

# A Review of The Metastable Omega Phase in Beta Titanium Alloys: The Phase Transformation Mechanisms and Its Effect on Mechanical Properties

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## Abstract:

Since its discovery in 1954, the omega ( $\omega$ ) phase in titanium and its alloys has attracted substantial attention from researchers. The  $\beta$ -to- $\omega$  and  $\omega$ -to- $\alpha$  phase transformations are central to  $\beta$ -titanium alloy design, but the transformation mechanisms have been a subject of debate. With new generations of aberration-corrected transmission electron microscopy and atom probe tomography, both the spatial resolution and compositional sensitivity of phase transformation analysis have been rapidly improving. This review provides a detailed assessment of the new understanding gained and related debates in this field enabled by advanced characterization methods. Specifically, new insights into the possibility of a coupled diffusional-displacive component in the  $\beta$ -to- $\omega$  transformation and key nucleation driving forces for the  $\omega$ -assisted  $\alpha$  phase formation are discussed. Additionally, the influence of  $\omega$  phase on the mechanical properties of  $\beta$ -titanium alloys is also reviewed. Finally, a perspective on open questions and future direction for research is discussed.

**Keywords:** metastable omega phase titanium alloys

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## 1. Introduction

Titanium (Ti) is a metal with a high strength-to-weight ratio and good corrosion resistance,<sup>1</sup> making it a useful material for a wide range of applications in diverse industries.  $\beta$  titanium ( $\beta$ -Ti) alloys, a subset of Ti alloys, are particularly useful because they display a wide range of microstructures and mechanical properties. The high strengths attainable by  $\beta$ -Ti alloys allow them to be used in critical applications, such as the use of the Ti-5553 alloy in the landing gear of the Boeing 777.<sup>2–5</sup>  $\beta$ -Ti alloys can also be tuned to have a low elastic modulus for biomedical applications because the low modulus prevents stress-shielding of the bone and decreases the likelihood of implant failures.<sup>6,7</sup> Thus,  $\beta$ -Ti alloys are widely adopted in both high-strength structural applications and lower-strength biomedical applications because their mechanical properties can be tuned to meet the needs of specific applications through judicious control of phase transformations.

$\beta$ -Ti alloys contain alloying elements that stabilize the body-centered-cubic (bcc)  $\beta$  phase at room temperature (RT) by destabilizing the hexagonal close-packed (hcp)  $\alpha$  phase, which is stable at RT in the case of pure Ti. These elements are called  $\beta$ -stabilizers, and, along with lowering the  $\beta$  transus temperature, they cause the Ti-alloy phase diagram to be either the isomorphous or the eutectoid-type.<sup>1,3,8</sup> Vanadium (V), molybdenum (Mo), niobium (Nb), tantalum (Ta), hafnium (Hf), and rhenium (Re) are  $\beta$ -isomorphous stabilizers, and chromium (Cr), manganese (Mn), iron (Fe), cobalt (Co), nickel (Ni), copper (Cu), and tungsten (W) are  $\beta$ -eutectoid stabilizers. Two examples—the isomorphous Ti-Mo and the eutectoid Ti-Cr phase diagrams—are shown in Figure 1. Additionally, aluminum (Al), oxygen (O), nitrogen (N), and carbon (C) are  $\alpha$  stabilizers, and tin (Sn) and zirconium (Zr) are neutral to both  $\alpha$  and  $\beta$  phases.

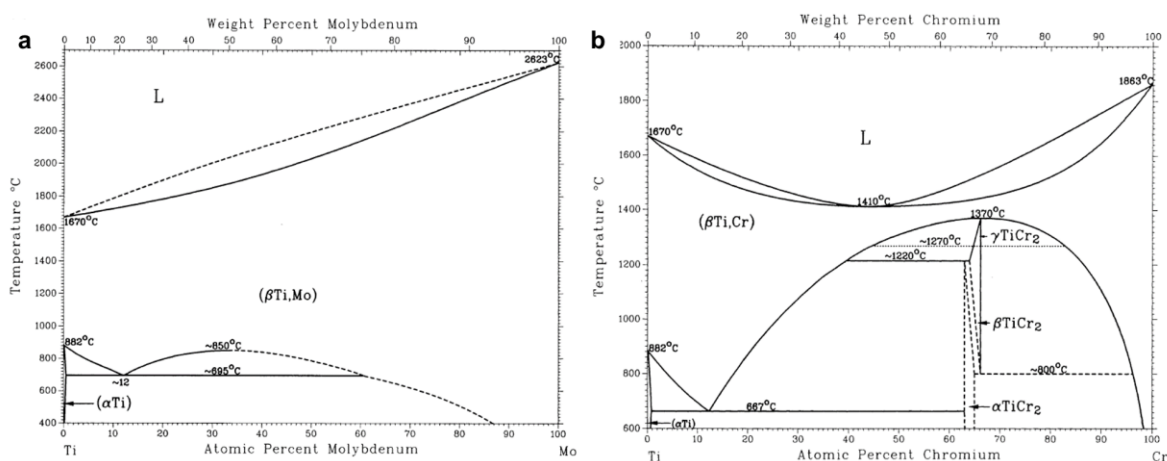


Figure 1. Phase diagrams of (a) an isomorphous-type Ti-Mo alloy and (b) a eutectoid-type Ti-Cr alloy. Adopted from J. D. Cotton, R. D. Briggs, R. R. Boyer, S. Tamirisakandala, P. Russo, N. Shchetnikov and J. C. Fanning, "State of the Art in Beta Titanium Alloys for Airframe Applications," JOM, vol. 67, no. 6, pp. 1281–1303, May 2015.

Processing  $\beta$ -Ti alloys allows a wide variety of microstructures to be obtained through  $\beta$ -to- $\alpha$  or  $\beta$ -to- $\alpha''$  transformations.<sup>1</sup> While these transformations are commonly used to improve  $\beta$ -alloy mechanical properties, they can be strongly influenced by another transformation that has been of significant interest to researchers: the  $\beta$ -to- $\omega$  transformation.

Discovered in 1954 by Frost et al.,<sup>9</sup> the  $\omega$  phase is a hexagonal crystal structure that can exist in pure Ti and zirconium (Zr) at high pressures,<sup>10,11</sup> in shock-deformed polycrystalline Ta and Ta-W alloys,<sup>12</sup> or in  $\beta$ -Ti alloys.<sup>3,13</sup>  $\beta$ -Ti alloys have been used to study the  $\omega$  phase, which can occur in these alloys at a pressure of 1 atm; because these alloys are thermodynamically favorable and dynamically stable at this pressure, they can be studied without the need for specialized high-pressure equipment.<sup>13–15</sup> As microscopy techniques have improved over time, investigations into the  $\omega$  phase have become more detailed. A surge in interest in the  $\omega$  phase has coincided with the use of atom probe tomography (APT) to study the composition of  $\beta$ -Ti alloys on the sub-nanometer scale, and recent work has seen an increase in both understanding and debate in the field.

Despite the comprehensive amount of work in this field, many questions about the  $\omega$  phase still exist, and a review of the current status of research is appropriate. This work aims to provide a summary of the current knowledge of the  $\omega$  phase, including current areas of debate and opportunities for further research. All alloy compositions given in this article are in weight % unless specified otherwise.

## **2. Crystal structure and orientation relationships**

The  $\omega$  phase has a hexagonal structure that can form by the collapse of a pair of (111) planes in a bcc crystal structure into a single plane.<sup>1,13,14,16,17</sup> This collapse occurs when linear defects of vacancies and crowdions in the  $\langle 111 \rangle$  direction align so that the  $\omega$  structure is created from the bcc structure, changing the stacking pattern of the (111) planes from ABCABC in the bcc structure to AB'AB' in the  $\omega$  structure.<sup>18,19</sup> Because there are four  $\langle 111 \rangle$  directions in the bcc unit cell, four variants of the  $\omega$  phase can be formed.<sup>1,20,21</sup> The  $\omega$  phase has three atoms in the primitive unit cell and the lattice points are (0,0,0), (2/3,1/3,1/2), and (1/3,2/3,1/2), belonging to the space group D1/6<sub>h</sub>, P6/mmm.<sup>10,16</sup> The stacking sequence is ABAB, but the  $\omega$  phase is not close-packed.<sup>14</sup> Figure 3 shows the bcc ( $\beta$  phase), hcp ( $\alpha$  phase), and hexagonal  $\omega$  phase structures. The crystallographic orientation between the  $\beta$ ,  $\alpha$ , and  $\omega$  phases has been determined as:

$$\begin{aligned} (\bar{1}010)_{\omega} \parallel (10\bar{1}0)_{\alpha} \parallel (211)_{\beta} \quad & \langle 0001 \rangle_{\omega} \parallel \langle 11\bar{2}0 \rangle_{\alpha} \parallel \langle 111 \rangle_{\beta} \\ \langle 1\bar{2}10 \rangle_{\omega} \parallel \langle 0001 \rangle_{\alpha} \parallel \langle 011 \rangle_{\beta} \quad & (0001)_{\omega} \parallel (\bar{1}2\bar{1}0)_{\alpha} \parallel (111)_{\beta} \\ (\bar{1}\bar{1}20)_{\omega} \parallel (0001)_{\alpha} \parallel (0\bar{1}1)_{\beta} \end{aligned}$$

These relationships have been verified through multiple independent studies.<sup>1,12,20,22–37</sup> A representation of the  $(-1010)_{\omega} \parallel (211)_{\beta}$  relationship is shown in Figure 2.

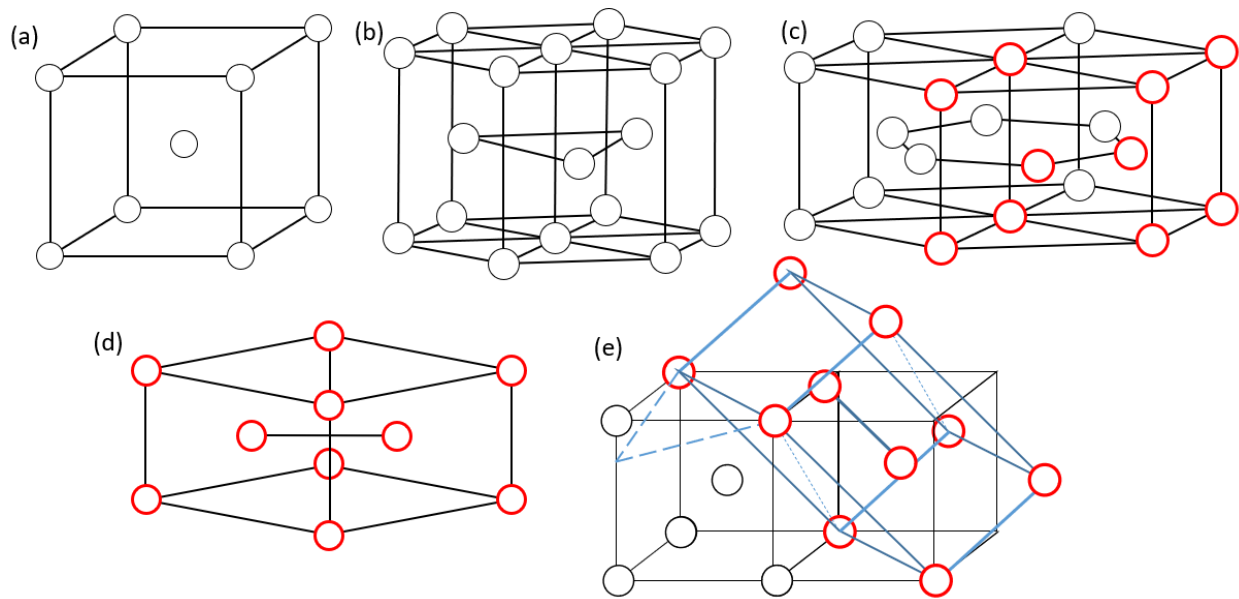


Figure 2. The (a) bcc  $\beta$  phase, (b) hcp  $\alpha$  phase, (c) hexagonal  $\omega$  phase with the primitive unit cell highlighted in red, (d)  $\omega$  phase primitive unit cell with corresponding highlighted atoms, and (e) relationship between the  $\beta$  and  $\omega$  phase unit cells, with the primitive  $\omega$  unit cell in blue.

The  $\omega$  phase in a Ti16V alloy was reported to have a lattice parameter ‘a’ value of 460pm and a ‘c’ value of 282pm.<sup>13,16</sup> Calculating the c/a ratio from these parameters gives a value of 0.613 and a calculated atomic packing factor (APF) of 0.68. The  $\omega$ -phase APF was calculated assuming the radii of each atom in the unit cell was equivalent. In the case of the  $\alpha$  phase, an ‘a’ value of 295pm, a ‘c’ value of 468pm, a c/a ratio of 1.588, and an APF of 0.74 has been reported.<sup>8</sup> Even though the reported  $\omega$  phase lattice parameters are for Ti-16V and Aurelio and Guillermet<sup>38</sup> and Bönisch et al.<sup>39</sup> found that  $\omega$  phase lattice parameters change with alloy content, the c/a ratio is consistently 0.613.<sup>13,38</sup> In Zr alloys, however, Hatt and Roberts found that the  $\omega$  phase c/a ratio was 0.622,<sup>34</sup> and a shock-induced  $\omega$  phase in polycrystalline Ta had a c/a ratio of 0.611.<sup>12</sup>

The  $\omega$  phase APF calculation illustrates that if the plane collapse in the  $\beta$ -to- $\omega$  transformation occurred in pure Ti, the overall volume of the material should not change because the APF for the  $\beta$  phase is also 0.68.<sup>1</sup> The APF also helps clarify that even though the  $\omega$  phase is sometimes misidentified in the literature as hcp,<sup>8,35,40</sup> it is not close-packed.

Selected area electron diffraction (SAED) patterns are commonly used to analyze the presence of the  $\omega$  phase.  $\omega$  phase diffraction spots can be imaged along the  $[110]_{\beta}$  or  $[113]_{\beta}$  zone axes, appearing at the  $1/3\langle 112 \rangle_{\beta}$  or  $2/3\langle 112 \rangle_{\beta}$  positions. Figure 3 shows an example diffraction pattern of the  $\beta$  and  $\omega$  phases. These patterns have allowed the crystallographic relationships between the  $\beta$  and  $\omega$  phases (detailed above) to be verified, such as the  $(1010)_{\omega}$  planes having a d-spacing of 3 times that of the  $(121)_{\beta}$  planes. These patterns have also allowed the morphology of the  $\omega$  phase to be studied using dark-field imaging. Two morphologies of the  $\omega$  phase—ellipsoidal and cuboidal—are observed in Ti alloys (see Figure 3).<sup>18,41,42</sup> The morphology of the  $\omega$  phase depends on alloying elements and processing and is covered in more detail in Section 3.3.

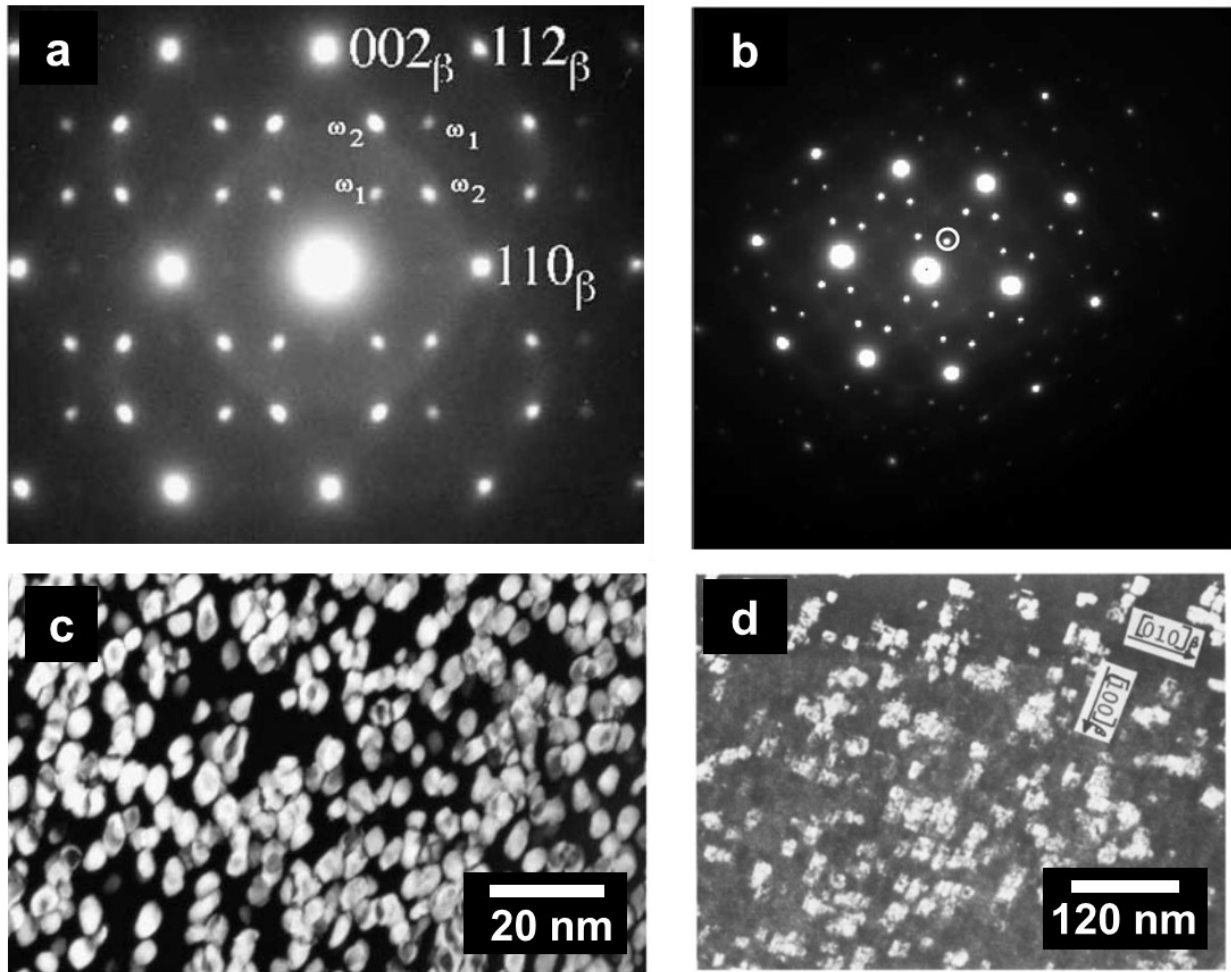


Figure 3. Examples of SAED patterns containing the  $\beta$  and  $\omega$  phases taken from the (a)  $[110]_{\beta}$  and (b)  $[113]_{\beta}$  zone axes, as well as dark-field images obtained using the  $\omega$  reflections in SAED patterns showing the (c) ellipsoidal and (d) cuboidal morphologies of the  $\omega$  phase. ((a, c) adapted from F. Prima et. al., *Scripta Materialia*, vol. 54, pp. 645–648, 2006. (b) from J. I. Qazi et. al., *Materials Science and Engineering C*, vol. 25, pp. 389–397, 2005. (d) from V. Chandrasekaran et. al. *Metallography*, vol. 11, pp. 183–198, 1978).

### 3. Classification of $\omega$ phases

The plane collapse that creates the  $\omega$  phase in  $\beta$ -Ti alloys is initiated by one of three mechanisms: pressure/plastic deformation, quenching from above the  $\beta$ -transus, or thermal energy. Compositional differences in the  $\beta$  phase can facilitate these mechanisms,<sup>20,43,44</sup> which create different types of the  $\omega$  phase. While these types have the same crystal structure (outlined above), they are differentiated in the literature to help the reader distinguish between the formation mechanisms. These distinctions are useful because the formation mechanism can affect the size, morphology, and composition of the  $\omega$  phase. The generally recognized types of  $\omega$  phase are the (1) **deformation-induced  $\omega$  phase**, (2) **athermal  $\omega$  phase ( $\omega_{\text{ath}}$ )**, and (3) **isothermal  $\omega$  phase ( $\omega_{\text{iso}}$ )**. A detailed analysis of the state of the knowledge of each type of  $\omega$  phase is given below.

### 3.1 Deformation-induced $\omega$ phase

The deformation-induced  $\omega$  phase can be studied in pure Ti and Zr through the application of high static pressures because the structure remains stable after the pressure is removed.<sup>10,11,45–48</sup> This is true of  $\beta$ -Ti alloys as well as pure  $\alpha$ -Ti.  $\beta$ -Ti alloys have four stress-induced transformations: the  $\alpha'$  and  $\alpha''$  martensites, the  $\omega$  phase, and mechanical twins.<sup>17</sup> Low concentrations of  $\beta$ -stabilizing elements favor  $\alpha'$ - and  $\alpha''$ -martensitic transformations (decreasing the  $M_s$  temperature). Higher concentrations of  $\beta$ -stabilizing elements suppress the  $\alpha'$  and  $\alpha''$  transformations, allowing the  $\omega$  phase and twins to form instead,<sup>17,49</sup> with the  $\omega$  phase forming in both the  $\beta$  matrix and inside of the twins.<sup>49,50</sup> If a  $\beta$ -stabilizer concentration is reached that is high enough, the  $\omega$ -phase transformation will also be suppressed and only twins will form.<sup>17,51</sup> Clear cutoffs for low, medium, and high  $\beta$ -stabilizer concentrations are not well defined in the deformation-induced  $\omega$ -phase literature. Comparing  $\beta$  stability between studies of the deformation-induced  $\omega$  phase can be difficult because different alloys are often used, such as Xing and Sun's Ti-23Nb-0.7Ta-2Zr-0.2O (at%) alloy<sup>52</sup> and Ahmed et al.'s Ti-10V-3Fe-3Al alloy.<sup>50</sup>

One way to compare alloys between studies is through the Mo Equivalency (Mo-Eq) equation.<sup>8,53</sup> Mo-Eq is a way to compare different  $\beta$ -Ti alloys with different alloying elements and concentrations by finding the equivalent concentration of molybdenum that, if alloyed instead, would provide an equivalent amount of  $\beta$  stability. This is done by dividing 10 wt.% Mo by the minimum wt.% of each stabilizing element needed to create a completely  $\beta$ -stable alloy (where 100% of the  $\beta$  phase is retained upon quenching from above the  $\beta$  transus) and multiplying the division with the wt.% of each stabilizing element, then summing the results.<sup>53</sup> This gives the Mo-Eq equation:

$$Mo - Eq = Mo + 0.67 * V + 0.44 * W + 0.28 * Nb + 0.22 * Ta + 2.9 * Fe + 1.6 * Cr - 1.0 * Al$$
where the elemental symbol is the amount of that element in weight percent.<sup>53</sup> While not a perfect way to compare  $\beta$ -Ti alloys because some of the elements are isomorphous stabilizers and some are eutectoid stabilizers, Mo-Eq provides a reference point for comparison. Even using Mo-Eq, it is still difficult to compare  $\beta$  stability between alloys.

In general, a low  $\beta$ -stabilizer concentration seems to refer to alloys with a Mo-Eq of less than 10.<sup>54</sup> The  $\beta$  phase stability range where the  $\omega$  phase forms is a Mo-Eq of approximately 10–12.5.<sup>50–52,54,55</sup> A range of ~12 to 15 supports deformation-induced twin formation, with or without the  $\omega$  phase.<sup>55,56</sup> A Mo-Eq higher than ~15 supports slip as the dominant deformation mechanism.<sup>54</sup> These ranges are only general guidelines for  $\beta$ -Ti alloys and exceptions to these ranges can be found, such as twins and  $\omega$  phase forming along with  $\alpha''$  martensite in a cold-rolled Ti-24Nb-4Zr-8Sn alloy (Mo-Eq of ~6.7)<sup>49</sup> and  $\omega$  phase forming in a Ti-3Nb (Mo-Eq of 0.86) alloy after high-pressure torsion (HPT).<sup>57</sup>

However,  $\beta$ -stabilizer concentration is not the only variable that affects deformation-induced  $\omega$  phase formation. Grain size,<sup>26</sup> processing route,<sup>50</sup> and the existence of deformation twins<sup>48</sup> have been shown to affect the volume fraction of the  $\omega$  phase. In static-pressure experiments, impurities such as O or N increased the pressure required to initiate deformation-induced  $\omega$  phase formation.<sup>45</sup>

Severe shear plastic deformation during HPT has been shown to induce formation of the  $\omega$  phase in Ti and its alloys. The torsion that the samples experience lowers the incubation time for  $\omega$  phase formation in Ti by approximately 27 hours compared to static pressure alone<sup>58</sup> and reduces the amount of pressure needed to generate the  $\omega$  phase.<sup>59</sup>  $\beta$ -stabilizer concentration affects the amount of  $\omega$  phase in the material after HPT processing.<sup>60,61</sup> Annealing temperature before HPT processing can also affect the volume fraction of the  $\omega$  phase formed.<sup>57</sup> However, Tane et al.<sup>62</sup> and Panigrahi et al.<sup>63</sup> demonstrated that HPT processing can lead to the formation of a deformation-induced  $\omega$  phase in pure Ti and Ti-16.1Nb alloys. The starting microstructure can affect the number of turns needed to start the  $\omega$  phase

transformation<sup>60</sup> as well as the amount of  $\omega$  phase in the material formed after HPT.<sup>64</sup> HPT can also induce an  $\alpha$ -to- $\omega$  phase transformation<sup>58,64</sup> through atomic shuffles in the  $(0001)_\alpha$  planes.<sup>60</sup> However, the  $\alpha$ -to- $\omega$  transformation is not guaranteed to form in all Ti alloys during HPT.<sup>64</sup> Wang et al.<sup>26</sup> and Edalati and Horita<sup>65</sup> found that grain size is an important variable that influences the  $\omega$ -phase formation in both  $\beta$ -Ti and pure Ti, with smaller grains lowering the amount of  $\omega$  phase in the material, although temperature was also a variable in the Edalati and Horita study.<sup>65</sup>

Multiple morphologies are reported for the deformation-induced  $\omega$  phase. In compression testing, long, thin lamellar  $\omega$  phases were generated.<sup>50</sup> During hot deformation in the  $\beta + \alpha$  region, a thin lamellar-like  $\omega$  phase was also reported to form.<sup>35</sup> Impact testing formed a plate-like  $\omega$  phase.<sup>66,67</sup> Cold-rolling also produced plate-like  $\omega$  phase.<sup>49</sup> In polycrystalline Ta and Ta-W alloys, the shock-generated  $\omega$  phase had a zigzag structure with multiple interconnected  $\omega$ -phase variants.<sup>12</sup> Also, alloying Ta with the  $\beta$ -stabilizer W increased both the volume fraction and the size of the  $\omega$  phase.<sup>12</sup>

### 3.2 Athermal $\omega$ phase

The athermal  $\omega$  phase forms as well-dispersed particles during the quenching of  $\beta$ -Ti alloys from above the  $\beta$  transus temperature when the  $(111)_\beta$  plane collapse occurs as a result of the instability of the  $\beta$  phase with respect to a specific mode of phonon.<sup>21,68,69</sup>  $\omega_{\text{ath}}$  particles are less than 10 nm in size—usually between  $\sim 2$  and  $5$  nm<sup>1,20,28,49,70–72</sup>—and are generally reported as spheroidal<sup>20,35,50,71,73</sup> or ellipsoidal.<sup>28,35,70,72</sup> Because quenching does not allow diffusion to occur,  $\omega_{\text{ath}}$  theoretically has the same composition as the surrounding  $\beta$  matrix,<sup>74</sup> with higher volume fractions forming in  $\beta$  alloys with  $\beta$  stabilizer concentrations close to the lower limit of  $\beta$ -phase retention.<sup>13,31,75</sup>

As a result of the small size of  $\omega_{\text{ath}}$ , local strain fields around  $\omega_{\text{ath}}$ , and/or short-range correlated displacements in the  $\beta$  lattice,  $\omega_{\text{ath}}$  is observed to lead to diffuse streaking in SAED patterns rather than discrete diffraction spots.<sup>34,68,69,76,77</sup> Diffuse streaking in selected area diffraction patterns are also observed when there is an incomplete collapse of the  $(111)_\beta$  planes during the  $\beta$ -to- $\omega$  transformation. Instead of being located at  $(0,0,0)$ ,  $(1/3,2/3,1/2)$ , and  $(2/3,1/3,1/2)$ , the atoms have positions at  $(0,0,0)$ ,  $(1/3,2/3,1/2+\delta)$ , and  $(2/3,1/3,1/2-\delta)$ , where  $0 < \delta < 1/6$ . The value of  $\delta$  depends on the degree of plane collapse, with smaller  $\delta$  values corresponding to lower  $\beta$ -stabilizer concentrations.<sup>14,78</sup> Although the existence of the partially collapsed  $\omega$  phase has been debated,<sup>68</sup> high-resolution STEM (HRSTEM) images have definitively proved its existence.<sup>20,25,43</sup> Sometimes referred to as “rumpled”  $\omega$ ,<sup>12</sup> “rumpled-plane”  $\omega$ ,<sup>34</sup> trigonal  $\omega$ ,<sup>14</sup> “modulated”  $\omega$ ,<sup>79</sup> or “embryonic”  $\omega$ ,<sup>20,25,27,80</sup> these “incommensurate” structures have also been reported with the fully collapsed “commensurate”  $\omega_{\text{ath}}$  after quenching.<sup>20,69,76,78</sup> However, it is important to note that these  $\omega$  structures can also form during heat treatments, causing diffuse streaking in selected area diffraction patterns of quenched and isothermally aged samples.<sup>27,81</sup> Figure 4 shows an example of the incommensurate  $\omega$  phase as seen in high resolution transmission electron microscopy (HRTEM), along with graphical representations of the incommensurate and commensurate  $\omega$  structures.

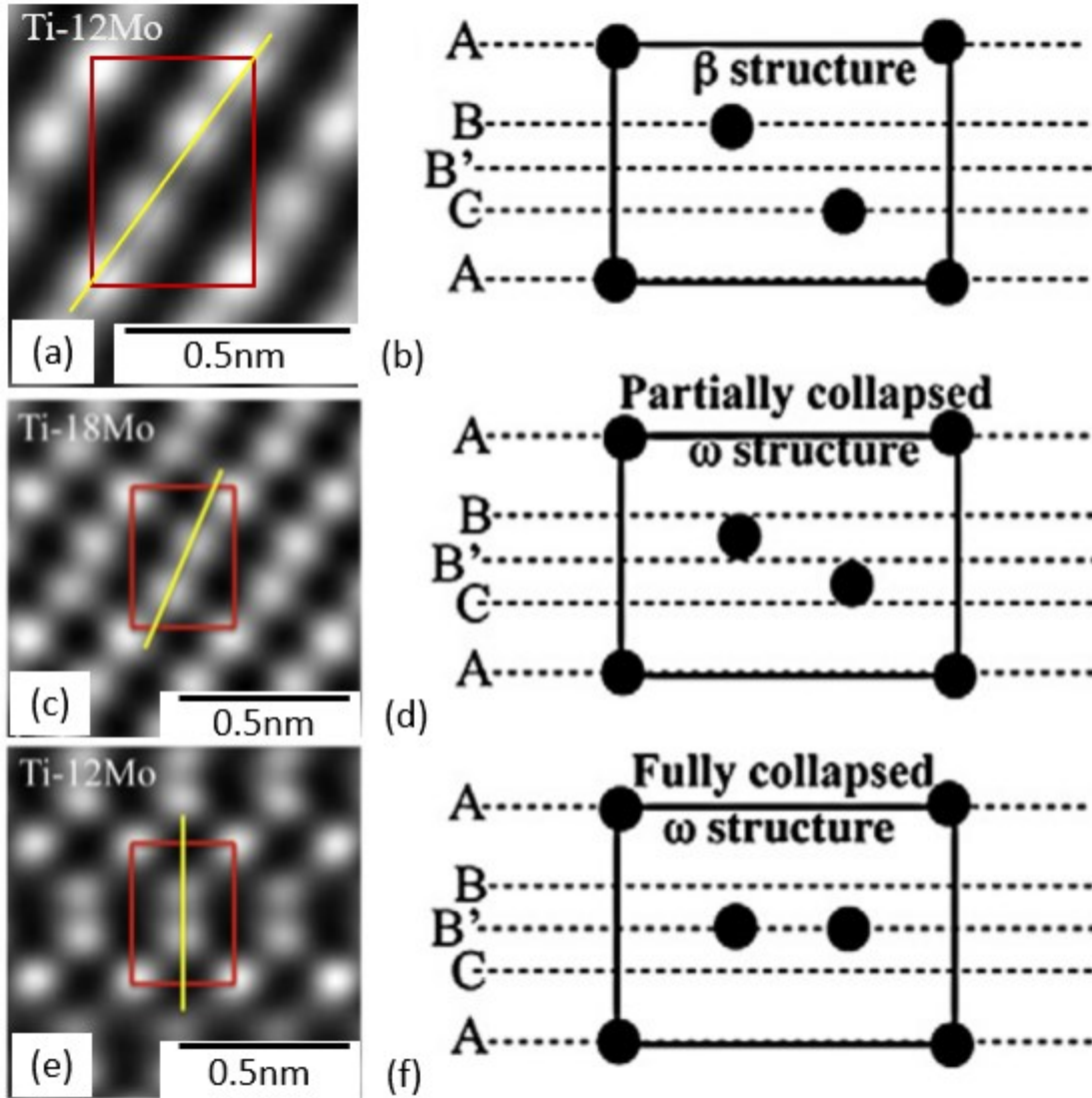


Figure 4. (a) An HRTEM image and (b) a schematic of the  $\beta$  structure, (c) an HRTEM image and (d) a schematic of the partially collapsed “incommensurate”  $\omega$  structure, and (e) an HRTEM image and (f) a schematic of the fully collapsed “commensurate”  $\omega$  structure in Ti-Mo alloys. (All HRTEM images adopted from Zheng, Y., Banerjee, D., and Fraser, H.L. *Scripta Materialia*, V. 116, 2016, pp. 133. All schematics adopted from Devaraj et al. *Acta Materialia*, V. 60, No. 2, 2012, pp. 604).

The different types of  $\omega$  phase (deformation-induced, athermal, and isothermal) are differentiated via their formation pathway (deformation, quenching, heat treatment). Because the incommensurate  $\omega$  phase can form through either quenching or heat treatment, labeling it as solely an athermal or isothermal type of  $\omega$  phase is not sufficient and the literature to date has been inconsistent with its nomenclature. Therefore, in the remainder of this article, the authors use the terminology “athermal  $\omega$  phase” or “ $\omega_{\text{ath}}$ ” to mean a commensurate  $\omega$  phase that forms through quenching and “isothermal  $\omega$  phase” or “ $\omega_{\text{iso}}$ ” to mean a commensurate  $\omega$  phase that forms during isothermal aging. To refer to incommensurate  $\omega$  phase formed through the athermal pathway, “incommensurate athermal  $\omega$  phase” or “incommensurate  $\omega_{\text{ath}}$ ” is used. Similarly, “incommensurate isothermal  $\omega$  phase” or



“incommensurate  $\omega_{iso}$ ” is used to refer to incommensurate  $\omega$  phase formed through the isothermal pathway.

### **3.3 Isothermal $\omega$ phase**

The isothermal  $\omega$  phase precipitates homogeneously in metastable  $\beta$ -Ti alloys during low-temperature heat treatments that favor the  $\beta$ -to- $\omega$  transformation over the  $\beta$ -to- $\alpha$  transformation.<sup>8,13,69,82–85</sup> Depending on the alloy, the lowest temperature that initiates the  $\beta$ -to- $\omega_{iso}$  transformation is between 150 °C<sup>24</sup> and 280 °C.<sup>86</sup> Most researchers use a temperature range of 300 °C–400 °C to generate the isothermal  $\omega$  phase,<sup>23–25,27,30,32,33,41,80,86–88</sup> and 500°C is the highest temperature reported to form  $\omega_{iso}$ .<sup>39,89</sup> Using in situ synchrotron diffraction experiments, Bönisch et al.<sup>39</sup> found that the  $\omega$  phase lattice parameters remained relatively constant in both Ti-28.5Nb and Ti-36Nb as they were heated from 300 °C to 500 °C.

$\omega_{iso}$  has been found to nucleate at defects in the  $\beta$  matrix left by  $\omega_{ath}$  after  $\omega_{ath}$  reverts to  $\beta$  phase during heat treatment.<sup>24,90</sup>  $\omega_{iso}$  has also been found to grow from  $\omega_{ath}$ .<sup>21,22,74</sup> Locations in the  $\beta$  matrix with lower concentrations of  $\beta$ -stabilizers due to nanoscale chemical fluctuations are also favorable sites for  $\omega_{iso}$  nucleation.<sup>20,25,43</sup> Because of the relationship between  $\omega_{ath}$  and  $\omega_{iso}$ , Prima et al. have proposed that the volume fraction of  $\omega_{iso}$  formed in the  $\beta$  phase can be indirectly controlled using  $\omega_{ath}$ .<sup>90</sup>  $\omega_{iso}$  nucleation is also influenced by aging temperature, as Prima et al. had a higher volume fraction of  $\omega_{iso}$  form after aging at 310 °C than at 250 °C.<sup>24</sup>

During the growth of  $\omega_{iso}$  particles, alloying elements diffuse from the  $\omega$  phase into the surrounding  $\beta$  matrix.<sup>1,13,14,86,91</sup> APT is a powerful technique for investigating sub-nanometer scale, spatially resolved composition of precipitate phases,<sup>92</sup> and APT studies of multiple binary  $\beta$  Ti alloys clearly show that the  $\omega$  phase is solute-lean (shown in Figure 5).<sup>20,32,43,70,93</sup> For example, in a binary Ti-9.9Mo (at%) alloy, the ellipsoidal  $\omega$  phase was found to have a Mo concentration near 2 atomic %.<sup>20,93</sup> More complex alloys have also been studied using APT, showing that  $\omega_{iso}$  is depleted of both  $\beta$ - and  $\alpha$ -stabilizing elements.<sup>22,27,30,71,80,82,83,87,94,95</sup>

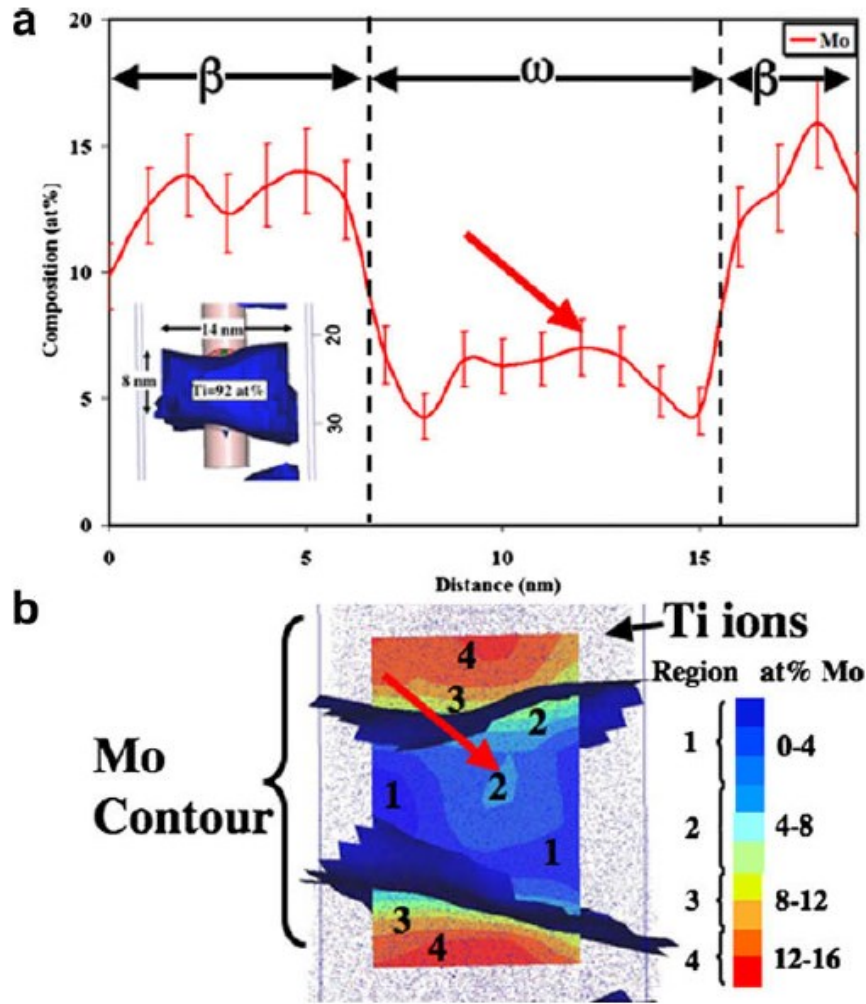


Figure 5. (a) A Mo composition plot taken from a 1-D cylinder showing the Mo composition difference between the  $\beta$  matrix and an  $\omega$ -phase particle and (b) the associated Mo contour plot. (Figure adopted from A. Devaraj, et al., "Three-dimensional morphology and composition of  $\omega$  precipitates in a binary titanium-molybdenum alloy," *Scripta Materialia*, vol. 61, pp. 701–704, June 2009).

The size of  $\omega_{iso}$  particles depends on the time spent at the heat treatment temperature because the particles coarsen with time. A range of 2–100 nm has been reported for  $\omega_{iso}$  particles generated using various temperatures and times.<sup>20,23,25,28,42,70,71,74,82,83,88,93,96</sup> Moffat and Larbalestier<sup>96</sup> have also shown that higher aging temperatures produce larger precipitates for the same amount of time. Because of the small size of  $\omega_{iso}$ , transmission electron microscopy (TEM) is the most effective microscopy technique for detecting the  $\omega$  phase in materials, although some studies have detected the presence of the  $\omega$  phase using X-ray diffraction.<sup>31,89,94,97</sup>

Heat treatment temperature and time also affects the volume fraction of  $\omega_{iso}$  formed.<sup>98,99</sup> For example, holding a Ti-12wt% alloy at 450°C for 50 hours forms a higher volume fraction of  $\omega_{iso}$  than holding the same alloy at 400 °C for 250 hours.<sup>98</sup> The combination of temperature and time that produces the highest volume fraction of  $\omega_{iso}$  also changes with alloy composition, shown by Hickman when a Ti-8Mo

(at%) alloy reaches a maximum  $\omega_{iso}$  volume fraction of 0.78 after 175 hours at 350 °C while a Ti-10Mo (at%) alloy formed a maximum  $\omega_{iso}$  volume fraction of 0.56 after 8 hours at 400 °C.<sup>99</sup>

The morphology of the  $\omega$  phase is determined by the elastic strain energy in the crystal that is due to the difference between the radii of alloying elements and Ti<sup>21,93</sup> or by elastic interactions between the  $\omega_{iso}$  particles.<sup>100</sup>  $\omega_{iso}$  particles in high-misfit systems, such as Ti-Cr, have a cuboidal morphology with the flat surfaces parallel to the  $(110)_\beta$  planes.<sup>1,93,96</sup>  $\omega_{iso}$  particles in low-misfit systems, such as Ti-Mo, have an ellipsoidal morphology in which the major axis is parallel to the  $\langle 111 \rangle_\beta$  directions<sup>1,36,93,96</sup> as a result of the elastic anisotropy of the  $\beta$  phase, and an average ratio of the major to the minor axis of 3.56.<sup>93</sup> With enough time, the ellipsoidal  $\omega_{iso}$  particles in low-misfit systems become cuboidal because the elastic interactions between the particles affect the morphology with increasing growth of  $\omega_{iso}$ .<sup>100</sup> Blackburn and Williams<sup>36</sup> show this transformation from ellipsoidal morphology to a cuboidal morphology of  $\omega_{iso}$  in a high-misfit Ti-V alloy. Figure 3 above shows dark-field TEM images of  $\omega_{iso}$  particles with the two morphologies. Furthermore, for a low-misfit Ti-6554 alloy, the morphology of  $\omega_{iso}$ —which grew from an ellipsoidal incommensurate isothermal  $\omega$  phase—was observed to evolve from more lath-like to a mixed plate- and rod-like morphology as the annealing time increases at 300 °C.<sup>27</sup>

Because of the effects of  $\omega_{iso}$  on mechanical properties (covered in detail below), methods have been developed to avoid precipitation of  $\omega_{iso}$ . Precipitation of the isothermal  $\omega$  phase can be avoided through the addition of alloying elements that increase the stability of the  $\beta$  phase<sup>15,89,101</sup> or the  $\alpha$  phase (such as oxygen or aluminum).<sup>29,101–103</sup> Heat treatment temperatures above 500 °C can be used to precipitate the  $\alpha$  phase without precipitating the  $\omega$  phase,<sup>13,69,84</sup>. However, the  $\omega$  phase might still be precipitated if the heating rate to reach temperatures above 500 °C is slow enough to allow  $\omega$  precipitation or if the  $\alpha''$  phase was present before heating and a fast heating rate is used.<sup>40,104,105</sup> Increasing the amount of time the alloy stays at lower heat treatment temperatures (100 °C–500 °C) can cause the  $\alpha$  phase to grow into the  $\omega$  phase, leaving only the  $\alpha$  crystal structure and no  $\omega$  crystal structure;<sup>23,25,27,42</sup> in some alloys, however,  $\omega_{iso}$  can coexist with  $\alpha$  after heat treatments of 192 hours.<sup>25,30</sup>  $\omega$  phase formation can also be suppressed by large quantities of grain boundaries or dislocations that prevent  $(111)_\beta$  plane collapse.<sup>84</sup>

#### **4. $\beta$ to $\omega$ phase transformation mechanism**

$\omega_{ath}$  and deformation-induced  $\omega$  are formed through a displacive mechanism via atomic shuffling. Driven by quenching or stress, the  $(111)_\beta$  plane collapse occurs without diffusion. The displacive collapse is reversible, as De Fontaine et al. showed through in situ TEM  $\omega_{ath}$  formation and reversion in the temperature range of –171 °C to 5 °C, which prevented diffusion.<sup>68</sup> The combination of temperature increase from electron irradiation and oscillations from inelastic scattering in a TEM can also reverse the displacive collapse.<sup>73</sup>

In comparison, the isothermal  $\omega$  phase is known to form via a diffusion mode. That is, elemental partitioning occurs along with the  $(111)_\beta$  plane collapse, creating an  $\omega$  phase with a lower energy than the  $\omega$  phase generated through displacive collapse.<sup>15</sup> While the isothermal  $\omega$  phase has been shown to grow rapidly without diffusion for short times (60 seconds) at the start of nucleation,<sup>21,71,106</sup> the rejection of the  $\beta$  stabilizers is important for isothermal  $\omega$  phase growth because it stabilizes the  $\omega$  phase by lowering the energy of the structure and making the  $\omega$  transformation irreversible.<sup>68</sup> Quenching before the heat treatment and heating during isothermal aging are necessary to make the  $\beta$ -to- $\omega$  transition more energetically favorable than the  $\beta$ -to- $\alpha$  transition, with faster quenching rates increasing the number of  $\omega_{iso}$  precipitates formed.<sup>96,103,107</sup> This is attributed to the relationship between  $\omega_{ath}$  and  $\omega_{iso}$  because  $\omega_{ath}$

locations in the  $\beta$  matrix have been shown to act as nucleation sites for  $\omega_{iso}$ .<sup>14,90</sup> However, the exact mechanism behind  $\omega_{ath}$ -assisted precipitation of  $\omega_{iso}$  is not known. Additionally, a slow heating rate is found to allow nucleation and growth of  $\omega_{iso}$  particles from the  $\beta$  phase during heating,<sup>40,104,105</sup> although Barriobero-Vila et al.<sup>105</sup> have shown that if  $\alpha''$ -martensite is present, a faster heating rate increases the volume fraction of  $\omega_{iso}$  compared to slower heating rates by activating the  $\alpha''$ -to- $\omega_{iso}$  transformation along with the  $\beta$ -to- $\omega_{iso}$  transformation.

Current debate about the  $\beta$ -to- $\omega$  transformation centers around the athermal and incommensurate  $\omega_{iso}$  phases (which are early stages of  $\omega_{iso}$ ). Nag et al.<sup>43</sup> and Devaraj et al.<sup>20</sup> found evidence that incommensurate  $\omega_{iso}$  particles formed in pockets of slight Mo-depletion in a binary Ti-Mo alloy during quenching, and Li and Min<sup>78</sup> found incommensurate  $\omega_{ath}$  in Mo-lean regions in a Ti-15Mo alloy. Li and Min also suggest that the shear modulus is softened in the Mo-lean regions of Ti-Mo alloys, which allows the (111) $_{\beta}$  planes to collapse more easily.<sup>78</sup> These results seem to agree with predictions by Gullberg et al.<sup>108</sup> and earlier findings by McCabe and Sass<sup>109</sup> that  $\omega_{ath}$  is Ti rich, as well as with observations by Fan and Miodownik<sup>44</sup> that the  $\omega$  phase forms in solute-lean  $\beta$  phase areas more readily. They also support findings by Nag et al.<sup>110</sup> that suggested  $\omega_{ath}$ /incommensurate  $\omega$  phase particles were associated with fluctuations in stabilizer concentrations. More recent work by Li et al.<sup>25,27</sup> supports this as well, finding that the incommensurate  $\omega_{iso}$  particles in Ti-6Cr-5Mo-5V-4Al were Mo lean. However, they clarified that the Mo-lean pockets were created by second-order spinodal decomposition of the  $\beta$  phase and that the incommensurate  $\omega_{iso}$  phase forms via a displacive mechanism within these Mo-lean regions.<sup>25</sup> Tane et al.<sup>111</sup> came to a similar conclusion with their work in a Ti-V alloy system, showing that the  $\omega$  phase can form without diffusion in quenched-in V-lean pockets while aging at room temperature. In contrast, Ahmed et al.<sup>40</sup> and Ng et al.<sup>71</sup> have found no change between the matrix composition and the composition of the athermal  $\omega$  phase. Coakley et al.<sup>87</sup> also did not observe the formation of  $\omega_{ath}$  in Mo-lean pockets in Ti-5Al-5Mo-5V-3Cr. Sun et al.<sup>106</sup> and Mantri et al.<sup>112</sup> both found no compositional variation in a Ti-12Mo alloy associated with the formation of  $\omega_{ath}$ , but both recorded sharp diffraction spots—this suggests a lack of the incommensurate  $\omega$  phase, which Nag et al.,<sup>43</sup> Devaraj et al.,<sup>20</sup> and Li et al.<sup>27</sup> all observed forming in the Mo-lean pockets.

## **5. $\omega$ -assisted precipitation of the $\alpha$ phase**

Another important phase transformation is the  $\omega$ -to- $\alpha$  phase transformation.  $\omega_{iso}$  is known to affect the precipitation of the  $\alpha$  and  $\alpha''$  phases.<sup>21,37,84</sup> The finely distributed  $\alpha$  platelets formed by  $\omega$ -assisted precipitation were noted by Ohmori et al.<sup>37</sup> to have a significantly deviated habit plane compared to the coarse  $\alpha$  plates formed without  $\omega$ -assistance. It should be clarified that  $\omega_{ath}$  and the incommensurate  $\omega_{iso}$  phases do not assist in  $\alpha$  formation<sup>27,32,113,114</sup> but they can coexist with the  $\alpha$  phase.<sup>27,69</sup> The commensurate  $\omega_{iso}$  phase promotes the formation of refined  $\alpha$  precipitates, thereby providing attractive mechanical properties for high-strength applications.<sup>22</sup> A fine-grained  $\alpha$ -phase also formed in HPT samples composed of the deformation-induced  $\omega$ -phase when heated above 320 °C, although the mechanical properties of this  $\alpha$ -phase have not been investigated.<sup>63</sup>

Understanding  $\omega$ -assisted precipitation of the  $\alpha$  phase is key for tuning processing conditions and/or alloy content to achieve desired mechanical properties. In the last 20 years, this topic has been an active area of research, and APT has expanded the field of knowledge considerably.

In high-misfit systems where the  $\omega$  phase has a cuboidal morphology, ledges between the  $\beta$  and  $\omega$  phases drive the  $\omega$ -to- $\alpha$  transformation.<sup>32</sup> In low-misfit systems, the nucleation sites for the  $\alpha$  phase have been the subject of debate for years. Historically, three locations of  $\alpha$  precipitation have been considered:

1. The  $\alpha$  phase precipitates at some distance from the  $\omega/\beta$  boundary.<sup>85</sup>
2. The  $\alpha$  phase precipitates in the core of the  $\omega$  phase.<sup>42</sup>
3. The  $\alpha$  phase precipitates at the  $\omega/\beta$  boundary.<sup>40</sup>

Azimzadeh and Rack<sup>85</sup> proposed that Al diffuses away from the  $\omega$  phase into the  $\beta$  matrix in a low-cost  $\beta$ -Ti alloy, providing an  $\alpha$  phase nucleation point at some distance from the  $\omega$  phase, where it then grows toward—and looks like it is precipitating “in association” with—the  $\omega$  phase. Against this view, Prima et al.<sup>42</sup> have presented concurrent  $\omega/\alpha$  phase images that suggest the  $\alpha$  phase forms in the core of the  $\omega$  precipitate. The  $\alpha$  phase might prefer nucleating in the core of the  $\omega$  phase as a result of the low amounts of  $\beta$  stabilizers present in the  $\omega$  phase compared to the heavily stabilized surrounding  $\beta$  phase, and the transformation would most likely be a purely displacive reaction.<sup>24,42</sup> TEM images of  $\alpha$  particles with  $\omega$  super cells at both boundaries—which could provide evidence that the  $\alpha$  phase nucleates in the  $\omega$  phase core—have been presented by Ahmed et al.<sup>40</sup> Ahmed et al.’s<sup>40</sup> TEM images could also provide evidence for the  $\alpha$  phase nucleating at the  $\omega/\beta$  boundary because the  $\alpha$  particles were coherent with both the  $\beta$  and  $\omega$  particles. HRTEM images by Furuhashi et al.<sup>115</sup> also suggest that the  $\alpha$  phase nucleates at the  $\omega/\beta$  boundary, although the  $\alpha$  phase growing toward the boundary cannot be ruled out. Li et al.<sup>25</sup> presented images similar to Furuhashi et al.<sup>115</sup>, but they determined that the  $\alpha$  phase did not nucleate in the  $\beta$  phase and grow toward the boundary through composition analysis. If the  $\alpha$  phase had nucleated in the  $\beta$  phase, the  $\beta$  stabilizer concentrations in the  $\alpha$  phase would be expected to decrease with time, which does not match experimental observations.<sup>25</sup> Chemical driving force calculations by Shi et al.<sup>116</sup> indicate that the cuboidal  $\omega/\beta$  interface is the most favorable nucleation site for  $\alpha$  phase, and subsequent HRSTEM work showed the presence of the  $\alpha$  phase at the cuboidal  $\omega/\beta$  interface as predicted. Li et al.<sup>27</sup> and Zheng et al.<sup>32</sup> have also presented images of the  $\alpha$  phase that nucleated at the  $\omega/\beta$  boundary, and the 3-D APT results in the work of Zheng et al.<sup>32</sup> show the association of the  $\omega$  and  $\alpha$  phases.

Another much-disputed topic is the dominant nucleation driving force for  $\omega$ -assisted  $\alpha$  formation. Recent studies proposed that the elastic stress associated with the coherent  $\omega/\beta$  interface, compositional variation between  $\omega_{iso}$  and the surrounding  $\beta$  phase, and/or dislocations migrating to or forming at the  $\omega/\beta$  boundary could provide additional driving force to promote the precipitation of the  $\alpha$  phase.<sup>21,22,116</sup> A combination of those reasons could also influence  $\alpha$  precipitation. To ascertain the key nucleation driving force for  $\omega$ -assisted  $\alpha$  formation, the transitional stage before and after  $\alpha$  forms must be identified. Li et al.<sup>27</sup> captured the transitional stage by analyzing samples aged for 2, 4, 8, 12, 16, 24, 32, 48, and 192 h, respectively, at 573 K using HRTEM and APT. They found that high elastic stress and O-rich regions present around the coherent incommensurate- $\omega/\beta$  interface (at 12 h) do not assist  $\alpha$  formation (see Figure 6).<sup>27</sup> The  $\alpha$  phase forms only when the coherent interface becomes semi-coherent, suggesting that increased interfacial energy serves as the dominant factor in triggering  $\omega$ -assisted  $\alpha$  precipitation.<sup>27</sup> In addition, O trapped by ledges at the semi-coherent interfaces could provide an additional nucleation driving force for  $\omega$ -assisted  $\alpha$  formation. These results are supported by the chemical driving force calculations of Shi et al.<sup>116</sup> which indicate that a semi-coherent  $\beta/\omega$  interface provides a lower energy barrier to  $\alpha$  phase nucleation than a coherent interface. However, while Shi et al.<sup>116</sup> indicate that local enrichment of O at the  $\omega/\beta$  interface could provide another chemical driving force for  $\alpha$  nucleation, they also argue that the energy associated with other factors could outweigh the contribution of O and that more study is needed. Similarly to Shi et al.,<sup>116</sup> Nag et al.<sup>110</sup> concluded that coherency strains and local enrichment of Zr both played a role in the nucleation and growth of  $\alpha$  precipitates at the  $\omega/\beta$  interface. Zheng et al.<sup>22</sup> also conclude from CALPHAD studies that composition variance and coherency stress between the  $\beta$  and  $\omega$  phases both play a role in  $\alpha$  nucleation. Thus, there is a lack of current consensus and more experimental

evidence combined with modeling studies are required to determine the key nucleation driving force behind  $\omega$ -assisted  $\alpha$  formation.

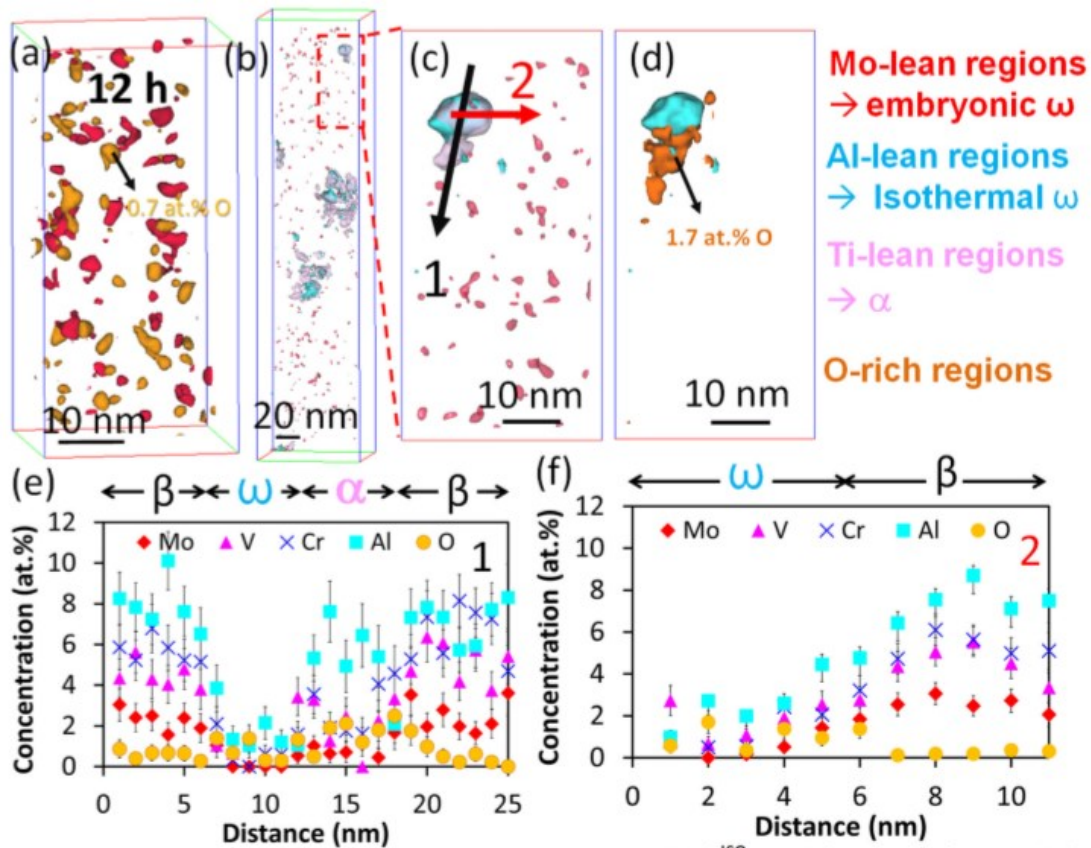


Figure 6. APT reconstructions of a Ti-6554 sample showing the relative positions of orange O-rich regions compared to (a) Mo-lean regions in red (embryonic  $\omega$  phase) and (b) Al-lean regions in blue (isothermal  $\omega$  phase) and Ti-lean regions in pink ( $\alpha$  phase). A close-up region of (b) is shown in (c) and (d) highlighting the Al-lean and Ti-lean regions in (c) and the same Al-lean regions along with the O-rich regions in (d). The concentration profile in (e) was made using Arrow 1 in (c), and the profile in (f) was made using Arrow 2 in (c).

Furthermore, conclusions about O's role in  $\alpha$  phase nucleation have led to debate on O's significance in  $\omega$ -assisted  $\alpha$  nucleation. While O increases the range of  $\alpha$  phase stability in Ti alloys,<sup>1</sup> it also suppresses the formation of  $\omega_{iso}$ . O suppresses  $\omega_{iso}$  by pinning the linear defects that initiate the  $(111)_\beta$  plane collapse<sup>29</sup> and/or by increasing the relative energy of the  $\omega$  phase and the energy barrier for the  $\beta$ -to- $\omega$  transition.<sup>117</sup> Therefore, Li et al.'s<sup>30</sup> analysis that O-rich regions were near the  $\omega$  phase, and that those O-rich regions could drive  $\alpha$ -phase nucleation, was novel. Li et al.'s<sup>30</sup> claims are supported by Coakley et al.'s<sup>118</sup> results of a 27% higher concentration of O at the  $\omega/\beta$  boundary compared to the  $\beta$  matrix. O-rich regions have also been found near Mo-lean embryonic  $\omega$  particles,<sup>80</sup> and Coakley et al.<sup>87</sup> observed O-enrichment at the  $\omega/\beta$  boundary starting after 1 hr of 300 °C heat treatment of Ti-5Al-5Mo-5V-3Cr. Within  $\omega$ -phase particles, Niinomi et al.<sup>94</sup> have found a higher concentration of O than in the surrounding  $\beta$  matrix, although the association between O, the  $\omega$  phase, and the  $\alpha$  phase was not reported. Contradictory to those results, Zheng et al.<sup>22,32,95</sup> did not find a significant difference in O concentration

moving from the  $\beta$  phase to the  $\omega$  phase. Instead, compositional differences of the other alloying elements and strains at the  $\omega/\beta$  boundary are presented as the driving force—instead of O—by these studies.

## 6. Challenges in $\omega$ phase composition analysis

The differing results about O's role in  $\alpha$  phase formation, along with the differing results about the composition of athermal and incommensurate  $\omega_{iso}$  phase outlined above, could be because of the varying alloy compositions used by the different studies and lack of worldwide standardized approaches for APT data analysis procedures, especially when it comes to analyzing minor impurity element concentrations. Comparing the alloys used in 22 studies by Mo-Eq, 15 different Mo-Eq's were investigated as shown in Table 1. It is clear that comparisons of mechanisms are not usually made between alloys with similar Mo-Eq's. For example, Ahmed et al.<sup>40</sup> compared their  $\omega_{ath}$  composition results from a Ti-5Al-5Mo-5V-2Cr-1Fe (Mo-Eq of 9.45) alloy to Devaraj et al.'s<sup>20</sup> results for a Ti-18Mo alloy (Mo-Eq of 18). Along with different Mo-Eq's, Ahmed et al.'s<sup>40</sup> alloy is more complex than the binary Ti-18Mo alloy. Stabilizer types are not consistent either, with some studies using alloys of four or more alloying elements of isomorphous- and eutectoid-type stabilizers, while others use binary isomorphous systems. Additionally, trace impurity elements are also found in the alloys which are not included in the Mo-Eq calculation. The presence of these elements, while small, could affect the phase transformations and are not always included in the APT data analysis.

Table 1. Molybdenum equivalencies of alloys used in APT studies.

Alloy	Mo equivalency	Trace elements present	Reference
Ti-6Al-4V	-3.32	Fe, C, N, O, H	44
Ti-18Mo	18	O	20,43,72,93,95
Ti-18Mo-5Al	13	O	95
Ti-10V-6Cu*	11.32	Not reported	71
Ti-5Al-5Mo-5V-3Cr	8.15	Fe, C, N, O, H	22,82,87
Ti-5Al-5Mo-5V-3Cr-0.5Fe	12.8	Not reported	28
Ti-5Al-5Mo-5V-2Cr-1Fe	9.45	O	40,119
Ti-6Cr-5Mo-5V-4Al	13.95	O	25,27,30,80
Ti-1Al-8V-5Fe	18.86	C, N, O	5
Ti-29Nb-13Ta-4.6Zr	10.98	O	94
Ti-38Nb-2Ta-3Zr	11.08	Not reported	83
Ti-24Nb-4Zr-8Sn	6.72	O	120–123
Ti-24Nb-3Mo-3Zr-2Sn	9.72	H	124
Ti-12Mo	12	O	72
Ti-20V	13.4	O	32
Ti-12V-2Fe-1Al	12.84	Not reported	125
Ti-34Nb-7Zr-7Ta	11.06	O	110

\*Coefficient of 0.77 used for Cu calculated using the method described by Bania.<sup>53</sup>

Along with alloy composition, the APT experimental procedure can affect the reported compositions and spatial accuracy of APT results. Coakley et al.<sup>118</sup> have addressed this by comparing their results to those of Li et al.,<sup>25</sup> explaining how higher sample temperatures and laser energies make quantitative concentration analysis more difficult and increase the potential for surface diffusion, which affects spatial accuracy. Other studies, however, have not addressed how experimental procedures can affect results. This can be seen in Ahmed et al.'s<sup>40</sup> comparison to Devaraj et al.'s<sup>20</sup> work; Ahmed et al.<sup>40</sup>

did not mention that they used pulsed-laser-assisted APT and that Devaraj et al.'s<sup>20</sup> work used pulsed-voltage APT. In APT studies of the  $\omega$  phase, the biggest differences in experimental procedure are the pulsing mode (pulsed laser vs. pulsed voltage), operating temperature, and laser pulse energies. The APT experimental parameters used for a selection of  $\omega$  phase studies in  $\beta$ -Ti alloys are shown in Table 2. Unfortunately, not all settings are reported in the literature for pulsed-laser-assisted APT.

Table 2. Experimental procedure settings used for 19 different APT studies of the  $\omega$  phase.

APT mode	Operating temp.	Evaporation rate	Voltage pulse fraction	Laser wavelength	Pulse rate	Pulse energy	Laser spot size	Ref.
Voltage	60 K	0.2–1.0%	20%					22,32
Voltage	60 K	0.5–0.7%	20%					95
Voltage	70 K	0.2–1.0%	20%					20,43,71 ,93,110
Voltage	70 K	0.2–1.0%	30%					28
Voltage	80 K	0.5–0.8%	10%					94
Laser	60 K		0.005 ions/pulse	355 nm		20 pJ		5
Laser	50 K		0.005 ions/pulse	355 nm	250 kHz	70 pJ	2 $\mu$ m	40
Laser	50 K		0.005 ions/pulse		250 kHz	70 pJ	2 $\mu$ m	25
Laser	50 K		0.005 ions/pulse			80 pJ		27
Laser	50 K					80 pJ		30,80
Laser	40 K			355 nm		50 pJ		82
Laser	40 K			512 nm		200 pJ		82
Laser					250 kHz	50 pJ		125

Both voltage pulsing and laser pulsing of APT has been used to analyze  $\beta$ -Ti alloys. In both voltage and laser modes, higher specimen temperatures or higher laser energies can provide better yield but poorer spatial resolution as a result of surface migration. Coakley et al.<sup>87</sup> found that higher laser pulse energies reduced the volume fraction of the  $\omega$  phase that is identified in a sample, as shown in Figure 7. Coakley et al.<sup>87</sup> have also found that laser direction can influence detection of the  $\omega$  phase in Ti samples, with  $\omega$ -phase particles on the same side as the laser being more difficult to detect.



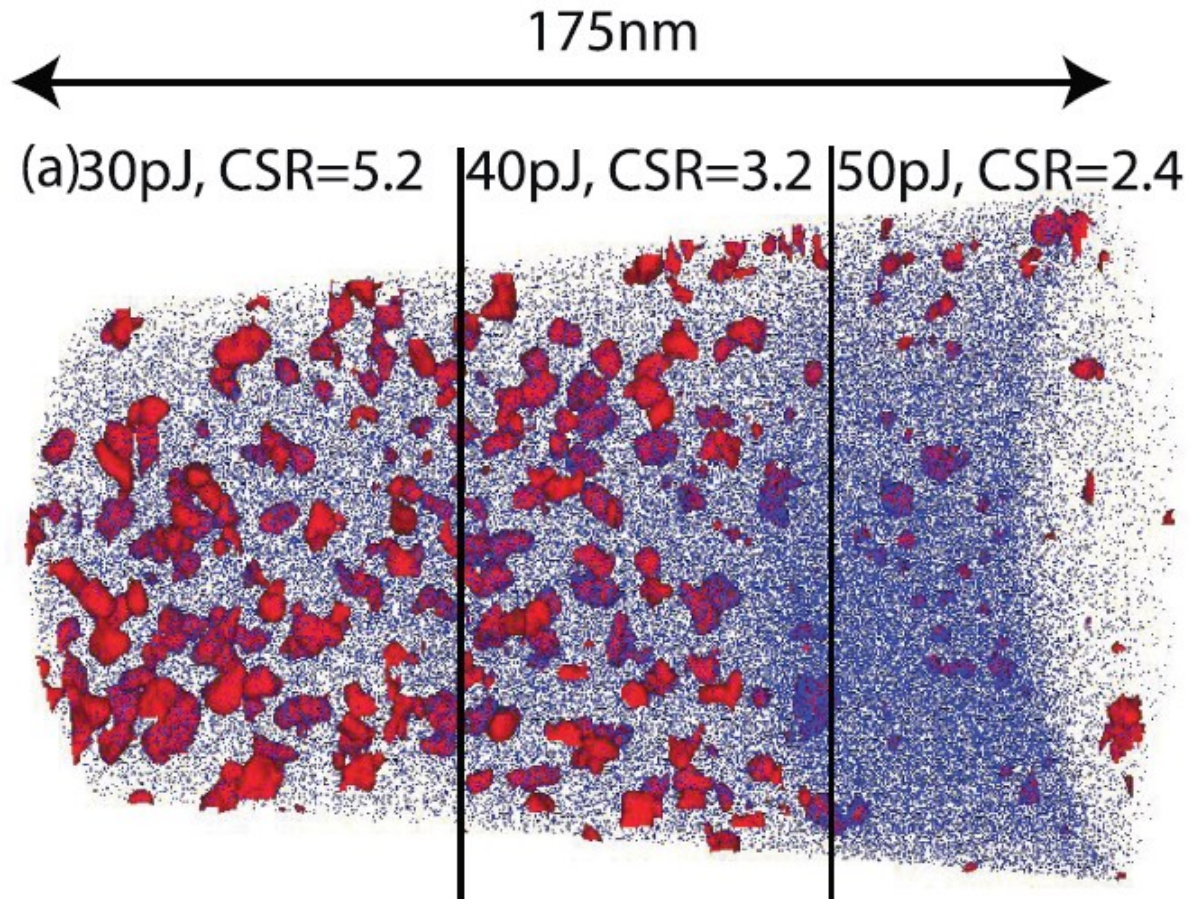


Figure 7. An APT sample reconstruction showing how higher laser energy reduces the volume fraction of the  $\omega$  phase detected in  $\beta$ -Ti alloys through a stepped increase in laser energy during a single APT experiment. The red volumes indicate the Ti-rich  $\omega$  phase. Adopted from J. Coakley, A. Radecka, D. Dye, P. A. Bagot, T. L. Martin, T. J. Prosa, Y. Chen, H. J. Stone, D. N. Seidman and D. Isheim, "Characterizing nanoscale precipitation in a titanium alloy by laser-assisted atom probe tomography," *Materials Characterization*, vol. 141, pp. 129–138, 2018.

Finally, when it comes to using APT to analyze the composition of  $\omega$ ,  $\beta$ , and  $\alpha$  phases, the procedure selected for ranging the mass-to-charge spectra peaks of minor impurity elements such as O can induce a significant amount of statistical variability. For example, the  $O^{+1}$  peak at 16 Da can overlap with the  $Ti^{3+}$  peak, requiring a detailed peak deconvolution.  $TiO$  molecular peaks are often minor and may not be indexed if the mass-to-charge spectra ranging is not conducted using a log scale for the counts. Such variability in data analysis procedures must be accounted for to facilitate accurate interpretation of the roles of minor impurity elements such as O in key phase transformation mechanisms in alloys.

Thus, the different alloy compositions, variabilities in APT experimental parameters used, and APT mass-to-charge spectra ranging procedures used in the research for  $\omega$  phase transformation in  $\beta$  titanium alloys can make it difficult to conclusively use only the current body of work to resolve the debates around the  $\omega$ -phase transformation.

## **7. Effect of the $\omega$ phase on mechanical properties**

A single crystal  $\omega$  phase is difficult to grow without high pressures, so the mechanical properties of a single crystalline  $\omega$  phase have been determined using simulations and polycrystalline material. Tane et al.<sup>62</sup> used HPT to create a polycrystalline  $\omega$  microstructure to determine the mechanical properties of the  $\omega$  phase because the elastic tensor was calculated using simulations but had never been verified experimentally. The  $\omega$  phase was anisotropic, with the  $\langle 0001 \rangle$  direction modulus equal to  $201 \pm 7.4$  GPa and the  $\langle 11-20 \rangle$  direction modulus being 70 GPa lower at  $129 \pm 3.9$  GPa.<sup>62</sup> The experimental  $\omega$ -phase elastic tensor agreed well with the simulated elastic tensor calculated using the tight-binding model<sup>126</sup> (shown in Table 3), and an experimental isotropic elastic modulus was calculated to compare to the  $\beta$  phase.<sup>62</sup> While direct comparison to pure Ti with the bcc structure at RT is not possible, the  $\beta$  values at 1293K are included. Also included are a range of property values compiled from multiple  $\beta$  alloys ( $\beta$  III,  $\beta$  C, Ti-1023, Ti-13V-11Cr-3Al, Ti-15V-3Cr-3Al-3Sn, Ti-8Mo-8V-2Fe-3Al, Ti-15Mo-5Zr, Ti-15Mo-5Zr-3Al, Ti-8V-5Fe-1Al, and Ti-16V-2.5Al) at RT, both as-quenched and aged, since processing can significantly affect the mechanical properties of  $\beta$  alloys. More detail on aging treatments and corresponding modulus values for specific alloys can be found in the Materials Properties Handbook: Titanium Alloys<sup>8</sup>.

Table 3. The elastic and shear modulus of the  $\omega$  and  $\beta$  phases.

Phase	Temperature (K)	$E_{iso}$ (GPa)	$G_{iso}$ (GPa)	Reference
$\omega$ (simulation)	0	155	61	<sup>62</sup>
$\omega$ (polycrystalline)	RT	$152.8 \pm 0.6$	$60.1 \pm 0.3$	<sup>62</sup>
$\beta$	1293K	65	23	<sup>127</sup>
$\beta$ (as-quenched)	RT	78-103	34-43	<sup>8</sup>
$\beta$ (Aged)	RT	100-124	40.7-43.3	<sup>8</sup>

While it is difficult to compare these moduli because of the variability in temperatures, the experimental  $\omega$  phase elastic modulus is approximately 30 GPa higher than the highest modulus reached for  $\beta$ -Ti alloys, and the shear modulus is approximately 17 GPa higher than the highest  $\beta$ -Ti shear modulus. Therefore, the elastic and shear modulus of the  $\omega$  phase should contribute to an increase in the elastic modulus of  $\beta$ -Ti alloys when the  $\omega$  phase is present. Hsu et al.<sup>128</sup> and Ho et al.<sup>129</sup> have observed this, recording increases in the bending elastic modulus of  $\sim 35$  GPa and  $\sim 20$ – $40$  GPa, respectively. Coakley et al.<sup>87</sup> and Hsu et al.<sup>97</sup> also report an elastic modulus increase as indicated by the increase in slope in the elastic region of their tensile and bending tests, given in Figure 8.<sup>129</sup> However, this apparent increase is not observed for athermal  $\omega$  microstructures.  $\omega_{ath}$  tends to be neglected in studies on the mechanical influence of the  $\omega$  phase because it is generally thought not to affect the mechanical properties of  $\beta$ -Ti alloys.<sup>8,21,69</sup> Unfortunately, because  $\omega_{ath}$  cannot be prevented during quenching of metastable or near- $\beta$  alloys without changing the alloy composition to stabilize the  $\beta$  phase,<sup>28,130</sup> the mechanical properties of  $\beta$  alloys with and without  $\omega_{ath}$  (such as Ti-15Mo with  $\omega_{ath}$  and Ti-15Mo without  $\omega_{ath}$ ) cannot be compared.<sup>21</sup> It is possible that  $\omega_{ath}$  does affect mechanical properties—one study by Ho<sup>130</sup> reports an increase in microhardness from 263 Hv to 337 Hv associated with the presence of  $\omega_{ath}$ . No extensive studies on the mechanical effects of the incommensurate  $\omega$  phase have been done so far.

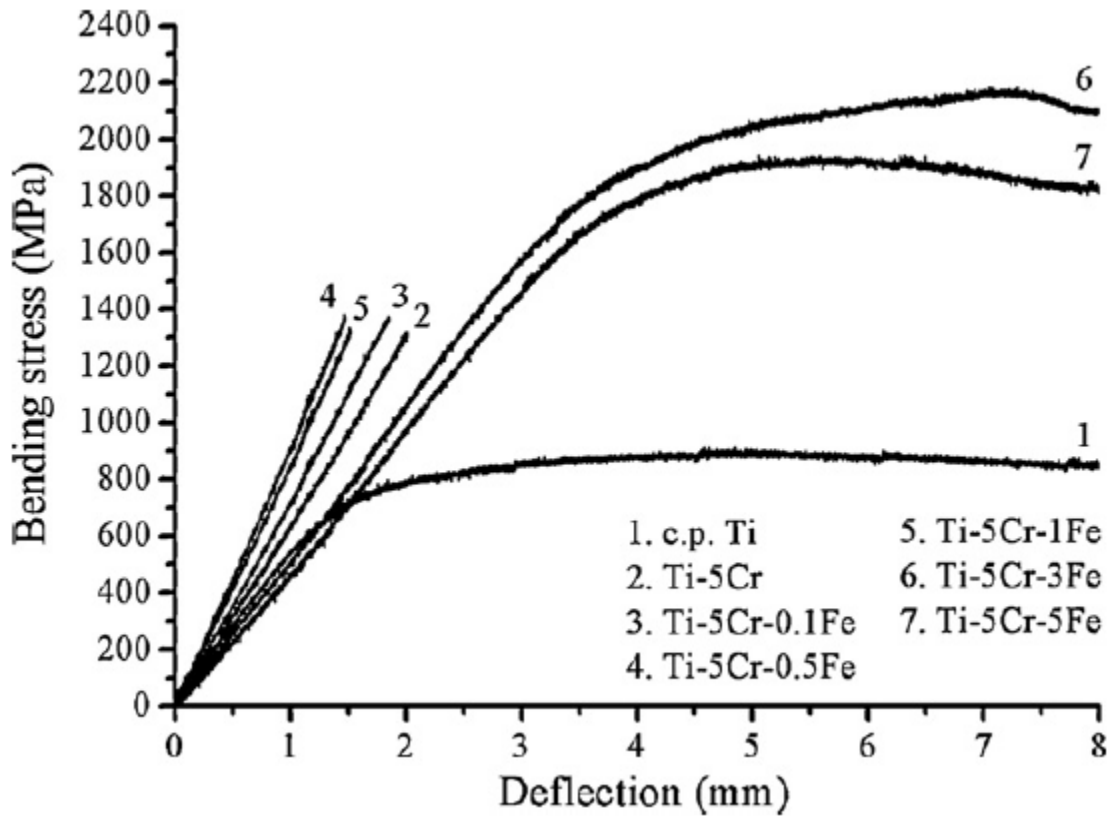


Figure 8. Bending test results showing the elastic modulus increase in  $\beta$ -Ti with  $\omega$ -phase microstructures. Adopted from Ho, W.-F., Pan, C.-H., Wu, S.-C., et al. "Mechanical properties and deformation behavior of Ti-5Cr-xFe alloys," *Journal of Alloys and Compounds*, V. 472, Nos. 1–2, 2009, pp. 546–50.

Along with an increase in the elastic modulus, the  $\omega$  phase increases the microhardness of  $\beta$ -Ti alloys. Multiple studies have reported an increase in microhardness after precipitation of the isothermal  $\omega$  phase, regardless of alloy composition.<sup>74,76,87,89,97,104,128,131–134</sup> Although Jones et al.<sup>69</sup> attribute the microhardness increase to precipitation of the nanoscale  $\alpha$  phase, Coakley et al.<sup>74</sup> counter that the observed hardness increase from 280 Hv to 400 Hv was due to the rapid precipitation of  $\omega_{iso}$  during a 400 °C heat treatment.  $\omega$  phase microstructures have also exhibited higher microhardness than martensitic  $\alpha''$  microstructures by approximately 100 Hv,<sup>97</sup> although the hardness of  $\alpha''$  martensite is not wholly intrinsic and is partly due to sliding interfaces. Hida et al.<sup>76</sup> found that hardness increased with precipitation of the commensurate  $\omega$  phase, as shown in Figure 9. The results in Figure 9 also support the assumption stated above that the incommensurate  $\omega$  phase does not affect the mechanical properties of  $\beta$  alloys because the incommensurate  $\omega$  phase did not appear to affect microhardness.<sup>76</sup> Figure 9 is also a typical example of the scale of hardness increase, as most hardness values were reported to start around 300 Hv and increase to 400–550 Hv.<sup>74,76,89,97,128,132–134</sup>

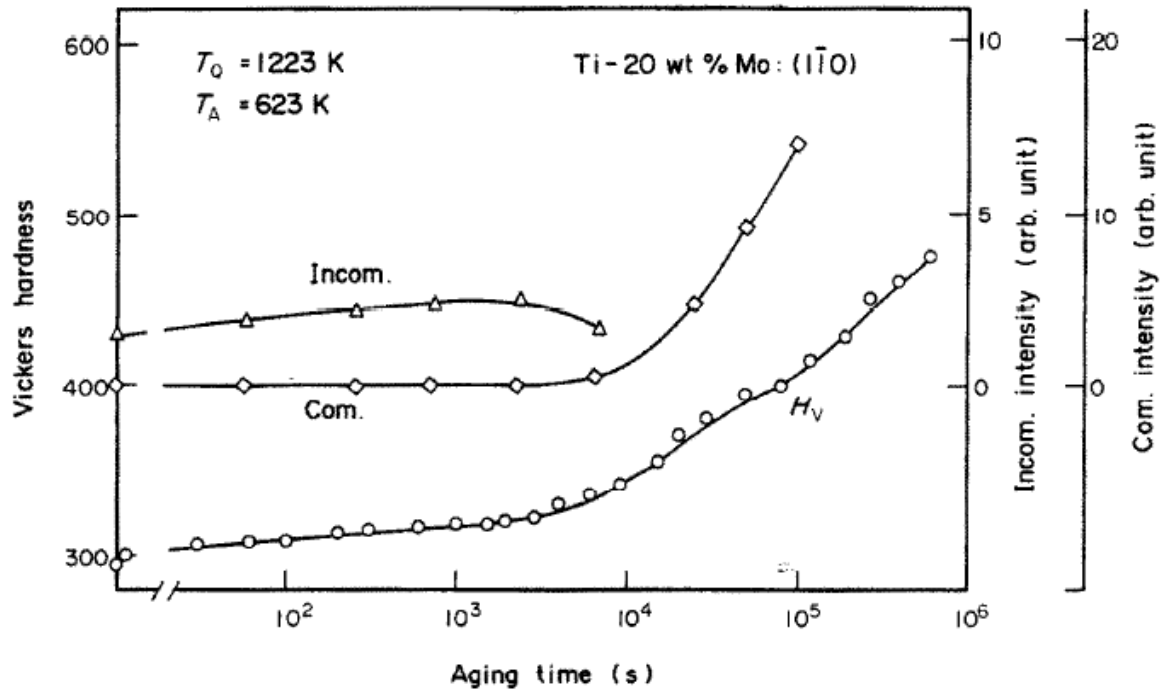


Figure 9. The effects of incommensurate and commensurate  $\omega$  phases on the microhardness of a Ti-20Mo alloy. Adapted from M. Hida, E. Sukedai and H. Terauchi, "Microscopic approaches to isothermal transformation of incommensurate  $\omega$  phase zones in Ti-20wt%Mo alloy studied by XDS, HREM and EXAFS," *Acta Metallurgica*, vol. 36, no. 6, pp. 1429–1441, 1988.

The  $\omega$ -phase microstructure also increases the tensile strength of alloys. Chandrasekaran et al.<sup>135</sup> found that precipitating  $\omega_{iso}$  in a Ti-15Cr (at%) alloy increased the yield strength of the alloy by  $\sim 186$  MPa, and Mantri et al.<sup>112</sup> found that  $\omega_{iso}$  precipitation increased the UTS of Ti-12Mo by  $\sim 525$  MPa. Feeney and Blackburn<sup>136</sup> found that  $\beta + \omega$  microstructures in a Ti-11.5Mo-6Zr-4.5Sn alloy exhibited strengths up to 1500 MPa. Williams et al.<sup>137</sup> found that the yield strength of Ti-10Mo (at%), Ti-25V (at%), Ti-8Mn (at%), and  $\beta$  III alloys increased when the  $\omega$  phase was present and decreased when the  $\alpha$  phase became the dominant phase. Alloys that deform via the  $\omega$  transformation also exhibit higher yield strengths than alloys that deform via twins.<sup>51</sup>

The increase in elastic modulus and tensile strength due to the  $\omega$  phase is accompanied by embrittlement. In tension, embrittlement is shown by a decrease in elongation to failure ( $\epsilon_f$ ) when  $\omega$  microstructures are present, as Zhao et al.,<sup>31</sup> Mantri et al.,<sup>112</sup> Williams et al.,<sup>137</sup> Wang et al.,<sup>125</sup> and Feeney and Blackburn<sup>136</sup> documented in alloys that had precipitated the isothermal  $\omega$  phase. The decrease can be seen the most clearly in Zhao et al.'s<sup>31</sup> Ti-18V alloy in Table 4 and is also demonstrated by the drop in  $\epsilon_f$  from  $\sim 35\%$  to  $< 5\%$  with  $\omega_{iso}$  precipitation in Mantri et al.'s<sup>112</sup> Ti-12Mo alloy. Li et al.<sup>88</sup> has shown that  $\epsilon_f$  in tensile deformation is dependent on the presence of the isothermal  $\omega$  phase, with higher  $\omega_{iso}$  volume fractions corresponding to lower elongation to failure values. A drop in  $\epsilon_f$  from 6.1% to  $\sim 1.7\%$  in a Ti-24Nb-4Zr-8Sn alloy was associated with the presence of isothermal  $\omega$  phase microstructures.<sup>131</sup> Chandrasekaran et al.<sup>41</sup> also observed a drop in  $\epsilon_f$  from 17% to 2% when  $\omega_{iso}$  was precipitated. When the  $\omega$  phase was removed from the  $\beta$  matrix using heat treatments, the  $\epsilon_f$  increased from 2% to 6%.<sup>135</sup> These embrittling effects are only true for  $\omega_{iso}$  microstructures because  $\omega_{ath}$  does not embrittle alloys like  $\omega_{iso}$ ; Wang et al.<sup>138</sup>

and Sun et al.<sup>106</sup> observed no embrittling effects in binary Ti-Mo alloys. Chandrasekaran et al.<sup>135</sup> have also found that  $\omega_{\text{ath}}$  microstructures reach an  $\epsilon_f$  of 13%, while  $\omega_{\text{iso}}$  microstructures reached 2%.

Table 4. The strengths and  $\epsilon_f$  of  $\beta$  and  $\beta + \omega$  phase microstructures.<sup>31</sup>

Alloy	Microstructure	approx. 2% Yield Stress (MPa)	approx. UTS (MPa)	approx. $\epsilon_f$ (%)
Ti-18V	$\beta + \omega_{\text{ath}}$	730	830	28
Ti-18V	$\beta + \omega_{\text{ath}} + \text{deformation } \omega$	750	1080	6
Ti-20V	$\beta + \omega_{\text{ath}}$	880	910	17
Ti-20V	$\beta + \omega_{\text{ath}} + \text{deformation } \omega$	900	1050	10
Ti-22V	$\beta$	910	900	11
Ti-22V	$\beta + \text{deformation } \omega$	950	980	10

Bending tests also illuminate the embrittling effects of the  $\omega$  phase. Hsu et al.<sup>97</sup> and Ho et al.<sup>129</sup> found that alloys with  $\omega$ -phase microstructures fractured during bending at significantly lower deflections than alloys containing the  $\omega + \alpha$  or no  $\omega$  phase microstructures. This can be seen in the low deflections at the fracture of  $\omega$ -phase microstructures (Alloys 2–5) in Figure 8, compared to Alloys 1, 6, and 7, which completed the bending test without fracture.<sup>129</sup> Cleavage facets were found on the fracture surfaces of the  $\omega$ -phase microstructures,<sup>97,129</sup> which are characteristic of decreased  $\epsilon_f$ .

The embrittling effects of the  $\omega$  phase are thought to be because dislocations stop or bend around  $\omega$  particles, leaving high dislocation densities in the  $\beta$  matrix.<sup>137</sup> When dislocations do move through the  $\omega$  phase, they shear the  $\omega$  phase, creating slip bands that can lead to crack nucleation at low macroscopic strains,<sup>139</sup> as shown in Figure 10. Recently, Lai et al.<sup>83</sup> have shown that high densities of nanometer-sized ( $\sim 1.23$  nm)  $\omega$  phases suppress TWIP and TRIP effects, and Wang et al.<sup>125</sup> have shown that  $\omega_{\text{iso}}$  changes the deformation mode from deformation twinning to slip+deformation twinning to slip+stress-induced  $\omega$  phase transformation to brittle fracture as it grows in size. TWIP and TRIP deformations have been shown to lead to a desirable balance of strength and  $\epsilon_f$  in  $\beta$  alloys,<sup>140</sup> and the  $\omega$  phase suppression of these effects leads to localized plasticity in dislocation channels and reduction of  $\epsilon_f$  values.<sup>83</sup> Sun et al.<sup>106</sup> have used low heat-treatment temperatures and 60 second heat treatment times to avoid embrittlement while strengthening their Ti-12Mo alloy through precipitation of  $\omega_{\text{iso}}$ . Their study pointed to diffusion being another cause of embrittlement because the processing route was meant to precipitate  $\omega_{\text{iso}}$  without allowing significant diffusion between the  $\omega$  and  $\beta$  phases.<sup>106</sup> When diffusion did not occur, strengthening of the alloy from  $\omega_{\text{iso}}$  was achieved but the true strain at the fracture was still over 40%. When diffusion occurred, the tensile strength of the alloys was observed to be highest, but the embrittling effects were also seen manifested by the reduction of true strain at fracture to  $\sim 3\%$ .<sup>106</sup>

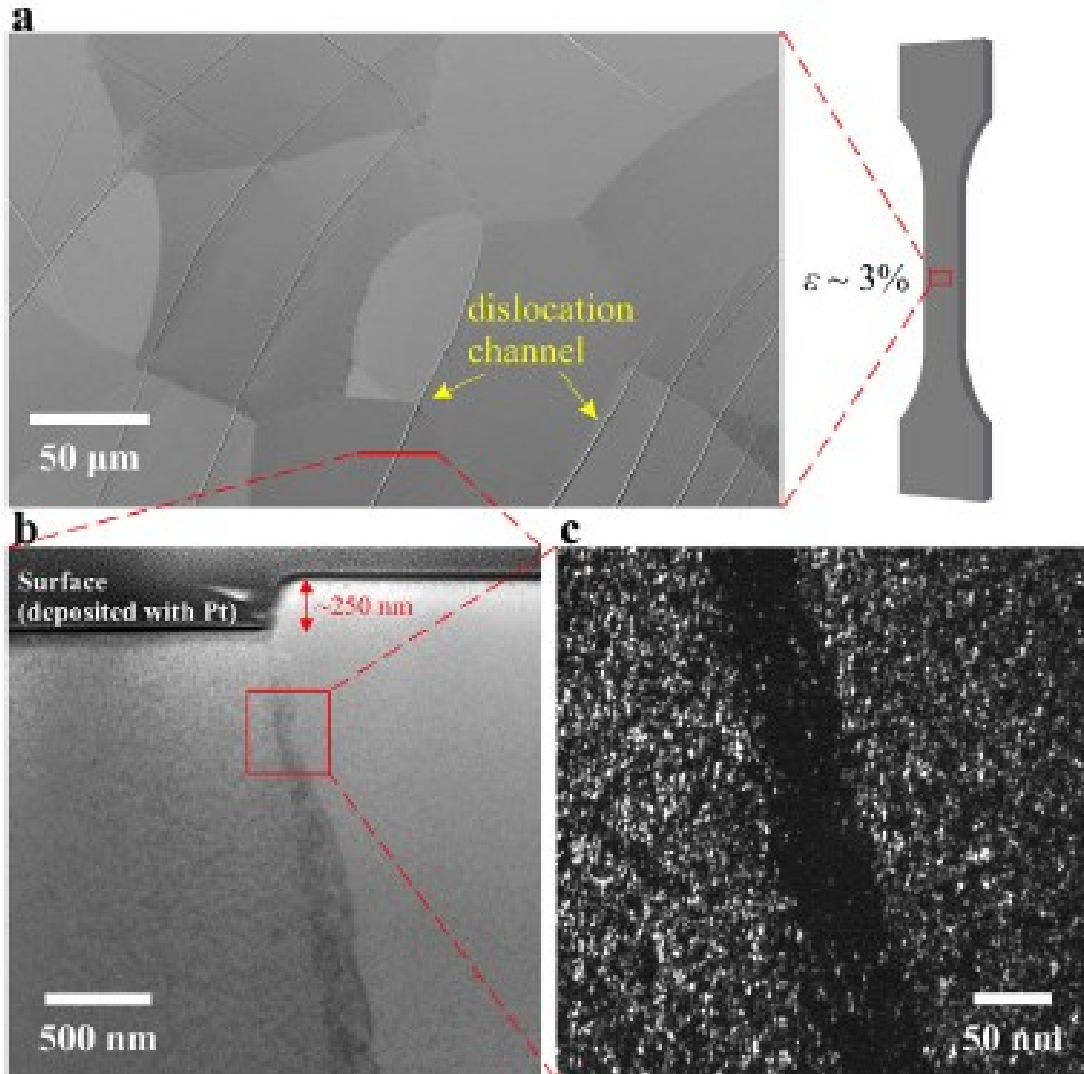


Figure 10. TEM micrographs of deformed  $\beta$  alloys showing dislocation channels that are  $\omega$ -phase free. Adapted from M.J. Lai, T. Li, and D. Raabe, “ $\omega$  phase acts as a switch between dislocation channeling and joint twinning- and transformation-induced plasticity in a metastable beta titanium alloy,” *Acta Materialia*, vol. 151, pp. 67–77, 2018.

Because the  $\omega$  phase allows crack nucleation at low macroscopic strains, it can be detrimental to fatigue properties.<sup>139</sup> However,  $\omega_{iso}$  microstructures have been found to retard fatigue crack propagation significantly more than  $\alpha$ -phase microstructures.<sup>141</sup> Nakai et al.<sup>133</sup> have also found that the maximum cycle stress of their TNTZ alloy increased by  $\sim 200$  MPa after precipitation of the  $\omega$  phase through a combination of cold-rolling and aging. In Feeney and Blackburn’s<sup>136</sup> study, the fracture toughness of the  $\beta$ -Ti alloy was independent of  $\omega$  particle size once the particles were larger than  $\sim 10$  nm.

Although crack nucleation and embrittlement are why processing routes and alloy compositions are designed to avoid precipitation of the  $\omega$  phase,  $\omega$ -assisted  $\alpha$ -phase microstructures have attractive mechanical properties. The  $\omega$  phase can also influence the mechanical properties of  $\beta$  alloys by preventing the formation of the martensitic  $\alpha''$  phase,<sup>16,23</sup> which is detrimental to the formability of  $\beta$  alloys.<sup>142</sup>

The tensile behavior of  $\beta$  alloys is heavily affected by the size, morphology, and distribution of  $\alpha$ -phase particles in the  $\beta$  matrix, and fine-grained  $\alpha$ -phase microstructures have exhibited high strengths with higher  $\varepsilon_f$  than  $\omega$ -phase microstructures.<sup>143</sup> Azimzadeh and Rack<sup>85</sup> found that the hardness of a Ti-6.8Mo-4.5Fe-1.5Al alloy (TIMET LCB alloy) was highest with a combination  $\omega$  phase and  $\alpha$  phase microstructures. Hsu et al.<sup>97</sup> and Ho et al.<sup>129</sup> found that alloys with  $\omega + \alpha$  phase microstructures could withstand more deflection before fracture in bending tests than alloys with only  $\omega$  phase microstructures.  $\omega$ -assisted  $\alpha$ -phase microstructures have been found to be weaker than  $\omega_{iso}$  microstructures, as Williams et al.<sup>137</sup> showed when the yield strength of Ti-10Mo (at%) dropped by  $\sim 275$  MPa with precipitation of the  $\alpha$  phase; however, the strength was still relatively high at  $\sim 965$  MPa, and the  $\varepsilon_f$  increased from 6% to 8%. The mechanical property benefits of the  $\omega$ -assisted  $\alpha$  phase are driving research of processing routes that encourage  $\omega_{iso}$  to form the  $\omega$ -assisted  $\alpha$  phase.<sup>22,27,32,40,42,113</sup>

## 8. Conclusions and future directions for research

With  $\beta$ -Ti's useful attributes of a high strength-to-weight ratio, good corrosion resistance, and a low elastic modulus, it is important to understand the phase transformations that affect its mechanical properties. An important transformation is the  $\beta$ -to- $\omega$  transformation, both because of its direct effects on the mechanical properties of  $\beta$ -Ti alloys and its influence on other phase transformations.

Quenching can precipitate both commensurate and incommensurate  $\omega$  phases. Low-temperature heat treatments precipitate the isothermal  $\omega$  phase as well-distributed ellipsoidal or cuboidal particles. Temperature and time can be used to influence the amount, size, volume fraction, and composition of  $\omega_{iso}$ .  $\omega_{iso}$  is known to assist in the formation of a fine-grained  $\alpha$ -phase microstructure. Cuboidal  $\omega_{iso}$  particles have ledges that act as nucleation points for the  $\alpha$  phase. The deformation-induced  $\omega$  phase can increase the strength of  $\beta$  alloys and has the potential to be used in HPT processing.

$\beta$ -Ti alloys with  $\omega$ -phase microstructures exhibit higher strengths, microhardness, and elastic moduli than  $\beta$ -Ti alloys without the  $\omega$  phase. The elongation to failure of  $\beta$  alloys also decreases when  $\omega$ -phase microstructures are present, which limits the application of  $\omega$ -phase microstructures. Recent work has suggested that the composition of the  $\omega$  phase is linked to the embrittling effects and that there is potential to reach high strengths using the  $\omega$  phase without significantly compromising  $\varepsilon_f$ ; however, determining the underlying reason for the embrittling effect of the isothermal  $\omega$  phase is still a future challenge.

Other unresolved questions in the  $\omega$ -phase field are the exact role that oxygen plays in  $\alpha$ -phase nucleation, how ellipsoidal  $\omega_{iso}$  particles assist  $\alpha$ -phase nucleation, and whether commensurate and incommensurate athermal  $\omega$  phases preferentially nucleate at stabilizer-lean pockets in  $\beta$  alloys. These questions are currently driving research in this field.

To investigate these questions, studies of binary alloy systems, such as Ti-Cr, could be used to see how alloy chemistry affects formation of the  $\omega$  phase and subsequent formation of the  $\alpha$  phase. Research into how eutectoid stabilizers affect the composition of the  $\omega$  phase is another avenue that merits investigation, especially because APT composition studies have mostly focused on isomorphous systems. Work on eutectoid stabilizers could also help with understanding  $\omega$ -phase formation in complex alloy systems that contain both isomorphous and eutectoid stabilizers. Also, taking a relatively well-understood system such as Ti-Mo and systematically increasing its complexity with the addition of other alloying elements (such as Fe and Al) would also be valuable for understanding how each element individually affects  $\omega$  phase formation. Modeling has also been a powerful tool to understand the driving forces of the  $\beta$ -to- $\omega$  phase transformation, and could be applied to determine the effects of individual alloying elements

on this transformation. Using modeling together with experimental results could help achieve a fundamental understanding of the different effects of alloying elements on  $\omega$  phase formation, in particular the differences in isomorphous and eutectoid  $\beta$ -stabilizers.

Studies to find evidence of O-enrichment at the  $\omega/\beta$  boundary in more alloy systems would also be useful. In conjunction with this, studying how alloy composition and APT experimental settings and data analysis procedures affect O concentration and location in  $\beta$ -Ti alloys could also help clarify the reasons for O-enrichment reported at the  $\omega/\beta$  boundary in some studies and not in others. Consistent standards for APT analysis could allow for a more systematic approach to investigating the influence of O on the  $\omega$ -assisted  $\alpha$  phase transformation in the future. Adopting consistent standards would also help in comparing the results of different studies, as currently it is difficult to systematically compare APT results from different research groups without addressing the differences in analysis as discussed in this work.

In general, more work on how the  $\omega$  phase influences fatigue behavior of  $\beta$ -Ti alloys is needed. Specifically, crack initiation and propagation studies would be meaningful contributions to the body of work on the  $\omega$  phase.

Moving forward in  $\omega$ -phase research, work to resolve these debates will continue helping researchers tune processing pathways and alloy compositions to achieve desired microstructures and mechanical properties in  $\beta$ -Ti.

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