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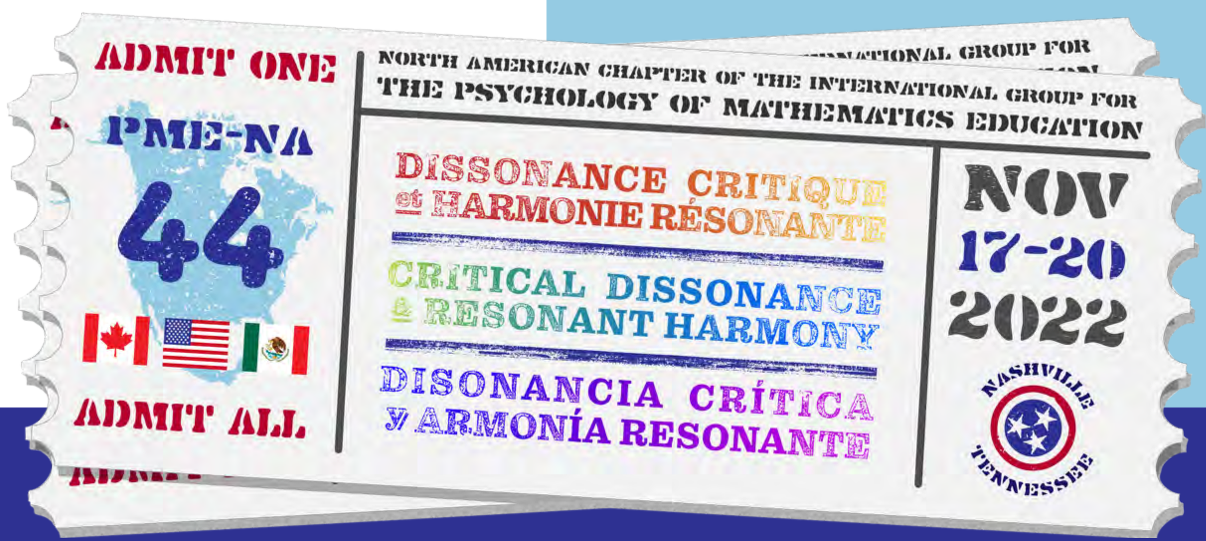
CRITICAL DISSONANCE AND RESONANT HARMONY



EDITED BY

Alyson E. Lischka
Elizabeth B. Dyer
Ryan Seth Jones
Jennifer N. Lovett
Jeremy Strayer
Samantha Drown

MIDDLE TENNESSEE STATE UNIVERSITY

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Critical Dissonance and Resonant Harmony

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Editors:

Alyson E. Lischka
Middle Tennessee State University
Alyson.Lischka@mtsu.edu

Elizabeth B. Dyer
University of Tennessee – Knoxville
edyer@utk.edu

Ryan Seth Jones
Middle Tennessee State University
Ryan.Jones@mtsu.edu

Jennifer N. Lovett
Middle Tennessee State University
Jennifer.Lovett@mtsu.edu

Jeremy Strayer
Middle Tennessee State University
Jeremy.Strayer@mtsu.edu

Samantha Drown
Middle Tennessee State University
DrownS@rcschools.net

UNDERSTANDING PRESERVICE TEACHERS' NOTICING OF ONLINE TEACHING

Jinqing Liu	Orly Buchbinder	Merav Weingarden
University of New Hampshire	University of New Hampshire	University of New Hampshire
jinqing.liu@unh.edu	orly.buchbinder@unh.edu	merav.weingarden@unh.edu

The rapid move to online teaching brought about by the global pandemic highlighted the need for the educational research community to develop new conceptual tools for characterizing these environments. In this paper, we propose a conceptual framework Instructional Technology Triangle (ITT) which extends the instructional triangle of teachers, students, and content to include technology as a mediating mechanism. We use the ITT framework to analyze noticing patterns in the written reflection of a prospective secondary teacher, Nancy, who, over the course of one semester taught online four lessons integrating reasoning and proof . The fluctuations in Nancy's noticing patterns, in particular, with respect to technology, shed light on her trajectory of learning to teach online and the role of reflective noticing in this process. We discuss implications for teacher preparation and professional development.

Keywords: Teacher Noticing, Preservice Teacher Education, Online and Distance Education.

Objectives

The pivotal role of technology in teaching mathematics has been widely recognized and thoroughly researched over the last decades (Ball et al., 2018; Bray & Tangney, 2017; Clark-Wilson et al., 2020; Hillmayr et al., 2020). These studies concerned teachers' classroom practice with technology, teachers' professional development (PD) for teaching mathematics with technology and impacts of technology on mathematics teaching and learning (see Clark-Wilson et al., 2020 for an extensive review). Most of these studies were conducted in a traditional face-to-face setting. Some teaching online has been addressed, but mainly in teacher PD and Massive Open Online Courses (MOOC) (e.g., Taranto et al., 2020). However, the complete and widespread shift to online teaching brought about by the global pandemic caught teachers, students and educational communities off guard (Seaton et al., 2022). In particular, the move to online teaching revealed its the uniqueness in the technology landscape. It also highlighted the need for the educational research community to develop new conceptual and analytical tools for characterizing teaching and learning that occurs in online environments.

In this paper, we propose a conceptual framework *Instructional Technology Triangle (ITT)* which extends the seminal notion of the instructional triangle, which conceptualizes instruction as interactions between teacher and students around particular content, situated in certain environments and time (Cohen et al., 2003; Lampert, 2001). In the context of online teaching and learning, technology becomes both the environment enabling the educational process to occur and the necessary mediator between teachers, students, and content. The ITT framework captures these relationships by placing technology within the instructional triangle.

We developed the ITT framework to capture and characterize noticing patterns of prospective secondary teachers (PSTs) as they reflected on video recordings of their own online teaching. The PSTs participated in the capstone course *Mathematical Reasoning and Proving for Secondary Teachers* where they designed and taught lessons that integrated reasoning and proving within the regular mathematics curriculum (Buchbinder & McCrone, 2020). Due to the pandemic, in Fall 2020, the school teaching component moved online. The PSTs taught via *Zoom*, uploaded the recording to *Canvas* Learning Management System, where they watched and reflected on their teaching. The analysis focused on PSTs' noticing, which is an essential professional skill (Buchbinder et al., 2021; Mason, 2002; Kosko et al., 2021; Sherin et al., 2011). As we analyzed our PSTs' noticing, we were struck by the prevalence of technology-related aspects, their variety, and their change over time. To capture the richness of these data, we developed the ITT framework.

In this paper, we introduce the ITT framework and illustrate its use by analyzing one PSTs' (Nancy, a pseudonym) noticing patterns in four online lessons. We aim to *describe and characterize Nancy's noticing patterns as a way to understand how her learning to teach online evolved over time*. In addition, we triangulate data from multiple sources, such as Nancy's lesson plans, written essays and course materials, to understand possible reasons behind fluctuations in her noticing patterns, in particular with respect to the role of technology.

Conceptual Framework

Reflective Noticing

The concepts of noticing and reflection have been closely intertwined in educational literature dealing with teacher learning and professional growth (Seidel et al., 2011; Moore-Russo & Wilsey, 2014). Amongst many definitions of teacher noticing (Dindyal et al., 2021), in this study, we adopt Stockero's (2021) definition of noticing as comprised of *attending to* aspects of the classroom situation and *interpreting* them. In this study, we consider PSTs' noticing of their own teaching, as they reflect on their own classroom teaching, and use the term *reflective noticing* to describe a process that combines the tacit nature of attending and the goal-oriented nature of reflection (Buchbinder et al., 2021; Liu & Buchbinder, in press).

Teachers' reflective noticing happens with a specific context and is affected by many factors, such as individuals' knowledge, belief, experience (Schoenfeld, 2011), identity (Oyserman, 2009), and instructional practices (Liu et al., 2021; Sherin et al., 2009). Cross Francis et al. (2021) found that teachers' mathematical knowledge for teaching, efficacy, belief, emotions, and identity influenced teachers' noticing of and reflection on students' mathematics thinking.

Reflective noticing supports teachers' professional development because it engages teachers in regulating their attention resources to the teaching aspects that they perceive as important and allowing them to prioritize these aspects. Learning from reflection requires careful and critical deliberation on practice, connecting to theoretical ideas and contemplating takeaways for the future (Anderson, 2019; Wilson, 2008). Thus, it is not surprising that PSTs' reflection on the video-recordings of their own teaching was found to be beneficial to PSTs' professional learning (Buchbinder et al., 2021; Liu & Buchbinder, in press; Walshe & Driver, 2019).

The Instructional Technology Triangle Framework

The instructional triangle is a heuristic structure for describing and characterizing teacher-student-content relationships (Friesen & Osguthorpe, 2018). It highlights the nature of teaching as mutual interactions among teachers, students, and content in environments (Cohen et al., 2003; Herbst & Chazan, 2012; Lampert, 2001; Yeo & Webel, 2017). In a face-to-face setting where teachers or students interact with technology, technology can be understood as a tool or a part of the environment. In an online environment, it is impossible to avoid treating technology as a key factor in learning and instruction. Content is represented through technology, and teachers and students interact with technology and interact with each other through technology (Seaton et al., 2022). Technology is what mediates between teachers-students-content, making the educational process possible. We extend the instructional triangle framework by adding technology as an explicit component of instruction (Figure 1) to foreground the critical role of technology in an online teaching environment.

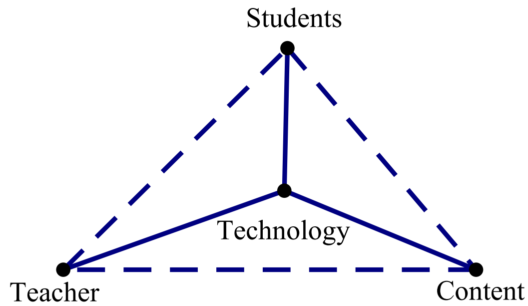

Figure 1: Instructional Technology Triangle Framework

The Instructional Technology Triangle (ITT) framework (Figure 1) considers four nodes: teacher, student, technology, and content and six sets of pair-wise relationships: student-content, student-technology, teacher-technology, teacher-student, teacher-content, content-technology. In the context of reflective noticing, we define the following ten categories. **Teacher -** considers teacher's reflection on their personality, behavioral characteristics like a voice pitch, thoughts, and self-impressions. **Students** - refers to noticing students' behavior and personalities, classroom participation and interactions with peers. **Content** refers to teacher reflection on the mathematics of the lesson and/or a rationale for including a certain content in the lesson. **Technology** involves reflecting on technology as a tool, without relating it to teaching moves or students' mathematical thinking. Noticing of **Teacher-Content** refers to teacher reflection on how they taught a content or made an instructional decision related to content. **Teachers-Students** noticing indicates attending to interactions between the teacher and the students. **Student-Content** describes reflecting on students' mathematical thinking while interacting with or responding to a mathematical question or task. **Content-Technology** refers to teacher's reflection on how technology is useful or not in representing a particular mathematical content; while **Teacher-Technology** describes teacher noticing of manage technology for effective teaching. **Student-Technology** captures the teacher's noticing of students' interaction with technology. We illustrate these categories in depth in the results section.

Methods

The Setting

This study is a part of the larger project that designed a capstone course *Mathematical Reasoning and Proving for Secondary Teachers* and studied how PSTs' mathematical and pedagogical knowledge develops during the course (Buchbinder & McCrone, 2020). The four course modules focused on the following proof themes: (1) direct proof and argument evaluation; (2) conditional statements, (3) quantification and the role of examples in proving, and (4) indirect reasoning. In each module, the PSTs strengthened their subject matter knowledge of the proof themes, learned about students' proof-related (mis)conceptions and then designed and taught to small groups of students from local schools a 50-minute lesson that integrates a particular proof theme with an ongoing topic from the school curriculum. Then, the PSTs viewed the 360-video recording of their lesson and reflected on it (Buchbinder et al., 2021). In Fall 2020, due to the pandemic, the PSTs taught their four lessons online, via *Zoom*.

Nancy was a senior mathematics education major in a high school certification track. She had robust mathematical knowledge and educational orientation, as evidenced in her high GPA, and strong performance on pre-and post-course assessment on Mathematical Knowledge for Teaching Proof and Dispositions toward Proof survey (Buchbinder & McCrone, 2021). We chose to analyze Nancy's reflective noticing since it was elaborated and rich in detail.

Data Sources and Analytic Techniques

The main data source for this paper is Nancy's reflection reports she completed on *Canvas* Learning Management System 2-3 days after teaching the lesson. For this report, Nancy watched the video of her lesson, and used the commenting feature to write a reflective comment about every five minutes, about 8-9 comments per lesson. These comments provide information about Nancy's noticing in the moment of watching the video. We analyzed Nancy's reflective comments using the Instructional Technology Triangle coding scheme. Each comment was examined for the presence of a particular code; if needed, long comments were divided into shorter thematic units and assigned separate codes. The first two authors coded the data, discussed and resolved any discrepancies. Next, we examined the distribution of codes across the coding categories and four lessons (Table 1) in conjunction with data from supplemental sources: Nany's lesson plans, reflective essays submitted along with the *Canvas* comments, and a summative essay about the course. We also relied on the course syllabus and instructional materials (the second author was the course instructor) to construct narratives explaining Nancy's noticing patterns as she participated in the sociocultural contexts of the capstone course and of the online student-teaching embedded in it.

Results

Table 1 summarizes the distribution of codes of the ITT framework identified in Nancy's reflective comments across the four lessons. The total number of codes per lesson is specified in the heading; the modal response in each column is highlighted. We briefly describe each lesson and identify the key patterns in Nancy's noticing, followed by an interpretive analysis of these patterns.

Table 1: The Distribution of Nancy's Noticing Across ITT Coding Categories and Lessons

	Lesson 1 (*N* =14)	Lesson 2 (*N* =13)	Lesson 3 (*N*=12)	Lesson 4 (*N* =16)	Total (N)
Teacher-content	36%	15%	8%	31%	13
Teacher-Student	7%	23%	17%	25%	10
Teacher-Technology	14%	8%	17%	13%	7
Student-content	14%	0%	25%	0%	5
Technology	0%	15%	8%	6%	4
Teacher	21%	0%	0%	6%	4
Student-Technology	0%	8%	0%	13%	3
Content-Technology	0%	8%	0%	6%	2
Student	0%	8%	0%	0%	1
Content	0%	0%	8%	0%	1

Lesson 1. Direct Proof and Argument Evaluation: Supplementary and Vertical Angles.

Nancy started by engaging students in a discussion of "what makes a good two-column proof?" Next, she facilitated an exploration of the Vertical Angles Theorem (VAT) where she used *GeoGebra* to manipulate intersecting lines and had students observe the changing measures of vertical angles. Nancy guided students to notice that the vertical angles remained congruent.

In this lesson, Nancy was the only one engaging with GeoGebra while students observed and made conjectures verbally. Then, Nancy guided students through writing two-column proof of the VAT first with specific numeric values of the angles and then using variables.

Table 1 shows that Nancy mostly noticed the teacher-content relation (36%), and her own actions (teacher, 21%). This pattern is not surprising considering that this is the first time Nancy taught a lesson online and her first time watching her own teaching and reflecting on it. For example, Nancy noticed her speaking habits, e.g., “I am saying ‘um’ a lot,” and her fast speaking pace and reflected on the need to be aware of and improve her speech: “I need to slow down and not rush through explanations. I'm naturally a fast talker and when I get nervous, I tend to talk even faster … I need to work on slowing it down in the future.” Nancy seldom returned to these points in the following lessons, suggesting that the heightened focus on herself was due to the novelty of video reflecting experience.

Based on the prevalence of teacher-content codes (36%), Nancy’s main concern in this lesson was making sure the lesson ran smoothly and achieved the teaching goals. These included engaging students in mathematical discourse to collectively construct a two-column proof of the VAT, develop criteria for a “good” two-column proof, and have students recognize there are multiple ways to prove a theorem. Nancy constantly monitored her progress towards these goals and justified her instructional decisions. For example, Nancy noticed her effective teaching practice of developing a list of ideas for proof before writing the two-column proof as a way to address a common student challenge of completing a proof. She wrote:

> As we worked through the proof, I asked the students questions and tried to guide their thinking. I brought their attention to our ideas that we developed earlier on, and they were able to see that we could use two of those expressions to help us with our proof. I think this is important because oftentimes when faced with a proof, it can be challenging to come up with the next step, but if you have already come up with ideas before writing the proof, it can make writing the proof a little easier.

Nancy also criticized some of her teaching choices like having a pre-labeled diagram which, in her words, “defeated the purpose” of having “the students realize that we needed to generalize the angles to prove all cases”. Nancy also identified areas of improvement, like managing discussions in ways that increase student mathematical engagement.

> One thing that I noticed during this period was that I didn't really ask the students for their input about generalizing the proof. I just told them that we need to generalize the proof to make sure that we show all cases of vertical angles. After doing this lesson one of my revisions is to do less telling and have the students try coming to the conclusion on their own.

Fourteen percent of Nancy’s comments concerned teacher-technology, which is not surprising given this was her first time teaching ever, and online. Nancy was familiar with *GeoGebra* software but struggled to show multiple screens on Zoom: “One teaching move that I found challenging occurred when I wanted to keep the problem up, but also wanted to show the list of ideas that the students were coming up with”. Student interactions with technology did not show up in Nancy’s noticing possibly because she was the only one manipulating GeoGebra in this lesson.

Lesson 2: Conditional Statements: Isosceles and Equilateral Triangles

Nancy used *Prezi* with multiple examples to introduce the concepts of conditional statement and its converse. Then, students used *GeoGebra* to explore three conditional statements about triangles. For each statement, students were to determine if it is true or false (if false, construct a counterexample), then write the converse and determine whether the converse is true or false.

After the first lesson Nancy seemed to gain comfort with teaching online and shifted the mode of student engagement by having them interact with *GeoGebra*. This is reflected in the shifts in Nancy's noticing patterns toward increased focus on interaction with students (23%). She wrote:

> While going over the conditional statement "if a number is divisible by 10, then it is divisible by 5" I asked the students if they thought the statement was true or false. One student responded that he thought it was false. Instead of saying that his answer was incorrect, I asked him why he thought that and as he started to explain, I realized he was talking about a different conditional statement. I'm glad that I asked him to clarify because otherwise I wouldn't have known that he was talking about a different statement.

By following up on students' answer, rather than discarding it as incorrect, Nancy inferred from the student explanation that he offered a correct response to a different question. Nancy then was able to steer this interaction towards the lesson goals.

An important trend of Nancy's noticing in lesson 2 is the presence of all technology-related codes. Nancy reflected how she transitioned from one technology to another, e.g., "I think the transition from the *Prezi* presentation to *GeoGebra* was pretty smooth and I think I did a nice job at explaining the key aspects of GeoGebra" (technology code, 15%); how the use of technology benefited her teaching, e.g., "I really like having exit tickets because it gives me really good feedback about what the students learned and how they felt with different parts of the lesson" (teacher- technology 8%). Nancy also noticed students' difficulty interacting with each other on *Zoom* (student-technology, 8%):

> When I told the students that they could work together, no one did. I think zoom makes this hard because it's not like you can turn to your neighbor and discuss. Instead, if you want to talk, you end up talking in front of everyone which can make people nervous. Also, as a student it can seem daunting to start up a discussion with your peers. This is something I need to keep in mind for the future.

The patterns of Nancy's noticing are consistent with her use of diverse technological tools (*Prezi, GeoGebra, Google forms*) and with having students interact with *GeoGebra* directly.

Lesson 3: Quantification and the Role of Examples: Triangle Similarity Theorems

Nancy first used *Google Slides* to introduce the concepts of universal and existential statements and the role of examples in proving/disproving them. She had students find counterexamples to three universal statements (e.g., All isosceles triangles are similar) and find examples proving two existential statements (e.g., There exist two right triangles that are similar). After briefly reviewing three similarity theorems as a group, the students spent most of the class time working on Side-Side-Side (SSS) similarity proof, each typing their work on a designated slide in a shared *Google Slides* document.

One unique feature of Nancy's noticing in lesson 3 is the focus on students' learning of content (25%). For example, she noted that "the students recognized that they needed to use SSS but were unable to connect/use proportions to relate the corresponding sides"; that "a lot of them [students] didn't understand how to include proportions into the proof," and that "the students were able to come up with some counterexamples." Nancy monitored students' progress when they typed their answers into *Google Slides,* but she did not have access to their *GeoGebra* screens. Nancy's reflective comments concerned her interactions with students (17%) and with technology (17%). For example, she reflected on the affordances of *Google Slides*: "I, again, liked the use of the google slides here, because I was able to see the students' progress through the similarity proof and get an understanding of what parts they found confusing."

Lesson 4: Indirect Reasoning: Coordinate Proofs

For this lesson, Nancy adapted her cooperating teacher's lesson plan about analytic geometry proofs by integrating indirect reasoning in it. The main activity was the game "The Quadrilateral Detective," where students determined the type of quadrilateral given the coordinates of its vertices. The students also created statements involving indirect reasoning, such as "The quadrilateral cannot be a kite because otherwise it would not have parallel sides". Nancy solved one task together with the students as an illustration. To avoid the complexity of students typing algebraic symbols, Nancy suggested students write their proofs on paper and post a picture of their work into the shared *Google Slides* document.

Despite Nancy's best intentions, not all parts of the lesson proceeded as planned. She seemed to manage better during the teacher-led parts of the lesson and reflected abundantly on her instructional decisions (teacher-content, 31%). For example, "One teaching move that I liked during this lesson was creating a theme for the lesson…" ; "One teaching move that I think was good for this portion was that I had already graphed and typed out the solution to case 1 in the presentation/desmos…"; "I think it was a successful teaching move to include some exposition about indirect reasoning at the end of the lesson instead of the beginning…"

Table 1 shows that from Lesson 1 to Lesson 3 Nancy's noticing shifted from more teacher-centered to more student-centered. But in lesson 4 Nancy mostly noticed her teaching of content (teacher-content, 31%), while also reflecting on her interactions with students (teacher-student, 25% – the largest percentage of this category across the four lessons). Specifically, Nancy reflected on her attempts to press students for explanations, e.g., "why he put his x in a certain spot"; on her calling on students to "get them to participate;" on how "open ended discussion allowed the students to express their answers and explain their reasonings" and on how "providing students enough time to work on their ideas leads to richer discussions."

Nancy also reflected on how she and her students used technology (teacher-technology, 13%; student-technology, 13%). One of the aspects she noticed was the difference between her expectations about students' use of technology and the reality. Nancy expected students to write their solutions to the Quadrilateral Detective Task on paper, scan, and post to the shared *Google Slides* document. Instead, the students created new slides within that document and started typing in their solutions. While Nancy sought to optimize student use of technology by providing space to post their work, the students chose an inefficient and time-consuming approach. Nancy noticed this and reflected on ways for making improvements for future practice.

Discussion and Conclusions

In this paper, we presented the Instructional-Technology Triangle (ITT) framework, which foregrounds technology as an essential element of online learning environments, and a primary mediator between teachers, students, and content. The ITT framework contributes to the literature advocating the need for extending the basic instructional triangle (Cohen et al., 2003) to represent additional elements of instruction in general (Herbst & Chazan, 2012) and with respect to technology (e.g., Yeo & Webel, 2017). The ITT framework was developed when teaching via online videoconferencing (e.g., Zoom, Google Meet, Microsoft Teams, VooV Meeting, etc.) became a norm during the pandemic in response to the seeming lack of theoretical and analytical tools for conceptualizing teaching and teacher noticing in the online context. In this context, teaching technology is not just another element of instruction, but a necessary medium that makes this instruction possible. The content is represented through technology, and both teachers and students interact with technology and through technology. This may include navigating *multiple* types of technology (e.g., video conferencing platform, digital interactive whiteboards, virtual manipulatives) simultaneously by teachers as well as students. Thus, technology started playing an essential role in determining the quality of instruction and the quality of student learning experience. The ITT framework represents our attempt to capture this unique role of technology in the online setting.

Our study also contributes to the body of knowledge on noticing and reflection by demonstrating the utility of the ITT framework in analyzing Nancy's reflective noticing and its development over time. Nancy's first teaching experience happened to be online, and she had to adjust to it quickly. Due to her strong mathematical content knowledge, Nancy invested most energy in designing interactive activities, choosing appropriate technological tools, and interacting with students. The novelty of the experience contributed to the heightened focus on herself and on her teaching in the first lesson (Table 1). As the semester progressed, Nancy seemed to become more comfortable and confident, as evidenced in her delegating responsibility to students by having them interact with technological tools like *GeoGebra, Desmos* and *Google Slides*. Accordingly, Nancy's noticing patterns shifted in lessons 2 and 3 toward increased focus on interactions between students, content, and technology (Table 1). These patterns shifted again in lesson 4 when Nancy enacted the cooperating teacher's modified lesson plan and encountered a mismatch between her expectations of student engagement and how the lesson unfolded. The ITT allowed us to create a nuanced representation of Nancy's reflective noticing and the changes in it. The observed patterns are consistent with the noticing literature (e.g., Buchbinder et al., 2021, Sherin & van Es, 2005; Stockero, 2021) on teachers' initial inclination to focus on their actions, followed by increased noticing on student mathematical thinking. Many researchers highlight the importance of noticing students' mathematical thinking by teachers (e.g., Barnhart & van Es, 2015; Cross Francis et al., 2021; Sherin & van Es, 2009). Despite its importance, it may be unwise to use it exclusively as an indicator of quality of teacher noticing and run the risk of overlooking other aspects. As Spangler (2019) advocates, "We as teacher educators need to demonstrate the curiosity and intellectual humility that allows us to understand how and why something a teacher did or said came from a place that made sense to them" (p. 2). In this paper, we attempted to interpret Nancy's noticing from a place that made sense to her, and to foreground her rationality in the context of her teaching practice (Herbst & Chazan, 2003).

Stepping back from our study, we believe that even as the schools begin to reopen, online education will retain a substantial presence in a variety of forms, such as teacher preparation, professional development, and situations where remote learning is a necessity for some reason. The ITT framework can be broadly applied for conceptualizing and analyzing teacher-student-content interactions in an online setting, responding to the need to attend to contexts in which the teaching and learning takes place, the experiences of the participants and to their individual voices (Liu et al., 2020; PME-NA 44 Conference Theme, 2022).

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