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Robustness of interdependent scale-free networks based on link addition strategies

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ABSTRACT

It is well known that interdependent networks are more vulnerable to cascading failure than single and isolated networks. In this report, we propose a new scheme to improve the robustness of interdependent scale-free network under degree-based deliberate attacks by adding links to enhance the connectivity of the interdependent network. Our proposal details 14 link addition strategies using two link importance functions. To verify the feasibility of the proposed strategies, we synthesize three different types of two-layer interdependent Barabási–Albert networks where nodes from each layer are, after ranked by their degrees, bijectively inter-coupled by assortative coupling (AC), disassortative coupling (DC), or random coupling (RC). We find that when the number of attacked nodes in the system is small, the harmonic closeness (HS-IDD) link addition strategy has the highest efficiency. Among them, S indicates that the information fusion method of each layer of the network is addition, IDD refers to the degree difference of network. With the increase of the attack proportion, the degree (DS-IDD) link addition efficiency gradually increases, and the effect is more obvious in DC and RC. Besides, through comparing different strategies, under DC and RC, the link addition strategy for the product is more effective than the sum-based strategy. However, under AC, this phenomenon is not obvious. The results show that our proposed approach can enhance the robustness of interdependent networks, and that our method provides a valuable reference for the control and prevention of cascading failures in existing interdependent networks.

1. Introduction

In recent years, with the rapid development of the social economy and continuous progress in science, complex networks have greatly facilitated people's lives. However, with the increase of the network's scale and complexity, even though an edge or a node fails, the entire network can be affected, often causing a global collapse. We call this phenomenon cascading failure [1]. The phenomenon of cascading failure has attracted widespread attention from different disciplines,

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such as transportation network [2,3], power network [4–7], communication network [8,9], water supply network [10], biological network [11], and many more [12–16]. However, the real-word network is no longer just a single network but is composed of two or more network layers that rely on each other to be functional. Although the independency links is useful for the working system's efficiency under each sub-networks' cooperation, it is more vulnerable to node attacks or node failures than a single complex network [17]. Therefore, how to improve the robustness of interdependent networks has become an important issue in the study of network dynamics. Therefore, how to improve the robustness of interdependent networks has become an important issue in network dynamics research. Among them, the BA scale-free network is often more likely to attract the attention of scholars because of its lower robustness to deliberate attacks [18,19].

In 2010, Buldyrev S.V et al. for the first time, proposed the interdependent network theory and analyzed its cascading failure process. They find that the coupling strength between networks reduces the robustness of the network, and interdependent networks composed of sub-networks with a wider degree distribution have worse robustness [17]. Thus, the topology of the sub-network is related to the robustness of the network. On this basis, a large number of scholars began to study the cascading failure process of complex networks from the perspective of interdependent networks [20–24]. Based on the analytical framework to track the dynamic process of cascading failures of interdependent networks, Gao et al. proposed a general framework to analyze the percolation of n interdependent networks [20,21,23] and the past studies of the robustness of a single network is a specific case when $n = 1$. Combined with the observed characteristics of real networks, Liu et al. established a general theoretical framework to analyze the cascading failures of interdependent directed networks in consideration of existing networks' directivity. Then, based on the general idea of research on interdependent networks, the characteristics of multilayer networks have been studied extensively in recent years [25–27].

The failure of interdependent networks is mainly reflected in the failure of interdependent nodes and the load failure caused by the redistribution of loads. At present, the strategies for the cascading failure of interdependent networks can be categorized into hard strategies and soft strategies [28,29]. Among them, the soft strategy refers to adjusting a load of network elements under the inherent network structure, reducing the possibility and scope of cascading failure by increasing the network's capacity. The hard strategy is to resist cascading failure by changing the structure of the network itself. Since the interdependent edge's failure has nothing to do with the load of the sub-network, a hard strategy is a better choice for the interdependent network.

Among the soft strategies to deal with complex network cascade failure, aiming at the redundancy characteristics of the edges in the real-word network, Chen et al. proposed a weighted complex network cascade failure nonlinear model considering overload, which greatly reduced the impact of load failure and improved the robustness of the network [30]. Zhang and Yin not only studied the interdependent network from the perspective of network controllability but also proposed different backup strategies for network nodes or interdependent edges to reduce the impact of overload faults [31]. Chen et al. considered the degree information of the node's nearest neighbor and next-nearest neighbor and define a multi-level neighbor information index to rank the importance of nodes in the network [32]. The protection of important nodes can also resist cascading failures. The soft strategy can improve the tolerance of the network to cascading failures under different network structures. However, the protection of critical nodes can only reduce the possibility of network damage rather than improve the network's robustness. The cost of changing network elements is high, which is also not easy to be implemented. Therefore, it is not easy to comprehensively improve the robustness of the network through soft strategies.

Among the hard strategies, most of them consider adding edges on a single-layer network. Without changing the original network functions and original services, the robustness of the network is improved by adding edges such as, random addition strategy [33–35], low degree addition strategy [34,35], low betweenness addition strategy [34], and algebraic connectivity based addition strategy [36]. Based on these strategies, Cao et al. took the node's betweenness as the object of consideration and founded that the edge addition strategy with high betweenness is more efficient [33]. Ji et al. considered the interdependence of interdependent networks and proposed two newly connectivity link addition strategies, and performed simulations under different topology-interdependent network models to verify the effectiveness of the proposed model [37]. However, most of the works consider too few node features, and there are few studies on how to add edges to interdependent networks more effectively. Besides, the impact of edge addition on the robustness of the network may be related to coupling preference, which means that under different edge addition strategies, the behavior of interdependent networks is more complicated than that of a single network. In order to explore the relationship between different edge addition strategies and network robustness under deliberate attacks, this paper fully considers the degree, betweenness, and harmonic closeness of the network as well as the characteristics of the interdependent network. We propose 14 addition strategies by two edge importance functions and then simulate the BA–BA interdependent network with the coupling modes AC, DC, and RC. The simulation results show the feasibility of the proposed strategy, which could greatly improve the network's robustness. The research results have a guiding role in optimizing interdependent networks' structure in actual networks and improving the robustness of interdependent networks against cascading failures.

2. Methods

In this part, we mainly review the modeling process of interdependent networks, and establish BA–BA interdependent networks with different coupling preferences. Then, we study the cascading failure of interdependent networks from different angles. Finally, some common border addition strategies are introduced and compared with those proposed by us.

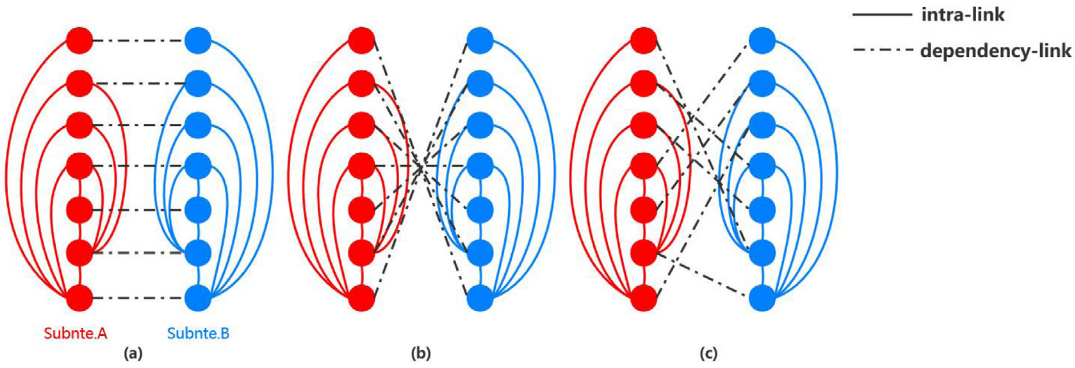


Fig. 1. A case to illustrate interdependent networks with AC (a), DC (b), and RC (c)..

2.1. Interdependent network model

In order to model the interdependent networks, this paper adopts the "one-to-one correspondence" model proposed by Buldyrev et al. That is, the nodes in the sub-network have a one-to-one correspondence. Since most real networks have scale-free characteristics, such as air network, Internet, power grid, etc., we assume that a subnetwork of a interdependent network are of the same size and are both BA scale-free networks, and all nodes in the A network are only connected to a node in the B network. In this paper, we consider the BA scale-free network coupled in three ways as follows:

1. Assortative coupling (AC). Sort the degrees of the nodes in the two sub-networks in descending order, and then connect them in order from high to bottom. As shown in Fig. 1(a).

2. Disassortative coupling (DC). Sort the degrees of the nodes in the A network in descending order and B network in ascending order, and then connect them in order from high to bottom. As shown in Fig. 1(b).

3. Random coupling (RC). Randomly choose a node without a dependency link in network A and B to connect. As shown in Figure Fig. 1(c).

2.2. Cascading failure model

Most networks in the actual network have load effects, and there are dependencies among the subnets of interdependent networks. This paper also considers the influence of interdependent edges and loads on interdependent networks. The steps for cascading failure are as follows:

Step 1: Assuming that the capacity of node i in the network is proportional to the load, calculate the load and capacity of the nodes in the entire network as follows [38]:

$$C_{A_i} = (1 + a_i) l_{A_i}, \tag{1}$$

$$l_{A_i} = (d_{A_i})^{\lambda}, \tag{2}$$

where, a_i is the tolerance coefficient of node i and is greater than 0, l_{A_i} is the load of node i in network A, d_{A_i} is the degree of node i in network A, λ is the load regulation parameter.

Step 2: Sort the degree values of the nodes in sub-network A in descending order, and attack the node with the largest degree in sub-network A.

Step 3: Redistribute the load of the failed node in the sub-network A to determine whether a new failed node is generated, if there is, remove the node from the sub-network A, if not, go to step 4.

Step 4: Remove nodes that are not in the largest connected slice in subnet A, and remove the corresponding interdependent nodes in B.

Step 5: Redistribute the load of the failed node in the sub-network B to determine whether a new failed node is generated, if there is, remove the node from the sub-network B. If not, go to step 6.

Step 6: Remove nodes that are not in the largest connected slice in subnet B, and judge whether there is any new interdependent node failure in sub-network A, if so, go back to step 3; otherwise, go to step 7.

Step 7: Judge whether the number of attack nodes meets the requirements. If so, the cascading failure model will end; otherwise, go to Step 2

2.3. Connectivity link addition strategies

This paper mainly considers the impact of different edge addition strategies on the robustness of interdependent networks under the same edge addition budget. We will explain each edge addition strategy, and for each link addition strategy, we have the following regulations: 1. Self-loop edges are not allowed. 2. Recalculate the degree and betweenness of the node at each step.

2.3.1. Edge addition strategy for single layer network

Both LD and LB are strategies for adding links in a single-layer network. Considering that high degree (high betweenness) nodes will always be attacked, both LD and LB strategies are adding links between nodes with the lowest degree or betweenness.

LD: After each cascade failure occurs, calculate the degree of nodes in the network, and then sort the degree of nodes in descending order, add links to the lowest degree unconnected nodes, and repeat this process until the number of added links is satisfied.

LB: After each cascade failure occurs, calculate the betweenness of nodes in the network, and then sort the betweenness of nodes in descending order, add links to the lowest degree unconnected nodes, and repeat this process until the number of added links is satisfied.

2.3.2. Interdependent network edge addition strategy using IDD

The degree of similarity of interdependent networks are related to the robustness of the network. The more similar sub-networks are, the higher the robustness of their interdependent networks are. In response to this feature, Kazawa et al. proposed the IDD-LD, IDD-LDP, IDD-LDS edge addition strategies based on IDD [39]. IDD refers to the degree difference of the network. It is assumed that node i in network A and node j in node B are interdependent nodes. For node i in A, its $IDD\,(i)$ is $k_A - k_B$.

IDD-LD: After each cascade failure occurs, the IDD and degree of all nodes in the A network and the B network are calculated. Add a link between a pair of unconnected nodes with a negative IDD and the smallest degree. If no pair of nodes have negative IDD, a link is added between randomly selected unconnected nodes. Repeat this process until the number of added links is satisfied.

IDD-LDS: Calculate the IDD and the sum of degree of all nodes in the A network and B network at each step. Except that the degree sum is zero, a link is added between a pair of unconnected nodes with the smallest degree sum and a negative IDD. If no pair of nodes have negative IDD, a link is added between randomly selected unconnected nodes. Repeat this process until the number of added links is satisfied.

IDD-LDP: Calculate the IDD and the product of degree of all nodes in the A network and B network at each step. Except that the degree product is zero, a link is added between a pair of unconnected nodes with the smallest degree product and a negative IDD. If no pair of nodes have negative IDD, a link is added between randomly selected unconnected nodes. Repeat this process until the number of added links is satisfied.

2.3.3. Link adding strategy using IDD, degree, betweenness and harmonic closeness

Considering the higher the similarity of the network, the higher the robustness. The edge addition strategies proposed in this paper all consider the case where the IDD is negative. If there is no node with a negative IDD in the network, then randomly add a link among unconnected nodes. In order to obtain an effective edge addition order, comprehensively considering the degree, betweenness and harmonic closeness of the node, this paper defines two edge importance functions, which consider the sum and product of the information of interdependent nodes to calculate the priority of adding edges. The equation is as follows:

$$P\left(A_iB_i\right) = \alpha\left(\frac{k_{A_i}}{\sum k_{A_i}} + \frac{k_{B_i}}{\sum k_{B_i}}\right) + \beta\left(\frac{b_{A_i}}{\sum b_{A_i}} + \frac{b_{B_i}}{\sum b_{B_i}}\right) + \gamma\left(\frac{h_{A_i}}{\sum h_{A_i}} + \frac{h_{B_i}}{\sum h_{B_i}}\right), \tag{3}$$

$$P\left(A_iB_i\right) = \alpha\left(\frac{k_{A_i}k_{B_i}}{\sum k_{A_i}\sum k_{B_i}}\right) + \beta\left(\frac{b_{A_i}b_{B_i}}{\sum b_{A_i}\sum b_{B_i}}\right) + \gamma\left(\frac{h_{A_i}h_{B_i}}{\sum h_{A_i}\sum h_{B_i}}\right) \tag{4}$$

where α, β, γ are adjustable parameters, which is the weight of adjustment degree, betweenness, and harmonic closeness respectively. $k_{A_i}, b_{A_i}, h_{A_i}$ respectively represent the degree, betweenness, and harmonic closeness of node i in sub-network A. k_{B_i}, b_{B_i}, and h_{B_i} respectively represent the degree, betweenness, and harmonic closeness of node i in sub-network B. $\sum k_{A_i}$, $\sum b_{A_i}$, and $\sum h_{A_i}$ respectively represent the sum of the degree, betweenness, and harmonic closeness of node i in sub-network A. $\sum k_{B_i}$, $\sum b_{B_i}$, and $\sum h_{B_i}$ respectively represent the sum of the degree, betweenness, and harmonic closeness of node i in sub-network B. The betweenness of node i reflects the ratio of the number of the shortest paths throughout the node to the total number of shortest paths between all pairs of nodes, which is defined as:

$$b_{A_i} = \sum \frac{x_{A_mA_n(A_i)}}{x_{A_mA_n}} \tag{5}$$

where $x_{A_mA_n}(A_i)$ represents the number of shortest paths from node A_m to node A_n through node A_i, and $x_{A_mA_n}$ represents the total number of shortest paths from node A_m to node A_n. The harmonic closeness h_{A_i} of node A_i is defined as:

$$h_{A_i} = \frac{1}{N-1}\sum \frac{1}{d_{ij}} \tag{6}$$

Where N represents the number of nodes in the sub-network. d_{ij} represents the shortest distance from node i to node j.

The addition of interdependent edges is added in ascending order of priority. From Eqs. (3) and (4), fourteen kinds of addition strategies can be easily obtained, which are divided into seven categories, as shown in Table 1. Finally, we

Table 1
Edge addition strategies under different values of parameters.

α	β	γ	Information	Fusion	Strategy
>0	0	0	k	Sum	DS-IDD
0	>0	0	b	Sum	BS-IDD
0	0	>0	h	Sum	HS-IDD
>0	>0	0	k, b	Sum	DBS-IDD
>0	0	>0	k, h	Sum	DHS-IDD
0	>0	>0	b, h	Sum	BHS-IDD
>0	>0	>0	k, b, h	Sum	DBHS-IDD
>0	0	0	k	Product	DP-IDD
0	>0	0	b	Product	BP-IDD
0	0	>0	h	Product	HP-IDD
>0	>0	0	k, b	Product	DBP-IDD
>0	0	>0	k, h	Product	DHP-IDD
0	>0	>0	b, h	Product	BHP-IDD
>0	>0	>0	k, b, h	Product	DBHP-IDD

believe that the capacity of the node is moderate to discuss the robustness of the network. So we assume that the a_i = 0.5, λ = 1.2. For most nodes, when the degree of the node is high, we believe that its betweenness and harmonic closeness are also high, so we do not consider the influence of degree, betweenness and harmonic closeness of a certain range on network robustness. For simplicity, we consider $\alpha = 1$ in DS-IDD and DP-IDD strategies; $\beta = 1$ in BS-IDD and BP-IDD strategies; $\gamma = 1$ in HS-IDD and HP-IDD strategies; $\alpha = 1$ and $\beta = 1$ in DBS-IDD and DBP-IDD strategies; $\alpha = 1$ and $\gamma = 1$ in DHS-IDD and DHP-IDD strategies; $\beta = 1$ and $\gamma = 1$ in BHS-IDD and DHP-IDD strategies; $\alpha = 1$, $\beta = 1$ and $\gamma = 1$ in DBHS-IDD and DBHP-IDD strategies.

Network parameters are shown in Table 1.

3. Results

This paper mainly discusses the influence of different edge addition strategies on the robustness of BA–BA interdependent networks. Our simulation data comes from 20 different BA–BA interdependent networks with $N = 500$ and $\langle K \rangle = 2$, repeated the experiment 50 times. The error bars are not plotted because they are smaller than the resolution of the curves. P is the proportion of attacked nodes. The edge addition ratio f_{ab} proposed in this paper satisfies the following formula:

$$f_{ab} = M'/(M_a + M_b), \tag{7}$$

where M' is the number of edges added, M_a represents the sum of edges in subnet A, M_b represents the sum of edges in subnet B.

We mainly analyze the robustness of BA–BA interdependent networks with different coupling preferences in different edge addition strategies. First of all, we consider that the f_{ab} = 20%. Through the simulation diagram, we can see the effectiveness of our proposed edge addition strategy. Compared with the single-layer network edge strategy, we can see that our proposed strategy is generally better than the LB strategy, and is also better than the LD strategy in certain intervals, because the strategy we proposed can affect the similarity between two networks, and research shows that the similarity between sub-networks can affect the robustness of interdependent networks, which is the main reason why the proposed strategy is better than LB and LD. As shown in Fig. 2, when the coupling form of the interdependent network is AC and the edge addition strategy is not considered, the system collapses when $P = 0.3$. When the system considers the edge addition strategy, we can see that the robustness of the system has been greatly improved. When $P < 0.3$, that is, there are fewer attacked edges, the HS-IDD strategy is more effective in the BA–BA interdependent network with the coupling form of AC. This is because the more efficient nodes have higher network centrality. In this case, when the edges between nodes with smaller h are added, the network connectivity is greatly improved, and the network is combined into a new component. But as the number of attacked nodes increases, such nodes are more likely to be affected by the remaining invalid edges, so the effect of adding edges becomes weaker. In DS-IDD and BS-IDD, for the BA network, there are many edges connected between low-degree (betweenness) nodes and high (betweenness) nodes. When P increases to a certain value, these edges are likely to be removed. Therefore, as P increases, such a strategy is more efficient, and it will make the network form more small components. It can be seen from the figure that when P increases to about 0.35, the G value obtained by DS-IDD and BS-IDD will be greater than other edge addition strategies. The results show that in the presence of a large number of attacked edges, DS-IDD and BS-IDD edge addition makes the interdependent BA network more robust.

From Figs. 2(b) and 2(c), we can see that when the BA–BA network is coupled by RC or DC, the HS-IDD strategy is more effective than other strategies. This is because in the case of RC or DC networks, the harmonic closeness of high-degree nodes is generally large, and low-degree nodes connected to high-degree nodes also have high-harmonic closeness, so it is easy to cause the high degree nodes in network B to fail when attacking low-degree nodes in network A. Therefore,

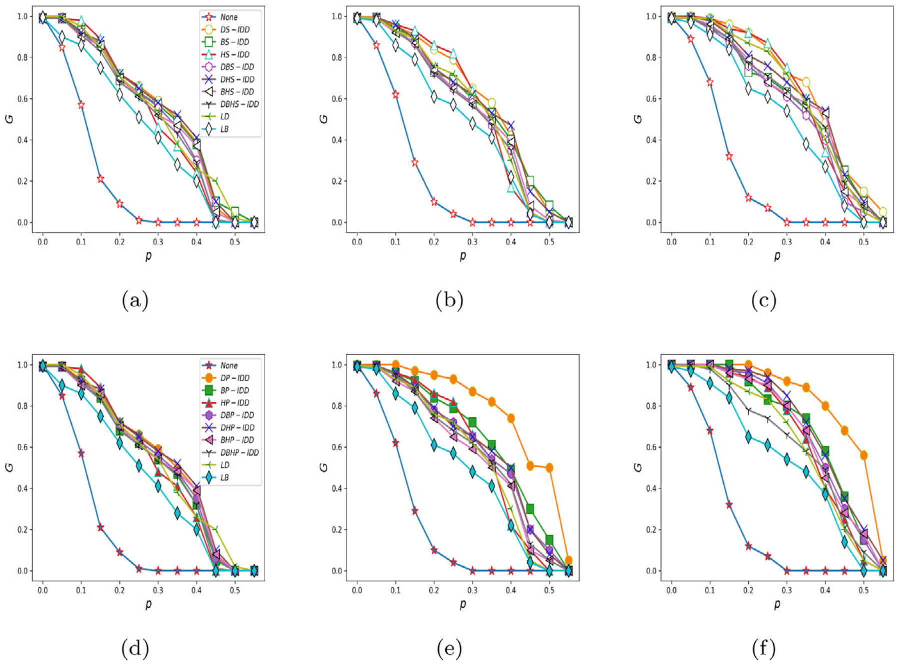


Fig. 2. The relative size of the giant component G is shown as a function of the fraction P of removed nodes under add edge strategies concerning the sum of the information between net works with AC (a), DC (b), and RC (c), and under the ones concerning the product of the information between networks with AC (d), DC (e), and RC (f), f_{ab} = 20%.

in this case, the strategy can improve the robustness of the network better. Similarly, when p is small, for DS-IDD that only reflects the degree of information in the network, the edge between nodes with a high degree of recovery may be restored, but because there are few nodes in the giant component, its improvement of robustness is lower than HS-IDD strategy.

From Fig. 2(d), we can find that when the BA–BA network is coupled by AC, the law of all strategies is similar to the strategy where the information fusion method is sum. However, when the BA–BA network is coupled by DC or RC, when the information fusion method is product, we can see from Figs. 2(e) and 2(f) that the edge addition strategy of DP-IDD is more effective. Under RC and DC, if the information fusion method is production, it will cause the order of the result of the production to be different from that of the addition. For some interdependent links with large differences in terminal node degrees, the failure of low-degree nodes may cause large-scale cascade failure. This means that the effect of the edge addition strategies depends on the connected edges with large differences in node information. On the contrary, under RC and DC, for the interdependent links with similar degrees of end nodes, the high node will almost cause the failure of another network under the condition of priority attack, which means the effectiveness of attacking such edge and edge addition strategies are irrelevant.

In addition, we can see from the figure that regardless of the information fusion method, the betweenness strategy is relatively inefficient in the BA network with coupling preferences of AC, DC, and RC. Because the betweenness mainly reflects the number of shortest paths through the node. However, the lack of one path can be replaced by other paths, which does not affect the connectivity of the network, and for nodes with low degrees, the betweenness may be 0. So there is no other strategy to use betweenness information to add edges and the edge addition strategy using betweenness is not as effective as others.

Through the analysis of various edge addition strategies, we can see that the way of fusing information has greatly improved the robustness of the network.

In order to study the relationship between the coupling preference between networks and the way of information fusion, we compare the attack strategy with the sum and product of information between networks under AC, DC and RC conditions. As shown in Fig. 3, an interesting phenomenon is that under DC and RC, the attack strategy for the product of information between networks is more effective than the attack strategy for the sum of information between networks. In addition, the edge addition strategy has more obvious observations in terms of degree and betweenness. This is because in the case of adding information, the edge addition strategy focuses more on the nodes with a larger centrality measure, while ignoring the information of their corresponding interdependent nodes.

The results show that when the coupling form is AC, since the nodes of similar degree in network A and network B are connected to each other, the increase or multiplication of information between the networks hardly changes the order of the adding edges. Therefore, it can be clearly seen from Fig. 3 that in the BA network interdependent with AC,

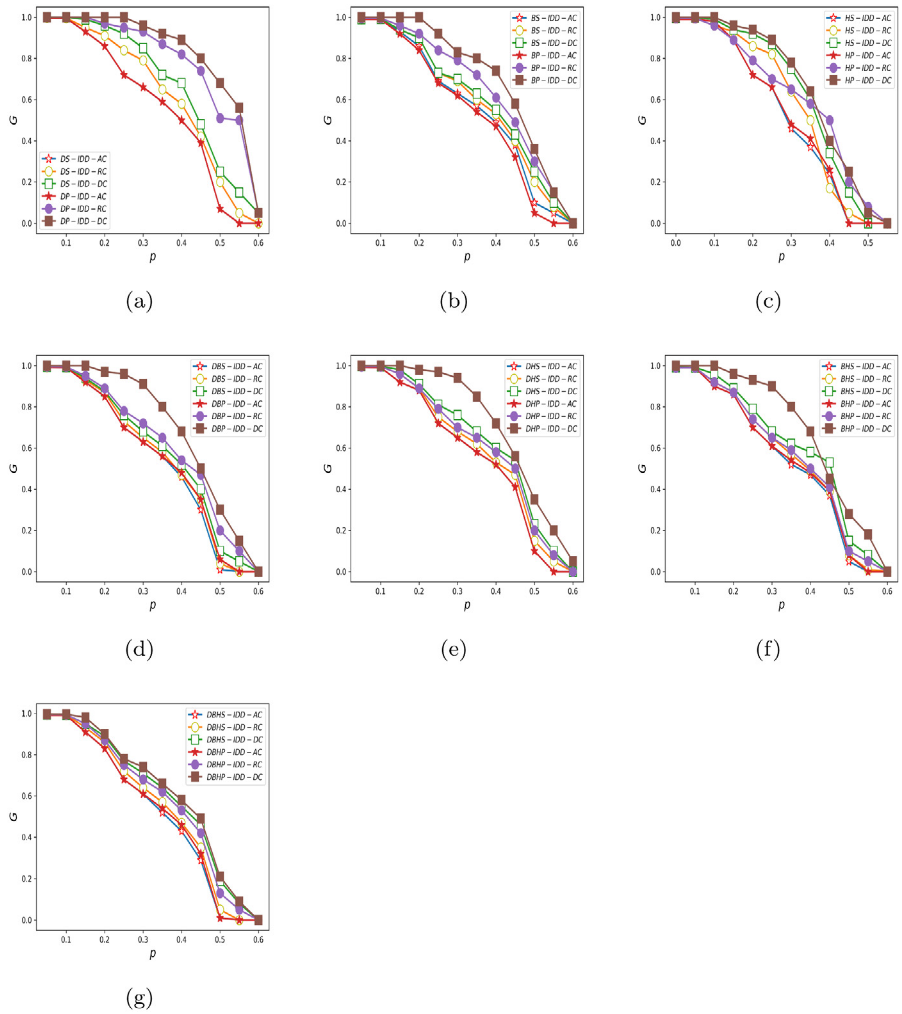


Fig. 3. The relative size of the giant component G is shown as a function of the fraction P of removed nodes in interdependent BA networks with AC, DC and RC under DS-IDD and DP-IDD (a), BS-IDD and BP-IDD (b), HS-IDD and HP-IDD (c), DBS-IDD and DBP-IDD (d), DHS-IDD and DHP-IDD (e), BHS-IDD and BHP-IDD (f), DBHS-IDD and DBHP-IDD (g), f_{ab} = 20%.

there is little difference in the edge addition strategy of information fusion between networks into sum and product. And when several information fusions are considered comprehensively, the edge addition strategy of the sum and product of information between networks is also similar.

In addition, it can be seen from Fig. 3 that regardless of the edge addition strategy, the edge addition strategy of the interdependent BA network with the coupling mode of DC is the most efficient, and the interdependent BA network with the coupling mode of AC has the lowest efficiency. This is because in the case of AC, attacking few nodes can also cause great cascading failure, and the speed of adding edges cannot keep up with the speed of cascading failures. On the contrary, in DC, attacking a node with a large centrality measure in network A is prone to failure due to more edges and neighboring nodes with higher attack priority, but there are many key nodes in B that are not prone to failure. The whole process is in a relatively slow cascade failure process, so the edge addition strategy is more effective.

In order to explore the impact of the number of edges added on the robustness of the interdependent network, and whether the law of edge addition strategy is contingent, we increase f_{ab} to 40%. The specific simulation results are shown in Fig. 4. It is easy to see the superiority of our proposed strategy, and it also has advantages compared with traditional LB and LD strategies. From the figure, we can see that as f increases, the robustness of the system is improved. Compared with f = 20%, we find that the impact of each strategy on the robustness is not accidental, and the difference between the strategies is more obvious. However, with the increase of P, we found that adding more edges could not resist cascading failures more effectively, because the connected slices generated by adding edges were smaller than the impact caused by cascading failures. Thus, it is no longer recommended to continue adding edges.

Similarly, we compare the attack strategy with the sum and product of information between networks under AC, DC and RC conditions. The results are shown in Fig. 5. We can clearly see that with the increase of f_{ab}, the robustness of

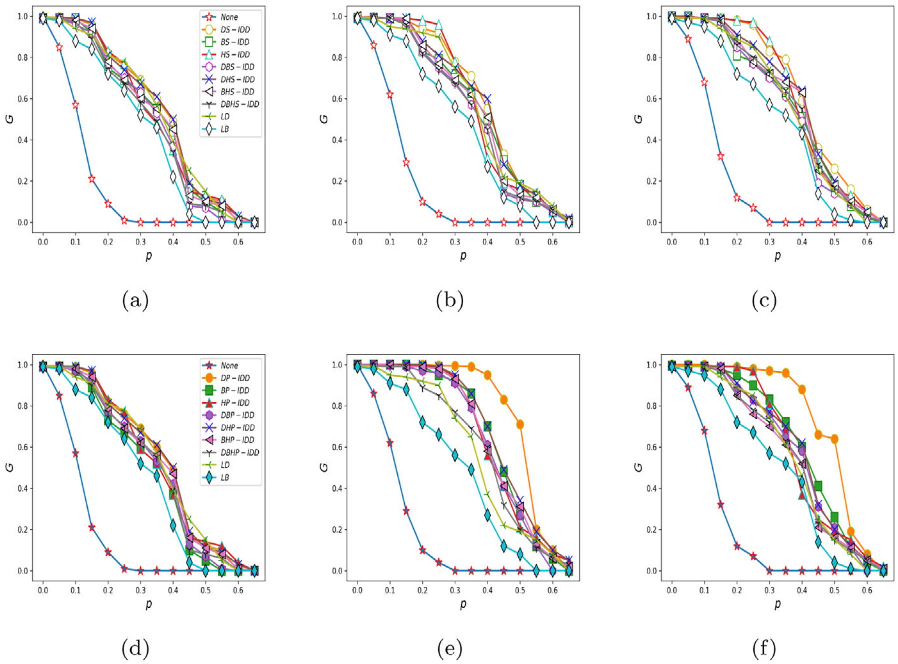


Fig. 4. The relative size of the giant component G is shown as a function of the fraction P of removed nodes under add edge strategies concerning the sum of the information between net works with AC (a), DC (b), and RC (c), and under the ones concerning the product of the information between networks with AC (d), DC (e), and RC (f), f_{ab} = 40%.

the network has been improved, and the order of influence brought about by various strategies is almost unchanged. The effect of the same coupling type network under two different information fusion methods becomes larger. To further explore the influence of the edge addition ratio on the BA–BA scale-free interdependent network, we separately propose the HS-IDD strategy to study the impact of different edge addition ratios on the robustness of the system, because we believe this strategy is more effective. The results is shown in Fig. 6. We can see that the robustness of the network is positively correlated with the edge addition ratio. However, when the system robustness is very strong or extremely poor, the increase in the ratio of edges will have a smaller impact on the robustness of the system. In this case, it is not recommended to increase the ratio of edges to improve the robustness of the network.

Considering that the system scale as an uncertain factor may affect our experimental results, we assume that the proposed method will be simulated under the premise that the system scale is 100, 300, and 500 respectively. To compare and verify with the previous article, the selected networks are all BA interdependent networks with a coupling form of AC and an average degree of 2. The specific simulation diagram is shown in Fig. 7.

From Fig. 7, we can see that the size of the network has almost no effect on the effects of each strategy, which is consistent with the results obtained in Refs. [33,39,40], which also reflects the accuracy of our experimental results. In this part, we choose a new indicator AUC (A) to describe the overall performance of the network. From this value, as the network scale increases, the overall performance of the network gradually increases, but it has no effect on the pros and cons of each strategy.

Taking into account the cascading failure phenomenon on the actual network, we take the power outage that occurred in Italy in 2009 as an example to study the influence of the edge addition strategy proposed in this article on the cascading failure process [40]. In the Italian power grid, each power plant and substation are regarded as a node in the network, with 310 nodes and 161 edges, including 97 source nodes, 113 load nodes, and 100 junction nodes. The Internet responsible for communication has 39 nodes and 58 edges. The interdependent edges of the Internet and the power grid are only established on load nodes, and a communication node can only depend on a node in power grid. The connection probability of the two is positively correlated with the distance between them. Due to the privacy of the information communication network, it is difficult to obtain a specific topological diagram of dependencies, so we connect the edges according to the edge connection method in ref [17]. The specific network topology is shown in Fig. 8.

The cascading failure simulation results of the strategy proposed in this paper under the Italian power grid are shown in Fig. 9. From the simulation diagram, we can see that when the information fusion method is addition, when the proportion of attacks is small, the HS-IDD strategy has the best edge addition efficiency. As the proportion of attacks increases, the marginal efficiency of the DS-IDD strategy and the LD strategy gradually increases. When the information fusion method is multiplication, the efficiency of most edge addition strategies shows an increasing trend, and the efficiency of the edge addition strategy of DP-IDD is the highest. The simulation analysis shows that the cascading failure analysis under the

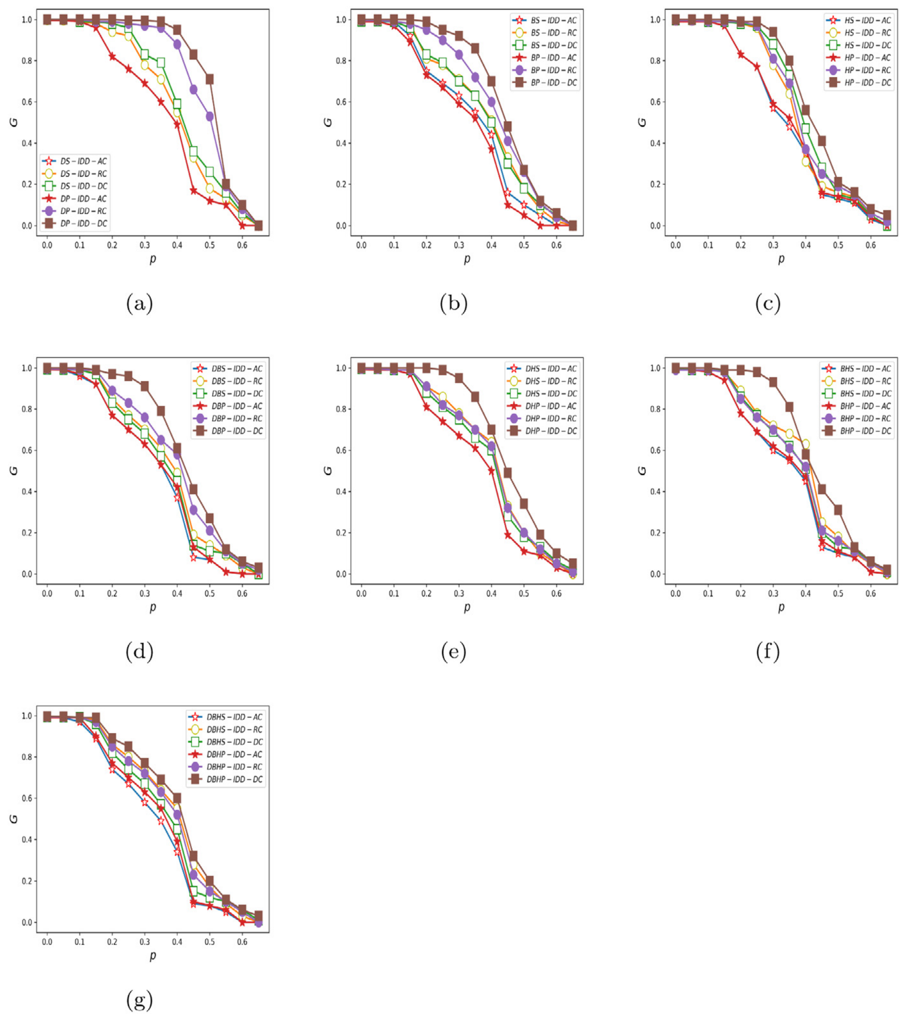


Fig. 5. The relative size of the giant component G is shown as a function of the fraction P of removed nodes in interdependent BA networks with AC, DC and RC under DS-IDD and DP-IDD (a), BS-IDD and BP-IDD (b), HS-IDD and HP-IDD (c), DBS-IDD and DBP-IDD (d), DHS-IDD and DHP-IDD (e), BHS-IDD and BHP-IDD (f), DBHS-IDD and DBHP-IDD (g), f_{ab} = 40%.

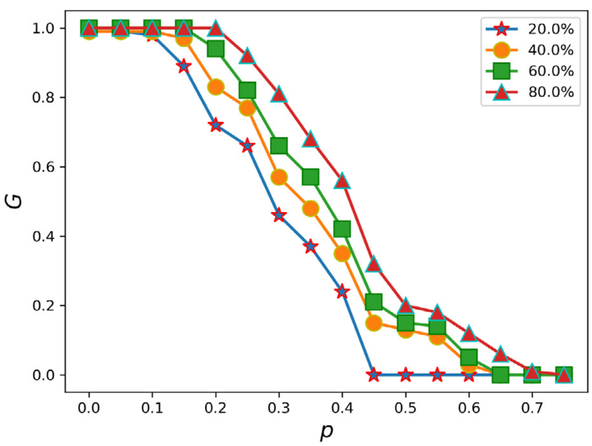
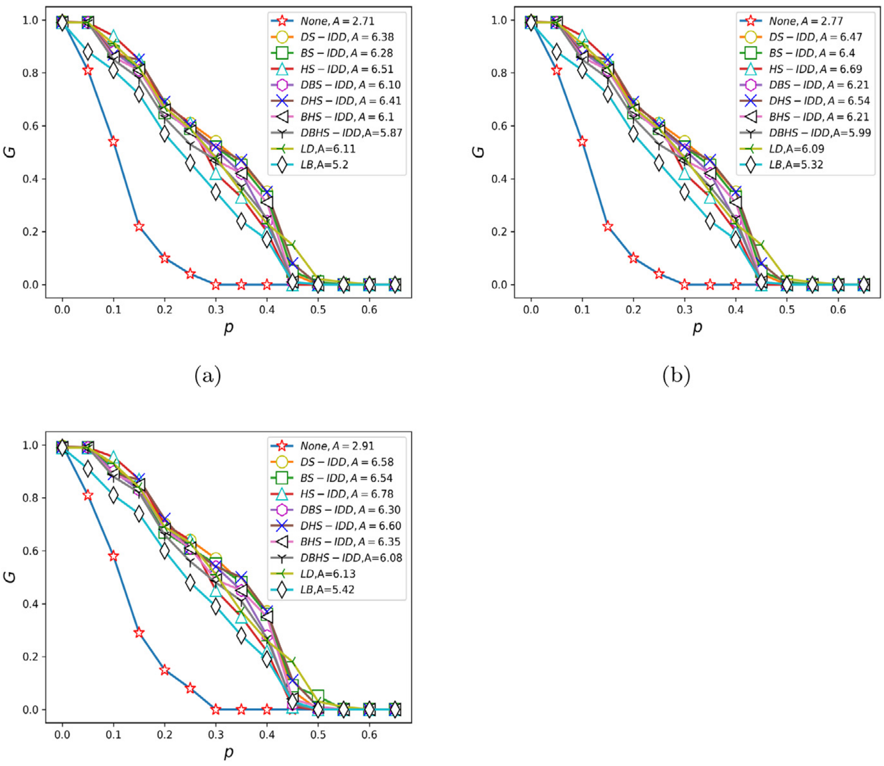


Fig. 6. The impact of HS-IDD strategy on the robustness of AC-coupling BA scale-free network under different edge addition ratios.



(a) (b) (c)

Fig. 7. Network invulnerability diagrams under different network scales, f_{ab} = 20% **(a)** Network size is 100. **(b)** Network size is 300 **(c)** Network size is 500.

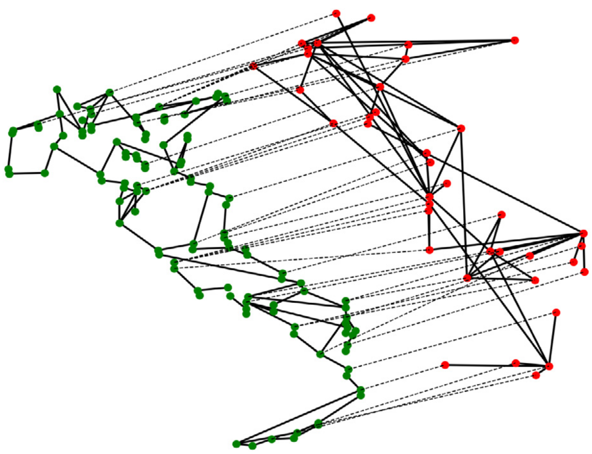
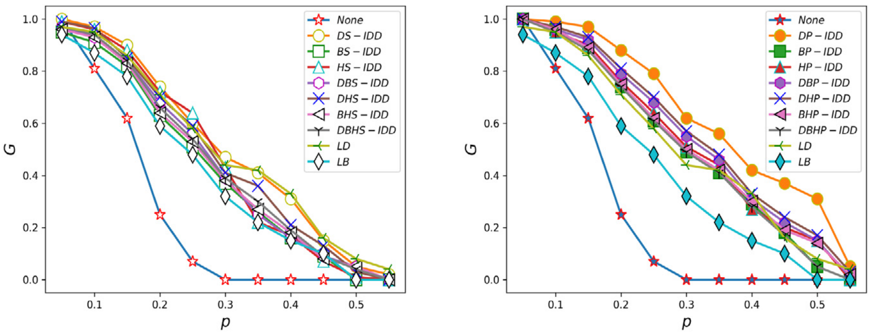


Fig. 8. Schematic diagram of the Italian blackout. The green nodes represent load nodes in the Italian power grid, and the red nodes represent Internet nodes for communication. The interdependence between the networks is represented by dashed lines, and the interconnections within the networks are represented by solid lines.

Italian power grid is similar to the BA–BA interdependent network with the coupling mode of RC. However, the effect difference between the strategies becomes smaller. This is because there is a one-to-many situation in the Italian power grid and the power law exponent is small. The simulation results show that the proposed strategy is equally effective in resisting cascading failures in actual networks.



(a) (b)

Fig. 9. Diagram of damage resistance under different edging strategies in Italian power grid, f_{ab} = 20%.

4. Discussion

This paper proposes 14 edge addition strategies by two edge importance functions, and analyze the edge addition strategy in the BA–BA interdependent network of AC, DC and RC. Comparing with the traditional single-layer network link-addition strategy LD and LB, we have the following conclusions:

(1) In the case of deliberate attacks, compared with the BA–BA scale-free interdependent network that does not consider the edge addition strategy, our proposed strategy can greatly improve the robustness of the network. Compared with the traditional single-layer network edge-added strategy LB and LD, we find that our proposed strategy is generally better than the LB strategy, and in some cases better than the LD strategy. This is because the edge-added strategy we proposed can increase the similarity of the two sub-networks, and the effect of the LB strategy is slightly lower. Because the betweenness mainly reflects the number of shortest paths through the node. However, due to the lack of one path, it can be replaced by other paths, which does not affect the connectivity of the network to a certain extent. This is consistent with the performance of other strategies in this article.

(2) The simulation results show that the HS-IDD has the highest efficiency when the number of attacked nodes in the system is small. With the increase of the attack ratio, the DS-IDD gradually increases, and the effect is more obvious in DC and RC. This is because the harmonic closeness of high-level nodes is generally large, so low-level nodes connected to high-level nodes also have high-harmonic closeness. Therefore, when attacking low-level nodes in the A network, it is easy to cause the height in the B network fails, so this strategy can improve the robustness of the network. Through the analysis of various edge addition strategies, we can see that the way of fusing multiple information has greatly improved the robustness of the network. And DS-IDD, HS-IDD, DP-IDD, HP-IDD are better than other strategies in effect. The edge addition strategy on betweenness is inferior in effect.

(3) When the information fusion method is multiplication, the effect of the edge addition strategy will be better. In the interdependent BA network with the coupling form of AC, the difference between the sum of information and the product edge addition strategy between networks is not obvious, but in DC With RC, the edge addition strategy for the product of information between networks is more effective than the attack strategy for the sum of information between networks. This is because the increase or multiplication of information between networks will change the order of edges being added. In this case, edges with similar network topology elements in the network are more likely to be added, and the similarity between sub-networks is improved.

(4) With the increase of the edge addition ratio, the robustness of the system has been greatly improved, and the effect of the same coupling form of the network under the two different information fusion methods has become larger, but the order of influence brought about by various strategies is almost unchanged.

In this paper, we mainly study the impact of the proposed strategy on the robustness of the BA–BA interdependent network with a sub-network size of 500, which has certain limitations. However, considering that the scale-free network is a very large branch of the actual network, we believe that our proposed strategy has certain desirability. At the same time, it is commendable that our conclusions are general to BA scale-free networks, since they are almost independent of the size of the network. In the follow-up research, we will conduct network dynamics research for networks of different natures, and introduce actual networks of different sizes for discussion. In summary, we believe that the research results have guiding significance for the identification and protection of critical edges and the design of interdependent scale-free networks.

CRediT authorship contribution statement

Chao-Yang Chen: Designed the research, Executed the experiments, Prepared the figures, Wrote the manuscript, Analyzed the results, Edited this paper, Reviewed the manuscript. **Yang Zhao:** Designed the research, Executed the experiments, Prepared the figures, Wrote the manuscript, Analyzed the results, Edited this paper, Reviewed the manuscript. **Huanmei Qin:** Designed the research, Analyzed the results, Edited this paper, Reviewed the manuscript. **Xiangyi Meng:** Designed the research, Analyzed the results, Edited this paper, Reviewed the manuscript. **Jianxi Gao:** Designed the research, Analyzed the results, Edited this paper, Reviewed the manuscript.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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