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Risk-averse and strategic prosumers: A distributionally robust chance-constrained MPEC approach

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Abstract

Under the ruling of FERC Order 2222, prosumers who own distributed energy resources are expected to play an essential role that can significantly affect the market outcomes. This paper assesses the market power potential of a risk-averse prosumer by designating it as a Stackelberg leader in the market. We formulate the problem as a mathematical program with equilibrium constraints with a distributionally robust chance-constrained framework to account for the renewable generation uncertainty. The numerical results demonstrate that the prosumer's strategy depends on the magnitude of renewable generation uncertainty and the degree of risk aversion, which jointly affect the perceived quantity of its renewable resources. The risk-averse Stackelberg case always yields a higher payoff for the prosumer compared to the Cournot and perfect competition cases. Moreover, it is more potent for the prosumer to exercise buyer's market power rather than seller's market power. The situation worsens when the prosumer is less risk averse. This highlights the importance of understanding the role of the prosumer acting either as a consumer or as a producer and the risk faced by the prosumer when evaluating its interaction with the wholesale market.

Keywords Prosumers · Mathematical program with equilibrium constraints · Distributionally robust chance constraint · Stackelberg · Market power

1 Introduction

The power sector is undergoing rapid transformations in terms of available technologies and architecture. Driven by a need for decarbonization, sustainability, and resilience, we have witnessed a remarkable move towards advancing and deploying

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distributed renewable resources as well as harnessing price-responsiveness of energy demand. These include demand response, storage and other flexible resources, which altogether form the broader concept of distributed energy resources (DERs). This paradigm shift towards a more engaged demand side challenges the conventional top-down power grid architecture based on the supply side and calls for a new market design for the power sector (e.g., FERC Order 745 and 2222). In particular, as new agents, such as "prosumers" with the ability of concurrent generation and consumption, are introduced to the power sector, their presence is expected to alter economic incentives, which might create opportunities for manipulations, thereby undermining efficiency of the power market.

While behind-the-meter households or end-prosumers may have limited access to the main grid, the ruling under the FERC 2222 allows the integration of multiple DERs owned by different entities with different sizes and diverse technologies to participate in the regionally organized wholesale energy, capacity, and ancillary services markets alongside traditional resources [1]. We refer to those entities with direct access to the main grid as "prosumers" in order to distinguish them from "end-prosumers." This would also affect decisions on the expansion of transmission and distribution networks, and investments in power generation facilities [2].

The transactions between DER owners and the wholesale power market are empowered by the presence of aggregators at both the transmission and distribution levels. Examples include community choice aggregators, which are popular in California and other states. Aggregators could be responsible for operation of generation assets, e.g., solar panels, storage, and electric vehicle charging, over wide geographical areas and diverse types of households that constitute a substantial distributed generation and energy management capability [3, 4]. This provides an economic leverage for prosumers participating in the wholesale power markets far beyond ordinary customers as they are capable of manipulating considerable resources over time and space, which at the same time, also fundamentally changes the business models within the electricity market [5]. To align incentives with the desired outcomes of the power market, it is imperative to understand how this new entity, the prosumer, might impact market outcomes given current market conditions.

An emerging concern is that DERs' growing presence in the market might render them market power potential.[1] Indeed, wind producers in some regions (e.g., western Denmark) have already gained dominant positions in power markets, possibly acting as price-makers instead of price-takers [9]. Over the past decade, strategic offering of wind producers as price-makers has been extensively studied in the literature [9–12]. Those studies investigate typical monopoly/oligopoly of the producer side to raise the market price, which can be regarded as "single-sided" market

[1] A recent thread of literature has also focused on the role of aggregators as middle-men in the power sector that operate DERs on behalf of owners and their interactions with the wholesale power market [3, 4]. The heterogeneity in terms of geographical placement and type of resources owned by the prosumers grants them a competitive advantage, as the information is likely to be private, only known by prosumers. This also calls for a careful examination of DERs market power potential in the market [6], and other studies have shown that even low levels of wind penetration could enable strategic manipulation, leading to efficiency loss [7, 8].

power, similar in the case of conventional thermal generators. By contrast, owners of DERs or prosumers could potentially exert "double-side" market power: that is, the prosumer selling excess power to the grid may attempt to raise the price as a monopolist/oligopolist, whereas it may try to lower the price as a monopsonist/oligopsonist purchasing power from the grid. To the best of our knowledge, most studies on prosumers' market power only focus on monopoly/oligopoly of the producer side [13, 14]. A few exceptions are [15, 16], which examine prosumers' market power from both producer and consumer perspectives. However, these studies do not consider inherent uncertainty of prosumers' renewable sources. Thus, our contribution is to develop a model for studying market power of risk-averse strategic prosumer when subjecting them to uncertainty of DER outputs.

Regression-based analysis is a common approach used by researchers when empirically examining the extent of ex post market power. However, it is less useful for vetting the potential of market power when existing data are not yet available. On the other hand, game-theoretical models based on bottom-up formulations have extensively been used to evaluate ex ante electricity market outcomes (see, for example, [17, 18]). The strength of these models is that they allow representing the interactions among different market participants, especially new ones, while considering market rules and other institutional settings. A number of studies have applied this approach to address outcomes of a power market considering information asymmetry related to aggregators. For instance, [19] examines the impact of a demand response (DR) aggregator operating a green energy management system in the wholesale market by implementing a quantity-based (Cournot) strategy. The paper, however, (1) does not reflect the dual nature of concurrent generation and consumption, and (2) is limited to Cournot strategy by the aggregator, which is just one of the several strategies at the aggregator's disposal. On the other hand, the Stackelberg game has long been used to model sequential-move games or leader-follower situations [20–27]. In a more recent work, a game in the electricity market is modeled in a Stackelberg setting, where the aggregator of DERs is the leader, and the grid operator along with other producers are the followers [28]. The DER in these studies is basically modeled as a supplier and is unable to reflect the buyer's market power that a prosumer can exhibit.

This paper is related to existing work [15, 16] but is different in significant ways. We extend the simultaneous-move game (Cournot assumption) of [15] to a sequential-move game (Stackelberg assumption) to examine the extent of double-sided market power exercised by a risk-averse prosumer. When designating a prosumer as a leader in a Stackelberg framework, we assume that the prosumer who owns DERs could gain a non-negligible position in the market in the near future, a similar assumption of a leader-wind producer in [9–12].[2] This framework demonstrates the potential of prosumer's double-side market power, and hence the findings from our analyses can help guide market surveillance authorities. Similar to [15, 16], the model does not fixate the role of the prosumer as a producer (long position)

[2] In fact, a recent report concludes that by 2050, prosumers can produce twice as much power as nuclear production now in Europe (https://www.greenbird.com/news/utility-death-spiral).

or as a consumer (short position), but allows the solutions to decide what its role should be in order to maximize its profit. Moreover, we extend the models in [15, 16] by applying a distributionally robust chance-constrained framework to account for the uncertainty of the prosumers' renewable generation, leading to a stochastic mathematical program with equilibrium constraints (MPEC) for a leader-follower or Stackelberg setting.

The various conclusions under the assumption of risk-neutrality in [16] cannot be directly applicable to or generalized to the risk-averse situation when the impact of the renewable generation uncertainty and risk attitude is considered. A distributionally robust chance constraint approach, which is a more recent strand of research on uncertainty, has been applied to various instances such as the optimal power flow (OPF) model, where the chance constrained OPF limits the probability of violating transmission constraints [29]. To our best knowledge, our paper is the first attempt to develop a distributionally robust chance-constrained MPEC for a leader-follower setting, focusing on a risk-averse leader prosumer in the power sector. The non-convexity of the MPEC resulting from the bilinear terms in the upper-level problem and from the optimality conditions of the lower-level problem is overcome using Wolfe duality and disjunctive constraints. The problem is then recast as a mixed integer quadratic program (MIQP). Our findings demonstrate that the prosumer's decision depends on both renewable output uncertainty and prosumer's risk preference, which jointly affect the "perceived" quantity of its renewable resources. The Stackelberg case always yields a higher payoff for the risk-averse prosumer compared to the Cournot and perfect competition cases. Interestingly, the analysis reveals that it is more potent for the prosumer to exercise buyer's market power in a short position rather than seller's market power in a long position. The effect of market power becomes more significant when the prosumer is less risk averse. The findings are new and would provide valuable information to guide regulatory authorities to oversee prosumers in the near future.

The rest of this paper is organized as follows. In Sect. 2, the leader-follower formulation of the prosumer's problem is introduced. A case study based on the IEEE 24-bus system is implemented in Sect. 3. The outcomes of analyses altering the amount of renewable output are presented in Sect. 4. We conclude the paper in Sect. 5.

2 Models

We introduce in this section the prosumer's problem in the upper-level and the lower-level optimization problems faced by the grid operator. Throughout the paper, we denote I as the set of nodes and L as the set of transmission lines consisting of elements in ordered pairs of distinct nodes. F is the set of generation firms, while H is the set of generation units, and $H_{fi} \subset H$ is the set of generation units owned by firm f at node i. Greek letters render the corresponding dual variable.

We consider a Stackelberg framework in which the prosumer is the leader and the other market participants are followers. This Stackelberg game is formulated as an MPEC formally defined as in (1):

$$\text{minimize} \quad f(x, y) \tag{1a}$$

$$\begin{aligned} &\text{subject to} \\ &(x, y) \in Z \end{aligned} \tag{1b}$$

$$y \in \mathcal{S}(x) \tag{1c}$$

In this setup, $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$, $f : \mathbb{R}^{n+m} \to \mathbb{R}$, $F : \mathbb{R}^{n+m} \to \mathbb{R}^m$, and $Z \subseteq \mathbb{R}^{n+m}$. We also define a set-valued mapping $C : \mathbb{R}^n \to \mathbb{R}^m$, where $C(x)$ is a closed convex subset of $\mathbb{R}^m$ for each $x \in \mathbb{R}^n$, and let X be the projection of Z onto $\mathbb{R}^n$. $\mathcal{S}(x)$ is the solution to the variational inequality defined as $(v - y)^\top F(x, y) \geq 0, \quad \forall v \in C(x)$ [30].

An MPEC can be regarded as an optimization problem faced by a leader (upper-level problem), whose actions affect the equilibrium of a market (lower-level problem), which consequently affects the leader's objective. The following sections discuss the model in detail.

2.1 Upper-level prosumer's problem

We assume that the problem for a prosumer is the result from bundling a large number of individual DERs, thereby allowing them to interact with the bulk market directly, consistent with the situation stipulated by the recent FERC Order 2222 [1].[3]

The renewable generation output at node i is denoted by a random variable $\tilde{K}_i$, which is dependent on available natural resources in real time, e.g., solar and wind.[4] We assume that the distribution $\mathbb{P}_i$ of $\tilde{K}_i$ belongs to a set $\mathcal{P}_i$ of distributions, where the first and second moments are known, i.e., $\mathbb{E}(\tilde{K}_i) = K_i$ and $\mathbb{V}(\tilde{K}_i) = \sigma_i^2$, respectively, but without exact knowledge of the probability distributions. Meanwhile, the prosumer also owns a dispatchable or backup resource, e.g., on-site diesel generator, that supplies power g_i with an increasing and strictly convex cost function $C_i^g(g_i)$ and with capacity of G_i in order to hedge against uncertain output $\tilde{K}_i$. Specifically, we assume a quadratic cost function $C_i^g(g_i) = D_i^{g0} g_i + \frac{C_i^{g0}}{2} g_i^2$.[5] Yet, its supply would not be able to fully back up the intermittent renewable output.

[3] For instance, prosumers are allowed to participate in a day-ahead market but are subject to a fixed retail or contracted rate in real time, similar to the situation faced by several EU countries, e.g., Italy, Netherlands, and Belgium [31].

[4] Two concerns, privacy and truth-telling, are worth more discussion. We believe that neither should be a significant concern in the current context. Considering that DERs owners enter a contract with a prosumer, e.g., OhmConnect in [32], the contract will then specify the types of necessary private information from the owner and how the information will be handled. Moreover, given that the prosumer is tasked to maximize the joint profit of participants, non-truth-telling by end-prosumers would undermine its ability to maximize the joint profit on their behalf.

[5] We assume a diesel generator that exhibits an increasing marginal cost. Moreover, the backup on-site generation can represent or be generalized to various options. It could also be a battery system characterized by an increasing marginal cost as storing energy over time may incur additional costs due to round-trip (in)efficiency.

For the purposes of this study, the prosumer's benefit function of consuming electricity at node i is given by $B_i^l(l_i)$, where l_i corresponds to the self-consumption at each node. The benefit function $B_i^l(l_i)$ is assumed to be increasing and strictly concave. The monotonicity of $B_i^l(l_i)$ indicates that the prosumer's objective function is increasing in the level of consumption. Specifically, we assume a quadratic benefit function $B_i^l(l_i) = A_i^{l0} l_i - \frac{A_i^{l0}}{2B_i^{l0}} l_i^2$.[6] We posit that the prosumer maximizes its profit by deciding (1) the amount of power to buy from ($z_i < 0$) or sell to ($z_i > 0$) in node i at price p_i, (2) the amount of its own power consumption l_i, and (3) the amount of power to be generated from the backup dispatchable technology or g_i. We also assume that the prosumer is only allowed to sell/buy power locally, i.e., at each node, which is consistent with the future grid's layered structure [5].[7]

We then formulate a distributionally robust chance-constrained problem of the prosumer facing an uncertain renewable output $\tilde{K}_i$ as follows:

$$\begin{aligned} \underset{z_i, l_i, g_i}{\text{maximize}} \; & \sum_i \Big(p_i z_i + B_i^l(l_i) - C_i^g(g_i) \Big) \\ + & \sum_i \mathbb{E}\Big[P_i^c \Big(\tilde{K}_i - z_i - l_i + g_i \Big) \Big] \end{aligned} \tag{2a}$$

subject to

$$\inf_{\mathbb{P}_i \in \mathcal{P}_i} \mathbb{P}_i \Big(z_i + l_i - g_i - \tilde{K}_i \leq 0 \Big) \geq 1 - R_i \quad (\delta_i), \forall i \tag{2b}$$

$$g_i \leq G_i \quad (\kappa_i), \forall i \tag{2c}$$

$$l_i, g_i \geq 0 \quad \forall i \tag{2d}$$

where $R_i \in (0, 1)$ denotes a risk tolerance level of the prosumer. The three terms in the first line of the objective function (2) correspond to revenue (+) or cost (−) from transactions in the day-ahead wholesale market, benefit of consuming power, and generation costs incurred from backup resource, respectively. The second line gives the expected cost/revenue in real time, where P_i^c is the fixed retail rate or the contracted price between the prosumer and the utility at node i. Constraint (2b) is the distributionally robust chance constraint of the prosumer. It states that the sum of renewable output $\tilde{K}_i$ and self generation g_i net of transactions with the wholesale day-ahead market, i.e., z_i, has to be equal or greater than the self-consumption l_i with

[6] $B_i^{l\prime}$ is entirely separated and different from $B_i^{\prime}$ that represents willingness-to-pay of consumers in the wholesale market. One can subject them to a sensitivity analysis to understand their impacts on the outcomes. However, since the prosumer's strategy is mainly impacted by how much renewable generation it faces, and its net position as a net seller or a net buyer with respect to the wholesale market, our numerical example in Sect. 3 examines these two possibilities explicitly by changing the level of the mean renewable output.

[7] Had the prosumers been modeled to allow to sell surplus power from its local node i to other locations, it is expected to produce the same market outcomes [15].

probability $1 - R_i$ or greater for any distributions in $\mathcal{P}_i$. The dual variable for the distributionally robust chance constraint, δ_i, represents the marginal impact of risk tolerance level on the expected benefit of the prosumers in (2). Constraint (2c) limits the output g_i by its capacity G_i with the dual variable κ_i. Constraint (2d) states the non-negativity of the variables for self-consumption and backup generation.

2.2 Lower-level problem

We next introduce the lower-level problem, in which the grid operator takes supply bids from suppliers and demand bids from consumers/load serving entities and maximizes the bulk market's social surplus subjected to prosumer's decision z_i. Let x_{fih}, d_i, and y_i denote the power output produced by generation unit h at node i owned by firm f, the quantity demanded by consumers at node i, and the power injection/withdrawal at node i, respectively. We assume (1) an increasing and strictly concave benefit function $B_i(d_i) = P_i^0 d_i - \frac{P_i^0}{2Q_i^0} d_i^2$ for consumers, which is separated from that of the prosumer, $B_i^l(l_i)$ in (2), and (2) an increasing and strictly convex cost function $C_{fih}(x_{fih}) = D_{fih}^0 x_{fih} + \frac{C_{fih}^0}{2} x_{fih}^2$ for generation.

$$\underset{x_{fih}, d_i, y_i}{\text{maximize}} \sum_i B_i(d_i) - \sum_{f,i,h \in H_{fi}} C_{fih}(x_{fih}) \tag{3a}$$

$$\text{subject to} \quad x_{fih} \le X_{fih} \quad (\beta_{fih}), \forall f, i, h \in H_{fi}, \tag{3b}$$

$$\sum_i PTDF_{ki} y_i \le T_k \quad (\lambda_k^+), \forall k, \tag{3c}$$

$$-\sum_i PTDF_{ki} y_i \le T_k \quad (\lambda_k^-), \forall k, \tag{3d}$$

$$d_i - \sum_{f,h \in H_{fi}} x_{fih} - z_i = y_i \quad (\eta_i), \forall i, \tag{3e}$$

$$\sum_i y_i = 0 \quad (\theta) \quad , \tag{3f}$$

$$x_{fih} \ge 0 \quad (\varepsilon_{fih}), \forall f, i, h \in H_{fi}, \tag{3g}$$

$$d_i \ge 0 \quad (\xi_i), \forall i \tag{3h}$$

The lower-level problem is the social surplus maximization problem faced by the grid operator or independent system operator (ISO), as formulated in (3a). The lossless linearized DC flow is applied to modeling power flow in the transmission network using the power transfer distribution factor ($PTDF_{ki}$).[8] Constraints (3b)–(3d) limit generation capacity (X_{fih}), and transmission capacity (T_k). Constraint (3e) is the nodal balance with prosumer's transaction (z_i) embedded. The inclusion of this constraint shifts the demand of conventional consumers in the wholesale market operated by the ISO, given the decision z_i of the leader prosumer in the upper level. Specifically, it follows from (3e) that the inverse demand (or marginal benefit) function of conventional consumers can be represented as $B_i'(d_i) = B_i'(\sum_{f,h} x_{fih} + z_i + y_i)$ in equilibrium.[9] When the prosumer purchases $z_i (< 0)$ from node i, the effective "wholesale" demand increases or shifts to the right by the absolute value of z_i, reflecting the demand from both the conventional consumers and the prosumer. Similarly, if the prosumer sells $z_i (> 0)$ to node i instead of purchase, then the wholesale demand decreases or shifts to the left by z_i through (3e). Note also that the benefit function of the prosumer, $B_i^l(l_i)$, does not appear in the objective function of the ISO in (3a), but this does not affect the outcome in the lower-level problem because the leader prosumer's decision l_i and resulting $B_i^l(l_i)$ in the upper level are taken as given (and hence exogenous) by the ISO in the lower level. The balance between supply and demand is implied in (3f). Notice that the social surplus maximization problem does not include sales of each generation firm as a decision variable but rather decides on their output (x_{fih}) and the sales/purchases by the prosumer in node i (z_i). However, once x_{fih} and z_i are decided by the solution of the problem, the sales balance for the generation firm holds automatically and would be consistent with (3e) and (3f). Constraints (3g)–(3h) state the non-negativity of generation and consumption, respectively. Given that the lower level is a concave programming problem, the solutions can be represented by its optimality conditions as follows:

$$-C_{fih}'(x_{fih}) - \beta_{fih} + \eta_i + \varepsilon_{fih} = 0 \quad \forall f, i, h \in H_{fi}, \tag{4a}$$

$$B_i'(d_i) - \eta_i + \xi_i = 0 \quad \forall i, \tag{4b}$$

$$-\sum_k (\lambda_k^+ - \lambda_k^-) PTDF_{ki} + \eta_i - \theta = 0 \quad \forall i, \tag{4c}$$

$$0 \leq \beta_{fih} \perp x_{fih} - X_{fih} \leq 0 \quad \forall f, i, h \in H_{fi}, \tag{4d}$$

[8] Of course, the voltage variation at the distributional level could be a concern. Given that (1) our main interest lies at the wholesale level as well as (2) the fact that prosumers can aggregate over various DERs over space, each with a small capacity, which will have a limited impact on voltages in the medium voltage circuits, we therefore abstract from representing the distribution network. An example is OhmConnect, which recently announced a plan to link homes dispersed in California to form a 550-MW virtual power plant (VPP) of DERs [32].

[9] The interaction of the prosumer with the day-ahead wholesale energy market is modeled through shifting of demand curves of conventional consumers. An alternative way of modeling this situation is to horizontally aggregate consumers' and prosumers' demand curves. However, this aggregation might lead to kinked demand curves, which poses numerical difficulties, see [33] for example.

$$0 \leq \lambda_k^+ \perp \sum_i PTDF_{ki} y_i - T_k \leq 0 \quad \forall k, \tag{4e}$$

$$0 \leq \lambda_k^- \perp -\sum_i PTDF_{ki} y_i - T_k \leq 0 \quad \forall k, \tag{4f}$$

$$d_i - \sum_{f,h \in H_{fi}} x_{fih} - z_i - y_i = 0 \quad \forall i, \tag{4g}$$

$$\sum_i y_i = 0, \tag{4h}$$

$$0 \leq \epsilon_{fih} \perp x_{fih} \geq 0 \quad \forall f, i, h \in H_{fi}, \tag{4i}$$

$$0 \leq \xi_i \perp d_i \geq 0 \quad \forall i \tag{4j}$$

2.3 Distributionally robust chance-constrained MPEC formulation

This section describes the Stackelberg leader-follower formulation in the context of prosumers in the electricity market. Here, the upper-level problem is the prosumer surplus maximization, and the lower-level problem is a collection of complementarity or optimality conditions in the market derived from the grid operator's social surplus maximization problem. The resulting problem of the prosumer is cast as a distributionally robust chance-constrained mathematical program with equilibrium constraints (or DRCC-MPEC) in (5):

$$\begin{aligned} \underset{\Phi \cup \Omega \cup \Lambda}{\text{maximize}} \sum_i \Big(\eta_i z_i + B_i^l(l_i) - C_i^g(g_i) \Big) \\ + \sum_i \mathbb{E}\Big[P_i^c \Big(\tilde{K}_i - z_i - l_i + g_i \Big) \Big] \end{aligned} \tag{5a}$$

subject to

$$\inf_{\mathbb{P}_i \in \mathcal{P}_i} \mathbb{P}_i \Big(z_i + l_i - g_i - \tilde{K}_i \leq 0 \Big) \geq 1 - R_i \quad (\delta_i), \forall i \tag{5b}$$

$$g_i \leq G_i \quad (\kappa_i), \forall i \tag{5c}$$

$$\begin{aligned} & l_i, g_i \geq 0 \quad \forall i \\ & (4a) - (4j) \end{aligned} \tag{5d}$$

where $\Phi = \{z_i, l_i, g_i\}$, $\Omega = \{x_{fih}, d_i, y_i\}$, and $\Lambda = \{\beta_{fih}, \lambda_k^+, \lambda_k^-, \eta_i, \theta, \epsilon_{fih}, \xi_i\}$. The first line of the objective function is the net benefit of the prosumer with equilibrium

Table 1 Results under Stackelberg leader prosumer cases

Variables\scenarios	(a)	(b)	(c)
Mean renewable output [MW]	25	50	120
Prosumer's sale(+)/purchase(−) [MWh]	− 60.83	− 42.26	11.69
Prosumer's load [MWh]	99.35	101.14	105.42
Prosumer's backup generation [MWh]	15.19	12.21	5.11
Marginal cost of backup [$/MWh]	50.17	47.21	40.11
Imbalance settlement quantity [MWh]	1.67	3.33	8.00
Prosumer surplus [$K]	9.77	10.91	13.73
Total power demand [MWh]	2848.31	2851.68	2858.07
Total power production [MWh]	2909.13	2893.94	2846.38
Power price in node 1 [$/MWh]	45.32	43.83	40.68
Sales-weighted power price [$/MWh]	35.49	35.39	35.19
Grid operator's revenue [$K]	9.77	8.35	5.44
Producer surplus [$K]	41.86	42.77	44.76
Consumer surplus [$K]	255.86	256.30	257.17
Wholesale social surplus [$K]	307.50	307.42	307.37

power prices (η_i) derived from the dual variable associated with the nodal balance constraint (3e) in the lower-level problem. Constraints (5b)–(5d) indicate the operational constraints of the prosumer. Constraints (4a)–(4j), which are inherited from the optimality conditions of the lower-level problem, form the complementarity problem characterizing the equilibrium of the market.

The problem specified in (5) is difficult to solve due to at least three reasons. First, (5b) considers probability constraints over all possible distributions with given moments. Second, (5) is neither linear nor concave because of the bilinear term $\sum_{fi} \eta_i z_i$. Third, the feasible region is not convex because of the complementarity conditions. We address the first issue of uncertainty by replacing (5b) with a robust counterpart of the distributionally robust chance constraint indicated in (6) as in [34].[10]

$$z_i + l_i - g_i - K_i + \sigma_i \sqrt{\frac{1 - R_i}{R_i}} \leq 0 \quad \forall i \tag{6}$$

Moreover, regarding the second issue, we propose using the Wolfe's strong duality to concavify the bilinear term in the objective function (see Appendices A and B). We can then substitute the bilinear term in the objective function using equality (7):

[10] In fact, (6) is binding in equilibrium. Thus, for a given P_i^c, the second line of (5a) is equal to $\sum_i P_i^c \sigma_i \sqrt{\frac{1-R_i}{R_i}}$, which is a constant. The term $\sigma_i \sqrt{\frac{1-R_i}{R_i}}$ can be regarded as expected imbalance settlement quantity in real time. With fixed σ_i and R_i, expected real-time balancing is the same regardless of the prosumer's strategies (see also Tables 1, 2 and 3). However, note that a change in the renewable output uncertainty σ_i or prosumer's risk preference R_i indeed affects the outcomes through (6).

Table 2 Results under perfect competition cases

Variables\scenarios	(a)	(b)	(c)
Mean renewable output [MW]	25	50	120
Prosumer's sale(+)/purchase(−) [MWh]	− 67.74	− 47.06	12.54
Prosumer's load [MWh]	101.95	102.94	105.10
Prosumer's backup generation [MWh]	10.87	9.22	5.64
Marginal cost of backup [$/MWh]	45.87	44.22	40.64
Imbalance settlement quantity [MWh]	1.67	3.33	8.00
Prosumer surplus [$K]	9.75	10.89	13.73
Total power demand [MWh]	2847.05	2850.81	2858.15
Total power production [MWh]	2914.79	2897.87	2845.61
Power price in node 1 [$/MWh]	45.87	44.22	40.64
Sales-weighted power price [$/MWh]	35.52	35.42	35.19
Grid operator's revenue [$K]	10.30	8.72	5.40
Producer surplus [$K]	41.53	42.53	44.79
Consumer surplus [$K]	255.71	256.18	257.18
Wholesale social surplus [$K]	307.54	307.44	307.37

Table 3 Results under Cournot prosumer cases

Variables\scenarios	(a)	(b)	(c)
Mean renewable output [MW]	25	50	120
Prosumer's sale(+)/purchase(−) [MWh]	− 20.10	− 13.86	3.61
Prosumer's load [MWh]	84.03	90.46	108.46
Prosumer's backup generation [MWh]	40.59	29.93	0.07
Marginal cost of backup [$/MWh]	75.59	64.93	35.07
Imbalance settlement quantity [MWh]	1.67	3.33	8.00
Prosumer surplus [$K]	9.12	10.58	13.71
Total power demand [MWh]	2855.21	2855.77	2857.34
Total power production [MWh]	2875.31	2869.63	2853.73
Power price in node 1 [$/MWh]	42.23	41.92	41.07
Sales-weighted power price [$/MWh]	35.28	35.27	35.22
Grid operator's revenue [$K]	6.84	6.56	5.79
Producer surplus [$K]	43.76	43.95	44.50
Consumer surplus [$K]	256.87	256.86	257.07
Wholesale social surplus [$K]	307.37	307.37	307.36

$$\sum_i \eta_i z_i = \sum_i B_i'(d_i)d_i - \sum_k (\lambda_k^+ + \lambda_k^-)T_k \\ - \sum_{f,i,h\in H_{fi}} \left(C_{fih}'(x_{fih})x_{fih} + \beta_{fih}X_{fih} \right) \tag{7}$$

We finally linearize the complementarity conditions by disjunctive constraints [35] (see Appendix C). Consequently, DRCC-MPEC (5) is recast as a mixed integer quadratic program (MIQP).

2.4 Perfectly competitive and Cournot models

In contrast to the Stackelberg leader–follower formulation in Sect. 2.3, the cases of perfect and Cournot competition entail simultaneous moves among all entities with a different information structure. Particularly, these involve solving simultaneously the prosumer's problem in (2) and the ISO's problem in (3a). As discussed in Sect. 2.3, we derive a robust counterpart of the distributionally robust chance constraint for the prosumer's problem. Then, the overall problem can be solved by the collection of first-order conditions of the prosumer's and the ISO's problems. This forms a problem known as a mixed linear complementarity problem (MLCP). In the Cournot case, the prosumer is fully aware of the wholesale's demand function and able to manipulate power prices. Under perfect competition, the prosumer behaves as a price-taker. Perfect competition and Cournot formulations for the prosumer along with their theoretical properties and existence of solutions are discussed in [15].

3 Numerical example

3.1 Data and assumptions

The model is applied to the IEEE Reliability Test System (RTS 24-Bus) [36]. The topology of the system consists of 24 buses, 38 transmission lines, and 17 constant-power loads with a total of 2,850 MW. We aggregate 32 generators into 13 generators by combining those with the same marginal cost and located at the same node. Six generation units, however, are excluded from the dataset since they are hydropower units, which operate at their maximum output of 50 MW [37]. In order to analyze the impact of transmission congestion, the capacity of line 7, between nodes 3 and 24, in the test case is reduced to 150 MW. The cost of generation is represented by a quadratic function parameterized by D_{fih}^0 and C_{fih}^0 as the coefficient for the linear and quadratic terms, respectively. Furthermore, the prosumer, or the leader, located at node 1 is assumed to have the same demand function as consumers in that node. The prosumer owns a renewable generation unit that produces varying amounts of power contingent on available natural resources and a dispatchable unit as a backup option. The RTS 24-Bus case is first formulated as a least-cost minimization problem and solved with fixed nodal load in order to compute dual variables associated with load constraints. The dual variables together with an assumed price elasticity of − 0.2 are then used to calculate the demand parameters, P_i^0 and Q_i^0. The magnitude of price elasticity of demand is comparable with the literature [38].

We examine three scenarios in detail, varied by the levels of mean renewable output from the unit owned by the prosumer at node 1 with $R_1 = 0.9$. Mean renewable output $K_1 = \mathbb{E}(\tilde{K}_1)$ is assumed to have three levels, 25, 50, and 120 MW depending on the weather condition, with their uncertainties characterized by their standard deviation σ_1, which is set at 20% of the mean output (i.e., $0.2K_1$). The implicit assumption is that the renewable capacity is fixed, but its output is subject to different levels of expected output due to weather condition, such as cloud coverage that affects solar production. These levels are chosen carefully to show results in both short and long positions of the prosumer. We also discuss the overall impact on economic rent among entities.

3.2 Results

Table 1 summarizes market outcomes when the prosumer is formulated as a Stackelberg leader for three scenarios, 25, 50, and 120 MW of mean renewable generation.[11] We also report outcomes from perfect and Cournot competition in Tables 2 and 3, respectively. As indicated in the first row of Table 1, the prosumer changes from purchase (−) to sale (+) with increased levels of mean renewable output. For cases of 25 and 50 MW, the prosumer buys 60.83 MWh and 42.26 MWh, respectively, acting as a consumer or in a *short* position. As expected, the quantity of the purchases decreases as the prosumer's renewable output grows. In (c), where mean renewable output is equal to 120 MW, the prosumer lies in a *long* position in equilibrium and sells 11.69 MWh to the power market. The prosumer's purchase and sale quantities in Table 1 are in between those of perfect and Cournot competition indicated in Tables 2 and 3.

Prosumer's quantity demanded, or load, is indicated in the second row of Table 1. It increases as the prosumer has more renewable generation resources available and is equal to 99.35, 101.14, and 105.42 MWh for (a)–(c), respectively. Facing more mean renewable output with zero marginal cost by the prosumer (moving from (a) to (c)) implicitly shifts the market supply curve to the right, leading to an increase in electricity consumption. The prosumer's quantity demanded is also in between those of perfect and Cournot competition illustrated in Tables 2 and 3: less than 101.95 MWh in (a) of Table 2 for perfect competition yet higher than 84.03 MWh for the Cournot case in Table 3. The same observation emerges in (b). In (c) where the prosumer's mean renewable output is 120 MW, the prosumer's demand in the Stackelberg equilibrium remains in between those of perfect and Cournot competition. However, in this case, the prosumer's quantity demanded is bounded above by the Cournot case (108.46 MWh) and bounded below by perfect competition (105.10 MWh) and asymptotically approaches that of the perfect competition. We

[11] All the results presented in this section exclude revenue/cost in real time, which is a constant across different market structures.

address its implication when discussing profit earned by the prosumer. With more renewable available, the prosumer decreases generation from the dispatchable unit as it requires less generation from the backup unit in light of higher levels of mean renewable output. This effectively reduces the marginal cost of the backup unit as more renewable resources become available as shown in Table 1, where the marginal cost of the backup unit decreases from 50.17 \$/MWh in (a) to 40.11 \$/MWh in (c).

Turning to the prosumer surplus, it is always the highest in Stackelberg equilibrium, unlike aforementioned market outcomes that lay in between price-taker and Cournot strategies.[12] This implies that the leader prosumer can exercise "double-side" market power at the highest level: i.e., as demonstrated in Table 1, it acts as a monopsonist/oligopsonist, purchasing power from the grid in (a) and (b), while it behaves as a monopolist/oligopolist, selling excess power to the grid in (c). For instance, as noted in Tables 2 and 3, the prosumer's profit in (a) is 9.75 and 9.12 \$K for the prefect and Cournot competition cases, respectively, which are lower than 9.77 \$K for the Stackelberg case in Table 1. The difference, however, narrows as the prosumer faces more renewable output in (c), suggesting that it is easier for the prosumer to exert buyer's market power in a short position than seller's market power in a long position since the latter is likely to be offset by other conventional producers. Moreover, the prosumer benefits from facing more renewable generation output. This is because having more resources with zero marginal cost, the prosumer is able to rely less on backup unit (lower operating cost), and sell more to (buy less from) the market, thereby obtaining higher benefit. As seen in Table 1 (also in Fig. 2), the prosumer surplus follows an increasing trend of 9.77, 10.91, and 13.73 (\$K) for 25, 50, and 120 MW of mean renewable output, respectively.

The power price in node 1, where the prosumer resides, is directly affected by the available renewables: i.e., it drops with increases in the amount of renewables. For example, in Table 1, the power price at node 1 is \$45.32/MWh with the mean renewable output of 25 MW, which is reduced to \$43.83/MWh and further to \$40.68/MWh when the mean renewable generation is 50 and 120 MW, respectively. The same impact over the entire grid can also be observed as the sales-weighted power price is reduced with more renewable generation output.[13] As indicated in Table 1, the sales-weighted power price is reduced from \$35.49/MWh in column (a) to \$35.39/MWh and \$35.19/MWh in columns (b) and (c), respectively. The power

[12] As alluded to in Footnote 10, the expected real-time imbalance settlement, $\sum_i P_i^c \sigma_i \sqrt{\frac{1-R_i}{R_i}}$, is the same constant across different market structures with fixed P_i^c, σ_i and R_i. We exclude it when calculating prosumer surplus in the day-ahead market. An interesting observation pointed out by one of the referees is worth noting. When the difference between day-ahead and real-time imbalance prices is small, the prosumer may become less risk averse (i.e., larger R_i), anticipating that the costs incurred in the real-time imbalance settlement would not be significant even in a worse situation. This may lead the prosumer to purchase less from the day-ahead market. However, modeling prosumer's endogenous risk preference is beyond the scope of the paper and we leave it to our future work.

[13] This only includes the power purchases by the conventional consumers, i.e., $\frac{\sum_i p_i d_i}{\sum_i d_i}$; that is, the power purchases by the prosumer when it is in a short position are not included. However, it considers the sales by the prosumer when it is in a long position.

price in node 1 as well as the sales-weighted power price in the Stackelberg equilibrium is between those of perfect and Cournot competition. However, whether it is from above or below depends on its net position in the equilibrium.

As the sales-weighted power price decreases with higher levels of mean renewable output faced by the prosumer, the total demand in the market increases when more zero-marginal-cost resources become available. As shown in Table 1, the total power demanded follows an increasing trend of 2848.31–2851.68 and 2858.07 MWh from (a) to (c) as the prosumer's renewable output increases. With increased consumption (demand) and lower prices, the consumer surplus (in K$) increases monotonically from 255.86 to 256.30 and 257.17 in (a), (b), and (c). Similarly, with more renewables, the prosumer engages in less purchases from the market in a short position or sells more to the market in a long position, thereby mitigating the need for generation from conventional producers and reducing the total power generation from conventional units. This is illustrated in Table 1, where total power production from the wholesale market is decreased from 2909.13 MWh in the case of 25 MW renewable to 2893.94 MWh and 2846.38 MWh for 50 and 120 MW, respectively. Since less generation from conventional units means that the grid operator would need to move less power around in the network, the grid operator's revenue decreases with higher levels of renewable output by the prosumer. Table 1 illustrates the fact that the grid operator's revenue ($K) is reduced from 9.77 when renewable output is equal to 25 MW to 8.35 and 5.44 when mean renewable output is 50 and 120 MW, respectively. On the other hand, lower producer revenues induced by lower power prices is more than made-up by lower transmission charges paid to the grid operator, leading to an increase in profits. For instance, the producer surplus in Table 1 increases from 41.86 K$ in (a) to $42.77K and $44.76K for (b) and (c), respectively.

The wholesale social surplus in strategic cases of Tables 1 and 3 is lower than that under perfect competition in Table 2.[14] This is due to the effect of welfare distortion by market power in Stackelberg and Cournot cases. The equilibrium prices in strategic cases indeed deviate from the ideal perfectly competitive prices. The sales-weighted power prices are lowest (highest) in Cournot case under the short (long) position. For instance, the sales-weighted power price under the 25 MW scenario of (a) is $35.28/MWh, $35.49/MWh, and $35.52/MWh for Cournot, Stackelberg, and perfect competition cases, respectively. This is mainly because when the prosumer buys less from the main grid under the Cournot case, it effectively shifts the wholesale demand curve to the left, thereby lowering the power prices. A reversal of order among three cases is observed in (c) in Tables 1, 2 and 3.

[14] The seemingly indiscernible difference in the social surplus when a prosumer is in a short position is partially attributed to the modest size of the prosumer. Had the prosumer grown to be larger, the difference would enlarge. We note that our interest in this paper lies in the order of social surplus or other indicators among cases rather than the absolute difference. However, when the prosumer is in a long position, the gap in the wholesale social surplus may remain small even if the prosumer becomes larger. This would highlight again the greater potential of market power by the prosumer when it is in a short position.

Finally, the comparison between the Stackelberg case in Table 1 and the Cournot case in Table 3 deserves more attention. When the prosumer is in a short position and purchases power from the grid in (a) and (b), conventional consumers are worse off in Table 1 than in Table 3 by \$1.0K (= 256.87 − 255.86) and \$0.56K (=256.86 − 256.30), respectively, as they compete more with the prosumer for power in the Stackelberg case. To understand the impacts on conventional producers, we then calculate the "output-weighted power price," since the producer surplus is also tied to the power sales to the prosumer.[15] They are equal to \$32.37/MWh (\$32.95/MWh), \$32.63/MWh (\$33.01/MWh), and \$33.26/MWh (\$33.18/MWh) for the Stackelberg (Cournot) case for 25 MW, 50 MW, and 120 MW of mean renewable generation, respectively. The lower output-weighted power price in (a) and (b) under the Stackelberg case leads to lower producer surplus in Table 1 compared to that in Table 3. On the other hand, since the prosumer purchases more energy from the grid in (a) and (b) in Table 1, it creates additional congestion to the system with greater grid operator's revenue. The increase in congestion rent more than offsets the declines in consumer and producer surplus, leading to a higher wholesale social surplus in the Stackelberg case in Table 1 than that in the Cournot case in Table 3. Similar arguments apply to the results in (c) by changing the short position to the long position: that is, the decrease in congestion rent is more than compensated with the increases in consumer and producer surplus, leading to a higher wholesale social surplus in the Stackelberg case in Table 1 than that in the Cournot case in Table 3.

4 Comparative analyses

This section further analyzes the impact of prosumer's presence in power markets. First, we investigate the effects of prosumer's mean renewable output (K_i) on its net position in equilibrium. Next, we examine how the uncertainty of renewable output (σ_i) and the degree of prosumer's risk aversion (R_i) affect the market outcomes.

We compare the outcomes of the Stackelberg case to perfect competition and Cournot cases by varying the levels of mean renewable output K_1 in node 1. Figure 1 plots the prosumer's sale (+) or purchase (−) in perfect competition, Cournot, and Stackelberg cases against the zero-marginal-cost renewable output in x-axis from 25 to 120 MW. The horizontal dotted line crossing zero on the y-axis corresponds to the *island mode* where the prosumer is isolated from the grid. Figure 1 indicates that for the range of renewable output, where the prosumer is in a short position purchasing power from the grid, the quantity purchased under the Stackelberg case is between perfect competition and Cournot cases. The same phenomenon is observed for the long position when the prosumer sells power to the grid. In other words, the prosumer formulated as a Stackelberg leader reduces purchases (sales) in the short (long) position compared to the case

[15] The output-weighted power price is defined by $\frac{\sum_{f,i,h \in H_{fi}} p_i x_{fih}}{\sum_{f,i,h \in H_{fi}} x_{fih}}$, which is similar to but different from the sales-weighted power price reported in Tables 1, 2 and 3.

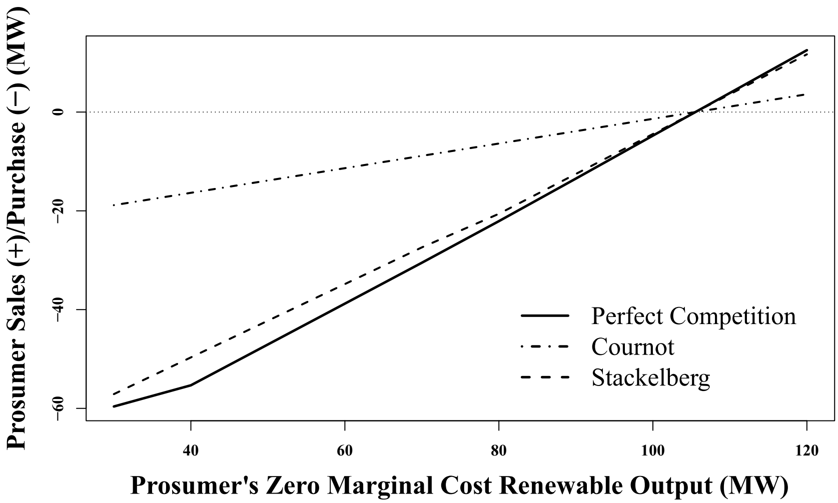

Fig. 1 Prosumer's sales (+) or purchase (−) under various levels of mean renewable output

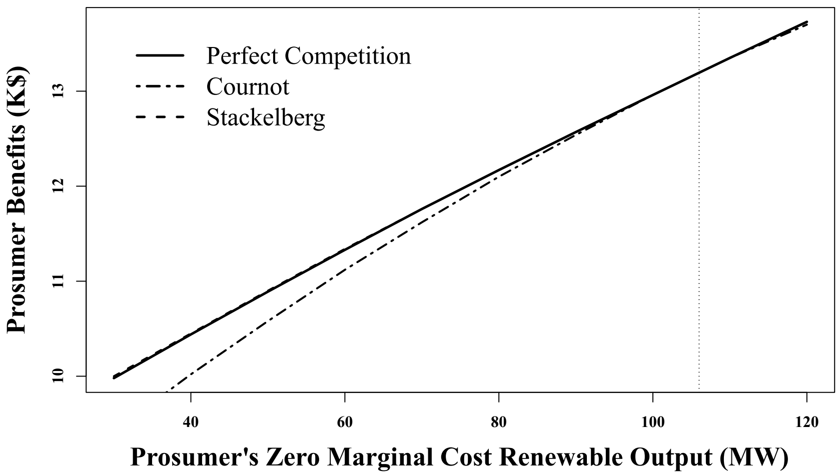

Fig. 2 Prosumer's profit under various levels of mean renewable output

of perfect competition but increases purchases (sales) in the short (long) position compared to the Cournot case. The results are broadly consistent with economic intuition in that the Cournot case could be the most aggressive one with agents in a simultaneous-move situation [39].

The prosumer's profit is also plotted in Fig. 2 against different levels of renewable output. Although the lines are not discernible between perfect competition and Stackelberg cases, in fact, for any level of renewable output, the prosumer's profit in the Stackelberg equilibrium is higher than those of perfect competition and Cournot cases as demonstrated in the tables in Sect. 3.2. This observation is

Table 4 Results under Stackelberg leader prosumer with different risk aversion parameters ($\sigma_1 = 10$ MW, $K_1 = 50$ MW)

Variables\scenarios	(a)	(b)	(c)
Risk aversion parameter (R_1)	0.10	0.50	0.99
Perceived output ($K_1^{perceived}$) [MW]	20	40	48.99
Prosumer's sale(+)/purchase(−) [MWh]	− 63.48	− 47.56	− 40.41
Prosumer's load [MWh]	99.09	100.63	101.32
Prosumer's backup generation [MWh]	15.61	13.06	11.91
Marginal cost of backup [$/MWh]	50.61	48.06	46.92
Prosumer surplus [$K]	9.60	10.59	11.02
Imbalance settlement quantity [MWh]	30	10	1.01
Total power demand [MWh]	2847.83	2850.72	2852.02
Total power production [MWh]	2911.31	2898.28	2892.42
Power price in node 1 [$/MWh]	45.53	44.25	43.68
Sales-weighted power price [$/MWh]	35.50	35.42	35.38
Grid operator's revenue [$K]	9.97	8.76	8.21
Producer surplus [$K]	41.73	42.51	42.85
Consumer surplus [$K]	255.80	256.17	256.34
Wholesale social surplus [$K]	307.51	307.44	307.41

corroborated with economic theory in that the leader prosumer in the Stackelberg case could perform better compared to the other two cases because of its first-mover advantage.

We next turn our attention to the uncertainty of renewable output and the degree of prosumer's risk aversion. Constraint (6), which associates prosumer's mean renewable output with the risk parameters, can be rewritten as follows:

$$z_i + l_i - g_i \leq K_i^{perceived} \quad \forall i \tag{8}$$

where $K_i^{perceived} = K_i - \sigma_i\sqrt{\frac{1-R_i}{R_i}}$ is dubbed "perceived" renewable output for given R_i and σ_i. Since $\sigma_i\sqrt{\frac{1-R_i}{R_i}}$ is non-negative, $K_i^{perceived} \leq K_i$ holds true. Here, $K_i^{perceived}$ may be interpreted as a "certainty equivalent" of uncertain renewable output, jointly affected by R_i and σ_i. In particular, the more risk averse the prosumer (i.e., smaller R_i) and/or the greater the uncertainty of renewable output (i.e., larger σ_i), the lower the perceived renewable output (i.e., $K_i^{perceived}$), thereby inducing the prosumer to act more conservatively.

Tables 4 and 5 summarize the market outcomes when changing the prosumer's risk aversion ($R_1 = 0.1, 0.5, 0.99$) and the uncertainty parameters ($\sigma_1 = 1, 5, 25$), respectively. We maintain $\sigma_1 = 10$ MW and $R_1 = 0.9$ at the baseline in Tables 4 and 5, respectively, while $K_1 = 50$ MW.

Columns (a), (b), and (c) in Table 4 correspond to the market outcomes of R_1 equal to 0.1, 0.5, and 0.99, respectively. The perceived output $K_1^{perceived}$ as reported in Table 4 increases from 20 MW in (a) to 48.99 MW in (c). With increases of R_1

Table 5 Results under Stackelberg leader prosumer with different uncertainty of renewable output ($R_1 = 0.9, K_1 = 50$ MW)

Variables\scenarios	(a)	(b)	(c)
Uncertainty of renewables (σ_1) [MW]	1	5	25
Perceived output ($K_1^{perceived}$) [MW]	49.67	48.33	41.67
Prosumer's sale(+)/purchase(−) [MWh]	− 39.87	− 40.93	− 46.24
Prosumer's load [MWh]	101.37	101.27	101.76
Prosumer's backup generation [MWh]	11.83	12.00	12.85
Marginal cost of backup [$/MWh]	46.83	47.00	47.85
Prosumer surplus [$K]	11.05	10.98	10.67
Imbalance settlement quantity [MWh]	0.33	1.67	8.33
Total power demand [MWh]	2852.12	2851.92	2850.96
Total power production [MWh]	2891.99	2892.86	2897.20
Power price in node 1 [$/MWh]	43.64	43.73	44.15
Sales-weighted power price [$/MWh]	35.38	35.39	35.42
Grid operator's revenue [$K]	8.17	8.25	8.66
Producer surplus [$K]	42.88	42.83	42.57
Consumer surplus [$K]	256.36	256.33	256.20
Wholesale social surplus [$K]	307.41	307.42	307.44

from 0.1 to 0.5, and to 0.99, the prosumer becomes less risk averse or more risk seeking, thereby purchasing less energy from the wholesale market, from 63.48 to 47.56 MWh, and to 40.41 MWh as shown in (a)–(c). The fact that less energy is purchased by the prosumer in (c) implies that more energy is available in the wholesale market, leading to lower sales-weighted power prices (i.e., \$35.38/MWh in (c) vs. \$35.50/MWh in (a)) and higher consumer surplus (\$256.34 K in (c) vs. \$255.80 K in (a)).

The final set of results concerning the impact of uncertainty in renewable output (σ_1) is reported in Table 5. The columns from (a) to (c) display the outcomes when increasing uncertainty σ_1 from 1 to 25 MW. With increases in σ_1, the perceived output $K_1^{perceived}$ decreases from 49.67 MW in (a) to 41.67 MW in (c). A smaller $K_1^{perceived}$ forces the prosumer to purchase more energy from the wholesale market in (c), and hence leaves less energy available to the consumers in the grid. This, in turn, leads to higher sales-weighted prices by \$0.04 from (a) to (c), even only marginally. As a result, consumers become worse off, subject to a decline of \$0.16 K (= 256.36 − 256.20) in consumer surplus.[16]

[16] The impacts of R_1 and σ_1 when the prosumer is in a long position, selling energy to the grid, can be analyzed in a similar way. In particular, a larger $K_1^{perceived}$ as in Table 4 encourages the prosumer to sell more energy, which is expected to lower sales-weighted power prices and makes consumers better off. On the other hand, a smaller $K_1^{perceived}$ as in Table 5 induces the prosumer to sell less energy, resulting in lower consumer surplus with higher sales-weighted power prices.

5 Conclusions

This paper focuses on the role of the risk-averse leader prosumer in the electricity market by formulating a distributionally robust chance-constrained MPEC approach and comparing the results to those where the prosumer is designated as a price-taker or a Cournot entity in the market.

The results indicate that market outcomes are affected by the risk-averse prosumer's strategy, the amount of mean renewable output, and the magnitude of renewable uncertainty. Facing relatively low (high) renewables, the prosumer behaves as a consumer (producer) and purchases power from (sells to) the main grid. The larger the magnitude of uncertainty in renewable output, the smaller the perceived renewables by the prosumer, which leads to more power purchase from (or less power sale into) the grid. Overall, the wholesale social surplus in the Stackelberg case lies in-between the perfect and Cournot competition cases. With a prosumer's long position under the relatively high amount of mean renewable output, the outcomes asymptotically approach that of the perfect competition case, whereas the gap is widened if the prosumer is in a short position. This suggests that it is easier or more potent for the prosumer to exercise buyer's market power in a short position than seller's market power in a long position since the latter is likely to be offset by other conventional producers. The situation worsens as the prosumer becomes less risk averse. The asymmetry of the prosumer's ability to exert market power will be more of concern to the future grid with an increasing growth of prosumers.

We highlight the importance of understanding the role of prosumers, acting as a producer or a consumer in equilibrium, when evaluating its interaction with and its impacts on the wholesale market. Our analysis demonstrates the effect of prosumers' "double-side" market power that can effectively create distortion, leading to lower wholesale social surplus compared to that of perfect competition. The findings are new and can provide valuable information to authorities, such as a market surveillance committee, to oversee prosumers appropriately in the future. Finally, the model is ready to include other energy products in the electric power sector, where prosumers can offer their services. For instance, a spinning reserve market or a market for fast-ramping products can be formulated in a similar way to [40]. As an extension of our work, data-driven methods can be applied to model uncertainty of renewable output using empirical distributions from historical data instead of the distribution moments.

Appendix A: Wolfe duality of lower-level problem

Since the lower-level problem is a concave program, we can obtain the optimal solution by solving its dual. Particularly, if we have a general concave program $\max_x\{f(x) : g(x) \leq 0\}$, the corresponding Wolfe dual is $\min_{x,\lambda}\{\mathcal{L}(x,\lambda) : \nabla_x \mathcal{L}(x,\lambda) = 0, \lambda \geq 0\}$, where $\nabla_x \mathcal{L}(x,\lambda)$ are the gradients of the

Lagrangian $\mathcal{L}(x,\lambda)=f(x)-\lambda^{\top}g(x)$. For a concave (or convex) programming problem, strong duality holds between the primal and Wolfe dual problems. Consequently, the Wolfe dual of the social welfare maximization problem in the lower level is:

$$\min_{\Omega\cup\Lambda} \quad \sum_i B_i(d_i) - \sum_{f,i,h\in H_{fi}} C_{fih}(x_{fih}) \tag{A.1a}$$

$$\begin{aligned}
&- \sum_{f,i,h\in H_{fi}} \beta_{fih}(x_{fih} - X_{fih}) - \sum_k \lambda_k^+ \left(\sum_i PTDF_{ki} - T_k \right) \\
&- \sum_k \lambda_k^- \left(\sum_i -PTDF_{ki} y_i - T_k \right) - \sum_i \eta_i d_i + \sum_i \eta_i z_i \\
&+ \sum_{f,i,h\in H_{fi}} \eta_i x_{fih} + \sum_i \eta_i y_i - \theta \sum_i y_i + \sum_{f,i,h\in H_{fi}} \varepsilon_{fih} x_{fih} \\
&+ \sum_i \xi_i d_i
\end{aligned} \tag{A.1b}$$

subject to

$$- C'_{fih}(x_{fih}) - \beta_{fih} + \eta_i + \varepsilon_{fih} = 0 \quad \forall f,i,h \in H_{fi}$$

$$B'_i(d_i) - \eta_i + \xi_i = 0 \quad \forall i \tag{A.1c}$$

$$- \sum_k (\lambda_k^+ - \lambda_k^-) PTDF_{ki} + \eta_i - \theta = 0 \quad \forall i \tag{A.1d}$$

$$\beta_{fih}, \varepsilon_{fih} \geq 0 \quad \forall f,i,h \in H_{fi} \tag{A.1e}$$

$$\lambda_k^+, \lambda_k^- \geq 0 \qquad \forall k \tag{A.1f}$$

$$\xi_i \geq 0 \qquad \forall i \tag{A.1g}$$

Note that non-negativity constraints are imposed on $\{\beta_{fih}, \lambda_k^+, \lambda_k^-, \varepsilon_{fih}, \xi_i\}$, which are associated with the inequality constraints in the primal problem. Otherwise, variables are unrestricted.

Given the concavity of the social welfare maximization problem and that strong duality holds, the original objective functions and (A.1a) have the same value. Thus:

$$
\begin{aligned}
&- \sum_{f,i,h\in H_{fi}} \beta_{fih}(x_{fig} - X_{fih}) + \sum_{f,i,h\in H_{fi}} \varepsilon_{fih} x_{fih} \\
&- \sum_k \lambda_k^+ \left(\sum_i PTDF_{ki} y_i - T_k \right) + \sum_i \xi_i d_i \\
&- \sum_k \lambda_k^- \left(\sum_i -PTDF_{ki} y_i - T_k \right) - \theta \sum_i y_i \\
&- \sum_i \eta_i d_i + \sum_{f,i,h\in H_{fi}} \eta_i x_{fih} + \sum_i \eta_i z_i + \sum_i \eta_i y_i = 0
\end{aligned} \tag{A.2}
$$

Using constraints (A.1b)–(A.1d), we further simplify (A.2). In particular, from (A.1b) we have:

$$
\begin{aligned}
&\left(-C'_{fih}(x_{fih}) - \beta_{fih} + \eta_i + \varepsilon_{fih}\right) x_{fih} = 0 \\
&\sum_{f,i,h\in H_{fi}} (-\beta_{fih} + \eta_i + \varepsilon_{fih}) x_{fih} = \sum_{f,i,h\in H_{fi}} C'_{fih}(x_{fih}) x_{fih}
\end{aligned} \tag{A.3}
$$

From (A.1c), we derive:

$$
\begin{aligned}
&\left(B'_i(d_i) - \eta_i + \xi_i\right) d_i = 0 \\
&\sum_i \left(-\eta_i + \xi_i\right) d_i = - \sum_i B'_i(d_i) d_i
\end{aligned} \tag{A.4}
$$

Fom (A.1d), we obtain:

$$
\begin{aligned}
&\left(- \sum_k (\lambda_k^+ - \lambda_k^-) PTDF_k + \eta_i - \theta \right) y_i = 0 \\
&\sum_i \left(\eta_i - \theta\right) y_i = \sum_{k,i} (\lambda_k^+ - \lambda_k^-) PTDF_k y_i
\end{aligned} \tag{A.5}
$$

Substituting (A.3), (A.4), and (A.5) into (A.2), we can rewrite the bilinear term $\sum_i \eta_i z_i$ as follows:

$$
\begin{aligned}
&\sum_i \eta_i z_i = \sum_i B'_i(d_i) d_i - \sum_k (\lambda_k^+ + \lambda_k^-) T_k \\
&- \sum_{f,i,h\in H_{fi}} \left(C'_{fih}(x_{fih}) x_{fih} + \beta_{fih} X_{fih} \right)
\end{aligned} \tag{A.6}
$$

Appendix B: Concavification of objective function in MPEC

We can then substitute (A.6) into the bilinear term $\sum_i \eta_i z_i$ in the original objective function of the MPEC. Along with other constraints, the objective function of MPEC, or the Stackelberg leader formulation for the prosumer, is then rewritten as follows:

$$\max_{\Phi\cup\Omega\cup\Lambda} \sum_i B_i'(d_i)d_i - \sum_k (\lambda_k^+ + \lambda_k^-)T_k$$
$$- \sum_{f,i,h\in H_{fi}} \left(C_{fih}'(x_{fih})x_{fih} + \beta_{fih}X_{fih}\right) \tag{B.1}$$
$$+ \sum_i \left(B_i^l(l_i) - C_i^g(g_i)\right) + \sum_i P_i^c\Big(K_i - z_i - l_i + g_i\Big)$$

Appendix C: MIQP reformulation

As the final step, we further remove non-convex terms caused by the complementarity conditions from the lower-level problem by utilizing disjunctive constraints. With binary variables $\Phi = \{\bar{r}_{fih}, r_k^+, r_k^-, r_{fih}, \hat{r}_i\}$ and positive big constants $\{M_1, M_2, M_3, M_4, M_5\}$, we reformulate the MPEC into an MIQP as follows:

$$\max_{\Phi\cup\Omega\cup\Lambda\cup\Psi} \sum_i B_i'(d_i)d_i - \sum_k (\lambda_k^+ + \lambda_k^-)T_k \tag{C. 1a}$$

$$- \sum_{f,i,h\in H_{fi}} \left(C_{fih}'(x_{fih})x_{fih} + \beta_{fih}X_{fih}\right)$$
$$+ \sum_i \left(B_i^l(l_i) - C_i^g(g_i)\right) + \sum_i P_i^c\Big(K_i - z_i - l_i + g_i\Big) \tag{C. 1b}$$

subject to

$$\sigma_i\sqrt{\frac{1-R_i}{R_i}} + z_i + l_i - g_i - K_i \le 0 \qquad \forall i$$

$$g_i \le G_i \qquad \forall i \tag{C. 1c}$$

$$l_i, g_i \ge 0 \qquad \forall i \tag{C. 1d}$$

$$-C_{fih}'(x_{fih}) - \beta_{fih} + \eta_i + \varepsilon_{fih} = 0 \quad \forall f, i, h \in H_{fi} \tag{C. 1e}$$

$$B_i'(d_i) - \eta_i + \xi_i = 0 \qquad \forall i \tag{C. 1f}$$

$$-\sum_k (\lambda_k^+ - \lambda_k^-)PTDF_{ki} + \eta_i - \theta = 0 \qquad \forall i \tag{C. 1g}$$

$$0 \le -(x_{fih} - X_{fih}) \le M_1\bar{r}_{fih} \qquad \forall f, i, h \in H_{fi} \tag{C. 1h}$$

$$0 \leq \beta_{fih} \leq M_1(1 - \bar{r}_{fih}) \qquad \forall f, i, h \in H_{fi} \tag{C. 1i}$$

$$d_i - \sum_{f,h \in H_{fi}} x_{fih} - z_i - y_i = 0 \qquad \forall i \tag{C. 1j}$$

$$0 \leq -\Big(\sum_i PTDF_{ki} y_i - T_k\Big) \leq M_2 r_k^+ \qquad \forall k \tag{C. 1k}$$

$$0 \leq \lambda_k^+ \leq M_2(1 - r_k^+) \qquad \forall k \tag{C. 1l}$$

$$0 \leq -\Big(-\sum_i PTDF_{ki} y_i - T_k\Big) \leq M_3 r_k^- \qquad \forall k \tag{C. 1m}$$

$$0 \leq \lambda_k^- \leq M_3(1 - r_k^-) \qquad \forall k \tag{C. 1n}$$

$$\sum_i y_i = 0 \tag{C. 1o}$$

$$0 \leq x_{fih} \leq M_4 r_{fih} \qquad \forall f, i, h \in H_{fi} \tag{C. 1p}$$

$$0 \leq \epsilon_{fih} \leq M_4(1 - r_{fih}) \qquad \forall f, i, h \in H_{fi} \tag{C. 1q}$$

$$0 \leq d_i \leq M_5 \hat{r}_i \qquad \forall i \tag{C. 1r}$$

$$0 \leq \xi_i \leq M_5(1 - \hat{r}_i) \qquad \forall i \tag{C. 1s}$$

$$\bar{r}_{fih}, r_k^+, r_k^-, r_{fih}, \hat{r}_i \in \{0, 1\} \tag{C. 1t}$$

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