Endogenous Risk Management of Prosumers by Distributionally Robust Chance-Constrained Optimization

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***Abstract*—Distributed renewable energy sources owned by prosumers have been considered an effective way to fortify grid resilience, and enhance sustainability. However, prosumers serve their own interests, which are affected by their risk preferences. Moreover, their objectives are unlikely to align with those of the society. In contrast to the conventional assumption of constant and exogenous risk preference, we propose an alternative modeling framework in which prosumers endogenously determine their risk attitudes through optimization in the power sector with a day-ahead market and real-time imbalance settlement. The decisions made by prosumers and other participants in the wholesale power market are expressed as a Stackelberg leader-follower game, resulting in a mathematical program with equilibrium constraints. The problem is formulated in a distributionally robust chance-constrained framework to account for renewable generation uncertainty. We show that the effect of uncertainty can be mitigated in the market *if* prosumers are allowed to optimally determine their risk attitudes. We also examine the case of price-taking prosumers, in which consistent implications are obtained. Therefore, our work highlights the fact that endogenizing risk preferences can be a useful tool for managing risk in the power market.**

***Index Terms*—Distributionally robust chance constraint, mathematical program with equilibrium constraints, power market, prosumer, risk.**

I. Introduction

A. Motivation

ELECTRICITY markets are evolving rapidly and fundamentally, in response to the growing need for renewable capacity and generation, mainly reinforced by the efforts to mitigate climate change and pursue sustainability. This has resulted in significant changes in the design and operation of modern power grids. As more smart meters and digital grid technologies become available, the industry witness more facilitation, and hence the willingness toward renewable power generation, such as a solar photovoltaic (PV) system, among traditional consumers. This trend, in conjunction with a variety of distributed energy resources (DER), such as electric vehicles (EV) and storage, has challenged the traditional supply-centric paradigm and illustrated a new reality focused on the demand side in electricity markets.

As the demand side of energy markets becomes increasingly engaged, the behavior and strategies of the once-idle demand-side participants begin to affect the functioning of electricity markets more significantly. In particular, we observe the emergence of *prosumers*, that is, agents capable of concurrent generation and consumption of power, as opposed to conventional consumers or suppliers that would traditionally participate in one side of the market only. As more conventional consumers become prosumers, the collective effect on the design and operation of electricity markets becomes paramount [1]. The engagements of consumers in the electricity markets are amplified by aggregators that integrate demand response (DR) and DER that offer bundled energy products to the market [2], [3]. This trend has recently been accelerated by FERC Order 2222, which paves the way for the increased adoption of DER technologies [4].

Prosumers with non-dispatchable renewable power sources face inherent uncertainty of natural resources, such as solar and wind. Hence, individuals' attitudes toward risk play an important role in the decision making of prosumers, similar to the situation facing investors in financial markets. Within the research communities of economics, finance, and engineering, the assumption of constant and exogenous risk preference has been standard practice [5], [6]. However, in recent years, such a standard assumption has been considered unrealistic because it occasionally fails to describe historical data, particularly in financial markets.[1] For instance, [8] argues that a model with

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[1] Related to this point, Alan Greenspan suggested in his speech at the Federal Reserve Bank of Kansas City that an increase in the market value of asset claims partly depends on changing investors' attitudes toward risk [7].

constant risk aversion cannot replicate the key statistical properties observed in real financial markets. The study demonstrates that a time-varying risk-aversion parameter that responds to unexpected excess returns can replicate historical data. In contrast, [9] develops a general equilibrium model in which heterogeneous preferences are endogenously determined in markets. They empirically show that the risk aversion parameters vary across households under different market conditions by comparing landed and landless households in Bangladesh.

Given the increasing influence of prosumers with uncertain renewable power sources, their behavioral and risk assessments have become a key focus, with possible ramifications for the outcomes of current electricity markets [10], [11]. Prosumer attitudes toward risk have a direct bearing on the quantities they supply and demand, and hence, overall market outcomes, given the rapid penetration of renewable energy sources. The degree of prosumer risk aversion is expected to respond and adapt to the surrounding economic environment and market conditions as in the case of investors in the financial field [8]. Therefore, a modeling framework that captures the decision making of prosumers in the context of endogenous risk attitudes under uncertainty is of particular importance.

B. Related Work and Contributions

Previous studies have addressed some dimensions of risk elements in electricity markets involving prosumers. In [12], the authors investigated the risks involved in community energy markets, with a particular focus on peer-to-peer and energy-sharing mechanisms. The authors developed a conditional value-at-risk (CVaR) model for household and community PV systems and formulated the problem in terms of a stochastic game. Similarly, the authors in [13] formulated a cooperative game theory framework for energy hubs and community energy systems using CVaR with a profit-sharing scheme. In [14], the authors proposed a decision method for energy bids in the day-ahead energy market, and evaluated the risks related to the different decisions of prosumers in the microgrid. [15] studied the role of distributed energy resources or on-site generation in the consumer's risk management strategy using a CVaR approach. The study shows that by swapping electricity with high price volatility for gas with low price volatility, even relatively inefficient technologies can reduce risk exposure. However, existing studies on the stochastic approaches for prosumers are scant and mostly assume constant and exogenous risk preference. These studies fall short of explicitly capturing the formation of the risk tolerance of prosumers and modeling how this factor affects their decisions and, consequently, market outcomes.

In general, common approaches to dealing with uncertainty are divided into two groups, i.e., stochastic optimization (or programming) and robust optimization. The stochastic optimization approach explicitly considers a probability distribution regarding uncertainty [16], while the robust optimization method assumes realizations of events within a deterministic uncertainty set [17]. Both methods have been extensively applied to the area of power systems [18], [19], [20]. In this study, we focus on stochastic optimization, specifically a chance-constrained stochastic program [21], [22]. Chance-constrained problems involve probabilistic constraints with an explicit risk tolerance (or reliability) level, which is an exogenously pre-defined parameter in the model. Utilizing this explicit risk factor, we propose an alternative model for endogenously adjusting the risk attitude via optimization.

One drawback of chance-constrained stochastic programs is the possibly limited availability of the exact distributional information. It is usually hard to identify the true probability distribution associated with uncertainty. A recent strand of research has addressed this issue, proposing stochastic programs with distributionally robust chance constraints, which consider a family of probability distributions with some known properties [23], [24], [25], [26].[2] Recent applications of this approach to the power sector include [28], [29], [30]. More recent advances in data-driven distributionally robust chance-constrained programs that combine the strength of machine learning and mathematical programming can be found in [31], [32], [33].

In a similar vein, we investigate a distributionally robust chance-constrained framework for endogenous risk management of prosumers. We model prosumers who own renewable generation systems and make decisions in a day-ahead wholesale power market, anticipating the effect of their decisions, especially regarding their risk preferences, on the other participants in the market. Specifically, in our model, the degree of the risk aversion of prosumers is endogenously determined by their profit-maximization problem.[3] To the best of our knowledge, this is the first study to derive a prosumer risk-aversion parameter (as a decision variable) by optimization, thereby advancing the modeling approach in addition to its management implications. Such a framework has been scarce, even in other fields, such as finance, with the exception of [34], which models an investor that endogenously chooses his/her risk preference by maximizing the probability of achieving wealth that grows faster than the target growth rate.

The situation confronting prosumers is expressed as a Stackelberg leader-follower game with a formulation of a mathematical program with equilibrium constraints (MPEC) [35], [36]. The problem is formulated in a distributionally robust chance-constrained framework to account for the uncertainty of the renewable generation of prosumers. Within this framework, similar to [37], prosumers who act as a leader maximize their surplus by adjusting their risk attitudes. In other words, we are interested in the extent to which prosumers can benefit from the market through their strategic behavior and ability to optimally manage their risk preferences. Our solution approach applies the Wolfe duality to the lower-level problem to concavify the bilinear term in the objective function of the upper-level problem. A theoretical analysis of solution properties of the model is also provided. Finally, a perfectly competitive case

[2]In the past decades, the robust optimization approach has also been extended to the distributionally robust methods. See, for example, [27] for a review of recent research progress in this field.

[3]Individual DER owners with different risk preferences may enter an agreement by signing onto a program and pay a premium to delegate an aggregator to manage their risk. In this case, the aggregator, as a proxy, would determine its risk attitude in a way that maximizes their aggregate profits.

is also presented to compare the results with the Stackelberg case. We demonstrate how optimally adjusted risk aversion by prosumers affects outcomes in the wholesale power market using the IEEE Reliability Test System (RTS 24-Bus) [38]. Thus, our contribution includes:

1) explicitly and endogenously modeling of prosumers' risk preference using a Stackelberg leader-follower framework formulated as a distributionally robust chance-constrained optimization,
2) analyzing the theoretical and solution properties of the model,
3) examining and comparing the market outcomes to the perfectly competitive cases when prosumers can manage their risk by optimally adjusting their risk tolerance, and
4) demonstrating that endogenous adjustment of risk aversion can be a useful tool to manage uncertainties.

This paper proceeds as follows. In Section II, we formulate a distributionally robust chance-constrained MPEC, in which leader prosumers determine their risk preferences. Section III further presents a case study based on an IEEE 24-bus system. Section IV concludes the study.

II. Model

A. Endogenous Risk Management

Consider a simple chance-constrained stochastic program of the following form:

$$\underset{x\in X}{\text{minimize}}\, \mathbb{E}\left[f(x,\xi,r)\right] \tag{1a}$$

subject to

$$\mathbb{P}\left[g(x,\xi)\le 0\right]\ge 1-r \tag{1b}$$

where $X\subset\mathbb{R}^n$, ξ is a random parameter with probability distribution $\mathbb{P}$ supported on $\Xi\subset\mathbb{R}$, $r\in[0,1]$ is an exogenous parameter for risk tolerance, $f:\mathbb{R}^{n+2}\to\mathbb{R}$, and $g:\mathbb{R}^{n+1}\to\mathbb{R}$. This formulation can be regarded as exogenous risk management in the sense that the risk tolerance is pre-specified by the modeler [21].

However, a decision maker in the real world may modify its attitude toward risk in different situations such as changing economic environments. In this paper, we propose an alternative formulation in which a decision maker adjusts its risk tolerance r by optimization:

$$\underset{x\in X, r\in[0,1]}{\text{minimize}}\, \mathbb{E}\left[f(x,\xi,r)\right] \tag{2a}$$

subject to

$$\mathbb{P}\left[g(x,\xi)\le 0\right]\ge 1-r \tag{2b}$$

This approach is viewed as endogenous risk management with an adjustable risk preference. It provides more flexibility for the decision maker, which would be beneficial. Since it is, in practice, challenging to obtain knowledge of the underlying probability distribution, we further consider a distributionally robust chance-constrained framework. Sections II-C and II-D detail the prosumer's problem in this context.

B. Market Framework

We examine the decision and outcomes in a day-ahead market. The grid operator maximizes the social surplus, collecting bids from generators and consumers. The prosumer maximizes its surplus in the day ahead, facing renewable output uncertainty in the real time. A shortage or excess of energy for the prosumer is finally settled at a real-time imbalance rate/price. Note that the real time is another layer, which is beyond the scope of our paper.

We consider a Stackelberg framework in which the prosumer is the leader and the other market participants are followers. This Stackelberg game is formulated as a mathematical program with equilibrium constraints (MPEC) defined as in (3a):

$$\underset{(x,y)\in Z}{\text{minimize}}\, f(x,y) \tag{3a}$$

subject to

$$0\le F(x,y)\perp y\ge 0 \tag{3b}$$

where $x\in\mathbb{R}^n$, $y\in\mathbb{R}^m$, $Z\subset\mathbb{R}^{n+m}$, $f:\mathbb{R}^{n+m}\to\mathbb{R}$, $F:\mathbb{R}^{n+m}\to\mathbb{R}^m$, and $\perp$ denotes complementarity, i.e., $0\le u\perp v\ge 0$ is equivalent to $u\ge 0$, $v\ge 0$ and $uv=0$.[4]

An MPEC can be regarded as an optimization problem faced by a leader (upper-level problem), whose actions affect the equilibrium of a market (lower-level problem), which consequently affects the leader's objective. The next section discusses this issue in detail.

C. Bi-Level Problem of Power Market With Prosumers

In this section, we describe the prosumer's problem in the upper- and lower-level problems faced by the grid operator. $\mathcal{I}$, $\mathcal{F}$, and $\mathcal{K}$ denote the sets of nodes, conventional generation firms, and transmission lines, respectively. Furthermore, $\mathcal{H}_{fi}$ denotes the set of generation units at node i owned by firm f. We also note that the Greek variables within parentheses to the right of the equation render the corresponding dual variable.

1) Upper-Level Problem: In this study, the prosumer makes a decision in a day-ahead wholesale power market, anticipating its effect on other participants. Therefore, the prosumer can be modeled as the leader in a Stackelberg game.[5] The prosumer at node i is assumed to possess non-dispatchable renewable capacity with a negligible short-run marginal cost. The output from renewable sources is denoted by a random variable $\tilde{K}_i$, which is uncertain because it depends on available natural resources such as solar and wind. We assume that the distribution $\mathbb{P}_i$ of $\tilde{K}_i$ belongs to a set $\mathcal{P}_i$ of distributions, and the only requirement we impose on $\mathbb{P}_i$ is that its first and second moments are known; that is, $\mathbb{E}[\tilde{K}_i]=K_i$ and $\mathbb{V}[\tilde{K}_i]=\sigma_i^2$, respectively, but without exact knowledge of the probability distributions. In contrast, the prosumer also owns a dispatchable or backup resource, for

[4] More formaly, an MPEC can be defined by using variational inequality [39]

[5] The individual "behind-the-meter" prosumers, such as the owner of a rooftop solar panel, might have limited access to the wholesale or bulk market and may be subject to fixed tariffs when selling their surplus power back to the grid. We assume that the prosumer (or aggregator) that we present here is a result of the aggregation of a large number of prosumers, thereby allowing them to interact with the bulk market directly.

example, an on-site diesel generator, that supplies power g_i with an increasing and strictly convex cost function $C_i^g(g_i)$ and capacity of G_i to hedge against uncertain output $\tilde{K}_i$. Specifically, we assume a quadratic cost function, $C_i^g(g_i) = D_i^{g0} g_i + \frac{C_i^{g0}}{2} g_i^2$. However, its supply would not be able to fully compensate for the intermittent renewable output.

Regarding the demand side, the prosumer's benefit function of consuming electricity at node i is given by $B_i^l(l_i)$, where l_i corresponds to the self-consumption at each node.[6] Benefit function $B_i^l(l_i)$ is assumed to be increasing and strictly concave. Specifically, we assume a quadratic benefit function, $B_i^l(l_i) = A_i^{l0} l_i - \frac{A_i^{l0}}{2B_i^{l0}} l_i^2$, for a relevant range of consumption. We assume that the prosumer is only allowed to sell or buy power locally, that is, at each node, which is consistent with the layered structure of a future grid [40]. Allowing the prosumer to sell power to other nodes will not change the outcomes as the increase/decline in revenue is entirely offset by the transmission charge under the framework of locatinal marginal prices [41].

We posit that the prosumer maximizes its surplus by determining four types of variables: i) its risk attitude/preference or tolerance $r_i \in [0,1]$, in which a smaller r_i indicates that the prosumer is more risk averse; ii) the amount of traded power z_i, buying from ($z_i < 0$) or selling to ($z_i > 0$) in node i at price p_i, iii) the amount of its own power consumption l_i and iv) the amount of power to be generated g_i from the backup dispatchable technology. We further formulate a distributionally robust chance-constrained problem for the prosumer facing an uncertain renewable output $\tilde{K}_i$ as follows:

$$\underset{r_i, z_i, l_i, g_i}{\text{maximize}} \sum_i \Big[p_i z_i + B_i^l(l_i) - C_i^g(g_i) \Big] + \sum_i \mathbb{E}\Big[P_i^c \Big(\tilde{K}_i - z_i - l_i + g_i \Big) \Big] \tag{4a}$$

subject to

$$\inf_{\mathbb{P}_i \in \mathcal{P}_i} \mathbb{P}_i \Big[z_i + l_i - g_i - \tilde{K}_i \leq 0 \Big] \geq 1 - r_i \quad (\delta_i), \forall i \tag{4b}$$

$$g_i \leq G_i \quad (\kappa_i), \forall i \tag{4c}$$

$$r_i \leq 1 \quad (\nu_i), \forall i \tag{4d}$$

$$r_i, l_i, g_i \geq 0 \quad \forall i \tag{4e}$$

The three terms in the first line of the objective function (4a) correspond to revenue (+) or cost (−) from transactions in the day-ahead wholesale market, the benefit of consuming power, and the generation costs incurred from backup resources, respectively.[7] The second line provides the expected cost/revenue in real time, where P_i^c is a fixed or contracted retail rate between the prosumer and utility for node i. Here, we envision a situation in which the prosumer is allowed to participate in a day-ahead market but is subject to a fixed retail rate in real time, similar to the situation faced by several EU countries such as Italy, the Netherlands, and Belgium [43]. P_i^c can also be regarded as a real-time imbalance price to finally settle a shortage or excess of energy for the prosumer.

Constraint (4b) is the distributionally robust chance constraint of the prosumer. It states that the sum of the renewable output $\tilde{K}_i$ and self-generation g_i, net of transactions with the wholesale day-ahead market, that is, z_i, is equal to or greater than the self-consumption l_i with probability $1 - r_i$ or greater for any distributions in $\mathcal{P}_i$. Because the renewable output $\tilde{K}_i$ is a random variable, its realization in real time inherently varies, depending on the weather condition. The prosumer who is conscious of risk would be concerned about bad scenarios with the realization of very low renewable output because it will incur costs to settle the imbalance of power in real time. This is analogous to investors who are cautious about market volatility and are concerned about possible losses owing to the realization of low returns on their financial assets. We posit that the prosumer decides their risk attitude by adjusting r_i in constraint (4b), where a smaller r_i means more risk averse. Constraints (4c)–(4e) specify the ranges of the four decision variables, that is, r_i, z_i, l_i, and g_i.

2) Lower-Level Problem: We further introduce the lower-level problem in which the grid operator takes bids from suppliers and consumers/load serving entities and maximizes the social surplus of the wholesale market, subjected to prosumer's decision z_i. Let x_{fih}, d_i, and y_i denote the power output produced by generation unit h at node i owned by firm f, the quantity demanded by consumers at node i, and the net power injection/withdrawal at node i, respectively. We assume an increasing and strictly concave quadratic benefit function $B_i(d_i) = P_i^0 d_i - \frac{P_i^0}{2Q_i^0} d_i^2$ for a relevant range of consumption, and an increasing and strictly convex quadratic cost function $C_{fih}(x_{fih}) = D_{fih}^0 x_{fih} + \frac{C_{fih}^0}{2} x_{fih}^2$ for generation. The lower-level problem is represented as the maximization of the social surplus (i.e., benefit minus cost) faced by the grid operator:

$$\underset{x_{fih}, d_i, y_i}{\text{maximize}} \sum_i B_i(d_i) - \sum_{f,i,h \in \mathcal{H}_{fi}} C_{fih}(x_{fih}) \tag{5a}$$

subject to

$$x_{fih} \leq X_{fih} \quad (\beta_{fih}), \forall f, i, h \in \mathcal{H}_{fi} \tag{5b}$$

$$\sum_i PTDF_{ki} y_i \leq T_k \quad (\lambda_k^+), \forall k \tag{5c}$$

$$-\sum_i PTDF_{ki} y_i \leq T_k \quad (\lambda_k^-), \forall k \tag{5d}$$

$$d_i - \sum_{f,h \in \mathcal{H}_{fi}} x_{fih} - z_i = y_i \quad (\eta_i), \forall i \tag{5e}$$

$$\sum_i y_i = 0 \quad (\theta) \tag{5f}$$

$$x_{fih} \geq 0 \quad (\varepsilon_{fih}), \forall f, i, h \in \mathcal{H}_{fi} \tag{5g}$$

$$d_i \geq 0 \quad (\xi_i), \forall i \tag{5h}$$

[6]The basic model for the prosumer's demand can be also found in [36].

[7]The interaction of the prosumer with the bulk day-ahead energy market is modeled by shifting the supply curves and sales decisions of conventional producers. An alternative way of modeling this situation is to horizontally aggregate the demand curves of consumers and prosumers. However, this aggregation might result in kinked demand curves, which pose numerical difficulties [42].

Constraints (5b)–(5d) limit the variables according to the generation capacity (X_{fih}) and the transmission capacity (T_k), along with the power transfer distribution factor ($PTDF_{ki}$). Constraint (5e) represents the nodal balance with the prosumer's transaction (z_i) embedded. The balance between supply and demand is implied by (5f). Constraints (5g)–(5h) represent the non-negativity of generation and consumption, respectively. Given that the lower level is a concave programming problem, the solutions can be represented by its optimality conditions, which form a mixed complementarity problem characterizing the equilibrium of the market:

$$-C'_{fih}(x_{fih}) - \beta_{fih} + \eta_i + \varepsilon_{fih} = 0 \quad \forall f, i, h \in \mathcal{H}_{fi} \tag{6a}$$

$$B'_i(d_i) - \eta_i + \xi_i = 0 \quad \forall i \tag{6b}$$

$$-\sum_k (\lambda_k^+ - \lambda_k^-) PTDF_{ki} + \eta_i - \theta = 0 \quad \forall i \tag{6c}$$

$$0 \le \beta_{fih} \perp x_{fih} - X_{fih} \le 0 \quad \forall f, i, h \in \mathcal{H}_{fi} \tag{6d}$$

$$0 \le \lambda_k^+ \perp \sum_i PTDF_{ki} y_i - T_k \le 0 \quad \forall k \tag{6e}$$

$$0 \le \lambda_k^- \perp -\sum_i PTDF_{ki} y_i - T_k \le 0 \quad \forall k \tag{6f}$$

$$d_i - \sum_{f,h \in \mathcal{H}_{fi}} x_{fih} - z_i - y_i = 0 \quad \forall i \tag{6g}$$

$$\sum_i y_i = 0 \tag{6h}$$

$$0 \le \varepsilon_{fih} \perp x_{fih} \ge 0 \quad \forall f, i, h \in \mathcal{H}_{fi} \tag{6i}$$

$$0 \le \xi_i \perp d_i \ge 0 \quad \forall i \tag{6j}$$

D. Distributionally Robust Chance-Constrained MPEC

Based on the upper- and lower-level problems in Section II-C, the prosumer's problem is now cast as a distributionally robust chance-constrained MPEC as follows:

$$\underset{\Phi \cup \Omega \cup \Lambda}{\text{maximize}} \ \sum_i \Big[\eta_i z_i + B_i^l(l_i) - C_i^g(g_i)\Big] + \sum_i \mathbb{E}\Big[P_i^c\Big(\tilde{K}_i - z_i - l_i + g_i\Big)\Big] \tag{7a}$$

subject to

$$\inf_{\mathbb{P}_i \in \mathcal{P}_i} \mathbb{P}_i\Big[z_i + l_i - g_i - \tilde{K}_i \le 0\Big] \ge 1 - r_i \quad \forall i$$

$$\text{(4c)–(4e), (6a)–(6j)} \tag{7b}$$

where $\Phi = \{r_i, z_i, l_i, g_i\}$, $\Omega = \{x_{fih}, d_i, y_i\}$, and $\Lambda = \{\beta_{fih}, \lambda_k^+, \lambda_k^-, \eta_i, \theta, \varepsilon_{fih}, \xi_i\}$. In the first line of the objective function, the net benefit of the prosumer is expressed using the equilibrium power price η_i (instead of p_i), which is the dual variable associated with the nodal balance constraint (5e) in the lower-level problem.

We first use the well-known result in [23] to convert the chance constraints into a set of deterministic convex constraints. Specifically, (7b) can be replaced by the following constraint:

$$z_i + l_i - g_i - K_i + \sigma_i \sqrt{\frac{1 - r_i}{r_i}} \le 0 \quad \forall i \tag{8}$$

where K_i and σ_i are the mean and the standard deviation of the random variable $\tilde{K}_i$, respectively, as defined earlier. Constraint (8) is convex since the only nonlinear term, $\sigma_i \sqrt{\frac{1-r_i}{r_i}}$, is convex with respect to r_i. This can be seen by writing out its second derivative: $\frac{1}{2r_i^4}(\frac{1}{r_i} - 1)^{-3/2} + \frac{1}{r_i^3}(\frac{1}{r_i} - 1)^{-1/2}$, which is clearly non-negative $\forall r_i \in (0, 1]$.

The term $K_i - \sigma_i \sqrt{\frac{1-r_i}{r_i}}$ in (8) can be interpreted as a "risk-derated" renewable output perceived by the prosumer, which is assumed to be non-negative:

$$K_i - \sigma_i \sqrt{\frac{1 - r_i}{r_i}} \ge 0 \quad \forall i \tag{9}$$

In our context, the term, $\sigma_i \sqrt{\frac{1-r_i}{r_i}}$, denoted by t_i, represents a "risk-averse reservation," also referred to as a "safety parameter" in the chance-constrained optimization literature.[8] This term can be regarded as the buffer amount of energy perceived by the prosumer in order to hedge against the uncertain renewable output in real time. A risk-averse prosumer with small r_i attempts to maintain sufficient reservation output t_i when making decisions in the day-ahead market, whereas a risk-neutral prosumer with $r_i = 1$ perceives the expected output K_i without any reservations, i.e., $t_i = 0$.

With the above reformulation, we can re-write Problem (7) as a deterministic optimization problem as follows:

$$\underset{\Phi \cup \Omega \cup \Lambda}{\text{maximize}} \ \sum_i \Big[\eta_i z_i + B_i^l(l_i) - C_i^g(g_i)\Big] + \sum_i P_i^c\Big(K_i - z_i - l_i + g_i\Big) \tag{10a}$$

subject to

$$z_i + l_i - g_i + \sigma_i \sqrt{\frac{1 - r_i}{r_i}} \le K_i \quad \forall i \tag{10b}$$

$$\sigma_i \sqrt{\frac{1 - r_i}{r_i}} \le K_i \quad \forall i$$

$$\text{(4c)–(4e), (6a)–(6j)} \tag{10c}$$

Note that the objective function in (10a) is equivalent to (7a) since the decision variables $(z_i, l_i, g_i)_{i=1}^{\mathcal{I}}$ are made before the uncertainty is realized; hence, they can be factored out of the expectation, making the objective function completely deterministic.

E. Existence

In the following, we show the existence of an optimal solution for the MPEC problem (10). To formally present the existence

[8] Note that at an optimal solution, (8) is always binding (i.e., equality), and hence, the second line of (7a) equals $\sum_i P_i^c \sigma_i \sqrt{\frac{1-r_i}{r_i}}$, indicating that the objective function in Problem (7) depends on r_i.

result, we let $\Xi(\mathbf{z})$ denote the set of $\{x_{fih}, d_i, y_i\}$ such that with a given vector $\mathbf{z}$, the same set of constraints are satisfied. We then have the following result.

Proposition 1: (Existence) Assume that for all $\{r_i, z_i, l_i, g_i\}$ that satisfies constraints (10b)–(10c) and (4c)–(4e), with $\mathbf{z} := (z_i)_{i=1}^{\mathcal{I}}$, Mangasarian Fromovitz constraint qualification (MFCQ) holds at all $\{x_{fih}, d_i, y_i\} \in \Xi(\mathbf{z})$. Then the MPEC problem (10) has a globally optimal solution.

Proof: First, it is easy to see that the MPEC problem is always feasible, with a feasible point of setting $r_i = 1$ for all i and all other variables to be 0. Next, we show that the feasible region of the MPEC is compact. While the consumption variables l_i and d_i are not explicitly bounded, based on the definition of the benefit functions $B_i^l(l_i)$ and $B_i(d_i)$, they will become negative when l_i and d_i become too large, which will never happen at an optimal solution. Hence, there are implicit upper bounds for l_i and d_i, for all $i = 1, \ldots, \mathcal{I}$. With l_i and g_i being bounded in (10b), so are the z_i's, which in turn leads to the boundedness of y_i, based on constraints (5e) and (5b). By the MFCQ assumption, the corresponding dual variables in the lower-level problem (6a)–(6j) are also bounded. The closedness of the MPEC feasible region is trivially true with all constraints being continuous. Hence, Problem (10) has a compact feasible region (or more precisely, a compact level set since the bounds on l_i and d_i are not explicit). Since the objective function (10a) is continuous, by the well-known Weirstrass extreme value theorem, a globally optimal solution exists for Problem (10). $\square$

Remark 1: The MFCQ assumption is not restrictive in our case. With downward-sloping consumers/prosumers' marginal benefit functions, the locational marginal prices η_i cannot be unbounded, which then implies the boundedness of the hub price θ and the congestion rents $\lambda_k^{+,-}$ based on constraint (6c), and the capacity rent β_{fih} by constraint (6a).

Remark 2: While we show the existence of a globally optimal solution of the MPEC problem, uniqueness may not hold. As seen in the objective function (10a), there may be multiple solutions such that the sum of the two terms $\sum_i [\eta_i(\mathbf{z}) - P_i^c] z_i$ and $\sum_i \Big[B_i^l(l_i) - C_i^g(g_i) + P_i^c\big(-l_i + g_i\big)\Big]$ remains the same.

F. Solution Method

While state-of-the-art nonlienar programming solvers, such as KNITRO and FILTER, could solve the MPEC directly, we are interested in finding a globally optimal solution with guarantees. It is challenging here because of the bilinear term $\sum_i \eta_i z_i$ in the objective function in addition to the complementarity constraints. To deal with the bilinear term, we propose a method that applies the Wolfe duality to the lower-level problem. Using the strong duality and constraints in the Wolfe dual formulation of the lower-level problem, we show that the bilinear term can be concavified as follows (see details in Appendix B):

$$\sum_i \eta_i z_i = \sum_i B_i'(d_i) d_i - \sum_k (\lambda_k^+ + \lambda_k^-) T_k - \sum_{f,i,h \in \mathcal{H}_{fi}} \Big[C'_{fih}(x_{fih}) x_{fih} + \beta_{fih} X_{fih}\Big] \quad (11)$$

With the above reformulation, we obtain the following MPEC:

$$\begin{aligned} \underset{\Phi \cup \Omega \cup \Lambda}{\text{maximize}} \; & \sum_i B_i'(d_i) d_i - \sum_k (\lambda_k^+ + \lambda_k^-) T_k \\ & - \sum_{f,i,h \in \mathcal{H}_{fi}} \Big[C'_{fih}(x_{fih}) x_{fih} + \beta_{fih} X_{fih}\Big] \\ & + \sum_i \Big[B_i^l(l_i) - C_i^g(g_i) + P_i^c\Big(K_i - z_i - l_i + g_i\Big)\Big] \end{aligned}$$

subject to

$$\text{(8), (9), (4c)–(4e), (6a)–(6j)} \quad (12)$$

which, excluding the complementarity constraints, is a concave programming problem, as the functions $B_i(\cdot)$, $C_{fih}(\cdot)$, $B_i^l(\cdot)$ and $C_i^g(\cdot)$ are all assumed to be convex or concave quadratic functions. To further deal with the complementarity constraints, under the assumption of MFCQ of the set $\Xi(\mathbf{z})$, we can use the well-known big-M reformulation technique to convert the complementarity constraints into mixed-integer based constraints [44], [45]. More specifically, for a generic complementarity constraint (in one dimension): $0 \leq u \perp v \geq 0$, we can equivalently re-write it as (given that u and v are bounded): $0 \leq u \leq Mz$, $0 \leq v \leq (1 - z)M$, $z \in \{0, 1\}$, with a sufficiently large constant M. Based on the above discussions of concavity and using the big-M technique, the MPEC problem (12) can be solved by off-the-shelf mixed-integer solvers, such as Gurobi, to obtain a globally optimal solution.[9]

III. Case Study

A. Data and Assumptions

The model was applied to an IEEE reliability test system (RTS 24-Bus) [38]. The topology of the system comprises 24 buses, 38 transmission lines, and 17 loads. We aggregate the 32 generators into 13 generators by integrating those with the same marginal cost and located at the same node. However, six generation units were excluded from the dataset because they are hydropower units, which operate at a maximum output of 50 MW [47]. To analyze the effect of transmission congestion, the capacity of line 7 between nodes 3 and 24 in the test case was reduced to 150 MW. The generation cost is represented by a quadratic function parameterized by D_{fih}^0 and C_{fih}^0 as coefficients for the linear and quadratic terms, respectively. Furthermore, the prosumer, or the leader, located at node 1 is assumed to have the same demand function as the consumers in that node. The prosumer owns a renewable generation unit that produces varying amounts of power (contingent on available natural resources), and a dispatchable unit as a backup option. The RTS 24-Bus case was first formulated as a least-cost minimization problem and solved with a fixed nodal load to compute dual variables associated with load constraints. Further, the dual variables, in conjunction with an assumed price elasticity of -0.2 are used to calculate the demand

[9]In our case study, the model was solved as a mixed-integer nonlinear program using Gurobi v9.1.2 [46], which found a global optimal solution within seconds on a MacBook Pro equipped with 2.8 GHz Quad-Core Intel Core i7 and 16 G RAM.

TABLE I
RESULTS: THE CASE WHERE P^c =\$20/MWH AND σ =20% (OF K)

Expected renewable output [MWh]	25	110
Prosumer's sale(+)/purchase(–) [MWh]	-59.50	9.96
Prosumer's load [MWh]	99.48	105.32
Prosumer's backup generation [MWh]	14.97	5.28
Power price at node 1 [\$/MWh]	45.21	40.77
Prosumer's risk-averse reservation [MWh]	0	0
Prosumer's optimal risk (r^*)	1	1
Expected revenue(+)/cost(–) of imbalance[a] [\$K]	0	0
Prosumer surplus [\$K]	9.85	13.65
Conventional consumption [MWh]	2,848.55	2,857.91
Conventional generation [MWh]	2,908.05	2,847.95
Grid operator's revenue [\$K]	9.67	5.52
Producer surplus [\$K]	41.92	44.71
Consumer surplus [\$K]	255.89	257.14
Wholesale social surplus [\$K]	307.49	307.36
Total day-ahead social surplus [\$K]	317.35	321.02

[a]: This represents the expected imbalance settlement, which is equal to $P^c t = 0$

TABLE II
RESULTS: THE CASE WHERE P^c =\$60/MWH AND σ =20% (OF K)

Expected renewable output [MWh]	25	110
Prosumer's sale(+)/purchase(-) [MWh]	-79.39	-79.39
Prosumer's load [MWh]	97.56	97.56
Prosumer's backup generation [MWh]	18.16	18.16
Power price at node 1 [\$/MWh]	46.80	46.80
Prosumer's risk-averse reservation [MWh]	25	110
Prosumer's optimal risk (r^*)	0.038	0.038
Expected revenue(+)/cost(–) of imbalance[a] [\$K]	1.5	6.6
Prosumer surplus [\$K]	10.06	15.16
Conventional consumption [MWh]	2,844.93	2,844.93
Conventional generation [MWh]	2,924.33	2,924.33
Grid operator's revenue [\$K]	11.19	11.19
Producer surplus [\$K]	40.96	40.96
Consumer surplus [\$K]	255.45	255.45
Wholesale social surplus [\$K]	307.60	307.60
Total day-ahead social surplus [\$K]	316.17	316.17

[a]: This represents the expected imbalance settlement, which is equal to $P^c t$. We exclude it from the total "day-ahead" social surplus, while including it in calculating prosumer surplus.

parameters, P_i^0 and Q_i^0. The magnitude of the price elasticity of demand is comparable to that in the literature [48]. Hereafter, we omit index i for the variables and parameters associated with the prosumer, focusing on node 1. Several cases are considered by varying the imbalance price P^c (\$20/MWh and \$60/MWh) and expected renewable output $K = \mathbb{E}[\tilde{K}]$ (25 MWh and 110 MWh) with uncertainty characterized by the standard deviation σ (20% and 80% of the expected renewable output K). For example, the uncertainty corresponding to σ =20% of K =110 MWh is equal to 22 MWh ($= 0.2 \times 110$). The results are then presented in the next section.

B. Main Results

The main results are summarized in Tables I–IV. Each table with the same layout contains the results pertaining to the prosumer, wholesale, and economic rent distributions. We further focus our discussion mainly on the behavior of a prosumer. Note that for prosumer surplus, we include the expected monetary value of the imbalance settlement, which is equal to $P^c t$, but exclude it when calculating the "day-ahead" total social surplus as the real-time settlement does not occur in the day-ahead market and it is just an expected transfer between two parties (i.e., cancelled out).

Table I reports the case where P^c =\$20/MWh and σ account for 20% (of K). Facing a relatively low price of P^c =\$20/MWh in real time, the prosumer finds it economically undesirable to maintain a larger "risk-averse reservation" in the day-ahead market. This is because the real-time settlement for energy imbalance is of less concern for the prosumer. Therefore, as demonstrated in Table I, the prosumer prefers less risk-averse attitude, even behaving risk neutrally by choosing r^* =1. The prosumer is in short (long) position when expected renewable output K is 25 MWh (110 MWh), buying 59.5 MWh from (selling 9.96 MWh into) the grid. With a more expected renewable output of K =110 MWh, the prosumer consumes more while selling excess energy into the day-ahead market, which results in a larger amount of prosumer surplus. Overall, the system benefits from a higher expected renewable output, thereby resulting in a greater total day-ahead social surplus.

TABLE III
RESULTS: THE CASE WHERE P^c =\$20/MWH AND σ =80% (OF K)

Expected renewable output [MWh]	25	110
Prosumer's sale(+)/purchase(-) [MWh]	-59.50	9.96
Prosumer's load [MWh]	99.48	105.32
Prosumer's backup generation [MWh]	14.97	5.28
Power price at node 1 [\$/MWh]	45.21	40.77
Prosumer's risk-averse reservation [MWh]	0	0
Prosumer's optimal risk (r^*)	1	1
Expected revenue(+)/cost(–) of imbalance[a] [\$K]	0	0
Prosumer surplus [\$K]	9.85	13.65
Conventional consumption [MWh]	2,848.55	2,857.91
Conventional generation [MWh]	2,908.05	2,847.95
Grid operator's revenue [\$K]	9.67	5.52
Producer surplus [\$K]	41.92	44.71
Consumer surplus [\$K]	255.89	257.14
Wholesale social surplus [\$K]	307.49	307.36
Total day-ahead social surplus [\$K]	317.35	321.02

[a]: Same as in Table I

Table II shows the outcomes when P^c =\$60/WMh and σ is 20% (of K). Under a relatively high price of P^c =\$60/WMh, the energy imbalance settlement in real time is of greater concern, thereby inducing the prosumer to behave conservatively with r^* close to zero by holding a considerable amount of a "risk-averse reservation" in the day-ahead market. The prosumer's reservation becomes as high as the expected renewable output, thereby resulting in a risk-derated output of 0. Consequently, the prosumer increases energy purchases from the grid in the day-ahead market even when 110 MWh of renewable output is expected. This is in contrast to Table I, in which the prosumer behaves risk neutrally, selling 9.96 MWh to the grid under K =110 MWh. The backup generation of the prosumer also increases compared with that in Table I. When the expected renewable output increases, the risk-averse prosumer adjusts their reservation by the same amount, thereby resulting in the same rent distribution in the day-ahead market for K =25 MWh and 110 MWh in Table II.

In Tables III and IV, σ increases from 20% to 80% (of K), while maintaining the same setup for P^c, as shown in Tables I and II. The observations in Tables I and II also emerge in Tables III and IV. This implies that the degree of uncertainty would not affect the market outcomes when the prosumer can endogenously determine or "internalize" their risk attitude to

TABLE IV
RESULTS: THE CASE WHERE P^c =\$60/MWH AND σ =80% (OF K)

Expected renewable output [MWh]	25	110
Prosumer's sale(+)/purchase(-) [MWh]	-79.39	-79.39
Prosumer's load [MWh]	97.56	97.56
Prosumer's backup generation [MWh]	18.16	18.16
Power price at node 1 [\$/MWh]	46.80	46.80
Prosumer's risk-averse reservation [MWh]	25	110
Prosumer's optimal risk (r^*)	0.390	0.390
Expected revenue(+)/cost(–) of imbalance^a [\$K]	1.5	6.6
Prosumer surplus [\$K]	10.06	15.16
Conventional consumption [MWh]	2,844.93	2,844.93
Conventional generation [MWh]	2,924.33	2,924.33
Grid operator's revenue [\$K]	11.19	11.19
Producer surplus [\$K]	40.96	40.96
Consumer surplus [\$K]	255.45	255.45
Wholesale social surplus [\$K]	307.60	307.60
Total day-ahead social surplus [\$K]	316.17	316.17

^{a}: Same as in Table II

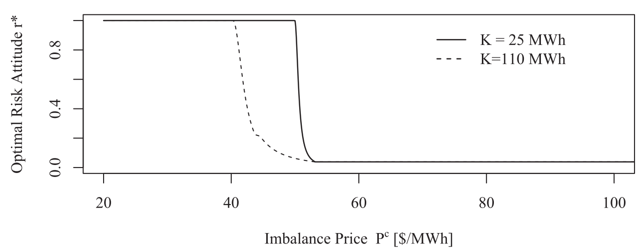

Fig. 1. Plot of optimal risk attitude r^* against imbalance price P^c under $\sigma =$ 20% (of K) and Stackelberg leader-follower cases.

achieve their maximized surplus. That is, the outcomes depend mainly on P^c rather than on σ when the risk preference can be optmized. Consequently, the outcomes in Table III (IV) are equivalent to those in Table I (II).

In Fig. 1, we further elaborate on the effect of imbalance price P^c on the optimal risk attitude r^* under $\sigma = 20\%$ (of K). Fig. 1 shows that the prices for energy imbalance settlement play a key role in the prosumer's decision of the optimal risk attitude, indicating that a relatively low imbalance price induces a less risk-averse behavior (i.e., greater r^*), and vice versa. When the imbalance price rises, the prosumer eventually switches from a risk-neutral to risk-averse attitude within a threshold range of P^c. As illustrated in Fig. 1, the threshold range of the imbalance price is higher when the prosumer faces expected renewable output of K =25 MWh compared to the case of K =110 MWh. This suggests that when facing a smaller magnitude of uncertainty (0.2×25 MWh), the prosumer can tolerate higher imbalance prices. Outside the threshold range of P^c, r^* is equivalent between K =25 and 110 MWh. Finally, the threshold range, in which the risk attitude of the prosumer is more responsive to the imbalance prices, depends on the (day-ahead) power prices at node 1. Had the power price at node 1 been higher, the threshold would also shift to a range bracketed with higher imbalance prices.

C. Relative Profit Loss

This section presents the results concerning the effect of a prosumer's decision about the risk attitude on its profit (or surplus). We define the "relative profit loss" in (13) to quantify the extent

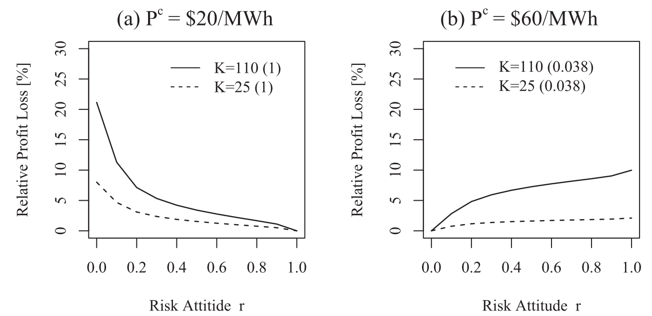

Fig. 2. Plot of relative profit loss against prosumer's risk attitude r under $\sigma =$ 20% (of K) and Stackelberg leader-followers cases.

of a prosumer's forgone profit when choosing a sub-optimal risk attitude r in comparison with the maximum profit under optimal r^* as follows:

$$relative\ profit\ loss = \frac{\pi(r^*) - \pi(r)}{\pi(r^*)} \times 100\% \qquad (13)$$

where $\pi(r^*)$ denotes a prosumer's profit under the optimal risk attitude r^* and $\pi(r)$ is its profit for a given level of r.

Fig. 2 plots the relative profit loss defined in (13) in y-axis against different values of risk tolerance (r) in x-axis for P^c equal to \$20/MWh in (a) and \$60/MWh in (b), respectively. All the scenarios shown in Fig. 2 assume that the degree of uncertainty σ is equal to 20% (of K). The values in parentheses correspond to the optimal r^* in each case.

Several observations have emerged. As demonstrated in Tables I and II and Fig. 1, a relatively low imbalance price of P^c =\$20/MWh makes the prosumer behave risk neutrally by choosing $r^* = 1$, while a relatively high price of P^c =\$60/MWh induces a risk-averse attitude with $r^* = 0.038$ close to 0. Relative profit loss depends on the deviation of r from the optimum r^*. In Fig. 2(a), given $r^* = 1$ under P^c =\$20/MWh, the relative profit losses display as decreasing functions in r. By contrast, the losses shown in Fig. 2(b) increase in r given that r^* is close to 0 under P^c =\$60/MWh. Additionally, the relative profit loss is larger for K =110 MWh compared with the case of K =25 MWh in both figures. This would make sense as getting wrong is likely more costly when the prosumer could have benefited substantially from a greater expected renewable output of K =110 MWh.

We further examine how σ, or the degree of renewable output uncertainty, affects the relative profit loss. Fig. 3 plots the relative profit loss against risk attitude r under various levels of σ with four combinations of K and P^c.[10] Fig. 3(a) and (b) show the cases of P^c =\$20/MWh, in which the optimal risk attitude of the prosumer is $r^* = 1$; that is, to behave risk neutrally. For a given sub-optimal r, when $r^* = 1$ is optimal, the prosumer worsens with a larger value of relative profit loss as the degree of uncertainty increases. In contrast, Fig. 3(c) and (d) illustrate the cases of P^c =\$60/MWh, in which the optimum for the prosumer is to choose the risk-averse attitude of $r^* < 1$. Contrary

[10]Note that condition (9) is violated below a threshold value of r under each scenario. We visualize only the relevant ranges in the figures.

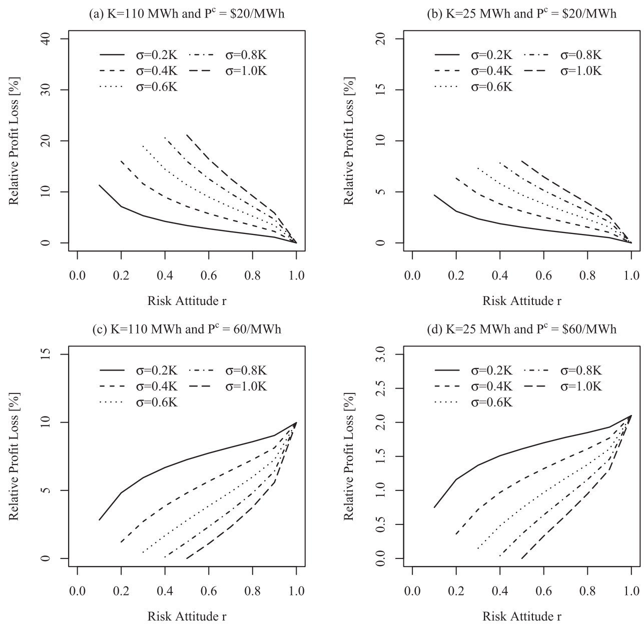

Fig. 3. Plot of relative profit loss ratios for various levels of σ under Stackelberg leader-follower cases.

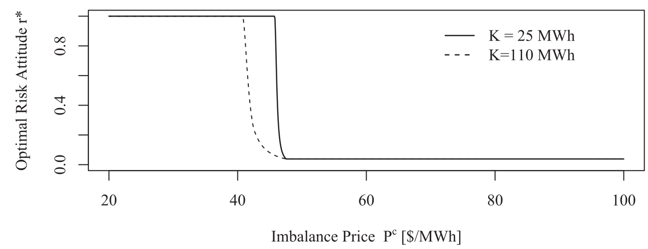

Fig. 4. Plot of optimal risk attitude r^* against imbalance price P^c under $\sigma = 20\%$ (of K) and perfect competition cases.

to the case of P^c =\$20/MWh, for a given sub-optimal r, the risk-averse prosumer is generally better off with a lower value of relative profit loss as the degree of uncertainty increases. Therefore, Fig. 3(a) and (b) imply that conservative risk attitude when P^c is lower is more costly in the face of a situation with greater uncertainty (larger σ and t), while the prosumer should have behaved risk neutrally (or $t = 0$) at the optimum. In contrast, as shown in Fig. 3(c) and (d), failure in adequately adjusting risk attitude when P^c is greater is more costly in the face of a smaller degree of uncertainty (smaller σ and t), resulting in an insufficient risk-averse reservation. Finally, the overall relative profit loss is greater under K =110 MWh than under K =25 MWh, intuitively indicating that the cost of getting wrong is greater with a larger K.

D. Case of Perfect Competition

In the case of perfect competition, the prosumer simply solves Problem (4) without the lower-level problem. Collecting the optimality conditions for both the prosumer and the grid operator, we can solve the resulting mixed complementarity problem to obtain the equilibrium outcomes. Fig. 4 plots the optimal risk attitude r^* against the imbalance price P^c under $\sigma = 20\%$ (of K) in the perfectly competitive case. The results illustrated in Fig. 4 are broadly consistent with those in Fig. 1 under the Stackelberg case in the sense that the threshold range of P^c is higher when the prosumer faces expected renewable output of K =25 MWh compared to the case of K =110 MWh. Moreover, when K =25 MWh, the prosumer abruptly switches from risk-neutral to risk-averse attitude in the threshold of P^c in both Figs. 1 and 4. However, one noticeable difference between these two figures for K =25 MWh is that the threshold of P^c is higher under the Stackelberg case than under the perfect competition case. This implies that the Stackelberg leader prosumer with market power can tolerate higher imbalance prices, whereas perfect competition induces more conservative attitute. Another discernible difference between these two figures is that when K =110 MWh, the transition of r^* from risk-neutral to risk-averse attitude is more gradual in Fig. 1 than in Fig. 4, suggesting that a leader position with market power allows the prosumer to soften more effectively the impact of the imbalance price through adjusting its risk preference.

IV. Conclusion

Entities like prosumers in the electric power sector increasingly confront various risks in the real world, including those induced by climate change, as exemplified by the Texas market in February 2021 [49]. Empirical evidence suggests that decision makers can modify their risk preferences in different situations, e.g., changing environments and volatile market conditions [50]. In the current context, a prosumer participating in the day-ahead wholesale power market can maximize its profit by optimally adjusting the risk preference by further considering uncertain renewable resources. This study proposes a distributionally robust chance-constrained mathematical program with equilibrium constraints (MPEC) approach to examine the prosumer's endogenous decision concerning their risk attitude. We overcome the non-concavity of the prosumer's objective function by applying Wolfe duality to the lower-level problem of the grid operator.

The model is applied to a case study based on the IEEE RTS 24-bus system. Our analysis indicates that when allowing a prosumer to endogenously decide the risk preference on behalf of individual DER owerns in the face of uncertain renewable output with energy imbalance settlement, the prosumer can effectively hedge market risk by behaving more conservatively upon perceiving a "risk-derated" output. However, a fine-tuned adjustment in risk preference is only required within a certain range of imbalance prices, as shown in Fig. 1. Below (Above) the threshold defined by the range, the risk neutrality (extreme risk aversion) is preferred. We demonstrate that the degree of uncertainty in renewable outputs may play a less significant role in determining market outcomes *if* the prosumer can internalize its risk attitude optimally based on the "risk-averse reservation." This insight suggests that endogenizing risk preference can be a useful tool for managing risk in the wholesale market. Finally, compared to the perfectly competitive case, prosumers, as a leader, can adjust their risk preference more comfortably, responding to the real-time imbalance price.

In summary, our contribution lies in developing a distributionally robust chance-constrained MPEC framework to model the endogenous decision making of risk preferences under uncertainty. Our analysis also highlights the importance of understanding the prosumer's risk aversion when evaluating its interaction with, and its effects on a power market. We believe that the framework developed herein as well as the concept of "risk-averse reservation" will become more important risk-management tools in near future when more DERs are introduced to the market. As an extension of our work, data-driven methods can be applied to model uncertainty of renewable output using empirical distributions based on Big Data (see, for example, [31]) although those would require more complex modeling with significant computational costs.

Appendix A
Nomenclature

1) Indices and Sets

$i \in \mathcal{I}$	Nodes.
$f \in \mathcal{F}$	Firms.
$h \in \mathcal{H}_{fi}$	Generating units at node i owed by firm f.
$k \in \mathcal{K}$	Transmission lines.

2) Parameters

$PTDF_{ki}$	Power transmission distribution factor for a unit of power transferred from node i to the hub through line k.
T_k	Thermal limit for line k (MW).
P_i^0, Q_i^0	Vertical and horizontal intercepts of demand curve at node i (\$/MWh).
D_{fih}^0, C_{fih}^0	Coefficients of marginal cost function for generation unit h at node i owned by firm f (\$/MWh).
X_{fih}	Production capacity for generation unit h at node i owned by firm f (MW).
A_i^{l0}, B_i^{l0}	Vertical and horizontal intercepts of prosumer's marginal benefit for consumption at node i (\$/MWh).
$\tilde{K}_i, K_i$	Prosumer's random and mean renewable output at node i (MWh).
σ_i	Standard deviation of prosumer's renewable output at node i (MWh)
D_i^{g0}, C_i^{g0}	Coefficients of marginal cost function for prosumer's dispatchable unit at node i (\$/MWh).
G_i	Production capacity of prosumer's dispatchable unit at node i (MW).
P_i^c	Fixed retail or contracted rate at node i (\$/MWh)

3) Primal Variables

r_i	Prosumer's risk attitude/tolerance variable at node i
t_i	Prosumer's risk-averse reservation at node i (MWh)
z_i	Prosumer's sales (+) or purchases (-) at node i (MWh).
l_i	Prosumer's demand at node i (MWh).
g_i	Power produced by prosumer's dispatchable unit at node i (MWh).
x_{fih}	Power generated by generation unit h at node i owned by firm f (MWh).
d_i	Consumer's demand at node i (MWh).
y_i	Power injection/withdrawal at node i (MWh).

4) Dual Variables

β_{fih}	Dual variable for capacity constraint of unit h at node i owned by firm f (\$/MWh).
λ_k^+, λ_k^-	Dual variables for limit of line k (\$/MWh).
η_i	Dual variable for supply and demand balance at node i (wholesale power price, p_i) (\$/MWh).
θ	Dual variable for total supply and demand balance (\$/MWh).
δ_i	Dual variable for prosumer's power balance at node i (\$/MWh).
κ_i	Dual variable for prosumer's dispatchable generation capacity at node i (\$/MWh).
ν_i	Dual variable associated with prosumer's risk tolerance cap at node i (\$)

Appendix B
Wolfe Dual of Lower-Level Problem

Cosider a general concave program $\max_x\{f(x)|g(x) \le 0\}$, where $x \in \mathbb{R}^n$, $f:\mathbb{R}^n \to \mathbb{R}$, and $g:\mathbb{R}^n \to \mathbb{R}^m$. A concave function f and convex functions g are assumed to be continuously differentiable. Then, the corresponding Wolfe dual is expressed as $\min_{x,\lambda}\{\mathcal{L}(x,\lambda)|\nabla_x\mathcal{L}(x,\lambda)=0,\lambda \ge 0\}$, where $\nabla_x\mathcal{L}(x,\lambda)$ are the gradients of the Lagrangian $\mathcal{L}(x,\lambda) = f(x) - \lambda^\top g(x)$ with multipliers $\lambda \in \mathbb{R}^m$. For a concave (or convex) programming problem, strong duality holds between the primal and Wolfe dual problems if the primal problem satisfies the Slater condition and has optimal solutions [51].

The Wolfe dual of the concave program (5a) in the lower level is represented as follows :

$$\min_{\Omega\cup\Lambda} \quad \sum_i B_i(d_i) - \sum_{f,i,h\in\mathcal{H}_{fi}} C_{fih}(x_{fih}) \tag{A-1a}$$

$$- \sum_{f,i,h\in\mathcal{H}_{fi}} \beta_{fih}(x_{fih} - X_{fih}) - \sum_k \lambda_k^+ \left(\sum_i PTDF_{ki} - T_k\right)$$

$$- \sum_k \lambda_k^- \left(\sum_i -PTDF_{ki}y_i - T_k\right) - \sum_i \eta_i d_i + \sum_i \eta_i z_i$$

$$+ \sum_{f,i,h\in\mathcal{H}_{fi}} \eta_i x_{fih} + \sum_i \eta_i y_i - \theta \sum_i y_i + \sum_{f,i,h\in\mathcal{H}_{fi}} \epsilon_{fih} x_{fih}$$

$$+ \sum_i \xi_i d_i$$

subject to

$$- C'_{fih}(x_{fih}) - \beta_{fih} + \eta_i + \epsilon_{fih} = 0 \quad \forall f,i,h \in \mathcal{H}_{fi} \tag{A-1b}$$

$$B'_i(d_i) - \eta_i + \xi_i = 0 \quad \forall i \tag{A-1c}$$

$$-\sum_k(\lambda_k^+ - \lambda_k^-)PTDF_{ki} + \eta_i - \theta = 0 \quad \forall i \tag{A-1d}$$

$$\beta_{fih}, \epsilon_{fih} \geq 0 \;\; \forall f, i, h \in \mathcal{H}_{fi}, \quad \lambda_k^+, \lambda_k^- \geq 0 \;\; \forall k, \quad \xi_i \geq 0 \;\; \forall i \tag{A-1e}$$

Note that non-negativity constraints are imposed on $\{\beta_{fih}, \lambda_k^+, \lambda_k^-, \varepsilon_{fih}, \xi_i\}$, which are associated with the inequality constraints in the primal problem (5a). Otherwise, the variables are unrestricted. From the strong duality of a concave program, the primal problem (5a) and the Wolfe dual problem (A-1) have the same optimal value. Thus, we obtain:

$$\begin{aligned}
&- \sum_{f,i,h\in\mathcal{H}_{fi}} \beta_{fih}(x_{fig} - X_{fih}) - \theta \sum_i y_i \\
&- \sum_k \lambda_k^+ \left(\sum_i PTDF_{ki} y_i - T_k \right) + \sum_{f,i,h\in\mathcal{H}_{fi}} \varepsilon_{fih} x_{fih} \\
&- \sum_k \lambda_k^- \left(\sum_i -PTDF_{ki} y_i - T_k \right) + \sum_i \xi_i d_i \\
&- \sum_i \eta_i d_i + \sum_i \eta_i z_i + \sum_{f,i,h\in\mathcal{H}_{fi}} \eta_i x_{fih} + \sum_i \eta_i y_i = 0
\end{aligned} \tag{A-2}$$

Using constraints (A-1b)–(A-1d), we can further simplify (A-2). In particular, from (A-1b), we have:

$$\begin{aligned}
&\big(-C'_{fih}(x_{fih}) - \beta_{fih} + \eta_i + \varepsilon_{fih}\big)x_{fih} = 0 \\
&\sum_{f,i,h\in\mathcal{H}_{fi}} (-\beta_{fih} + \eta_i + \varepsilon_{fih})x_{fih} = \sum_{f,i,h\in\mathcal{H}_{fi}} C'_{fih}(x_{fih})x_{fih}
\end{aligned} \tag{A-3}$$

From (A-1c), we derive:

$$\begin{aligned}
&\big(B'_i(d_i) - \eta_i + \xi_i\big)d_i = 0 \\
&\sum_i \big(-\eta_i + \xi_i\big)d_i = -\sum_i B'_i(d_i)d_i
\end{aligned} \tag{A-4}$$

Fom (A-1d), we obtain:

$$\begin{aligned}
&\Big(-\sum_k(\lambda_k^+ - \lambda_k^-)PTDF_k + \eta_i - \theta\Big)y_i = 0 \\
&\sum_i \big(\eta_i - \theta\big)y_i = \sum_{k,i}(\lambda_k^+ - \lambda_k^-)PTDF_k y_i
\end{aligned} \tag{A-5}$$

Substituting (A-3), (A-4), and (A-5) into (A-2), we can concavify the bilinear term $\sum_i \eta_i z_i$ as follows:

$$\begin{aligned}
\sum_i \eta_i z_i = &\sum_i B'_i(d_i)d_i - \sum_k(\lambda_k^+ + \lambda_k^-)T_k \\
&- \sum_{f,i,h\in\mathcal{H}_{fi}} \big(C'_{fih}(x_{fih})x_{fih} + \beta_{fih}X_{fih}\big)
\end{aligned} \tag{A-6}$$

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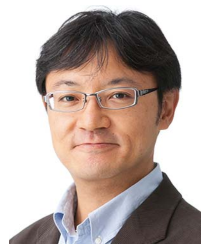

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