

## Overburden Normalization for In-Flight Centrifuge Miniature Cone Penetration Testing in Sand

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### ABSTRACT

Miniature cone penetration testing (CPT) is now widely used in centrifuge modeling. A common approach for interpreting the test results is to normalize the measured cone tip resistance,  $q_c$ , to the value that would be measured in the same soil at an effective vertical stress of 1 atm,  $q_{c1}$ . However, most overburden normalization methods are derived from CPTs performed in calibration chambers or from CPTs performed in the field in relatively uniform deposits. This paper summarizes measurements from in-flight miniature CPTs performed in clean sand centrifuge models where tip resistances were measured continuously to an effective vertical stress larger than 1 atm without any significant boundary effect or particle size effect. Based on these test results, the authors present a centrifuge CPT-specific overburden normalization for a wide range of sand relative densities, ~40%–90%. The in-flight miniature CPT data suggest that the overburden normalization exponent  $m$  generally is insensitive to relative density and is somewhat larger than the values computed using methods from the literature.

### INTRODUCTION

With the advance of geotechnical centrifuge modeling, in-flight miniature cone penetration testing (CPT) is now widely used. The cone penetration resistance obtained from those miniature cones also can be post-processed in a manner similar to that used for field CPT data to evaluate practical geotechnical problems:

$$q_{c1} = q_c \times C_N \quad (1)$$

$$C_N = \left( \frac{P_a}{\sigma'_v} \right)^m \quad (2)$$

where  $q_c$  is measured cone tip resistance,  $q_{c1}$  is cone tip resistance that would be measured in the same soil at an effective vertical stress of 1 atm,  $C_N$  is overburden normalization factor,  $P_a$  is atmospheric pressure (1 atm or 101.3 kPa),  $\sigma'_v$  is effective vertical stress,  $m$  is an exponent that depends on soil properties. In this study, the authors use  $q_{c1}$  as a representative parameter for cone penetration resistance instead of using other forms of normalized penetration resistance

(e.g., Olsen 1994; Cetin and Isik 2007; Robertson 2009; Jefferies and Been 2015) because  $q_{c1}$  has the simplest form for calculation and this value has also been widely used in geotechnical practice, e.g., liquefaction susceptibility, triggering, and post-liquefaction shear strength evaluation (e.g., Youd et al. 2001; Olson and Stark 2002, 2003; Idriss and Boulanger 2008).

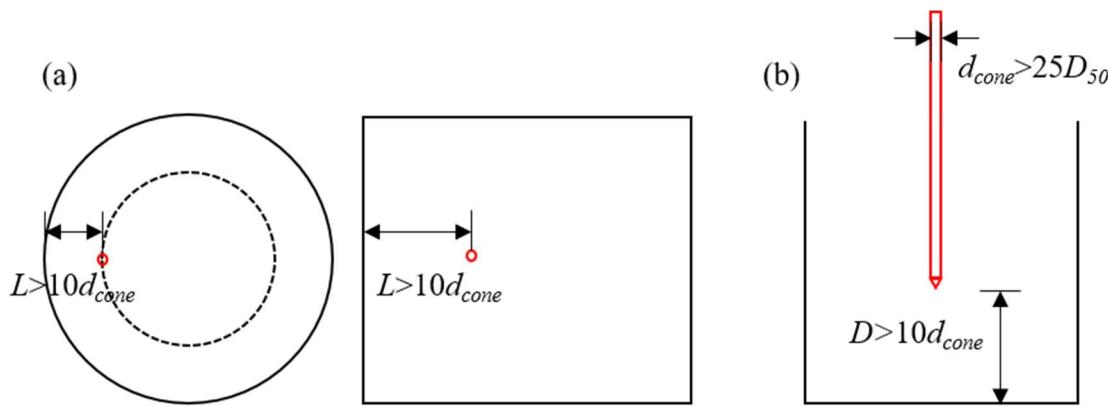
Numerous researchers have evaluated the parameter  $m$ . For example, Liao and Whitman (1986) suggested that  $m = 0.5$  based on their review of prior standard penetration test (SPT) studies (e.g., Bazaraa 1967; Peck et al. 1974; Seed 1979). The Bazaraa (1967) overburden normalization was based on SPT data from ten sites with varying geology. The Seed (1979) normalization was derived from SPT calibration chamber tests performed by Marcuson and Bieganousky (1977a,b). Although these overburden normalization methods were originally proposed for the SPT, they also have been widely used for CPT interpretation with  $m = 0.5$ .

Later, Idriss and Boulanger (2008) proposed that  $m = 0.784 - 0.521D_r$  based on the interpretation of CPT calibration chamber tests (where  $D_r$  is relative density). Indeed, there are other similar overburden normalization methods proposed for CPT that require additional measurements (e.g., Moss et al. 2006; Cetin and Isik 2007) or additional soil behavior type (SBT) interpretation (e.g., Robertson 2009). Unfortunately, most miniature CPT performed in centrifuge modeling do not include the required sleeve friction,  $f_s$ , measurement needed to employ these approaches. As this study focuses on CPT overburden normalization for centrifuge modeling, a functional form similar to Liao and Whitman (1986) and Idriss and Boulanger (2008) is preferred. Furthermore, the differences in the source of CPT data – full-scale CPT performed in the field or in a calibration chamber compared to miniature CPT performed in the centrifuge – may also result in some differences in overburden normalization. As a result, this study focuses only on miniature CPT performed in the centrifuge.

## CENTRIFUGE CPT DATA SCREENING CRITERIA

Centrifuge CPT results can be affected by proximity of the CPT to the model container boundaries, termed boundary effects, and by the ratio of cone diameter to median particle size (Salgado 2014). A schematic of the CPT in centrifuge modeling is illustrated in Figure 1. In the figure,  $L$  denotes the minimum distance from the CPT location to the cylindrical or rectangular container side wall and  $D$  denotes the minimum distance between the cone tip to the container base. As soil particles need to displace during penetration, rigid container boundaries may restrain particle movement, leading to unrealistically large penetration resistances. Bolton et al. (1999) suggested that  $L \sim 10 - 15$  cone diameters,  $d_{cone}$ , was needed to minimize side boundary effects in their study on Fontainbleau sand, which is a uniform fine silica sand. This conclusion can also be applied to evaluate the bottom boundary effect. As illustrated in Figure 1, we selected  $L/d_{cone}$  and  $D/d_{cone} > 10$  as the screening criterion for boundary effects.

In addition to boundary effects, the ratio of the cone diameter to median particle size,  $d_{cone}/D_{50}$ , also influences measured penetration resistance. As the particle size becomes larger relative to the cone size, cone tip resistance increases (Bolton et al. 1999). Bolton et al. (1999) studied three gradations of Leighton Buzzard sand and suggested that penetration resistance was unaffected by particle size for a fine gradation ( $D_{50} = 0.225$  mm) if  $d_{cone}/D_{50}$  was in the range of 28 – 85. For a medium Leighton Buzzard sand ( $D_{50} = 0.40$  mm), a similar observation was made when  $d_{cone}/D_{50}$  was in the range of 25 – 48. However, cone tip resistances were considerably larger in coarse Leighton Buzzard sand ( $D_{50} = 0.90$  mm) at shallow penetration depths. For the analysis presented in this paper, we selected  $d_{cone}/D_{50} \geq 25$  as a screening criterion because most centrifuge CPTs have been performed in sands with  $D_{50} < 0.4$  mm.



**Figure 1. Geometric/boundary screening criteria used in this study for CPT performed in centrifuge modeling containers: (a) plan view; and (b) section view.**

Lastly, to properly calculate the overburden normalization power parameter,  $m$ , the maximum effective vertical stress experienced during penetration should be larger than 1 atm, which allows a direct measurement of  $q_{cl}$  at  $\sigma'_v = 1$  atm. CPT results that fulfilled all of these screening criteria were compiled in the current study.

Following the compilation of CPT data, the next step is to define the depth range over which to extract tip resistance for each CPT sounding. The development depth is an important factor in this process. Development depth is defined as the depth below an interface between two layers (i.e., soil-soil, soil-water, or soil-air) that the cone must penetrate to fully develop the cone tip resistance (Salgado 2014). Similar requirements also have been proposed for driven piles. Meyerhof (1956, 1976) suggested that piles must penetrate into a soil layer a distance of about 10 times the pile diameter to fully mobilize the ultimate unit end bearing resistance. Gui et al. (1998) and Kim et al. (2017) also suggested that a significant cone diameter effect is observed until the development depth is exceeded, below which a “deep penetration” phase occurs. Based on these observations, Gui et al. (1998) suggested a normalized critical depth,  $(z/d_{cone})_{cr}$ , where  $z$  is vertical depth from soil surface, of about  $(5$  to  $10)d_{cone}$ , increasing with soil density and consistent with observations for driven piles. At this depth, normalized cone penetration tip resistance,  $Q = (q_c - \sigma_v)/\sigma'_v$ , where  $\sigma_v$  = total vertical stress and  $\sigma'_v$  = effective vertical stress, reaches a maximum. In this study, the normalized critical depth where  $Q$  reached its maximum was used to define the development depth, below which CPT results will be used to evaluate overburden normalization. In summary, the compiled CPT results fulfill the following criteria:

- (1)  $L/d_{cone} > 10$  (side boundary effect);
- (2)  $D/d_{cone} > 10$  (bottom boundary effect);
- (3)  $d_{cone}/D_{50} > 25$  (particle size effect);
- (4)  $q_c$  measured where  $(z/d_{cone}) > (z/d_{cone})_{cr}$ , i.e., in deep penetration phase; and
- (5) Maximum  $\sigma'_v > 1$  atm in deep penetration phase.

A large number of miniature CPTs performed in clean sand centrifuge models have been published (Lee 1990; Bolton and Gui 1993; Esquivel and Silva 2000; Dewoolkar et al. 2008; Kim et al. 2016; Kim et al. 2017; Carey et al. 2018; O’Hara and Martinez 2020; Sawyer 2020). However, only a limited number of these CPTs fulfill the criteria listed above. Table 1 summarizes the 40 CPTs that meet the screening criteria, as well as the cone and soil properties used in these studies.

**Table 1. Summary of miniature CPT and model soil properties used in this study.**

Reference	# of CPT profiles used	$d_{cone}$ (mm)	Saturation condition	Soil	$D_{50}$ (mm)	$C_u$	$G_s$	$e_{max}$	$e_{min}$	$D_r$ (%)
Lee (1990) <sup>1</sup>	4	10	Dry	52/100 Leighton Buzzard sand	0.23	1.69	2.65	0.93	0.59	54-64
Bolton and Gui (1993)	2	10	Dry	Fontainbleau sand	0.18	1.69	2.64	0.92	0.55	52-58
Dewoolkar et al. (2008)	2	12.7	Dry	Ottawa F-75 sand	0.18	2.00	2.65	0.80	0.49	80
Kim et al. (2016)	9	10, 13	Saturated	Silica sand	0.24	1.60	2.65	1.14	0.62	53-82
Kim et al. (2017)	17	10	Saturated and dry	Silica sand	0.24	1.60	2.65	1.14	0.62	40-83
O'Hara and Martinez (2020)	1	10	Dry	Ottawa F-65 sand	0.20	1.61	2.65	0.83	0.51	40
Sawyer (2020)	5	6, 10	Dry	100A	0.18	1.68	2.62	0.88	0.58	40-88

Note: <sup>1</sup>Limited the maximum depth of cone penetration to 250 mm to avoid any significant bottom boundary effects.

## EXAMPLE CALCULATION

Figure 2 presents an example calculation for the overburden normalization exponent  $m$ . In Figure 2(a),  $q_c$  is plotted against prototype depth. Considering the model thickness, the zone potentially affected by the bottom boundary is defined (i.e., within 10  $d_{cone}$  of the container base). The  $q_c$  profile clearly indicates a sharp increase within this zone as the cone approaches the container base. Figure 2(b) presents the  $(q_c - \sigma_v)/\sigma'_v$  profile. Here, the maximum  $(q_c - \sigma_v)/\sigma'_v$  value defines the boundary between shallow and deep penetration phases. In this example, tip resistances used for analysis were taken from the deep penetration phase above the zone of bottom boundary effect. Using measured  $q_c$ , the overburden normalization factor,  $C_N$ , can be calculated as:

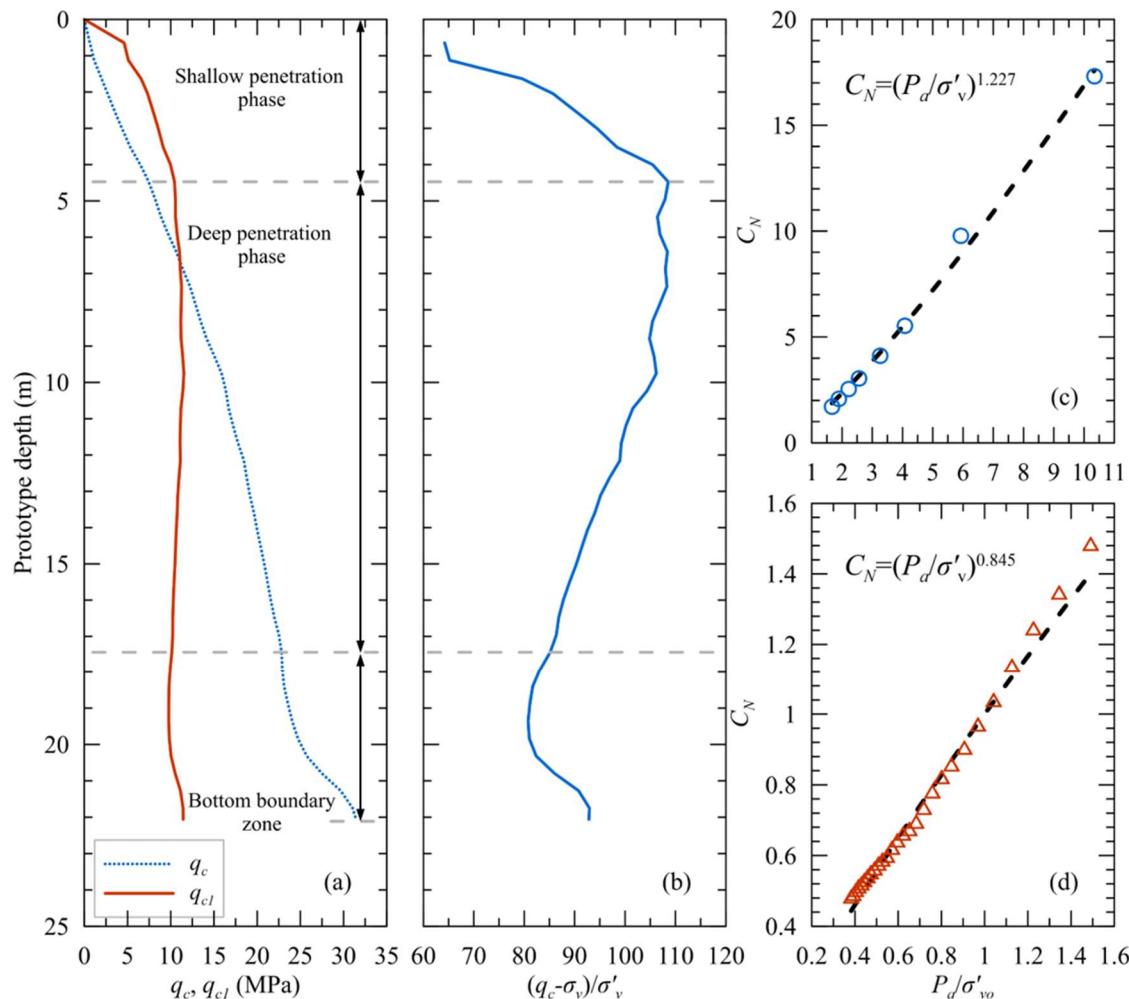
$$C_N = q_c \left( \text{at } \frac{P_a}{\sigma'_v} = 1 \right) / q_c \left( \text{at any } \frac{P_a}{\sigma'_v} \right) \quad (3)$$

The relationship between  $C_N$  and  $(P_a/\sigma'_v)$  then can be regressed using the functional form:

$$C_N = \left( \frac{P_a}{\sigma'_v} \right)^m \quad (4)$$

Figure 2(c) presents this regression for the shallow penetration phase (for comparison purposes only), while Figure 2(d) presents the regression for the deep penetration phase. As illustrated in Figure 2(c) and (d), the  $m$  value calculated from the shallow penetration phase (1.227) is much greater than that regressed from the deep penetration phase (0.845). As indicated above, tip resistance in the shallow penetration phase may be affected by factors like  $d_{cone}$ , and the penetration resistance may not be fully mobilized. Therefore, it is logical that  $m$  may be larger in the shallow penetration phase, resulting in unrealistic  $C_N$  values. In contrast,  $m$  values

computed in the deep penetration phase are solely a function of the sand properties (Boulanger 2003). Only  $m$  values calculated in the deep penetration phase are used in this study.



**Figure 2. Example calculation using the MWG8 CPT from Bolton and Gui (1998): (a) cone penetration resistance,  $q_c$  and  $q_{c1}$  [note:  $q_{c1}$  computed using regressed  $m$  value shown in (d)]; (b) normalized cone penetration resistance for defining the boundary between shallow and deep penetration phases; (c) relationship between  $C_N$  and  $(P_d/\sigma_v')$  in shallow penetration phase; and (d) relationship between  $C_N$  and  $(P_d/\sigma_v')$  in deep penetration phase.**

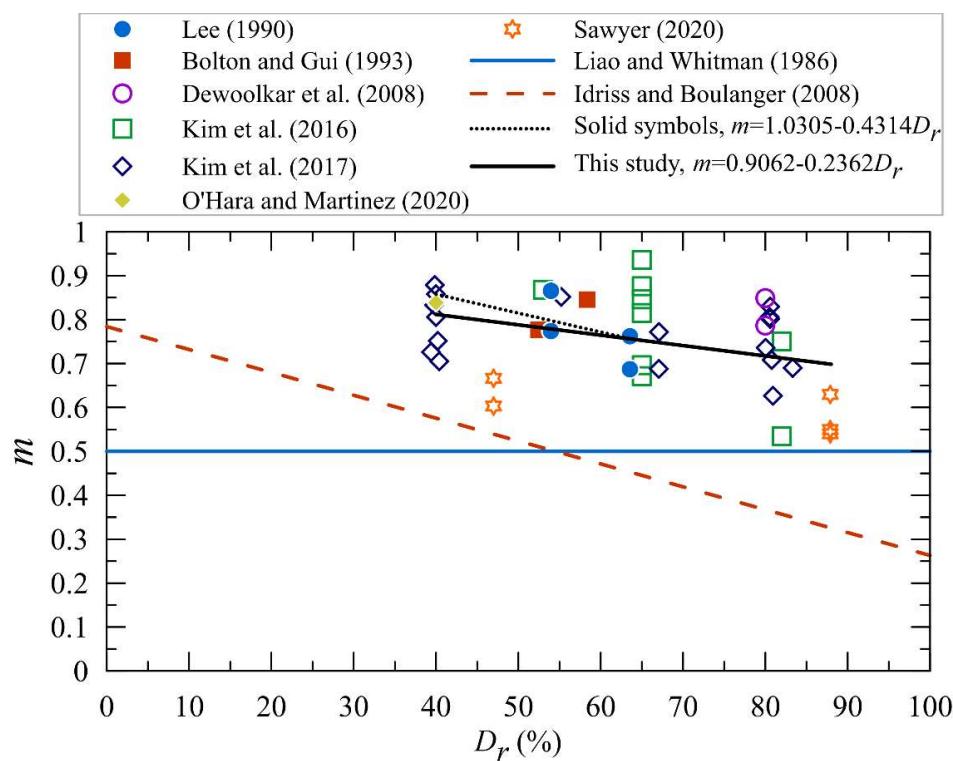
## RESULTS AND DISCUSSION

The example CPT shown in Figure 2 was performed in a centrifuge sand model with  $D_r = 58.4\%$  (Test MWG8; Bolton and Gui 1998). As indicated in the figure, the resulting  $m = 0.845$  is significantly larger than  $m = 0.5$  suggested by Liao and Whitman (1986). This regressed  $m$  value also exceeds the Idriss and Boulanger (2008) recommendation of  $m = 0.784 - 0.521D_r = 0.506$ . In this case, published overburden normalization methods may not apply to centrifuge miniature CPT and may significantly underestimate the true  $q_{c1}$  of the soil, leading to a lower strength interpretation.

Using the  $m$  values regressed for all of the CPT results listed in Table 1, Figure 3 presents a relationship between the overburden normalization exponent  $m$  and sand relative density. All of the regressed  $m$  values plot above published  $m$  values, indicating that a higher  $m$  value is more appropriate to interpret centrifuge miniature CPT results. A linear relationship between regressed  $m$  values and  $D_r$  can be defined as:

$$m = 0.9062 - 0.2362D_r \quad (5)$$

This relationship suggests that  $m$  decreases as relative density increases, similar to the Idriss and Boulanger (2008) relationship. However, the rate of the decrease is somewhat smaller than recommended by Idriss and Boulanger (2008), indicating that the regressed  $m$  values are only moderately sensitive to changes in relative density.



**Figure 3. Overburden normalization exponent  $m$  regressed from the CPT data in Table 2 as a function of relative density and compared to published overburden normalization methods.**

In addition to relative density, the relationship between regressed  $m$  and compressibility can be studied, where compressibility is represented by the critical state line slope in  $e - \log p'$  space ( $\lambda_{10}$ ; Jefferies and Been 1985). Table 2 summarizes available  $\lambda_{10}$  values for the sands listed in Table 1. As illustrated in Table 2, these three sands exhibit similar compressibility, with  $\lambda_{10}$  in the range of 0.034 to 0.074. A linear relation between  $m$  and  $D_r$  can be regressed for just those three low-compressibility sands (shown using solid symbols in Figure 3) as:

$$m = 1.0305 - 0.4314D_r \quad (6)$$

This relation, although slightly steeper (slope of -0.4314 compared to -0.2362), is quite similar to the Eq. (5) relationship (Figure 3). This similarity suggests that the compressibility may be similar for all sands listed in Table 1. The hypothesis of similar compressibility is further supported by the similarities in  $D_{50}$ ,  $C_u$ ,  $e_{min}$ , and  $e_{max}$  for all sands listed in Table 1. Thus, it may be reasonable to conclude that the proposed relationship can be applied to centrifuge model sands with similar properties as listed in Tables 1 and 2. Importantly, we note that this relationship (Eq. 5) applies only to penetration in the deep penetration phase performed in clean sand centrifuge models.

**Table 2. Summary of available critical state slope,  $\lambda_{10}$ , for sands listed in Table 1.**

Soil	$\lambda_{10}$	References
52/100 Leighton Buzzard sand	0.040-0.054	Jefferies and Been (2015)
Fontainbleau sand	0.040	Aghakouchak et al. (2015)
Ottawa F-65 sand	0.034-0.074 (0.054)	Values from: (1) ElGhoraiiby et al. (2020) undrained triaxial tests with $\sigma'_c < 1$ MPa; and (2) constant-volume direct simple shear tests performed by the authors

## CONCLUDING REMARKS AND FUTURE WORK

In summary, differences in the source of cone penetration test (CPT) data, namely full-scale CPT performed in the field or in a calibration chamber compared to miniature CPT performed in the centrifuge, have resulted in some differences in effective overburden stress normalizations. To investigate these differences, the authors collected penetration test results from miniature CPTs performed in-flight in clean sand centrifuge models. According to the screening criteria described in the paper, continuous cone tip resistances to an effective vertical stress larger than 1 atm without any significant boundary effect or particle size effect were evaluated. The data suggest that the overburden normalization exponent  $m = 0.9062-0.2362D_r$  may be more appropriate for in-flight miniature CPT instead of 0.5 or 0.784-0.521 $D_r$  that were derived from field and calibration chamber CPTs. This relationship applies to uniformly graded, clean, fine, relatively incompressible sands over a wide range of relative densities, ~40% to 90%, prepared in centrifuge models. As indicated by the relationship, the proposed overburden normalization exponent  $m$  is fairly insensitive to relative density and exceeds the values of  $m$  reported in the literature. However, the developed correlation is based on data with significant scatter and therefore not well constrained. Additional data will help refine the correlation.

## ACKNOWLEDGEMENTS

This work was supported by the U.S. National Science Foundation (Awards 1728199 and 1728172). Any opinions, findings, and conclusions or recommendations expressed herein are those of the authors and do not necessarily reflect the views of the National Science Foundation. The authors thank the researchers who collected the original data used in this study.

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