

Evolution of 4π -periodic Supercurrent in the Presence of In-plane Magnetic Field

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In the presence of a 4π -periodic contribution to the current phase relation, for example in topological Josephson junctions, odd Shapiro steps are expected to be missing. While missing odd Shapiro steps have been observed in several material systems and interpreted in the context of topological superconductivity, they have also been observed in topologically trivial junctions. Here, we study the evolution of such trivial missing odd Shapiro steps in Al-InAs junctions in the presence of an in-plane magnetic field B^θ . We find that the odd steps reappear at a crossover B^θ value, exhibiting an in-plane field angle anisotropy that depends on spin-orbit coupling effects. We interpret this behavior by theoretically analyzing the Andreev bound state spectrum and the transitions induced by the non-adiabatic dynamics of the junction and attribute the observed anisotropy to mode-to-mode coupling. Our results highlight the complex phenomenology of missing Shapiro steps and the underlying current phase relations in planar Josephson junctions designed to realize Majorana states.

Josephson junctions (JJs) fabricated on semiconductor structures with epitaxially grown superconductors have recently attracted attention due to their propitious characteristics^{1–9} and applications in quantum computing^{10–18}. In the presence of a Zeeman field^{19–21} or a phase bias^{22–24}, and a strong spin-orbit coupling (SOC) interaction, such high-quality JJs have shown signatures of topological superconductivity^{21–24}, which can host Majorana zero modes useful for fault-tolerant quantum computation^{25,26}. However, robust implementation and signatures of topological superconductivity remain ambiguous^{27–31}.

To harness the potential of topological superconductivity, it is essential to be able to identify unambiguously the topological character of the states in a JJ. Topological JJs exhibit a unique fractional Josephson effect which is inaccessible with DC measurements due to relaxation processes to the ground state. Consequently, detecting the fractional Josephson effect requires measurements on timescales shorter than the relaxation time^{32–37}; timescales that are accessible using microwave excitations^{38–42}.

When a microwave bias is applied to a JJ, the periodic modulation of the current bias becomes phase locked with the dynamics of the junction and results in constant voltage steps in the voltage-current characteristic known as Shapiro steps. The Andreev bound states (ABSs) of a conventional JJ in the short ballistic regime are 2π -periodic in phase ϕ , resulting in Shapiro steps at values of $n\frac{hf}{2e}$, where f is the frequency of the microwave drive, and n is an integer. When the current phase relation (CPR) is 4π -periodic, as expected for a topological JJs, the fractional Josephson effect results in Shapiro steps only at $n\frac{hf}{e}$, resulting in missing odd Shapiro steps. Missing Shapiro steps have been observed in different material

systems and are usually attributed to the presence of a topological state^{38–40,42–44}. In practice, even for a topological JJ, a 4π -periodic component CPR coexists with a 2π -periodic component in which case the absence of odd Shapiro steps depends on the details of the junction, and the frequency and power of the microwave radiation^{45–47}.

Recent work⁴⁸ has experimentally shown that topologically trivial JJs can also exhibit missing odd Shapiro steps as predicted previously by other theoretical works^{44,45,49–51}. This can happen when ABSs with a large probability of undergoing a Landau-Zener transition (LZT) at $\phi \sim \pi$, and a negligible probability of crossing into the continuum, are present. Other mechanisms responsible for missing Shapiro steps have also been proposed involving a bias-dependent junction resistance⁵², or the presence of multiband superconducting states⁵³. Therefore, the observation of 4π -periodic supercurrent $I_{4\pi}$ or missing Shapiro steps is a necessary signature of topological superconductivity but is not conclusive. Given that an in-plane magnetic field B^θ is one of the ingredients required to drive a JJ to a topological transition, understanding how missing Shapiro steps depend on B^θ is essential to distinguish a trivial JJ from its topological counterpart.

In this work, we present measurements on highly-transparent epitaxial Al-InAs JJs in the presence of an in-plane magnetic field B^θ and SOC effects, conditions associated with inducing topological superconductivity. For $B^\theta = 0$ mT, we observe missing odd Shapiro steps with no applied field due to the presence of a topologically trivial $I_{4\pi}$ as observed previously⁴⁸. As B^θ is increased, these missing Shapiro steps eventually reappear and no topological signatures are observed up to the junction critical field B_c^θ . The reappearance of the missing steps exhibits angle anisotropies that depend on the

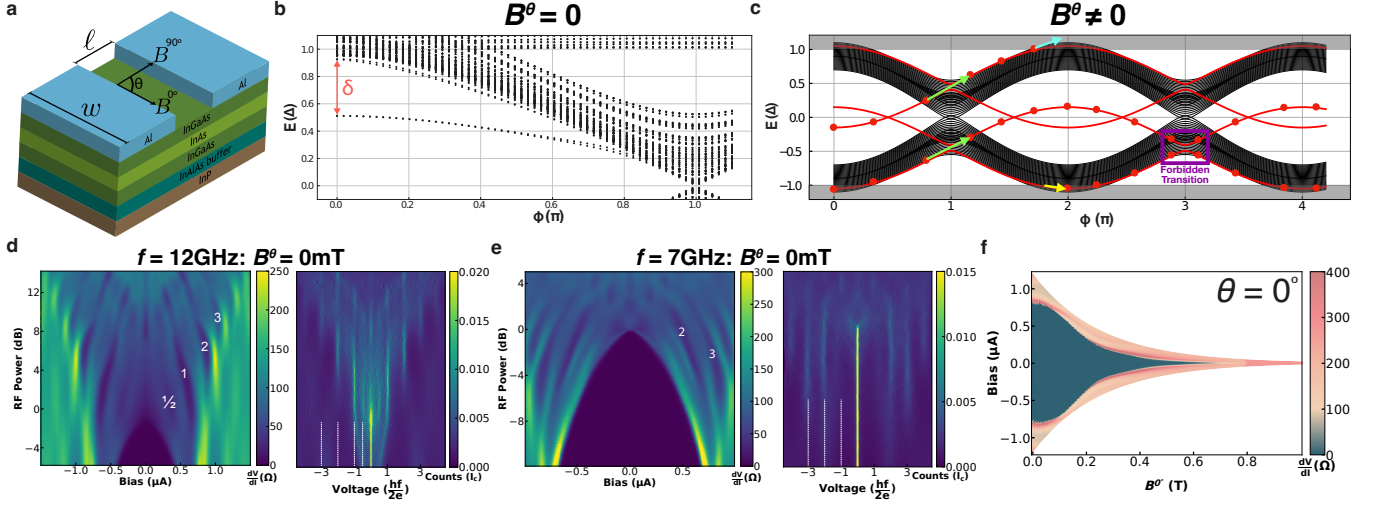


Figure 1: Josephson junction geometry, Andreev spectrum and characterization. **a** Schematic drawing of the material heterostructure with a junction of width w and length l made of Al superconducting contacts and an InAs surface quantum well. The 2D axis represents the direction of an applied in-plane magnetic field, where θ is the in-plane field angle such that B^{0° is the in-plane field along the junction and B^{90° is the in-plane field along the current. **b** Example of calculated energy spectrum of the Andreev bound states in a wide junction with no applied magnetic field. The results obtained are for a JJ with $w = 500$ nm and $l = 100$ nm, superconducting gap $\Delta = 300$ μ eV, and carrier density $n = 4 \times 10^{11}$ cm^{-2} . The long junction modes that contribute to $I_{4\pi}$ are separated by δ from the quasicontinuum at $E \sim \Delta$. **c** Energy spectrum in the presence of a finite magnetic field with the spin-split long junction modes (red). The dots on the modes indicate an occupied state. Arrows indicate possible mode to mode (green) and mode to continuum (light-blue/yellow) transitions. The dark-blue arrow indicates relaxation processes that fill low energy unoccupied states. The purple box defines a forbidden transition due to both states being occupied. **d, e** Differential resistance as a function of current bias, along with a histogram of the voltage distribution, all as a function of RF power, for frequency: **d** $f = 12$ GHz, and **e** $f = 7$ GHz with no applied field, $B^\theta = 0$ mT, for JJ1. The numbers correspond to the index of the Shapiro steps. **f** Differential resistance as a function of current bias and B^{0° for JJ1.

angle-dependent B_c^θ and carrier density associated with SOC interaction effects. Our results show the complex dependence of topologically trivial $I_{4\pi}$ on the applied in-plane field magnitude and direction, and SOC effects.

Fig. 1a presents the junction heterostructure studied. An InAs near-surface quantum well is grown between two layers of $\text{In}_{0.81}\text{Ga}_{0.19}\text{As}$ which is then capped with a thin layer of epitaxial Al grown *in situ*. Two JJs, JJ1 and JJ2, are fabricated on two different wafers grown under slightly different growth conditions (see Supporting Information). The junctions are defined using a selective wet etch of the Al and are $w = 4$ μm wide and $l \sim 100$ nm long. Given l of the junctions, the calculated mean free path to be $l_{\text{mfp}} \approx 150 - 250$ nm, and the superconducting coherence length $\xi \approx 530 - 630$ nm, the junctions are expected to be in the short ($l < \xi$) ballistic ($l < l_{\text{mfp}}$) regime.

To get insight into the dynamics of such highly transparent junctions, we first perform tight binding simulations of an Al-InAs junction using realistic parameters and calculate the energy spectrum of the ABSs shown in Fig. 1b (simulation details are provided in Supporting Information). The calculations of these wide junctions present a complex ABS spectrum with hundreds of modes. For a junction with width larger than the coher-

ence length ($w > \xi$), modes with momentum primarily along the transverse direction behave effectively as “long junction” modes⁴⁸. Consequently, these modes develop a detachment gap δ from the continuum when the phase difference across the junction ϕ is zero, as indicated in Fig. 1b. The number of long junction modes and their δ is sensitive to several factors (density n , w ,...). When the junction is highly transparent, the gap at $\phi = \pi$ is sufficiently small to allow Landau-Zener transitions (LZTs) when the system is diabatically driven^{45,48,49}. The combination of a large detachment gap and a small gap at $\phi = \pi$ for these long junction modes gives rise to a 4π -periodic contribution to the CPR, causing a topologically trivial junction to have both 2π - and 4π -periodic supercurrent channels^{46,47}. In the presence of a magnetic field in the plane of the junction, the Zeeman effect splits the ABSs and eventually leads to the closing of the detachment gap of the long junction modes, as seen in Fig. 1c. The 4π -periodic trajectory of long junction modes is then suppressed due to transitions to the continuum. Additionally, LZTs may occur between long junction modes and other modes with negligible detachments gaps, leading to transitions to the continuum mediated by conventional ABSs and suppressing $I_{4\pi}$.

To experimentally investigate such trivial 4π -modes,

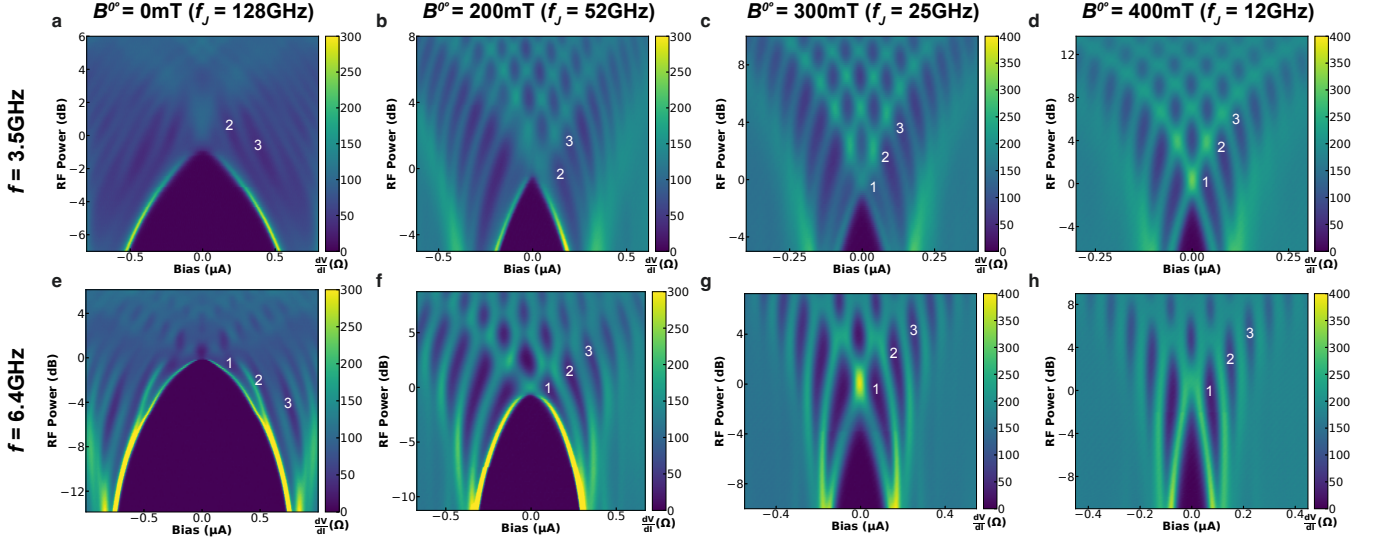


Figure 2: **Missing Shapiro step reemergence at finite in-plane magnetic field.** Differential resistance as a function of current bias and RF power at **a-d** $f = 3.5$ GHz and **e-h** $f = 6.4$ GHz for different B^{0° values for JJ1.

we examine the microwave response of JJ1 in a DC current-biased setup. The measurements are carried out at $T = 30$ mK where the junction exhibits no hysteresis, as seen in Supporting Fig. S2. In Fig. 1d, we present $\frac{dV}{dI}$ as a function of the DC current bias and RF power at $f = 12$ GHz in addition to a histogram of the voltage distribution. For this value of f , we can identify all the integer Shapiro steps along with subharmonic Shapiro steps. Subharmonic Shapiro steps are expected at high frequencies due to the anharmonicity associated with the forward skewness of the CPR in highly transparent junctions^{39,54–59}. The presence of a 4π -periodic supercurrent channel, with critical current $I_{4\pi}$, is expected to result in missing odd Shapiro steps^{44,45,48–51} when the energy of the photon irradiating the JJ, hf , is less than $hf_{4\pi} \approx 2eI_{4\pi}R_n$ ^{46,47}. Fig. 1e shows a similar Shapiro map for $f = 7$ GHz where we see that the first odd Shapiro step is missing indicating the presence of a finite $I_{4\pi}$ even though the JJ is in a topologically trivial regime. For JJ1, at $B^\theta = 0$ mT, we find $f_{4\pi} \sim 8.2$ GHz corresponding to $I_{4\pi} = 52.1$ nA. Considering the Josephson frequency, $f_J \equiv \frac{2eI_c R_n}{h}$, for JJ1, we get $f_{4\pi}/f_J \cong I_{4\pi}/I_c$ corresponding to 6.5% of the supercurrent being carried by a 4π -periodic supercurrent channel.

We next consider the dependence of the critical current I_c in JJ1 on a magnetic field, without a microwave bias, as seen in the differential resistance map in Fig. 1f where the in-plane magnetic field is applied along the junction, B^{0° . The critical field, $B_c^{0^\circ}$, is seen to be ~ 620 mT. Similar measurements performed at different θ values are presented in Supporting Fig. S3. The field dependence data show no topological signatures such as a minimum in I_c ²¹, indicating that the junctions are topologically trivial for all the values of B^θ up to the

critical field B_c^θ .

In Fig. 2, we present Shapiro maps for various magnetic field strengths applied along the junction for $f = 3.5$ GHz and $f = 6.4$ GHz. At $B^{0^\circ} = 0$ mT, the first Shapiro step is seen to be missing for both frequencies since $f < f_{4\pi}$. At $B^{0^\circ} = 200$ mT, the first step almost completely emerges for $f = 6.4$ GHz while still being missing for $f = 3.5$ GHz. At $B^{0^\circ} \sim 300$ mT, the first step starts emerging for $f = 3.5$ GHz, eventually completely appearing at $B^{0^\circ} = 400$ mT. This behavior implies a decrease of $I_{4\pi}$ as a function of in-plane field strength, consistent with the mechanisms described in Fig. 1c. We note that the data presented in Fig. 2 imply that $f_{4\pi}$ does not scale proportionally with f_J . In fact, the ratio $f_{4\pi}/f_J$ generally increases as a function of in-plane field strength. This indicates that the suppression of $I_{4\pi}$ is not simply proportional to the critical current I_c , implying that the response of diabatically driven long junction modes to an in-plane field is distinct from conventional “short junction” modes that make up the rest of the spectrum in 2DEG JJs and the entire spectrum in narrow junctions e.g., nanowire junctions.

Next, we consider the $I_{4\pi}$ dependence on the applied in-plane field direction, θ . A topologically non-trivial $I_{4\pi}$ is expected to be sensitive⁶⁰ to θ ; on the other hand, the angle dependence of a trivial $I_{4\pi}$ resulting from LZT is ambiguous and can depend on several contributing effects from Zeeman, orbital and SOC interactions. Fig. 3a and b show Shapiro maps with $f = 3.5$ GHz at $B^\theta = 200$ mT for $\theta = 30^\circ$ and $\theta = 90^\circ$. Unlike the $\theta = 0^\circ$ case presented in Fig. 2b, the first step appears to partially reemerge for $\theta = 30^\circ$ and completely reemerges for $\theta = 90^\circ$, which indicates an angle anisotropy of $I_{4\pi}$. To determine more

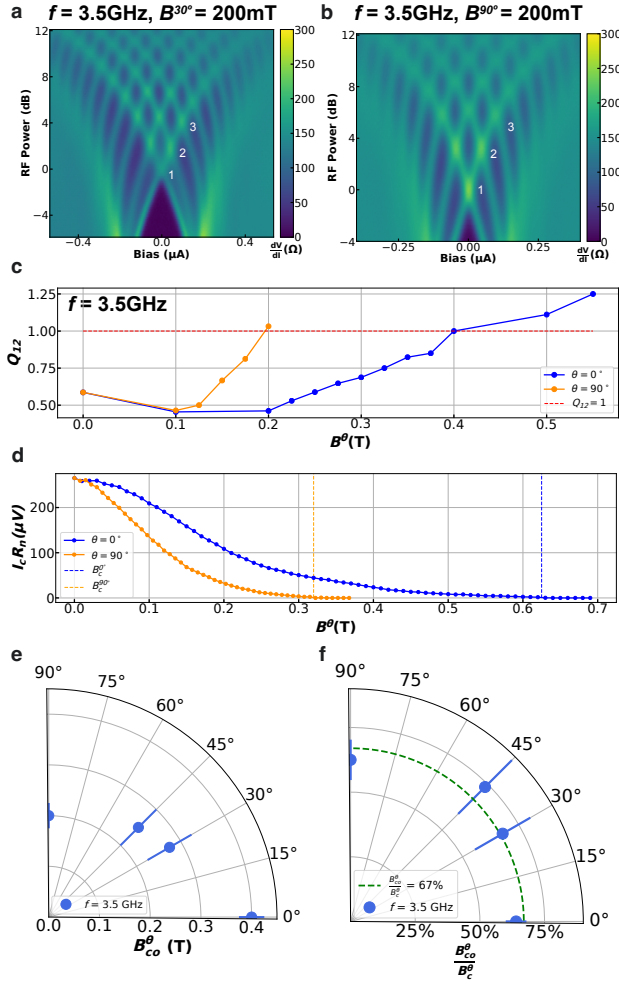


Figure 3: Angle dependence of reemergence of missing Shapiro step. Shapiro maps at $B^0 = 200$ mT for **a** $\theta = 30^\circ$ and **b** $\theta = 90^\circ$. **c** Calculated Q_{12} and **d** $I_c R_n$ as a function of in-plane magnetic field B^θ for in-plane field angles $\theta = 0^\circ$ and 90° . **e, f** The crossover field B_{co}^θ , field value at which missing Shapiro step first fully reemerges, presented in **e** units of Tesla and **f** normalized by the corresponding critical field B_c^θ , as a function of θ .

precisely the threshold value of B^θ above which the first step reappears, we calculate Q_{12} as a function of B^θ where the ratio $Q_{12} = \frac{s_1}{s_2}$ represents the strength of the first step with respect to the second found by binning the voltage distribution and calculating the max step size/bin count of the first (second) step, s_1 (s_2). More details about the extraction of Q_{12} from the data are provided in the Supporting Information. We then identify the crossover field B_{co}^θ for the in-plane angle θ as the value of B^θ for which $Q_{12} \approx 1$. Fig. 3c shows the evolution of Q_{12} with B^θ for $\theta = 0^\circ$ and $\theta = 90^\circ$. In both cases, the first step is suppressed up to the crossover value B_{co}^θ and is fully present for values $B^\theta > B_{co}^\theta$. The scaling of Q_{12} is seen to exhibit clear anisotropy with respect to B^θ : $\theta = 0^\circ$ shows a $B_{co}^0 \approx 400$ mT, whereas $\theta = 90^\circ$ shows a

$B_{co}^\theta \approx 200$ mT.

In Fig. 3e, we present a polar plot of B_{co}^θ (at $f = 3.5$ GHz) as a function θ . A large variation in crossover field is observed; however, we note that the critical field B_c^θ for $\theta = 0^\circ$ and 90° are significantly different ($B_c^0 = 620$ mT and $B_c^{90} = 320$ mT) as seen in Fig. 3d, similar to other Al-InAs junctions⁶¹. When normalized by their respective critical fields to account for the angle-dependence of B_c^θ , the crossover fields become quantitatively similar and in fact match a fit of $B_{co}^\theta/B_c^\theta = 67\%$ as seen in Fig. 3f. This suggests that the anisotropy observed in B_{co}^θ is likely due to the variation in critical field and implies that JJ1 has weak SOC effects.

One of the unique advantages of using a semiconductor-based system is the ability to have electrostatic tunability of the carrier density and SOC interaction using a gate. To study the trivial $I_{4\pi}$ dependence on such properties, we focus on JJ2 fabricated on the same heterostructure presented in Fig. 1 but equipped with a top gate. JJ2 is expected to have a stronger SOC interaction than JJ1 even at zero gate voltage ($V_g = 0$ V) due to the presence of a gate dielectric Al_2O_3 layer (see Supporting Information) that tends to increase the carrier density and consequently SOC interaction. JJ2 is markedly hysteretic at 30 mK due to thermal effects⁶², and so it is studied at 800 mK where it shows no hysteresis. At $B^\theta = 0$ mT and $V_g = 0$ V, JJ2 exhibits a missing first Shapiro step as seen in Supporting Fig. S8 even though at $T = 800$ mK the overall transparency is expected to be reduced. Further, Supporting Fig. S4 shows that JJ2 exhibits a similar B_c^θ anisotropy to that of JJ1. However, we note that for JJ2, B_c^θ also depends on V_g .

In Fig. 4, we present measurements performed on JJ2 at $f = 3.4$ GHz for $V_g = -5$ V and $+10$ V at different B^0 and B^{90} values. For $\theta = 0^\circ$, $V_g = -5$ V shows $B_{co}^0 = 125$ mT while $V_g = +10$ V shows $B_{co}^0 = 225$ mT. The difference between $V_g = -5$ V and $+10$ V is reconciled when considering $B_{co}^0/B_c^0(V_g)$, as seen in Fig. 4i, where both V_g values exhibit a $B_{co}^0/B_c^0(V_g)$ of $\sim 40\%$. For $\theta = 90^\circ$, the data presented in Fig. 4j show a B_{co}^{90}/B_c^{90} ratio of $\sim 57\%$ and $\sim 65\%$ for $V_g = -5$ V and $+10$ V, respectively. While the $\theta = 90^\circ$ case exhibits similar B_{co}^{90}/B_c^{90} values to that reported for JJ1, the $\theta = 0^\circ$ case shows a significant discrepancy for both V_g values. It is evident here that for JJ2, the angle anisotropy is not simply accounted for by considering B_c^θ and that other effects play a role in the suppression of $I_{4\pi}$, consistent with the expectation of JJ2 having stronger SOC effects in comparison to JJ1. In the following, we discuss the origin of such suppression of $I_{4\pi}$ and the observed angle anisotropy by considering the ABS spectrum.

Following the picture presented in Fig. 1c, we first con-

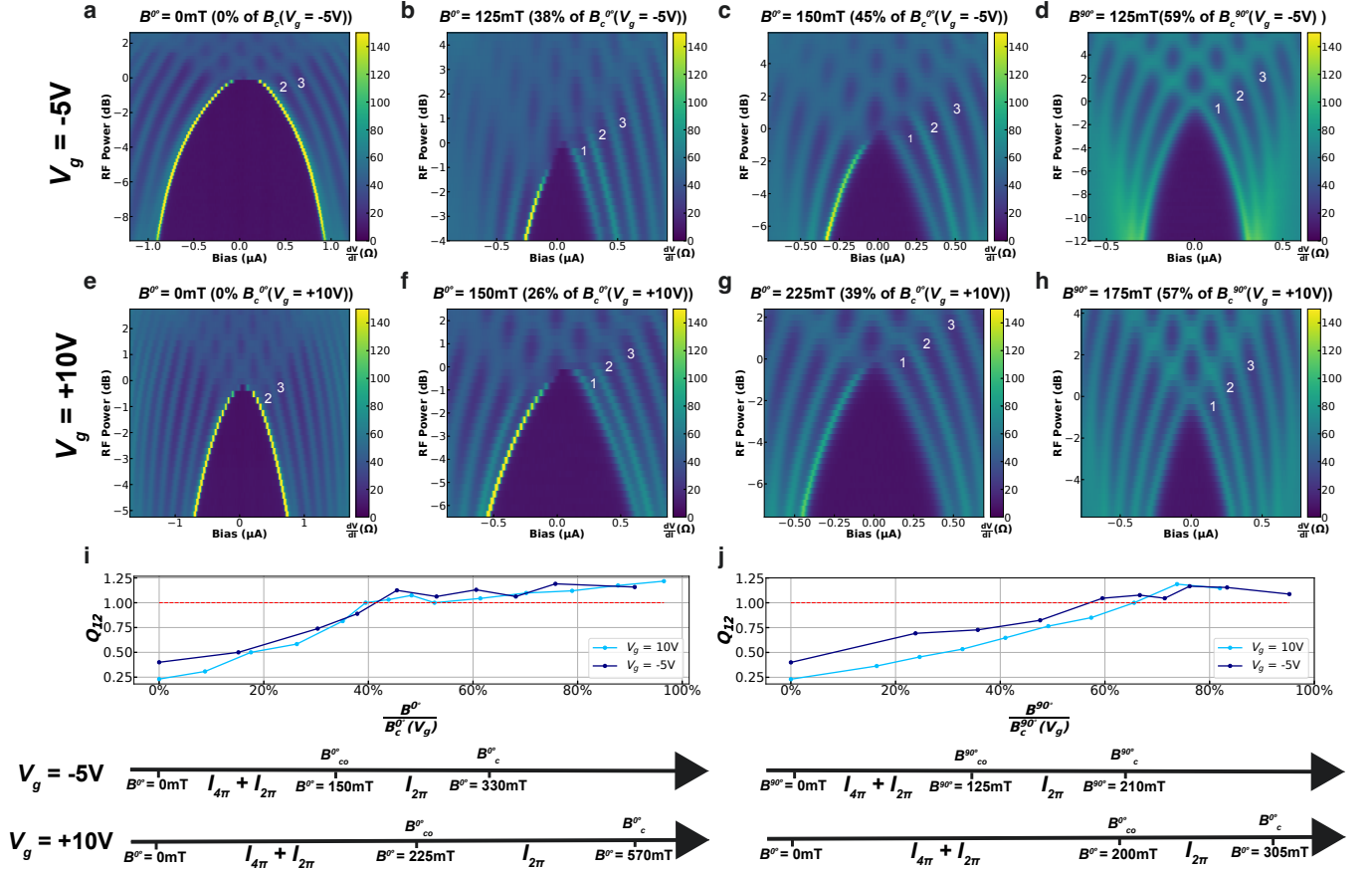


Figure 4: **Evolution of missing Shapiro steps at an applied gate voltage for JJ2.** Shapiro maps at $f = 3.4$ GHz for **a-d** $V_g = -5$ V and **e-h** $V_g = +10$ V for different B^0 and B^{90} values. **i, j** Q_{12} as a function of B^θ (normalized by the respective critical field for each θ and V_g value) for **i** $\theta = 0^\circ$ and **j** $\theta = 90^\circ$.

consider the suppression of $I_{4\pi}$ in terms of transitions between the long junction modes to the continuum, related mainly to the detachment gap δ . Using tight-binding simulations, we calculate the energy spectrum of the ABS spectrum in an InAs-Al junction. Fig. 5a shows a linear decrease in δ as a function of the Zeeman field Δ_Z^θ . The decrease in δ results in a higher probability of undergoing LZTs to the continuum, suppressing the 4π -component of the CPR. In the absence of SOC effects ($\lambda_{SOC} = 0$), corresponding to the black line in Fig. 5a, the suppression of δ as a function of B^θ shows no θ -dependence.

In the presence of strong SOC effects, the Fermi surface of the quantum well has an anisotropic response to an in-plane Zeeman field, creating an anisotropic suppression of δ in the ABS spectrum. For $\lambda_{SOC} = 7.5$ meV $\cdot$ nm, Fig. 5a illustrates that a larger Δ_Z^θ in the $\theta = 0^\circ$ (green line) direction is needed than in the $\theta = 90^\circ$ (orange line) direction to suppress δ by the same amount. However, Fig. 4i and j shows $B_{co}^{0^\circ}/B_c^{0^\circ} < B_{co}^{90^\circ}/B_c^{90^\circ}$. This indicates that the presence of strong SOC (as expected for JJ2) enhances the lack of correlation between the suppression of $I_{4\pi}$ and of δ .

We thus consider the suppression of $I_{4\pi}$ in terms of

mode-to-mode coupling. Due to the large number of ABS modes in our junctions, a result of the large width w , we have a very dense ABS spectrum. Consequently, we have several quasi-avoided crossings between ABSs and between ABSs and the continuum. In the presence of a Zeeman field, the ABS spectrum becomes even more complex, with more quasi-avoided crossings and new protected crossings. A fully microscopic description of the JJ would require the determination of the dynamics of a multi-level Landau-Zener problem. This is a problem that is computationally prohibitive to solve.

However, to gain a qualitative understanding, we can estimate the relevant multi-mode couplings by calculating the wave function overlap between a long junction mode at $\phi = \phi_i$ and all positive energy Andreev mid-gap states at $\phi = \phi_f$ far from the avoided crossing at $\phi = \pi$, as shown schematically in Fig. 5b. This allows to estimate the probability that an occupied ABS, when $\phi \approx \pi$, can either transition to an ABS with a large detachment from the continuum and therefore contribute to $I_{4\pi}$, or transition to an ABS with a small δ and therefore contribute solely to $I_{2\pi}$. We provide a detailed discussion of the calculations in the Supporting Information. In

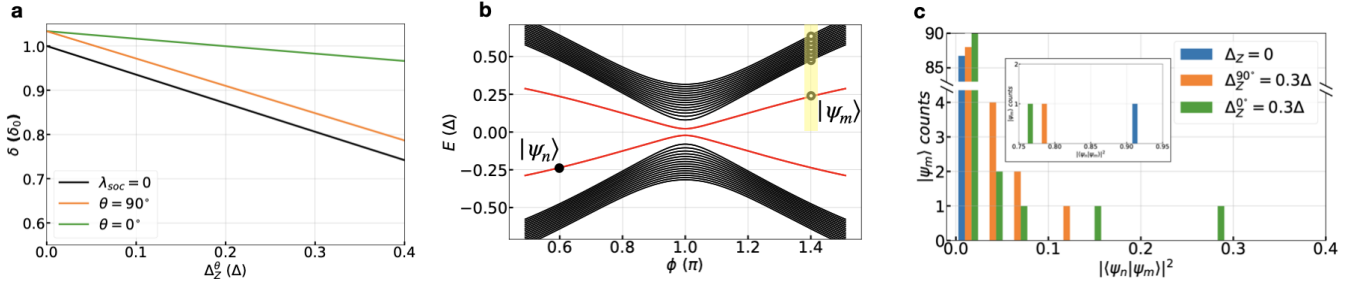


Figure 5: **Theoretical ABS spectrum analysis in the presence of a Zeeman field.** **a** Calculation of the detachment gap δ for a junction with $w = 500$ nm wide and $l = 100$ nm as a function of the Zeeman energy Δ_Z^θ for $\lambda_{SOC} = 0$ and for $\theta = 0^\circ$ and $\theta = 90^\circ$ with $\lambda_{SOC} = 7.5$ meV $\cdot$ nm. **b** Schematic of the Andreev bound state spectrum illustrating which states are used to calculate wavefunction overlaps. **c** Distribution of wavefunction overlaps with $\lambda_{SOC} = 7.5$ meV $\cdot$ nm between $\phi_i = 0.6\pi$ and $\phi_f = 1.41\pi$ for $\Delta_Z = 0$ and 0.3Δ for $\theta = 0^\circ$ and 90° . Inset: outlier overlaps where $|\psi_m\rangle$ is a long junction mode.

Fig. 5c, we present a histogram of the wave function overlaps $|\langle\psi_n(\phi_i)|\psi_m(\phi_f)\rangle|^2$ between a long junction mode $|\psi_n\rangle$ and modes $|\psi_m\rangle$ for $\phi_i = 0.6\pi$ and $\phi_f = 1.41\pi$. At $\Delta_Z = 0$, we observe a distribution localized at zero except for a single outlier shown in the inset. This outlier corresponds to an overlap with another long junction mode. At finite Δ_Z^θ and $\lambda_{SOC} = 7.5$ meV $\cdot$ nm, more states develop a non-zero overlap with the long junction mode evident from the histogram distribution. The histogram distribution also shows that the system is more sensitive to Δ_Z^θ in the $\theta = 0^\circ$ direction than the $\theta = 90^\circ$ direction with the $\theta = 0^\circ$ case exhibiting a broader distribution. These results suggest that the distribution of the overlaps between ABS states across $\phi = \pi$, through their effect on Landau-Zener transitions, play an important role in the anisotropy observed in Fig. 4i and j for JJ2, especially where a strong SOC interaction is present.

By studying the microwave response of an epitaxial Al-InAs JJ, we observe signatures of a 4π -periodic contribution to the CPR attributed to topologically-trivial LZT between long junction modes. With the application of an external magnetic field, the $I_{4\pi}$ is observed to be suppressed differently to $I_{2\pi}$ and eventually disappears at a crossover field. In a device with weak SOC (JJ1), we observe an isotropic suppression of $I_{4\pi}$ with an applied magnetic field when the device's angle anisotropy in B_c^θ is taken into account. In the gate tunable device (JJ2) with a significantly larger SOC, an anisotropic suppression of $I_{4\pi}$ is observed, which cannot be accounted for by the device's B_c^θ angle anisotropy. We attribute the anisotropy to SOC effects which introduce a non-trivial angle θ dependence in the coupling of long junction modes to other Andreev mid-gap states lacking a detachment gap, suggesting multi-level LZTs. Our results indicate that such anisotropy in in-plane magnetic field and dependence on SOC effects need to be considered when differentiating between topologically trivial and non-trivial $I_{4\pi}$ and requires other correlated signatures to make claims about topological superconductivity.

Supporting Information - Additional measure-

ments that support our findings as well as material growth, fabrication, measurement details and information about the theoretical model are provided in the Supporting Information section. This material is available free of charge via the internet at <http://pubs.acs.org>.

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