A Reference Governor for Linear Systems with Polynomial Constraints

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Abstract

The paper considers reference governor design for a class of linear discrete-time systems with constraints given by polynomial inequalities and constant reference commands. For such systems, we propose a novel algorithm to compute the maximal output admissible set. The reference governor solves a constrained nonlinear minimization problem at the initialization and then uses a bisection algorithm at the subsequent time steps. The effectiveness of the method is demonstrated by two numerical examples.

Key words: reference governors, nonlinear systems, control of constrained systems, polynomial methods.

1 Introduction

Reference governors are add-on schemes that, whenever possible, preserve the response of a nominal controller designed by conventional control techniques [1] while ensuring that output constraints are not violated. Conventional reference governor schemes are based on the so called maximal output admissible set (MOAS), i.e., the set of initial states and constant reference commands for which the ensuing response satisfies the constraints. Invariant inner approximations of this set represented by a finite number of inequalities can be easily computed when both the systems dynamics and output constraints are linear.

Of particular importance in control theory is the case when the nominal controller is based on feedback linearization (see, e.g, [2,3]) and the nonlinear dynamics are rendered linear by a coordinate transformation and an appropriately defined feedback law. However, when input or output constraints are present, even if they were linear to start with, they are generally transformed into nonlinear constraints on the transformed coordinates. Nonlinear constraints are much harder to manage with the reference governor techniques especially when they are not convex [4–6]. Therefore, in general, feedback linearization cannot be easily combined with a conventional reference governor [1] which assumes a linear model and linear constraints.

In this paper, the problem of designing a reference governor for linear systems with polynomial constraints and constant reference commands, r is addressed. For most of the paper, $r = 0$ is assumed and it is shown that the case $r \neq 0$ can be handled through coordinate change. The key idea is to embed the linear system into another higher dimensional linear system, the state of which, when correctly initialized, encompasses the state of the original linear system plus its higher order powers. Doing so, the polynomial inequality constraints required to design a reference governor become linear with respect to the extended state's coordinates.

The contributions of this paper are as follows. Firstly, we develop a procedure for computing the MOAS of a linear system with polynomial constraints. It is shown that this set is a subset of the MOAS for the aforementioned extended linear system with linear constraints and that it is representable by a finite number of inequalities under suitable conditions. The second contribution of this paper is the design of the reference governor utilizing thereby computed MOAS. This design, unlike conventional reference governors, exploits exponentially decaying reference dynamics so that the trajectory cannot hit a non convex polynomial constraint representing, e.g., an obstacle, and become stuck there. Then, the reference governor computation reduces to solving a constrained nonlinear minimization problem at the initial time instant and then using a bisection algorithm at the subsequent time instants. Finally, numerical examples are reported to illustrate the proposed approach.

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2 Preliminaries and problem statement

We first review properties of the Kronecker product [7] and then introduce the problem treated in this paper.

2.1 Kronecker products and polynomial systems base vectors

The Kronecker product of matrices A and B is denoted by $A \otimes B$. This product is non-commutative but associative and has the following useful mixed product property: if A, B, C and D are matrices of such size that one can form the matrix products AB and CD, then

$$(AB \otimes CD) = (A \otimes C)(B \otimes D).$$

Given a vector $x \in \mathbb{R}^{n_x}$ and $p \in \mathbb{N} > 1$, its powers $x^{p\otimes} \in \mathbb{R}^{pn_x}$ are defined recursively using the Kronecker product of two column vectors

$$x^{p\otimes} := \underset{i=1..p}{\otimes} x = x \otimes \left(\underset{i=1..p-1}{\otimes} x \right) = x \otimes (x \otimes \ldots (x \otimes x)) \quad (1)$$

$$= \left[x_1 x^{(p-1)\otimes} \;\; x_2 x^{(p-1)\otimes} \;\; \ldots \;\; x_{n_x} x^{(p-1)\otimes} \right]^T. \quad (2)$$

Since the product of two real numbers is commutative, it is not difficult to see that the vector $x^{p\otimes}$ possesses some redundant terms. We use the notation x^p to denote a base vector containing all monomials $x_1^{i_1} \ldots x_{n_x}^{i_{n_x}}$ for which $i_1 + \ldots + i_{n_x} = p$. Then, and as first remarked in [8], the dimension of the non-redundant vector x^p is given by

$$\sigma(n_x, p) = \frac{(n_x + p - 1)!}{p!(n_x - 1)!}. \quad (3)$$

For example, when $n_x = p = 2$, we have $x = [x_1, x_2]^T$, $x^{2\otimes} = [x_1^2, x_1x_2, x_2^2, x_2x_1]^T$ and $x^2 = [x_1^2, x_1x_2, x_2^2]^T$. Following [9], one can recursively compute the matrices $M_c(n_x, p) \in \mathbb{R}^{\sigma(n_x,p) \times pn_x}$ and $M_e(n_x, p) \in \mathbb{R}^{pn_x \times \sigma(n_x,p)}$ that determine the relation between the power vector with and without redundant terms:

$$x^p = M_c(n_x, p) x^{p\otimes}, \qquad x^{p\otimes} = M_e(n_x, p) x^p.$$

2.2 Problem statement

Consider a linear (pre-stabilized) discrete-time system with the model given by

$$x(k+1) = Ax(k) + Bv(k), \quad (4)$$

where $x \in \mathbb{R}^{n_x}$ is the state, $v \in \mathbb{R}^{n_v}$ is the reference governor output, and A is a Schur matrix. Let $x_v := [x^T, v^T]^T \in \mathbb{R}^{n_x+n_v}$ be the state and reference vector. The system (4) is subject to n_c polynomial constraints, which are expressed as

$$f_i(x_v) = \sum_{j=1}^{p} c_{i,j} x_v^j \leq h_i, \quad i \in \{1, \ldots, n_c\}, \quad (5)$$

where the row matrices $c_{i,j}$ are in $\mathbb{R}^{1 \times \sigma(n_x+n_v,j)}$. In this paper, we propose a reference governor strategy to ensure that the polynomial constraints (5) of a pre-stabilized linear system (4) are satisfied for all times while the reference governor output, $v(k)$, tends to the desired reference $r = 0$.

Remark 1 If r is constant and different from 0, one can perform a change of coordinates and then apply the proposed method to the error system between x and the equilibrium point $\bar{x}$ defined by $\bar{x} = A\bar{x} + Br$, where $\bar{x}$ exists and is unique since A is a Schur matrix. The polynomial constraints are reformulated in the error system coordinates as well before applying the proposed method. ⌟

3 Reference governor design

Given that the desired reference is $r(k) = 0$, we consider the input into (4), $v(k)$, being generated by:

$$v(k+1) = \beta v(k), \quad (6)$$

where $\beta \in]0, 1[$. Then, x_v evolves according to

$$x_v(k+1) = \Phi x_v(k), \quad (7)$$

where $\Phi = \begin{bmatrix} A & B \\ O_{n_v,n_x} & \beta I_{n_v} \end{bmatrix}$ is a Schur matrix since $\beta \in]0, 1[$.

Conventional reference governors for linear systems with linear constraints are based on the offline computation of a finitely determined inner approximation to the MOAS, called $\tilde{O}_\infty$ [1]. Then the reference v is updated on-line subject to the constraint $(x(k), v) \in \tilde{O}_\infty$. In the scalar reference governor case, the computational effort is generally small. The objective of this section is to extend these ideas to linear systems subjected to polynomial inequality constraints and propose procedures

- to compute (off-line) the MOAS O_∞ for system (7) with constraints (5), and
- to update the reference governor online based on this set.

Remark 2 Note that in the conventional reference and command governors [1], $\beta = 1$ is allowed. However, $\beta < 1$ is also used in [10] to handle time-varying reference commands and/or constraints, and non-constant reference evolution over prediction horizon is also assumed in extended command governor schemes [1]. Here, choosing β in $]0, 1[$ helps ensure that $v(k) \to r = 0$ as $k \to \infty$, which is otherwise difficult to guarantee if constraints are not convex as in cases treated in [4–6]. ⌟

3.1 Maximal output admissible set computation

For system (7) subject to constraints (5), the MOAS is defined as

$$O_{\infty,X} = \{x_v(0):\ f_i(x_v(k)) \le h_i, i = 1,\dots,n_c, k = 0,1,\dots\}, \quad (8)$$

where $x_v(k)$ is the state response of (7) at time instant k corresponding to the initial state $x_v(0)$. We call $O_{\infty,X}$ (and other MOASs in the sequel) finitely determined if the recursive procedure for its construction in [11] finitely terminates (i.e., there exists t^* such that $O_{t,X} = O_{t+1,X}$ for all $t \ge t^*$ where $O_{t,X}$ corresponds to imposing constraints on predicted response in (8) only for $k = 0,\cdots,t$); in this case, $O_{\infty,X}$ is representable by a finite number of inequality constraints. Note that the converse property does not hold, in general.

Remark 3 As Φ in (7) is Schur, the MOAS is computed in the sequel without requiring inner approximation [11]. ⌟

Let p be given and consider the following state augmentation:

$$X_v := \begin{bmatrix} x_v & x_v^2 & \dots & x_v^p \end{bmatrix}^T.$$

Let $j \in \{1,\dots,p\}$ and observe that:

$$\begin{aligned} x_v^j(k+1) &= M_c(n_x+n_v,p)x_v^{j\otimes}(k+1) \\ &= M_c(n_x+n_v,p)(\Phi x_v(k))^{j\otimes} \\ &= M_c(n_x+n_v,p)(\otimes_{i=1:j}\Phi)x_v^{j\otimes}(k) \\ &= M_c(n_x+n_v,j)(\otimes_{i=1:j}\Phi)M_e(n_x+n_v,j)x_v^j(k) \\ &:= \Phi^j x_v^j(k), \end{aligned} \quad (9)$$

i.e., the extended state vector of all monomials can be linearly propagated. Before stating our main result, we require the following two lemmas.

Lemma 1 *if Φ is a Schur matrix then Φ^j is a Schur matrix for all $j \in \mathbb{N}^*$.*

Proof. The proof follows from the fact that Φ is Schur if and only if $x_v = 0$ is a globally asymptotically stable equilibrium of (7). As a consequence, for all $j \in \mathbb{N}^*$, $x_v^j = 0$ is a globally asymptotically stable equilibrium of (9), which in turn implies that Φ^j is a Schur matrix. ☐

Let

$$\Phi_Z = diag(\Phi^j, j \in \{1,\dots,p\}) \quad , \quad H_Z = [h_1 \dots h_{n_c}]^T, \quad (10)$$

$$C_Z = (c_{i,j})_{i=1:n_c, j=1:\sigma(n_x+n_v,p)}, \quad (11)$$

and consider the following extended system:

$$Z(k+1) = \Phi_Z Z(k) \quad (12)$$

where $Z_j \in \mathbb{R}^{\sigma(n_x+n_v,j)}$ and where $Z = [Z_1^T,\dots,Z_p^T]^T \in \mathbb{R}^{\sum_{j=1\dots p}\sigma(n_x+n_v,j)}$ is subject to the constraints,

$$C_Z Z \le H_Z. \quad (13)$$

Remark 4 Consider a particular initial condition for (12) given by $Z_i(0) = Z_1^i(0) = x_v^i(0)$ for all $i \in \{2,\dots,p\}$. Then $Z_1(k) = x_v(k)$ for all $k > 0$, i.e., $Z_1(k)$ evolves according to our model (7). Furthermore, $Z_i(k) = Z_1(k)^i$ for all $k > 0$ and $i = 1,\cdots,p$. In this case, $f_i(x_v(k)) = C_Z Z(k)$ for all k, i.e., the polynomial constraints (5) exactly match the linear constraints (13). Thus adding polynomial constraints is an operation very similar to adding linear constraints, except that we had first to specify a basis to represent the polynomial constraint as linear constraints. For $Z_1 \in \mathbb{R}^{n_x+n_v}$, define

$$\varphi(Z_1) = [Z_1^\mathsf{T}, (Z_1^2)^\mathsf{T}, \dots, (Z_1^p)^\mathsf{T}]^\mathsf{T}.$$

The above discussion implies that

$$\varphi(\Phi^k Z_1) = \Phi_Z^k \varphi(Z_1)$$

for all Z_1. ⌟

In the sequel, we propose to compute the MOAS of our extended linear system; a subset (cross section) of this MOAS then provides the exact MOAS of the original system (7) with polynomial constraints (5).

Theorem 1 *The set $O_{\infty,Z} = \{Z(0):\ C_Z\Phi_Z^k Z(0) \le H_Z,\ k = 0,1,\dots\}$ is forward invariant. Furthermore, if Φ_Z is a Schur matrix and the set $\{Z:\ C_Z Z \le H_Z\}$ is compact with the origin in the interior, then $O_{\infty,Z}$ is nonempty and finitely determined.*

Proof. The proof follows immediately from the results in [11] applied to the case when the system output is the full state (hence observability condition of [11] is automatically verified). ☐

Note that $O_{\infty,Z}$ is the MOAS of the extended system neglecting the requirements that $Z^i = Z_1^i$. Now we establish and exploit the relation between $O_{\infty,Z}$ and $O_{\infty,X}$ suggested by Remark 4.

Theorem 2 *Let Φ_Z be a Schur matrix and the set $\{Z:\ C_Z Z \le H_Z\}$ be a compact set with the origin in the interior. Then $O_{\infty,X}$, the MOAS of (7)-(5), is representable by a finite number of inequalities and is forward invariant.*

Proof. Based on Remark 4, $O_{\infty,X} = \{Z_1 \in \mathbb{R}^{n_x+n_v}:\ \varphi(Z_1) \in O_{\infty,Z}\}$. Theorem 1, under our assumptions, implies that $O_{\infty,Z}$ is finitely determined; hence $O_{\infty,X}$ is representable by a finite number of inequalities. Furthermore, by Theorem 1, $O_{\infty,Z}$ is forward invariant. Now $Z_1 \in O_{\infty,X}$ implies $\varphi(Z_1) \in O_{\infty,Z}$ and, by Remark 4 and forward invariance of $O_{\infty,Z}$, $\varphi(\Phi^k Z_1) = \Phi_Z^k \varphi(Z_1) \in O_{\infty,Z}$ implying $\Phi^k Z_1 \in O_{\infty,X}$ thereby demonstrating the forward invariance of $O_{\infty,X}$. ☐

Theorem 2 requires that the set $\{Z:\ C_Z Z \le H_Z\}$ be compact. This property may not hold when the dimension of Z is large and the number of constraints n_c is small. It turns out the result of Theorem 2 still holds if (13) restricts $Z_1 = C_Z^1 Z$ to a compact set:

Theorem 3 *Let C_Z^1 and G_Z^1 be such that $C_Z Z \leq H_Z$ implies $G_Z^1 C_Z^1 Z \leq H_Z^1$ while the pair (Φ, C_Z^1) is observable and the set $\{Y: \; G_Z^1 Y \leq H_Z^1\}$ is compact with the origin in the interior. Then $O_{\infty,X}$ is representable by a finite number of inequalities and is forward invariant.*

Proof. From results in [11], the set $O_{\infty,Z_1} = \{Z_1: \; G_Z^1 C_Z^1 \Phi^k Z_1 \leq H_Z^1, \; k = 0,1,\ldots\}$ is compact and defined by a finite number of linear inequalities. Clearly, $O_{\infty,X} \subset O_{\infty,Z_1}$. Let $\tilde{C}_Z$, $\tilde{H}_Z$ be such that $Z_1 \in O_{\infty,Z_1}$ implies $\tilde{C}_Z \varphi(Z_1) \leq \tilde{H}_Z$ and the set $\{Z: \; \tilde{C}_Z Z \leq \tilde{H}_Z\}$ is compact with zero in the interior. Now augment the inequalities $\tilde{C}_Z Z \leq \tilde{H}_Z$ to (13) and compute the corresponding

$$O_{\infty,\bar{Z}} = \left\{ \bar{Z}: \; \begin{bmatrix} \tilde{C}_Z \\ C_Z \end{bmatrix} \Phi_Z^k \bar{Z} \leq \begin{bmatrix} \tilde{H}_Z \\ H_Z \end{bmatrix}, \quad k = 0,1,\ldots \right\}.$$

By Theorem 1, $O_{\infty,\bar{Z}}$ is finitely determined. By construction, $O_{\infty,\bar{Z}}$ has the same cross section as $O_{\infty,Z}$, i.e., $O_{\infty,X} = \{Z_1: \; \varphi(Z_1) \in O_{\infty,\bar{Z}}\}$. Indeed, if $\varphi(Z_1) \in O_{\infty,\bar{Z}}$, then $\varphi(Z_1) \in O_{\infty,Z}$ and $Z_1 \in O_{\infty,X}$. Vice versa if $Z_1 \in O_{\infty,X}$, then $\varphi(Z_1) \in O_{\infty,Z}$ and $\tilde{C}_Z \Phi_Z^k \varphi(Z_1) = \tilde{C}_Z \varphi(\Phi^k Z_1) \leq \tilde{H}_Z$ given that $\Phi^k Z_1 \in O_{\infty,X} \subset O_{\infty,Z_1}$ for all k due to forward invariance of $O_{\infty,X}$. Thus $\varphi(Z_1) \in O_{\infty,\bar{Z}}$. □

Remark 5 The proof of Theorem 3 leads to a procedure to compute $O_{\infty,X}$. First, linear constraints on Z_1 are found in such a way that the corresponding O_{∞,Z_1} is compact and finitely determined. Such linear constraints can be a combination of linear constraints on only Z_1 from (13) and redundant linear constraints overbounding possible values of Z_1 as constrained by (13). These linear constraints on Z_1 then induce linear constraints overbounding possible values of the extended state Z (as $Z = \varphi(Z_1)$); the latter linear constraints are added-in when computing $O_{\infty,\bar{Z}}$. Finally, $O_{\infty,X}$ is defined by the condition $\varphi(Z_1) \in O_{\infty,\bar{Z}}$. ⌟

3.2 Reference governor update

Let $O_{\infty,X}$ be the MOAS (8) considered in the previous subsection. Given an initial state $x(0)$, $v(0)$ is computed so that $(x(0), v) \in O_{\infty,X}$, e.g., as a solution to the following optimization problem:

$$\text{Minimize } v^{\mathsf{T}} v \text{ subject to } (x(0), v) \in O_{\infty,X}. \qquad (14)$$

Then, the reference to be applied at time $k > 0$ is:

$$v(k) = \beta v(k-1) + \lambda(k)(0 - \beta v(k-1)) \qquad (15)$$

where $\lambda(k) \in [0,1]$. The scalar variable $\lambda(k)$ can be determined using a bisection algorithm such that

$$\lambda(k) = \max_{\lambda \in [0,1]} \lambda \text{ subject to } (x(k), \beta(1-\lambda)v(k-1)) \in O_{\infty,X}. \qquad (16)$$

Note that given the feasibility of $\lambda = 0$ in (16), i.e., of $v' = \beta v(k-1)$ with $\beta \in]0,1[$ (due to our construction of MOAS), it immediately follows that $v(k)$, $x(k)$ tend to 0 as $k \to \infty$ as long as $v(0)$ is found such that $(x(0), v(0)) \in O_{\infty,X}$. The preview time of the bisection algorithm does not need to be 'heuristically' chosen since it is directly applied to the set of inequalities that defines the MOAS $O_{\infty,X}$. To reduce the computational burden and thus facilitate the practical application of the proposed method, we suggest to calculate 'off-line' both the MOAS and the initial value $v(0)$ for a grid of initial states. Doing so, only a simple bisection algorithm needs to be run 'on-line' to solve the optimization problem (16).

4 Numerical examples

4.1 Stall prevention of a civil aircraft

We consider the following aircraft longitudinal dynamics model based on [12] with $\cos(\alpha)$ approximated by 1:

$$\ddot{\alpha} = -\frac{d_1}{J} L(\alpha) + \frac{d_2}{J} u, \;\; L(\alpha) = l_0 + l_1 \alpha - l_3 \alpha^3,$$

where $d_1 = 4m$, $d_2 = 42m$, $J = 4.5 \times 10^5 Nm^2$, $l_0 = 2.5 \times 10^5 N$, $l_1 = 8.6 \times 10^6 N/rad$ and $l_3 = 4.35 \times 10^7 N/rad^3$. The angle of attack α is constrained by the stall limit as $-0.2 \times \frac{\pi}{180} \leq \alpha \leq 14.7 \times \frac{\pi}{180}$rad. The control input is the elevator force u and must satisfy $|u| \leq 4.10^5$ N.

Applying the dynamic inversion, $u = -k_p(\alpha + v) - k_d \dot{\alpha} + \frac{d_1}{d_2} L(\alpha)$ where $k_p = 5.2 \times 10^7$, $k_d = 7.6 \times 10^6$, and discretizing the system with the sampling period $T_s = 0.01s$, we obtain the (pre-stabilized) second order model (4) with

$$A = \begin{bmatrix} 0.9814 & 0.0072 \\ -3.3347 & 0.4940 \end{bmatrix}, \;\; B = \begin{bmatrix} 0.0186 \\ 3.3347 \end{bmatrix}.$$

This system is linear but the input inequality constraints are polynomial of order 3. Considering the extended state x_v and v defined by (6) with $\beta = .98$, we compute the MOAS O_{∞,Z_1} in Remark 5 only considering the linear constraint on α in this case. The MOAS O_{∞,Z_1} is finitely determined in $t^* = 75$ iterations of algorithm in [11] and is defined by 105 non-redundant linear inequalities. As this set is compact, we can compute bounds on all the components of the extended state and confirm, following Remark 5, that MOAS $O_{\infty,\bar{Z}}$ is representable by a finite number of inequalities. In fact, the $O_{\infty,Z}$ is determined in $t^* = 46$ iterations and is defined by 304 non-redundant linear inequalities. Figure 1 illustrates the constrained outputs responses obtained using (in blue) or not using (in magenta) the proposed reference governor when $\alpha(0) = 14\,deg$ and $\dot{\alpha}(0) = 0\,deg/s$. In the absence of a reference governor, the control input limits are violated. However, all the output constraints are satisfied with the implementation of the proposed reference governor strategy.

The projection of the MOAS set on the $(\alpha,\dot{\alpha})$ plane gives the set of initial conditions from which the state can be stabilized while respecting both the linear and polynomial constraints. Figure 2 shows this set and used a grid to calculate it. The constrained nonlinear minimization problem was solved at each point of the grid to determine whether or not the point belongs to this set.

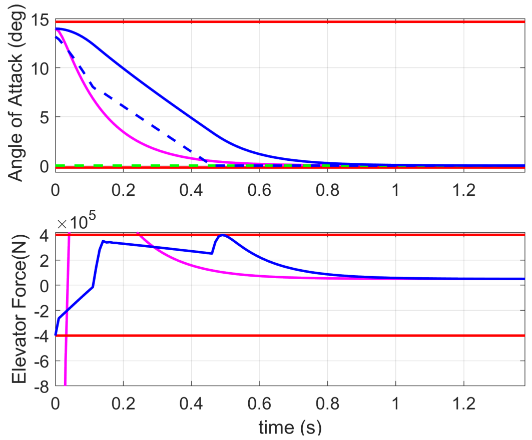

Fig. 1. Constrained outputs. Red: upper and lower limits. Magenta: when one applies the dynamic inversion without any reference governor, that is when $v=r=0$. Blue: when one uses the proposed reference governor. Dashed blue: evolution of the reference $v(k)$ of the reference governor. Dashed green: desired angle of attack.

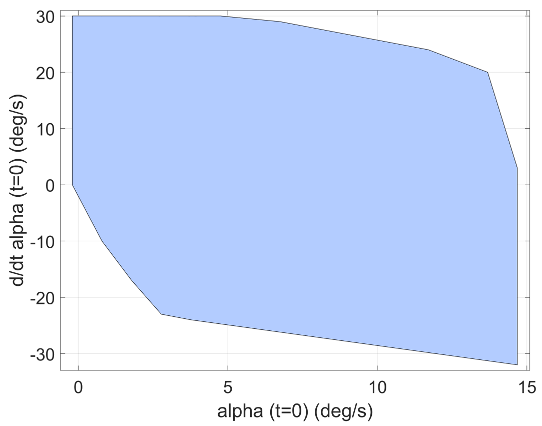

Fig. 2. Projection of the MOAS set

4.2 *Obstacle avoidance in the 2D plane*

The second numerical example shows the method's applicability to a controlled system subject to a non-convex polynomial constraint. We consider $\ddot{x}=u$ where $x,u\in\mathbb{R}^2$. This system is pre-stabilized using $u=-x-2\dot{x}+v$ where v is the reference governor input. Then, we discretize it with sampling period $T_s=0.1$s. The linear constraints are $|x|\leq 20$ and $|\dot{x}|\leq 5$. The nonlinear constraint $(x-x_{obs})^T(x-x_{obs})\geq r_{obs}^2$ is used to avoid a circular obstacle located at $x_{obs}=[10;0]$ with a radius $r_{obs}=2$. Considering the extended state x_v and v defined by (6) with $\beta=.98$, we first compute the MOAS O_{∞,Z_1}. This set is finitely determined in $t^*=15$ iterations and is defined by 108 non-redundant linear inequalities. Then, $O_{\infty,Z}$ is determined when we extend the state and add the nonlinear constraint. Here, $O_{\infty,Z}$ is determined in $t^*=343$ iterations and is defined by 863 non-redundant linear inequalities. Figure 3 illustrates the obstacle avoidance realized with the proposed reference governor strategy for several initial conditions $x(0)$ and when $\dot{x}(0)=[0;0]$. Figure 4 shows the evolution of the reference input vector used in this case. For example, when $x(0)=[12;-.01]$, x first follows the obstacle's boundary before reaching the origin. It should be noted that the proposed strategy does not use any waypoint [5] nor does it create a navigation field [6].

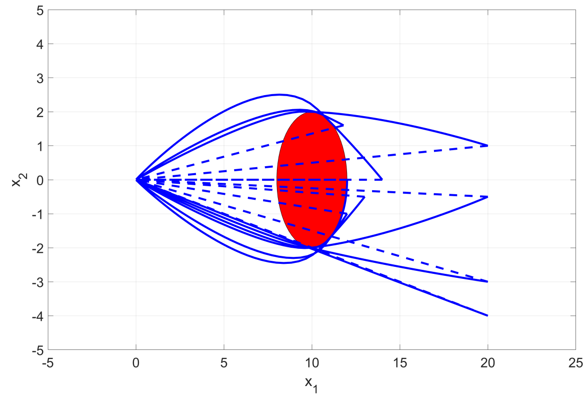

Fig. 3. Illustration of the obstacle avoidance. Red: obstacle. Dashed blue: without any reference governor, that is to say when $v=r=0$. Blue: when one uses the proposed reference governor.

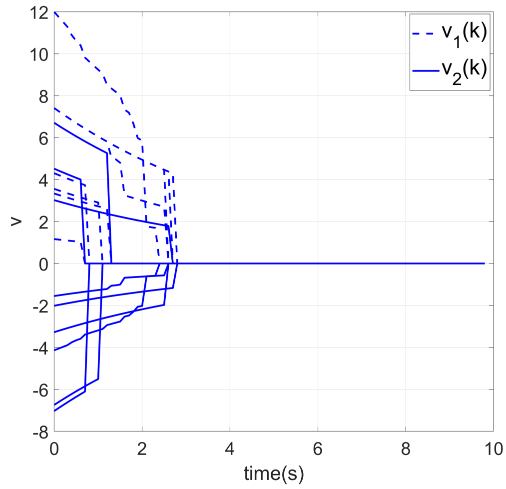

Fig. 4. Time evolution of the reference governor.

5 Concluding remarks

The developments in this paper are based on the observation that the propagation of some polynomial constraints through a LTI system can be accomplished by propagating some linear constraints through a higher dimensional LTI system. This permits extending the design of conventional reference governors to the class of LTI systems with polynomial constraints. Numerical results were reported to demonstrate the simplicity and practicality of the proposed method.

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