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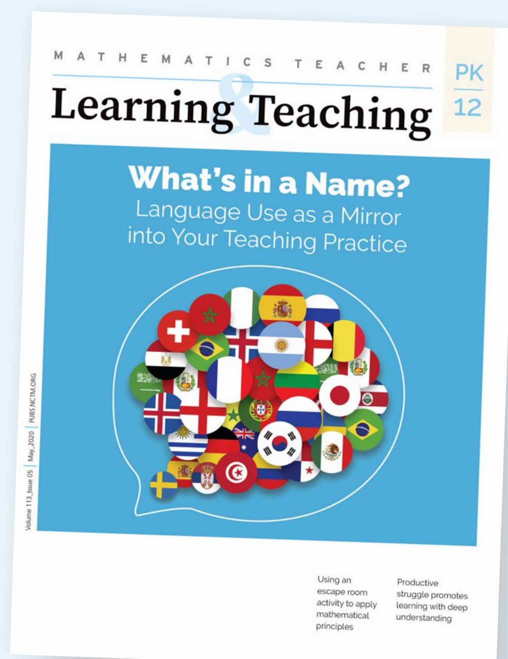

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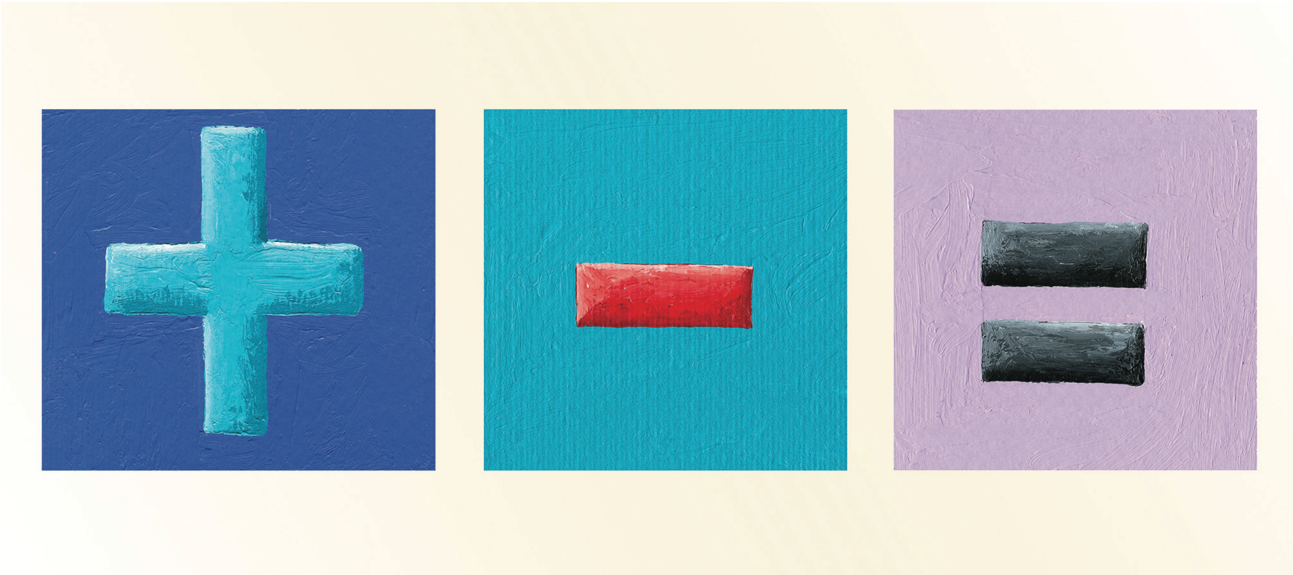

Beyond the Sign Rules

Are your students negative about integers? Help them experience positivity and joy doing integer arithmetic!

Jessica Pierson Bishop, Lisa L. Lamb, Ian Whitacre, Randolph A. Philipp, and Bonnie P. Schappelle

Consider the following integer arithmetic tasks:

$6 + \square = 4$

$-5 - -3 = \square$

$-2 + \square = 4$

$-8 - 3 = \square$

Which would be the most difficult for your students? Which would be the easiest? In our work with K–12 students, we were surprised to find that problems we initially thought would be challenging for students, like $-5 - -3 = \square$, were not. Further, problems we thought would be relatively easy, such as $-8 - 3 = \square$, were harder than we anticipated. We wondered what features of these problems made them more and less challenging and why. We answer these questions using data from interviews with 160 K–12 students. Specifically, we share problem-type categories for integer addition and subtraction that varied in difficulty and tended to evoke different types of reasoning (Lamb et al. 2018). In the same spirit as the well-known Cognitively Guided Instruction (CGI) problem-type categories for whole number operations (Carpenter et al. 2014), our categories for integer addition and subtraction tasks and our framework for Ways of Reasoning about integers provide an organizing structure for integer-specific content knowledge.

We want to highlight why this research has been useful to us. As former teachers, we often showed students a single way to solve problems—some of us preferred rules for operating on integers, and others used chips, or number lines, or contexts. Although these may have been helpful for students learning to operate on integers, we may have also inadvertently limited students' opportunities to reason about integers by proceduralizing the use of these tools in ways that may not have allowed for conceptual understanding. We believe that integers can be taught from a reasoning approach and found that the most successful students in our study had many ways to reason about problems. By reconceptualizing integers around Ways of Reasoning and problem types, we create opportunities for integers to be a conceptually oriented mathematical topic. Furthermore, we believe that this knowledge can support teachers to purposefully select problems for specific instructional goals and to elicit particular Ways of Reasoning.

PROBLEM TYPES

For many adults, the four problems in the introduction are the same in that each one can be solved by invoking a computational rule or procedure, perhaps one you learned in school as a student yourself. But for students, these problems were different, inasmuch as they used different strategies to solve them. For example, one student told us that $6 + \square = 4$ had no answer because 4 is smaller than 6 and you are adding, so the answer should be bigger than 6. But then she solved $-2 + \square = 4$ by counting up by ones from –2 to 4. And to solve $-5 - -3 = \square$, she took away three negatives from five negatives, which left two negatives. The different strategies and underlying reasoning we saw in students' solutions indicated that important distinctions exist among integer addition and subtraction problems beyond the operation itself. Below we share problem-type categories for integer addition and subtraction problems (what we call additive problem types) that tended to account for the different ways students reasoned about open number sentences.

We identified three broad categories of additive problem types for integers: *all-negatives*, *change-positive*, and *counterintuitive*. The first category, *all-negatives*, includes problems like $-5 - -3 = \square$, in which all the numbers (including the unknown, –2) in the problem are negative. For the second problem-type category, *change-positive*, we adopt the language used in the Cognitively Guided Instruction problem-type frameworks of start, change, and result values. In *change-positive* problems, the change value is positive. The problems $-8 - 3 = \square$ and $-2 + \square = 4$ are both *change-positive* because 3 and 6 (the unknown in the second problem), are positive numbers (the change is the number only and does not include the operation). We named the last category *counterintuitive problems* because these problems contradict the overgeneralizations that addition makes larger and subtraction makes smaller (Bishop et al. 2011; Karp, Bush, and Dougherty 2014). The problem $6 + \square = 4$ is a counterintuitive problem because the result (i.e., 4) of adding 6 to an unknown quantity is less than 6. These problem types are important because they reflect differences not only in students' strategies but also in problem difficulty. The flowchart shows additive problem types for integers (see figure 1).

In which problem-type category are the problems easiest? Hardest? We were surprised by the answers to these questions. In our interviews with 160 students across multiple grade levels, we found that the *all-negatives* category was the easiest type of problem, with correct answers given to 76.3 percent of these

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problems (Lamb et al. 2018). Students also did relatively well on the change-positive problems, answering 61.1 percent of those correctly, but answered only 48.4 percent of the counterintuitive problems correctly. We suspected that counterintuitive problems would be challenging but were surprised by how easily many students engaged with the all-negative problems.

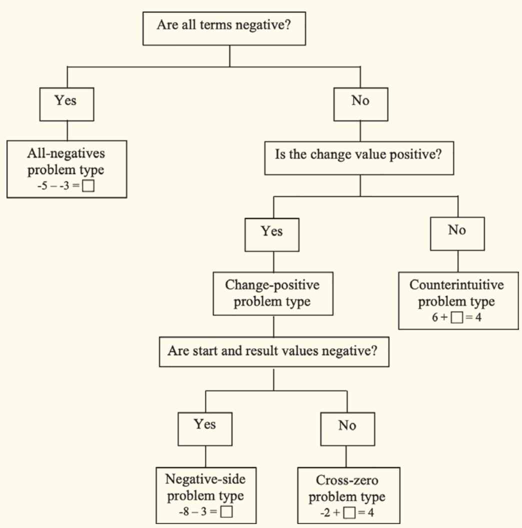
When looking closely at the change-positive problems, we noticed that some problems such as –2 + □ = 4 were easier than other problems like –8 – 3 = □. One second-grade child, Lynn, gave us insight into why. To solve this problem, Lynn wanted to count down but did not know which way to count with a negative starting value. She initially "counted down" saying, "Eight, seven [*raising one finger*], six [*raising a second finger*], five [*raising a third finger*]. Negative five." She then changed her mind saying, "I think it's switched. . . . I want to change this answer and count up now. Eight, nine [*raising one finger*], ten [*raising a second finger*], eleven [*raising a third finger*]. Negative eleven."

We suggest that Lynn was grappling with two competing ideas: She had to reconcile her understanding that subtraction should make smaller with the fact that –11 has a larger magnitude than –8. How could Lynn subtract 3 from –8 and end up with a "larger" value of –11? Her response highlights a challenge many students navigated when solving this problem and others like it: Students grappled with what *bigger* and *smaller* meant in terms of negative numbers and their resultant decisions about which way to count. This finding led us to further differentiate change-positive problems on the basis of their starting and ending values.

We classified change-positive problems with negative start and result values (e.g., –8 – 3 = □) as *negative-side* problems. This name reflects a common strategy of using the number line when solving –8 – 3 = □ by starting at –8 and moving 3 units left on the number line to end at –11. In the solution to this problem, the quantities *stayed on the negative side* of the number line. In contrast, using a number line to solve problems like –2 + □ = 4 involved *crossing zero* when moving from –2 to 4. Thus, we classified change-positive problems with start and result values on opposite sides of zero as *cross-zero* problems. Because cross-zero problems such as –2 + □ = 4 and 3 – 5 = □ involved start and result values on opposite sides of zero, and because students typically knew that negative numbers were smaller than positive numbers (Bofferding 2014; Whitacre et al. 2017), these types of problems did not appear to elicit as much debate over which way to count or whether the starting or ending values were larger (or smaller). In other words, students could successfully engage with 3 – 5 = □, for example, without contradicting the generalization that subtraction makes smaller because –2 (a negative number) is clearly less than 3 (a positive number). In contrast, negative-side problems such as –8 – 3 = □ and –9 + □ = –4 required students to confront whether –4 was larger or smaller than –9 and, therefore, whether the given problem contradicted the generalization of addition makes larger (or subtraction makes smaller).

To summarize, our problem-type framework comprises four categories: all negatives, two categories of change-positive (negative-side and cross-zero), and counterintuitive. Although integer addition and subtraction tasks are often categorized in terms of addition versus subtraction, we suggest that categorizing them differently is productive. The distinctions captured in our problem-type categories matter because they reflect differences in difficulty, differences in students' reasoning and strategy choice, and differences in the structures of problems (i.e., the locations, signs, and magnitudes of the values).

Fig. 1



This flowchart shows additive problem types for integers.

WAYS OF REASONING

In previous work, we identified five broad ways of reasoning (WoRs) about integer addition and subtraction into which we could classify students' responses: order-based, analogy-based, formal, computational, and emergent (Lamb et al. 2018). WoRs are general conceptualizations of signed numbers and operations (e.g., numbers as sequential and ordered in the case of order-based reasoning) that students draw on when they solve additive integer problems. The WoRs framework is a way to organize student thinking about integer addition and subtraction that accounts for the underlying views of number and operation leveraged in a student's strategy. Knowledge of WoRs can help teachers listen to, make sense of, elicit, and extend student thinking about integers. After describing the five WoRs (see table 1), we share examples through a series of videos.

Order-Based Reasoning

As an example of order-based reasoning, consider the response of Angie, who used a number line to solve $-8 - 3 = \square$ and $3 - \square = -6$ in video 1 (link online). For both problems, Angie treated the start and result values as locations on the number line and interpreted the change value as the amount to move. The operations of addition and subtraction indicated the direction of motion for Angie. For $-8 - 3 = \square$, Angie moved

Video 1 Using Order-Based Reasoning on a Number Line

Watch the full video online.

Table 1 Ways of Reasoning Framework

Ways of Reasoning		Description
Evidence of Engagement with Negative Numbers	**Order-based**	In this way of reasoning, one leverages the sequential and ordered nature of numbers. Strategies include use of the number line with motion and counting forward or backward by ones. For example, students may solve $-2 + \square = 4$ counting by ones from -2 to 4.
	Analogy-based	This way of reasoning is characterized by relating (signed) numbers to another concept or object and reasoning about negative numbers on the basis of behaviors observed in this other concept. Analogy-based reasoning is often tied to ideas about cardinality and understanding a number as having magnitude. For example, students may reason about $-8 - -1$ by treating -8 as eight of something that is "negative" (sad thoughts or bad guys). If -8 is eight negative things, we can remove one negative thing, leaving seven negative things, or -7.
	Formal	In this way of reasoning, signed numbers are treated as formal objects that exist in a system and are subject to mathematical principles that govern behavior. This way of reasoning includes generalizing beyond a specific case by making a comparison to another, known, problem and appropriately adjusting one's reasoning, or by applying properties of classes of numbers, such as generalizations about zero.
	Computational	In this way of reasoning, one uses a procedure, rule, or calculation. For example, some students changed the operation of a given problem along with the corresponding sign of the subtrahend or second addend (i.e., changing $6 - -2$ to $6 + 2$ or $5 + -7$ to $5 - 7$). Students often explained these changes by referring to rules.
Restricted to Whole Numbers	**Emergent**	This category of reasoning reflects preliminary attempts to compute with signed numbers; the domain of possible solutions appears to be restricted to whole numbers. The possible effect of operating with a negative number is not considered. A child may overgeneralize that addition always makes larger, and, as a result, claim that a problem for which the sum is less than one of the addends ($6 + \square = 4$) has no answer.

Adapted from Bishop et al. 2014; Lamb et al. 2018. Copyright NCTM.

from –8 to –11, explaining that subtracting 3 "goes back to the left even farther on the number line." Another common order-based strategy was counting by ones, which we described earlier in Lynn's reasoning. In general, order-based reasoning leverages an understanding of numbers as ordered and sequential and operations as ways to progress through those ordered sequences.

Analogy-Based Reasoning

Analogy-based reasoning involves the use of an analogy between signed numbers and some other concept, object, or idea. For example, students frequently compared negative numbers to positive numbers as seen in video 2 (link online), a compilation of multiple students solving problems including □ + –2 = –10, –5 + –1 = □, and –8 – □ = –2. In this video, students productively reasoned about adding and subtracting negative numbers on the basis of adding positive, or *regular,* numbers. One student explained his solution for –5 – –3 = □ by saying, "You would just take off the negatives and get 5 minus 3. And it would equal 2. Then you would put the negatives on the numbers again. That would give you negative 2." Other students compared negative numbers to contexts including owing money, elevation, and digging holes as seen in video 3 (link online). (These are contexts students used in our interviews, but we are not advocating the use of any particular context or model.)

Formal Reasoning

Students used formal reasoning when they treated negative numbers as mathematical objects that exist in a system governed by mathematical formalisms and properties. Students often conjectured about how operations with negative numbers functioned by generalizing or comparing the given problem to a related problem they could solve. Consider, for example, Alma's reasoning about –5 – –3 = □: "Because when you add, like when I'm adding negative 5 to negative 1, it's gonna be negative 6. [*The problem she had previously solved was –5 + –1 =* □.] So, it [–5 + –1] can't be the same as this [–5 – –3]. So, when you subtract negative 3 from negative 5, it's gonna be negative 2." Alma compared the sum of two negative numbers (–5 + –1) to the difference of two negative numbers (–5 – –3) and reasoned that the results of those operations would be "the other way around" because addition and subtraction are opposites. She continued, "It [the result to –5 – –3] can't be the same because it's [*circling the addition symbol in –5 + –1*] a different sign you're working with. . . . It's [–5 – –3] gonna work the other way around." (Watch Alma use formal reasoning on two problems in video 4 [link online].)

We also saw formal reasoning in Melissa's solution for 6 + □ = 4, as seen in video 5 (link online). Melissa productively leveraged a generalization that the sum of two positive numbers is larger than each addend, explaining, "You can't have a positive number [*pointing to the box*] to get a number [the sum] that's less than the first number. So, you would have to have a negative number right there [*pointing to the box*]." She noticed the operation and the signs and magnitudes of the given numbers and used these features to reason, more generally, about a class of problems—in this instance, problems in which addition does not make larger.

Video 2 Using Analogy-Based Reasoning by Treating Negative Numbers Like Positive Numbers

Watch the full video online.

Video 3 Using Analogy-Based Reasoning by Comparing Negative Numbers to Contexts

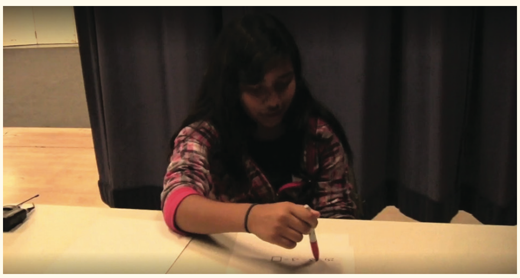
Watch the full video online.

Computational Reasoning

Computational reasoning involves using a procedure, rule, or calculation to solve an integers problem and was used with increasing frequency from elementary to high school. The most common rule was Keep, Change, Change (or KCC for short), so named as a memory device for the procedure to ***Keep*** the sign of the first number, ***Change*** the operation, and ***Change*** the sign of the second number (e.g., 6 – –2 is changed to 6 + $^{+}$2). Watch several students using variations of KCC in video 6 (link online).

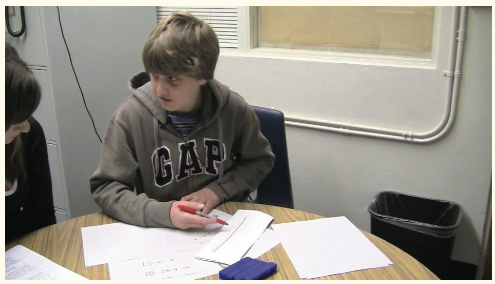
Although students often used procedures and rules efficiently and correctly to solve problems, they were typically unable to justify those procedures. Further, many students struggled with the idea of equivalent transformations that preserved the solution of a given equation. For example, if students transformed 6 – –2 to 6 + $^{+}$2 using KCC, we asked them if the answer to the original problem of 6 – –2 (before changing it) was also 8. Of the 19 seventh graders who changed 6 – –2 to 6 + $^{+}$2, 7 said the two expressions had different answers! (Watch students discussing whether 6 – –2 = 6 + $^{+}$2 in video 7 [link online]). After watching this clip, you might conclude that computational reasoning is less desirable than other WoRs. But that is not necessarily the case! After all, procedures and rules enable us to solve problems efficiently and flexibly. We are, however, highlighting that some students may learn to solve integer problems using procedures without understanding why these procedures are mathematically valid.

Video 4 Using Formal Reasoning through Logical Necessity



Watch the full video online.

Video 6 Using Computational Reasoning to Solve Problems Using the Keep-Change-Change Procedure

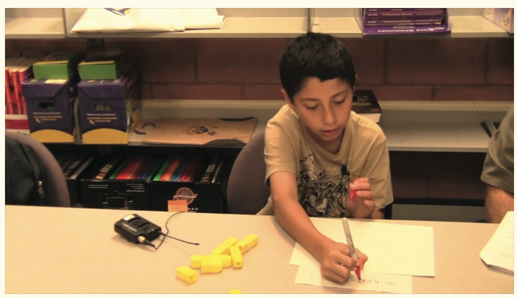


Watch the full video online.

Video 5 Using Formal Reasoning to Infer the Sign of the Unknown by Generalizing

Watch the full video online.

Video 7 Discussing the Equivalence of 6 – –2 and 6 + $^{+}$2



Watch the full video online.

Emergent Reasoning

The fifth way of reasoning, emergent reasoning, was often seen in young children's attempts to compute with integers. For example, many students overgeneralized the ideas that addition makes larger and subtraction makes smaller on the basis of their experiences with whole numbers and, thus, were unable to solve problems like 6 + □ = 4 and 5 – □ = 8. This reasoning was more prevalent with the second and fourth graders we interviewed, but a subset of seventh graders also maintained that these kinds of problems were not solvable (see video 8 [link online] for an example of a seventh grader using emergent reasoning). Although emergent reasoning frequently resulted in an answer of "not possible" or another incorrect answer, many of these strategies were sensible and provided a foundation from which more robust integer reasoning could emerge.

WHY PROBLEM TYPES AND WAYS OF REASONING MATTER

In this article, we shared frameworks for problem types and students' reasoning about integers. Furthermore, we found that all WoRs were used across grade levels. But there is more to the story: We found that specific problem types tended to evoke particular WoRs. Specifically, students were more likely to use analogy-based reasoning on all-negatives problems than on other problem types, order-based reasoning on change-positive problems, and emergent reasoning on counterintuitive problems (Lamb et al. 2018). Formal reasoning and computational reasoning did not follow any distinct patterns in our data. The videos themselves reflect these relationships: All but one problem in the videos illustrating analogy-based reasoning are all-negatives problems; all problems illustrating order-based reasoning are change-positive problems; and all problems illustrating emergent reasoning are counterintuitive. Although we presented these frameworks separately, we suggest that teachers integrate problem types and WoRs in their instruction.

Video 8 Using Emergent Reasoning to Conclude 5 – □ = 8 Is Impossible to Solve

Watch the full video online.

Purposefully Use Problem Types to Highlight Particular Ways of Reasoning

One way teachers can use these frameworks is to purposefully select problem types that are likely to elicit a particular WoR. For example, if a teacher wanted to engage students in a discussion about a single WoR, like order-based, they might pose a change-positive problem such as –2 + □ = 4. Or a teacher might pose two change-positive problems—a cross-zero problem like –2 + □ = 4 and a negative-side problem like –2 + 1 = □. Although both can be solved with order-based reasoning by counting up or using a number line, the problem –2 + 1 = □ challenges students to determine which way to count in a way that –2 + □ = 4 does not. In the discussion of strategies and answers to this problem pair, teachers might ask students whether we always count up (or move right) when adding a positive number. Or they might attach mathematical terminology and symbols to student ideas to help them differentiate order and magnitude—concepts that students confront when deciding what it means for a negative number to be "larger" (i.e., $-2 < -1$ but $|-2| > |-1|$). These conversations may help students to reason about what stays the same and what changes for integer addition and subtraction while extending their number systems from whole numbers to integers.

Purposefully Use Problem Types to Compare Multiple Ways of Reasoning

Alternatively, teachers may want to highlight various WoRs and promote fluency across WoRs. To do so, they might pose a single problem and elicit multiple WoRs for that problem. For example, we have seen students productively use analogy-based reasoning (using debt), order-based reasoning, and formal reasoning to solve –3 + 6 = □. Another way to elicit multiple WoRs is to pose problems from different problem-type categories. For example, a teacher might pose an all-negatives problem like –5 – –3 = □ and a change-positive problem like –5 – 3 = □ or –5 – □ = –8, and ask students to compare the open number sentences. Teachers can support students to identify features of problems (e.g., signs of the numbers, change value, and location of the unknown) and link those features to productive uses of WoRs. In the problem pairing just given, the class might consider which strategies are relatively easy to use for –5 – –3 (i.e., analogy-based reasoning) but more challenging to apply to –5 – 3 and why.

To support teachers to use problem types to elicit specific WoRs, we have paired each WoR with problems likely to elicit this WoR and follow-up questions to probe and extend that reasoning (see table 2). We encourage teachers to use this table to provide opportunities for students to use and discuss different WoRs and to explicitly promote the value of flexibly using multiple WoRs.

A common instructional approach for integers—one that we have used—is for teachers to emphasize one way of reasoning they want students to use when solving integer addition and subtraction problems. But we found that students in our study often used various ways of reasoning, depending on problem type, and the students who reasoned more flexibly were more successful. Flexibility and variety in reasoning involves making choices on the basis of the specifics of the problem. As a result, we do not believe that any one WoR should be exclusively used or taught. We recommend teaching with goals of cultivating ways of reasoning and flexibility and selecting problems according to the frameworks presented here.

Table 2 Problems to Pose

To Evoke This Way of Reasoning (WoR)	Use These Problems	Pose These Questions
All WoRs	All problem types	How did you think about this problem? Why did you decide to use this strategy? Was there a feature of the problem or numbers that encouraged you to use your strategy? (general questions that can be asked regardless of problem type or WoR)
Order-based	*Cross-zero* –3 + 6 = □ 4 – □ = –6	How did you know which way to count/move? Where to start/end?
	Negative-side –8 – 3 = □ –9 + □ = -4	Is –11 or –5 greater than or smaller than –8? In what way could you see –11 as larger than –8?
Analogy-based	*All-negatives* –7 – –5 = □ –8 + □ = –12	I noticed that you compared negatives to ________ (positives, owing, etc.). Why did you make this comparison? Did anyone try other approaches and decide they would be challenging to use? Which ones? Why do you think your approach was challenging to use for this problem?
Formal	*Counterintuitive* 6 + □ = 4 5 – □ = 8	You shared that you knew that the answer had to be negative. What features of this problem helped you to realize that the answer had to be negative?
Formal or Computational	*Counterintuitive* 6 – –2 = □ 4 + –7 = □	Is there a related problem that might help you solve this? Alternatively, share pairs of problems such as 6 – 2 and 6 – –2; 4 + 7 and 4 + –7; or –7 + 4 and 4 + –7, and ask, “How does knowing the answer to the first problem help you to answer the second problem?” (Bishop et al. 2016). Is there a rule that you can use to help you solve these problems? If a younger child asked you why the rule works, what would you say?

REFERENCES

Bishop, Jessica Pierson, Lisa L. Lamb, Randolph A. Philipp, Bonnie P. Schappelle, and Ian Whitacre. 2011. "First Graders Outwit a Famous Mathematician." *Teaching Children Mathematics* 17, no. 6 (February): 350–58.

Bishop, Jessica Pierson, Lisa L. Lamb, Randolph A. Philipp, Ian Whitacre, and Bonnie P. Schappelle. 2016. "Unlocking the Structure of Positive and Negative Numbers." *Mathematics Teaching in the Middle School* 22, no. 2 (September): 84–91.

Bishop, Jessica Pierson, Lisa L. Lamb, Randolph A. Philipp, Ian Whitacre, Bonnie P. Schappelle, and Melinda L. Lewis. 2014. "Obstacles and Affordances for Integer Reasoning: An Analysis of Children's Thinking and the History of Mathematics." *Journal for Research in Mathematics Education* 45, no. 1 (January): 19–61.

Bofferding, Laura. 2014. "Negative Integer Understanding: Characterizing First Graders' Mental Models." *Journal for Research in Mathematics Education* 45, no. 2 (March): 194–245.

Carpenter, Thomas P., Elizabeth Fennema, Megan Loef Franke, Linda Levi, and Susan B. Empson. 2014. *Children's Mathematics: Cognitively Guided Instruction*. 2nd ed. Portsmouth, NH: Heinemann.

Karp, Karen S., Sarah B. Bush, and Barbara J. Dougherty. 2014. "Thirteen Rules That Expire." *Teaching Children Mathematics* 21, no. 1 (August): 18–25.

Lamb, Lisa L., Jessica Pierson Bishop, Randolph A. Philipp, Ian Whitacre, and Bonnie P. Schappelle. 2018. "A Cross-Sectional Investigation of Students' Reasoning about Integer Addition and Subtraction: Ways of Reasoning, Problem Types, and Flexibility." *Journal for Research in Mathematics Education* 49, no. 5 (November): 575–613.

Whitacre, Ian, Lisa L. Lamb, Beti Azuz, Jessica Pierson Bishop, Randolph A. Philipp, and Bonnie P. Schappelle. 2017. "Integer Comparisons across the Grades: Students' Justifications and Ways of Reasoning." *Journal of Mathematical Behavior* 45 (March): 47–62.

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