration, Modeling, and Planning Agent Corpus. In addition to the CISC benchmark, we also ran the AutumnSynth algorithm on a subset of the Exploration, Modeling, and Planning Agent (EMPA) benchmark suite [Tsividis et al. 2021]. The EMPA suite consists of 27 distinct grid-world games, each of which contains two to five levels, making for a total of 90 different levels across all games. In our evaluation, we used the first level of each of the 27 games, so our final evaluation set contained 27 EMPA programs. Unlike CISC, EMPA models are not natively written in Autumn, which makes them a suitable benchmark for measuring how effective Autumn is at capturing general grid-world dynamics. Some EMPA stills are shown in Fig. 13 in the Appendix.

5.2 Scalability and Performance

We now discuss our experiments on CISC and EMPA evaluating the scalability of AutumnSynth.

5.2.1 CISC. For each of the 30 CISC programs, we manually constructed an input sequence of user actions and corresponding observations. Our objective when curating these input traces was to demonstrate all of the dynamics encoded by the model so that the synthesizer would be forced to compute a solution capturing all aspects of the grid environment. We ran the three AutumnSynth variants on these manually curated input traces for each of the 24 latent-state-containing programs, and ran just the Heuristic on the 6 non-latent-state-based programs since the three algorithms differ only in the latent state synthesis step. For the purpose of this section, we declared a success on a benchmark program if the synthesizer produced an Autumn program that generated the input sequence of observations given the input sequence of actions, even if the program was not semantically equivalent to the ground-truth benchmark. The results are shown in Fig. 9.

Notably, all three algorithm variants—Sketch, D&C Sketch, and Heuristic—are able to synthesize a program for most of the benchmark problems, with the Heuristic algorithm solving the most with 27 out of 30. The runtimes for the Heuristic algorithm range from just two minutes for some of the smaller benchmarks that have few on-clauses and short input traces (e.g. Disease with seven on-clauses and a 22-frame input trace) to up to 9 to 13 hours for larger benchmarks with many on-clauses or very long traces (e.g. Mario with 16 on-clauses or Sokoban with a 243-frame input trace). Unlike the Heuristic, both of the Sketch-based versions of the algorithm time out after 24 hours on several of the benchmarks. This is because those programs require large latent automata that in particular have many *hidden states*, which are additional accept states corresponding to a state-based update function. We provide further analysis of this result in Appendix A.1.1.

The fact that AutumnSynth is able to synthesize most of the CISC benchmarks, including those with many on-clauses and large latent automata, as well as from fairly long input traces with hundreds of frames, demonstrates the scalability of the algorithm.

	Model Name	# of A.	Max # of A. S.	Max # of A. T.	# of O.C.	Input Length (Frames)	Output Length (Program Lines)	Heuristic Runtime (min)	Sketch Runtime (min)	D&C Sketch Runtime (min)	Test Set Accuracy
No Latent State	Ants	0	0	0	3	17	24	177.4	207.3	417.3	100%
	Chase	0	0	0	7	27	34	11.3	14.0	21.4	100%
	Magnets	0	0	0	12	183	41	113.2	248.9	273.8	23.9%
	Space Invaders	0	0	0	12	55	50	820.7	849.5	⊥	100%
	Sokoban	0	0	0	9	243	43	521.8	836.7	1123.5	73.3%
	Ice	0	0	0	10	27	45	3.3	3.8	5.9	100%
Latent State	Lights	1	2	2	4	24	36	2.0	2.5	5.3	100%
	Disease	1	2	2	7	22	31	2.3	3.3	4.0	100%
	Grow	1	2	2	11	96	49	411.1	425.3	529.2	100%
	Grow II	1	2	2	11	96	-	×	×	×	×
	Sandcastle I	1	2	2	7	37	33	3.6	3.8	4.2	100%
	Sandcastle II	1	2	2	7	37	-	×	×	×	×
	Bullets	2	4	12	16	53	56	10.7	21.6	⊥	100%
	Gravity I	1	4	12	9	19	38	2.3	2.4	3.5	100%
	Gravity II	2	4	12	14	24	50	3.2	3.3	6.0	100%
	Gravity III	1	9	24	32	27	83	2.1	2.9	⊥	100%
	Gravity IV	1	8	56	17	43	54	2.7	3.2	7.3	100%
	Count I	1	3	4	6	22	31	2.1	2.2	3.8	100%
	Count II	1	5	8	10	39	39	2.3	2.7	11.0	100%
	Count III	1	7	12	14	69	47	2.4	⊥	⊥	100%
	Count IV	1	9	16	18	109	55	2.8	⊥	⊥	100%
	Count V	1	11	20	22	149	63	3.5	⊥	⊥	100%
	Double Count I	1	5	8	12	94	43	2.6	3.2	42.0	100%
	Double Count II	1	9	16	20	156	59	3.5	⊥	⊥	100%
	Wind	1	3	4	9	23	43	24.1	23.8	31.1	100%
	Paint	1	5	5	10	27	39	2.3	2.6	11.7	100%
	Mario	2	3	4	16	81	59	168.1	200.1	241.8	84.6%
	Water Plug	4	3	6	10	68	53	760.7	⊥	⊥	100%
	Mario II	2	4	6	16	81	-	×	×	×	×
	Coins	1	5	10	16	168	57	68.7	⊥	⊥	74.8%

Fig. 9. Results from running AutumnSynth on the CISC benchmark suite. ⊥ indicates timeout after 20 hours. The column header abbreviations signify the following: # of A. → # of Automata, Max # of A. S. → Max. # of Automaton States, Max # of A. T. → Max. # of Automaton Transitions, # of O.C. → # of On-Clauses. We further note that the output length was computed on the Heuristic-synthesized programs, and that the test set accuracies were computed by running the Heuristic-synthesized programs as well. This is because the Heuristic algorithm produced the most generalizable programs, though the Sketch algorithm matched it for the benchmarks that it also solved. See Appendix A.1.6 for additional evaluation details.

5.2.2 EMPA. As in the CISC evaluation, we manually constructed an input observation trace for each of the 27 programs in the EMPA suite and ran the AutumnSynth algorithm on those traces. Unlike CISC programs, EMPA games have a higher frame rate (20 frames per second versus 3 frames per second) that results in longer input traces and are further played on grids with significantly larger dimensions (e.g. 300 pixels by 110 pixels for the Aliens game versus the 16 pixels by 16 pixels of most CISC games), both of which cause longer synthesis runtimes. In addition, stylistic differences between EMPA games and CISC games required us to slightly modify the event and update function spaces used in the AutumnSynth algorithm in order to synthesize Autumn programs modeling EMPA environments, as well as modify some of the lower-level heuristics used in the algorithm (e.g. for object perception, object mapping, update function synthesis, etc.). We describe these modifications in greater detail in Appendix A.1.2.

In particular, one difference between the different corpora is that the vast majority (19 out of 27) of the EMPA games are non-deterministic, compared to very few of the CISC programs. Further, the type of random behavior present in EMPA is different from that in CISC: While the only non-determinism in CISC appears at the update function level (i.e. update functions may use the `uniformChoice` function but all trigger events in on-clauses are deterministic), EMPA programs also

	ID	Model Name	# of A.	Max # of A. S.	Max # of A. T.	# of O.C.	Input Length (Frames, Sec)	Output Length (Program Lines)	Heuristic Runtime	Sketch Runtime	D&C Sketch Runtime	Test Set Accuracy
No Latent State	1	Antagonist	0	0	0	4	162, 8.1s	48	54.1m	N/A	N/A	N.D.
	2	Avoid George	0	0	0	6	100, 5.0s	48	1.3h	N/A	N/A	N.D.
	3	Bees and Birds	0	0	0		66, 3.3s	41	12.9m	N/A	N/A	N.D.
	4	Boulder Dash	0	0	0	-	390, 18.5s	-	⊥	N/A	N/A	-
	5	Butterflies	0	0	0	3	193, 9.7s	193	1.6h	N/A	N/A	N.D.
	6	Chase	0	0	0	-	31, 1.6s	-	×	N/A	N/A	-
	7	Closing Gates	0	0	0	3	154, 7.7s	44	1.4h	N/A	N/A	97.5%
	8	Explore/Exploit	0	0	0	2	222, 11.1s	32	9.1m	N/A	N/A	100%
	9	Helper	0	0	0	4	267, 13.4s	41	48.9m	N/A	N/A	N.D.
	10	Preconditions	0	0	0	3	83, 4.2s	41	4.3m	N/A	N/A	100%
	11	Push Boulders	0	0	0	-	211, 10.6s	-	×	N/A	N/A	-
	12	Relational	0	0	0	5	318, 15.9s	44	45.5m	N/A	N/A	100%
	13	Sokoban	0	0	0	3	203, 10.1s	37	19.0m	N/A	N/A	100%
	14	Surprise	0	0	0	6	211, 10.6s	53	83.0m	N/A	N/A	N.D.
	15	Water Game	0	0	0	5	97, 4.9s	48	14.5m	N/A	N/A	98.9%
	16	Zelda	0	0	0	4	142, 7.1s	46	11.2m	N/A	N/A	N.D.
Latent State	17	Aliens	3	14	20	37	318, 15.9s	113	15.7h	⊥	⊥	N.D.
	18	Bait	1	2	1	5	131, 6.6s	50	19.5m	38.8m	23.9m	100%
	19	Corridor	-	-	-	-	170, 8.5s	-	×	×	×	-
	20	Frogs	-	-	-	-	274, 13.7s	-	⊥	⊥	⊥	-
	21	Jaws	2	2	1	10	95, 4.8s	62	57.2m	50.9m	1.0h	N.D.
	22	Lemmings	3	4	12	17	263, 13.2s	80	38.8h	⊥	29.4h	N.D.
	23	Missile Command	-	-	-	-	-	-	×	×	×	N.D.
	24	My Aliens	2	3	3	11	127, 6.4s	56	1.6h	1.5h	2.2h	N.D.
	25	Plaque Attack	3	11	11	35	235, 11.8s	112	12.1h	⊥	⊥	N.D.
	26	Portals	2	2	2	14	245, 12.3s	84	9.2h	9.7h	9.5h	N.D.
	27	Survive Zombies	2	7	7	19	138, 6.9s	78	2.7h	⊥	⊥	N.D.

Fig. 10. Results from running AutumnSynth on the EMPA benchmark suite. ⊥ indicates timeout after 40 hours, and N.D. indicates the program is non-deterministic, making it challenging to perform a test set accuracy experiment (see Section 5.3.2). The column header abbreviations mean the same as in Fig. 9. *See Appendix A.1.5 for note on Survive Zombies.

have a few kinds of dynamics triggered by *random events*. For example, enemy objects in EMPA shoot out rockets with some probability at every time step, a feature not supported by the original AutumnSynth algorithm tuned to the CISC benchmark. To account for this difference, we slightly modified the cause synthesis step of AutumnSynth to assign a single, arbitrary random event to certain types of update functions (e.g. bullet addition) for which a deterministic event cannot be found, instead of synthesizing latent state to explain those update functions. The modifications of the algorithm to better suit the new domain of EMPA programs are detailed in the Appendix. We emphasize that while AutumnSynth is flexible enough to handle different domains, such lower-level, domain-specific tweaks are generally needed.

The results of our evaluation are shown in Fig. 10. We find that AutumnSynth synthesizes a solution for 21 out of the 27 EMPA programs. As in the CISC evaluation, we run all three variants of the AutumnSynth algorithm on the latent-state-based benchmarks—which compose 9 out of the total 27—while only running the Heuristic version on the non-latent-state-based models. Since the majority of EMPA benchmarks display nondeterministic behavior, we had to manually inspect those programs to check that they matched the given input trace instead of performing an automatic check. Given the differences with respect to the original CISC programs that helped guide the development of the algorithm, it is notable that AutumnSynth manages to synthesize

	Model	Metrics	1 Trace	2 Traces	3 Traces	4 Traces	5 Traces	6 Traces	7 Traces
CISC	Magnets	Trace Len.	183	325					
		Runtime	32.8m	4.7h					
		Accuracy	23.9%	100%					
		O.C. / A.T.	12 / 0	12 / 0					
	Mario	Trace Len.	81	152	198				
		Runtime	36.6m	3.1h	13.8h				
		Accuracy	84.6%	87.1%	100%				
		O.C. / A.T.	16 / 4	18 / 6	20 / 6				
	Coins	Trace Len.	168	235	308	393	481	568	676
		Runtime	13.9m	35.1m	79.2m	3.0h	5.6h	9.6h	15.8h
		Accuracy	74.8%	77.5%	80.2%	82.8%	88.7%	96.0%	100%
		O.C. / A.T.	16 / 8	18 / 10	20 / 12	22 / 14	24 / 16	26 / 18	28 / 20
EMPA	Water Game	Trace Len.	97	140					
		Runtime	7.3m	10.9m					
		Accuracy	98.9%	100%					
		O.C. / A.T.	4 / 0	5 / 0					

Fig. 11. Results from running AutumnSynth on multiple traces taken from a sample of CISC and EMPA benchmarks (O.C. / A.T. means number of on-clauses/automata transitions). The new on-clauses learned for Coins correspond to automata transitions impossible to learn with fewer traces; see Section 5.3.1 for details.

models of many of the EMPA games, including some with very large numbers of on-clauses and latent states, such as Aliens, which has 14 latent states and 37 on-clauses.

5.2.3 Runtime Analysis. AutumnSynth scales roughly linearly with the number of non-latent-state-based on-clauses in the program, assuming that event search for each on-clause requires the average depth of exploration of the event space. In programs with latent state, however, event search can become a major bottleneck. This is due to the fact every state-based update function will require an exhaustive search of the entire event space before resolving that additional latent state is required. One future solution to this is to exploit the large degree of parallelism afforded by the decomposability of the algorithm. Specifically, event search may be performed independently and thus in parallel for each update function rather than sequentially, which is likely to markedly improve performance. Another bottleneck is automata synthesis: While the heuristic synthesizer is fast, the heuristics it uses may not apply to new scenarios, and the more generally-applying Sketch-based algorithms do not scale well. Since AutumnSynth is agnostic to the underlying automata synthesis algorithm used, we are optimistic these issues can be overcome by using more sophisticated automata synthesizers.

5.3 Quality of Synthesized Models

Next, we describe our experiments that help quantify the *quality* of the programs synthesized by AutumnSynth. Phrased differently, we are interested in verifying that the synthesized programs generalize reasonably well from the given input trace, as it is generally always possible for an inductive synthesis engine to simply produce a program that regurgitates the examples it was fed. To measure the generalization performance of the synthesized programs, we perform two kinds of experiments: (1) We construct new input traces from the benchmark programs and compute how often the synthesized and benchmark programs produce the same output observation sequences on these new traces, and (2) we run AutumnSynth on *multiple input traces* that are strung together, to show that an exact semantic match to the ground truth can be synthesized given enough data.

5.3.1 CISC. For each CISC program, we constructed a test set of additional user action traces and evaluated the synthesized and ground-truth programs on each trace. (For the three non-deterministic models in CISC, we fixed a random seed to enable accurate comparison between the programs over the test traces.) We measured the fraction of each new observation sequence until

the frame where it diverged from the ground-truth observation sequence (i.e. 1 if the sequences were exactly the same), and averaged these values to produce the percentages shown in the final column of Fig. 9. All but four—Magnets, Sokoban, Mario, and Coins—of the synthesized programs matched the corresponding ground-truth benchmark program on all of the test set traces. For Magnets and Sokoban, the reason behind their divergence was that some details of the model dynamics were not demonstrated in the input trace fed to the synthesizer. For example, a certain kind of diagonal motion that the blue magnet object can undergo was simply not shown in the input, so the synthesizer did not learn an on-clause describing it.

An incomplete input trace is also the reason that the Mario and Coins solutions generalized differently, but for those benchmarks, it is actually *impossible* to show all the dynamics of the model with just a single trace. Precisely, both the Mario and Coins models involved collecting coins that can be used to shoot bullets. Since there is a finite number of coins, it is impossible to show in one trace both that collecting all of the n coins in the model allows the agent to shoot n bullets in a row before clicking does nothing, and *also* show that collecting $k < n$ coins allows the agent to shoot just k bullets before clicking does nothing. Without this detail, the automata synthesis algorithm will not produce the ground-truth automaton, so we construct a few independent traces showing these dynamics. The AutumnSynth algorithm is trivially extended to take multiple traces as input by concatenating the traces into a single long trace and informing the synthesizer of the connecting times so it does not try to learn on-clauses describing those reset moments. We perform this experiment for a subset of three of the four programs discussed, with results shown in Fig. 11. It is clear that AutumnSynth scales to these extremely long inputs and produces the previously elusive ground-truth programs.

5.3.2 EMPA. Unlike CISC, the majority of the EMPA benchmarks contain non-deterministic behavior, and since the ground-truth programs are expressed in PyVGDL rather than Autumn, we cannot easily fix a random seed to compare the ground-truth programs with their synthesized Autumn counterpart on a set of test traces as was done for CISC. In addition, AutumnSynth does not actually infer the correct underlying probability distributions in these models but rather aims only to detect the presence of non-determinism, rendering attempts at exact, automatic comparison ineffective (see Appendix A.1.2 for further details). Consequently, we only performed the test set accuracy experiment for the few deterministic programs in the EMPA benchmark suite, with results shown in Fig. 10. All but two of the synthesized deterministic programs exhibit perfect generalization accuracy on the test traces. The Closing Gates program falls slightly below because its given input trace did not demonstrate a few, very minor details of the model that rely on perfectly timing arrow key presses within one-twentieth of a second, and hence are challenging to generate. This can be fixed by performing a multi-trace experiment with added traces containing the missing details (only a fraction of these details were demonstrated in the test set, due to the difficulty of producing them). The Water Game program was similarly synthesized using an input trace that did not show one small aspect of the model, though it was actually impossible to demonstrate that aspect with just one trace (the agent object could not be killed by the lake and reach the win position in a single run of gameplay). Rerunning AutumnSynth in the multi-trace setting with an added trace demonstrating this aspect produces the desired program, as shown in Fig. 11.

Next, in lieu of a test set experiment for the non-deterministic programs, we provide a general description of their quality. We first remark that the fact that the deterministic programs generalize well is itself a signal of the non-deterministic programs' quality since the synthesis algorithm is very similar in each case. We note, though, that many EMPA games possess both win conditions and lose conditions, which makes it impossible to demonstrate all conditions in a single trace. For example, the model of how the moveable agent dies in a game often describes that intersecting just

one type of enemy object causes the agent to die when in reality there are several enemy types that cause agent death. This may be fixed by running the synthesizer with multiple traces, each showing a different win or loss condition. Further, like in the CISC single-trace and deterministic EMPA experiments, sometimes some details present in the ground-truth model were omitted when creating the demonstration trace, so the synthesizer did not learn a model incorporating those. See Appendix A.1.4 for a more detailed analysis of the quality of these EMPA programs. Importantly, despite these limitations, these programs still represent impressive abstract compressions of the information in the long frame sequences given as input.

As a final observation, we mention that we often made new traces in the multi-trace experiments in the style of counterexample-guided synthesis (CEGIS): The synthesizer would produce an output that was slightly incorrect given n traces, so we would add an $(n+1)$-th trace to the input to correct this error, and then find that the newly synthesized program was still incorrect in some way, and repeat this process until the desired program was produced. This compatibility of AutumnSynth with CEGIS opens up interesting future directions related to online program synthesis.

6 RELATED WORK

6.1 The Apperception Engine

The Apperception Engine [Evans et al. 2020] is the most closely related work to our own in terms of motivation. Like AutumnSynth, the Apperception Engine aims to construct a causal theory of a domain from perceptual data. The Apperception Engine synthesizes theories in Datalog$^\ni$: a variant of Datalog extended with causal rules to capture causal dynamics. In their formulation, in order for a theory in Datalog$^\ni$ to explain or "make sense" of data, it must not only be consistent with the data, but also satisfy spatial, conceptual, static, and temporal *unity conditions*. These unity conditions collectively act as an inductive bias that requires that all the objects and predicates within a theory are connected or informative of one another, at least indirectly. Autumn, in contrast, is not a logic programming language, and as a result, is less conducive than Datalog$^\ni$ to declaratively specifying causal relationships. On the other hand, the unity conditions are satisfied by construction, either by AutumnSynth or by the Autumn language itself.

The Apperception Engine contributes a new formalism rather than innovation in the synthesis method; the synthesis problem is deferred entirely to an external solver. One consequence of this, combined with the fact that the unity conditions need to be solved for, is limited scalability. For example, Apperception takes 48 hours to synthesize a model of Sokoban from a sequence of 17 grid frames of 20 × 20 pixels. In contrast, AutumnSynth takes approximately 19 minutes on Sokoban EMPA (189 grid frames, 130 pixels × 90 pixels) and 8 hours (243 grid frames, 16 pixels × 16 pixels) on Sokoban CISC respectively—the runtime difference between CISC and EMPA is due to CISC having a larger space of library components to search through than EMPA, which did not require such a diverse search space due EMPA programs being generally more similar to one another. Further, like Apperception, AutumnSynth can be trivially extended to impute and retrodict missing observations in addition to prediction, by telling the synthesizer to ignore certain time steps in the update function and event synthesis procedures (like in multi-trace synthesis).

6.2 Automated Game Playing

Guzdial et al. introduce a method [Guzdial et al. 2017] that synthesizes the "game engine" of Super Mario Bros. in the form of if-else-rules, from observed image frames and a library of sprite components. Within the active area of automated game playing [Levine et al. 2013], this work is most distinctly related to our contribution in its motivation to discover an explicit world model from observations. Their approach is greedy heuristic search over the space of programs using

local syntactic modifications (e.g. rule addition or deletion). Aside from limitations of scalability, the major distinction from AutumnSynth is that this approach does not discover latent state or the types-structure of the program generally, and would be inapplicable to the majority of CISC.

6.3 The Abstraction and Reasoning Corpus (ARC)

AutumnSynth was directly inspired by the Abstraction and Reasoning Corpus (ARC), a popular benchmark and position paper by Chollet [Chollet 2019b]. ARC was designed to help spur the development of AI agents that can more effectively mimic human intelligence. Specifically, it targets the human ability to generalize from small data to realize abstract theories, which can then be used to reason about new scenarios. Each ARC problem displays a few pairs of colored grids, where the first grid (input) and second grid (output) are related in a particular way. The goal is to figure out the relationship between the input and the output grids, and apply it to a new input to predict the correct output. ARC has generated great interest in the AI community, with some of the best-performing solutions being program synthesis systems that model the theories as programs.

The Autumn language was designed to express similar kinds of intricate entity relationships as in ARC, but augmented with *time* so they more closely resemble real observations. While the running example we use in this paper is a video game and the EMPA benchmarks are all video games, Autumn was designed more generally to express visually-simplified distillations of real-world phenomena, like water causing plants to grow or wind-blowing snow (Water Plug and Wind benchmarks in CISC). As in ARC, learning models of these grid domains requires the synthesizer to identify complex dependencies between visual artifacts (and in the case of Autumn, non-visual inventions)—an ability innate to humans but not machines—while temporarily abstracting away the lower-level challenge of fuzzy, real-world perception.

6.4 Reactive Synthesis

There is extensive literature on the synthesis of reactive programs from temporal specifications. Early work such as that of Pnueli and Rosner [Pnueli and Rosner 1990] involved doubly exponential algorithms with little possibility of practical use. However, in 2012, Bloem et al. [Bloem et al. 2012] showed that this style of synthesis could be made tractable by restricting it to a subset of temporal logic known as GR(1). However, that work has been limited to synthesizing finite state automata from temporal specifications and it is not obvious how to apply it to inductive synthesis.

There is also extensive literature on synthesizing finite state models from examples, starting with the seminal work of Dana Angluin [Angluin 1987]. This work relied on an ongoing interaction with an oracle to provide examples until a correct automaton was found. The initial algorithm has served as a basis for a number of more sophisticated methods and has been used extensively for automated model creation. There are now several mature tools for induction of finite state machines from examples [Combe et al. 2010]. For example, LearnLib [Raffelt and Steffen 2006] is a popular automata synthesis library that has been used in a number of applications. Vaandrager [Vaandrager 2017] provides a survey of several recent methods and applications of automata learning. In addition, *symbolic automata synthesizers* [Drews and D'Antoni 2017], which learn more expressive automata in which the alphabet is given by a Boolean algebra and the transition labels are first-order predicates, are a very relevant future direction to explore for automata synthesis in AutumnSynth.

Abstraction is another technique that has been applied to learn more expressive functions. For example, Cho et al. [Cho et al. 2010] demonstrate the use of automata inference together with abstraction to discover botnet command and control protocols. Even with these extensions, however, none of these systems match the expressiveness of AutumnSynth, which can generate programs with unbounded lists of objects, each with their own finite state, and with the expressiveness afforded by syntax-guided synthesis.

7 FUTURE WORK

7.1 Transitioning to Learned Heuristics

The effectiveness of AutumnSynth at synthesizing models of a domain relies on a carefully-tuned inductive bias, which is currently curated by a user with knowledge of the domain. Explicitly, sources of bias include the proximity biases of the object mapping step (e.g. assuming objects do not move very far from their original positions in a single time step); the heuristics used to filter the abstract update function matrix in the update function synthesis step; the structure of the event search spaces (including the smaller co-occurring event and transition event spaces, which contain events more likely to be actual co-occurring and transition events than the full event space); the priority given to events composed of fewer atom events during event search; and the constraining of automata search via the closest co-occurring event, along with other automata search heuristics. In the future, we hope to explore *learning* some of these biases instead of having to hand-curate them. For example, we are interested in learning a library of higher-level Autumn functions useful for synthesis starting from a small set of lower-level components in the style of DreamCoder [Ellis et al. 2021], and learning some of the lower-level heuristics used for update matrix filtering.

7.2 Extensions to Broader Problem Settings

A key limitation of AutumnSynth is that it does not infer probabilistic programs, excluding the minimal handling of non-determinism implemented for the EMPA experiments. We hope to extend AutumnSynth to synthesize this broader space of Autumn programs in the future, a task that will require notable changes in formalism (e.g. integrating with a Bayesian synthesis paradigm). In addition, we are ultimately interested in applying AutumnSynth-style synthesis in the context of model-based reinforcement learning (RL). Programmatic world models, such as Autumn programs, offer the advantages that they can be learned from relatively fewer observations than in traditional RL, and that their symbolic structure allows them to be more effectively analyzed to aid in exploration and planning. Extending AutumnSynth so that Autumn programs are synthesized incrementally via *error correction* with each new observation (online learning), rather than re-running the full synthesizer at each time, is one important direction of interest. Finally, beyond grid domains, other theory induction domains in which the integrated automata and functional synthesis approach of AutumnSynth can apply include learning models of single-page web apps employing a mix of latent and visible state, a challenging problem in software testing.

8 CONCLUSION

We have presented a new functional reactive domain-specific language (Autumn) suitable for expressing and synthesizing non-trivial grid world programs, and a new algorithm (AutumnSynth) that induces functional reactive programs in this language from observation data. We have empirically evaluated our algorithm on a new benchmark suite of 30 Autumn programs that we call the Causal Inductive Synthesis Corpus (CISC), as well as on a third-party dataset of 27 grid-world-style games written in Python. Our evaluation shows that AutumnSynth is able to synthesize long programs describing complex dynamics and latent state, and is able to do so from long input traces containing hundreds of frames. Looking ahead, we expect AutumnSynth will provide a template for how to integrate functional and reactive synthesis in other theory induction domains.

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