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J. Li, Pol D. Spanos, J.B. Chen & Y.B. Peng (Eds)

Dynamic reliability of infrastructure networks using augmented Monte Carlo simulation

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Infrastructure networks offer critical services to modern society. They dynamically interact with the environment, operators, and users. Infrastructure networks are unique engineered systems, large in scale and high in complexity. One fundamental issue for their reliability assessment is the uncertainty propagation from stochastic disturbances across interconnected components. Monte Carlo simulation (MCS) remains approachable to quantify stochastic dynamics from components to systems. Its application depends on time efficiency along with the capability of delivering reliable approximations. In this paper, we introduce Quasi Monte Carlo (QMC) sampling techniques to improve modeling efficiency. Also, we suggest a principled Monte Carlo (PMC) method that equips the crude MCS with Probably Approximately Correct (PAC) approaches to deliver guaranteed approximations. We compare our proposed schemes with a competitive approach for stochastic dynamic analysis, namely the Probability Density Evolution Method (PDEM). Our computational experiments are on ideal but complex enough source-terminal (S-T) dynamic network reliability problems. We endow network links with oscillators so that they can jump across safe and failed states allowing us to treat the problem from a stochastic process perspective. We find that QMC alone can yield practical accuracy, and PMC with a PAC algorithm can deliver accuracy guarantees. Also, QMC is more versatile and efficient than the PDEM for network reliability assessment. The QMC and PMC methods provide advanced uncertainty propagation techniques to support decision makers with their reliability problems.

1 INTRODUCTION

Infrastructure systems (e.g., water/oil/gas, communications, power, and roads) are primarily designed to distribute critical services through interconnected components. Quantifying their reliability under uncertain contingencies is crucial for protective actions, mitigation and emergency operations against failures.

A fundamental issue in evaluating the dynamic reliability of infrastructure networks relates to uncertainty propagation from components to system states. Compared to engineered facilities that can be represented by logical structures of components, there is an additional dimension of complexity with infrastructure which arises from the interdependence among interconnected components (Zio, 2009). Also, the problem size with explicit identification of all possible system states increases exponentially with system size.

Classic approaches, such as event and fault trees (Devooght Smidts, 1992) as well as the Markov process (Papazoglou & Gyftopoulos, 1977), require significant resources for system state identification (Siu, 1994). Monte Carlo simulation (MCS), which directly simulates the system's responses to stochastic dynamic perturbations, remains a feasible approach. However, MCS-based methods may demand significant computational resources due to slow convergence, while only furnishing approximations.

In this paper, we will implement Quasi Monte Carlo (QMC) techniques to significantly increase the efficiency of the simulation model. Additionally, we will show how to equip crude MCS with approximation algorithms to achieve accuracy guarantees. Then, we will compare our methods with emerging alternatives, such as the probability density evolution method (PDEM) through computational experiments. Finally, we state conclusions and offer ideas for future research.

2 METHODOLOGY

2.1 *Dynamic reliability analysis through Monte Carlo simulation*

The fundamental issue of dynamic reliability evaluation of infrastructure networks is the non-linear stochastic dynamic analysis of the system responses against random inputs. There are two major approaches for stochastic dynamic analysis: the direct approach and the indirect approach.

The direct approach, developed by Ornstein (1919), directly analyzes stochastic dynamics based on the governing differential equation and characterizes the probabilistic distribution of the stochastic dynamic responses. It treats the stochastic dynamic analysis from the Lagrangian perspective, which follows individual dynamic responses as they transit over time domain. The indirect approach, initiated by the work by Einstein (1905), deals with the diffusion equation governing the probability density function (PDF) of the collection of stochastic dynamic responses. The indirect approach looks at the random processes from the Eulerian perspective, which fixates on a particular point in time domain and records the probabilistic characteristics of the stochastic dynamic responses passing through that point. Both types of approach originally motivated to describe Brownian movement.

The primary challenge the direct approach faces is that we can not analytically solve a stochastic differential equation (SDE). Uhlenbeck & Ornstein (1930) developed the idea of describing the excitation and response in terms of their probabilistic descriptions instead of explicitly solving the stochastic dynamic equation. The innovative use of moments combined with the development of auto- (or cross-) spectral densities of stationary random processes (Wiener, 1930), paved the way for modern stochastic dynamic analysis (Paez, 2011). The most widely studied random excitation is the stationary Gaussian process $X(t)$, which excites a linear system with a response $Z(t)$ (Crandall, 1963):

$$S_{ZZ}(\omega) = |H(\omega)|^2 S_{XX}(\omega) \quad (1)$$

where $S_{XX}(\omega)$ and $S_{ZZ}(\omega)$ are the spectral density of the excitation and the response, respectively, and $H(\omega)$ is the frequency response function of the linear system. With nonlinear systems, we require specialized analytical approximation treatments such as statistical linearization (Roberts & Spanos, 2003), equivalent linearization (Proppe et al., 2003), stochastic averaging (Roberts & Spanos, 1986), and path integral (Kougioumtzoglou & Spanos, 2012). Generally, these approximation techniques demand prior information about the stochastic dynamics to perform approximations. Moreover, the input-output relationship based methods only yield moments of system response.

Following the work by Einstein (Einstein, 1905), in the field of physics, Fokker (1914) and Planck (1917) derived the famous Fokker−Planck equation that describes the evolution of the PDF of the position and velocity of a Brownian particle excited by white noise. The Fokker−Planck equation is also known as the forward Kolmogorov equation in the mathematical community because he independently developed its rigorous mathematical basis (Kolmogorov, 1931).

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The Fokker−Planck−Kolmogorov equation transforms the problem of stochastic dynamic analysis into a deterministic partial differential equation. Although it provides an elegant mathematical framework, its application to engineering practice is restricted because it is hard to implement across large size engineering systems. Its computational complexity increases at least exponentially with the state-space dimension of the system (Proppe et al., 2003).

MCS remains a feasible method for stochastic dynamic analysis. We can use MCS to run possible scenarios of an infrastructure network under stochastic dynamic excitations. For each realization, all of the uncertain parameters are sampled and fed to the governing differential equation to compute the dynamic response of the system. Each independent system realization represents a possible response scenario for the system. We can characterize the probability distribution of the system response from the assembled collection of possible outcomes.

MCS also serves as a benchmark method to verify the accuracy of newly developed techniques in the area of stochastic mechanics (e.g. Li & Chen, 2004; Kougioumtzoglou & Spanos, 2012). MCS is versatile for practical application and insensitive to the number of dimensionality. Specifically, with infrastructure networks consist of interconnected components, the MCS is an object-oriented approach that allows explicit propagation of the uncertainty from excitation inputs to system response outputs.

The price for versatility and robustness is that MCS can be very inefficient. The convergence rate of crude MC method is $O(N^{-1/2})$. Additionally, the estimation accuracy is not guaranteed. We should have a stopping rule for MCS sampling with respect to a certain level of accuracy.

2.2 *Quasi Monte Carlo method*

The Monte Carlo (MC) method includes a broad spectrum of techniques that use randomly drawn input samples to approximate solutions, particularly in integration and simulation. Many integration problems can be reduced to integrals over the unit cube $I^d = [0,1]^d$

$$I[f] = \int_{I^d} f(\boldsymbol{\theta}) d\boldsymbol{\theta} \tag{2}$$

and its MC approximation is

$$\hat{I}[f] = \frac{1}{N}\sum_{n=1}^{N} f(\boldsymbol{\theta}_n) \tag{3}$$

where f is the function for integral, $\boldsymbol{\theta}$ is random variable in random space $\boldsymbol{\Omega}^d$, $\boldsymbol{\theta}_n$ is the n^{th} random sample. In many cases, the desired result of a simulation problem can be written as an expectation in the same form with Equation (3). For example, for the dynamic reliability evaluation of an engineered network G, we can set $f(\boldsymbol{\theta})$ to be a Bernoulli random variable with $f(\boldsymbol{\theta}) = 1(0)$ representing a 'safe' ('failure') state. Then the network reliability R can be approximated as the expectation of $f(\boldsymbol{\theta})$. Thus, such expectation approximation problem can also be reduced to the general integral by Equation (2). In the reminder of this paper, we use Equation (3) as a general formulation for MC approximation.

The purely random sampling technique for crude MCS suffers from the problem of clumping such that samples are concentrated in a range with high probability density, and close samples contribute limited credit to the estimation. Define the approximation error of Equation (3) relative to Equation (2) as

$$\varepsilon^*[f] = |I[f] - \hat{I}[f]| \tag{4}$$

Sample clumping can be mathematically described using the error bound equation invoking the Central Limit Theorem (CLT) (Feller 1971), which states that to ensure the approximation error being at most ε^* with confidence level δ^*, it requires N samples:

$$N = (\varepsilon^*[f])^{-2}\sigma^2[f]s(\delta^*) \tag{5}$$

where s is the confidence function for a normal random variable, and $\sigma^2[f]$ is the variance of $I[f]$,

$$\sigma^2[f] = \int_{I^d} (f(\boldsymbol{\theta}) - I[f])^2 d\boldsymbol{\theta} \tag{6}$$

Equation (5) indicates that $\varepsilon^* = O(\sigma[f]N^{-1/2})$.

The Quasi Monte Carlo (QMC) method improves the scaling exponent $-1/2$ by replacing random samples with quasi-random points that can provide space uniformity (Kuipers & Niederreiter, 1974; Hua & Wang, 1981). QMC method uses low-discrepancy sequences with correlations between the points to eliminate clumping. A sequence is low-discrepancy if

$$D_N \leq c(logN)^d N^{-1} \quad (7)$$

where c is a constant independent of N, d is the dimension of the sequence, and D_N is discrepancy, a measure of uniformity, for a sequence of N points $\{\boldsymbol{\theta}_n\}$ in the unit cube I^d (Caflisch, 1998). D_N is estimated using

$$D_N = sup_{J\in E}|\frac{\#\{\boldsymbol{\theta}_n \in J\}}{N} - m(J)| \quad (8)$$

where E is the set of all rectangular subsets, $\#\{\boldsymbol{\theta}_n \in J\}$ is the number of points that lie within J, and $m(J)$ is the exact volume of J. Low-discrepancy sequences possess desirable uniformity properties of numerical sequences (Kuipers & Niederreiter, 1974; Hua & Wang, 1981). Examples of low-discrepancy sequence include the one dimensional Van der Corput sequence as well as its multi-dimensional version, the Halton sequence (Halton, 1960), Sobol sequences (Sobol, 1976), and the number theoretical net (NT-net) (Hua & Wang, 1981).

The error of QMC method is justified by the Koksma-Hlawka inequality, which states that for a sequence $\{\boldsymbol{\theta}_n\}$ and a function f with bounded variation, the integration error $\varepsilon^*[f]$ is bounded as

$$\varepsilon[f] = V[f]D_N^* \quad (9)$$

where $V[f]$ is the integral variation of f over the unit cube, for the one dimensional case,

$$V[f] = \int_0^1 |\frac{df}{d\theta}|d\theta \quad (10)$$

or for d dimensions

$$V[f] = \int_{I^d} |\frac{\partial^d f}{\partial\theta_1 ... \partial\theta_d}|d\theta_1 ... d\theta_d + \sum_{i=1}^{d} V[f_1^{(i)}] \quad (11)$$

where $f_1^{(i)}$ is the restriction of the function f to the boundary $\boldsymbol{\theta}_i = 1$, D_N^* is defined the same way with D_N, with $J \in E^*$ and E^* is the set of all rectangular subsets with one vertex at 0. The Koksma-Hlawka inequality provides an upper bound for the estimation error of QMC. Although this bound can be conservative, the inequality implies that the integration error has a convergence rate of $O((logN)^d N^{-1})$, which is much faster than crude MC.

2.3 *Monte Carlo method with accuracy guarantees*

Mainstream applications of MC method for reliability evaluation of infrastructure networks lack performance guarantees on estimation error. The typical way to determine the sample size is through a trial and error process that aims to meet a target empirical variance measure such as the coefficient of variation (Paredes et al., 2019). However, such empirical approaches can be unreliable (Bayer et al., 2014), especially for reliability evaluation of systems with significant complexity. Although the Central Limit Theorem (CLT) (Feller, 1971) gives a relationship to determine the sample size with approximation error ε^* and confidence level δ^*, Equation (5) cannot be directly used. The exact value of the variance $\sigma^2[f]$ is unknown. We need to equip the MCS with accuracy guarantee schemes that deliver principled reliability approximation (Paredes et al., 2019).

There is a class of Probably Approximately Correct (PAC) approaches that deliver (ε, δ) approximations for MC applications (e.g., Huber, 2017, Dagum, 2000). The concept of PAC from artificial intelligence (Valiant, 1984), says that an (ε, δ) approximation of an expectation μ_Z returns an estimator $\hat{\mu}_Z$, such that

$$P(|\frac{\hat{\mu}_Z}{\mu_Z} - 1| \geq \varepsilon) \leq \delta \quad (12)$$

There is one PAC approximation for Bernoulli random variables named Gamma

Bernoulli approximation scheme (GBAS) (Huber, 2017). The basic idea of GBAS is to construct an estimator $\hat{\mu}_Z$ for the expectation μ_Z such that $\mu_Z/\hat{\mu}_Z \sim Gamma(k,k-1)$. The desired PAC guarantee given by Equation (12) is equivalent to $P(\mu_Z/\hat{\mu}_Z < (1+\varepsilon)^{-1} \ or \ \mu_Z/\hat{\mu}_Z > (1-\varepsilon)^{-1}) \leq \delta$. We can obtain the smallest k that provides such a guarantee from the *Gamma* distribution, for instance, for $\varepsilon = 0.05, \delta = 0.05$, the smallest k is 1550. A more general PAC algorithm is the Approximation Algorithm (AA) for general random variables in $[0,1]$ (Dagum, 2000). The AA first uses trial MCS experiments to roughly estimate the expectation and the standard deviation, and then it determines the desired sample size according to a stopping rule. The current paper focuses on demonstrating the implementation of the GBAS.

3 PROBABILITY DENSITY EVOLUTION METHOD

A competitive approach for dynamic reliability evaluation of structural and infrastructure systems is the Probability Density Evolution Method (PDEM) developed by Li & Chen (2004). PDEM is a general technique for stochastic dynamic analysis of engineering systems. It can be regarded as a hybrid approach that couples both the direct and indirect perspectives discussed in section 2.

The theoretical formulation of the PDEM is in the form of a partial differential equation that governs the evolution of the joint probability density function (PDF) of stochastic response variable $\boldsymbol{Z}$ and random variable vector $\boldsymbol{\Theta}$:

$$\frac{\partial p_{\boldsymbol{Z},\boldsymbol{\Theta}}(z,\boldsymbol{\theta},t)}{\partial t} + \sum_{i=1}^{d} \dot{z}_i(\boldsymbol{\theta},t)\frac{\partial p_{\boldsymbol{Z},\boldsymbol{\Theta}}(z,\boldsymbol{\theta},t)}{\partial z_i} = 0 \tag{13}$$

where $\dot{z}_i$ is the velocity of the i^{th} stochastic response, and d is its dimension. Equation (13) indicates that the transition of the probabilistic structure of a stochastic system relies on the change of the physical state of the system (Li & Chen, 2006). The joint PDF computed is a Eulerian metric. Also, the PDEM is derived based on the principle of probability preservation and the random event (Lagrangian) description of this principle (Li & Chen, 2006). In addition, Equation (13) is often solved in a Lagrangian manner using numerical techniques when it is applied to engineering systems. In the next section, we will perform computational experiments comparing the performance of PDEM, our proposed Quasi Monte Carlo (QMC) method, and the principled Monte Carlo (PMC) method.

Generally, we need to apply numerical techniques to solve Equation (13) given the initial condition

$$p_{\boldsymbol{Z},\boldsymbol{\Theta}}(z,\boldsymbol{\theta},t)|_{t=t_0} = \delta(z-z_0)p_{\boldsymbol{\Theta}}(\theta) \tag{14}$$

with $\delta(z-z_0)$ is the Dirac function, and an initial value z_0. $p_{\boldsymbol{Z}}(z,t)$ is computed by integrating the solved joint PDF $p_{\boldsymbol{Z},\boldsymbol{\Theta}}(z,\boldsymbol{\theta},t)$ over the random space,

$$p_{Z}(z,t) = \int_{\boldsymbol{\Theta}} p_{\boldsymbol{Z},\boldsymbol{\Theta}}(z,\boldsymbol{\theta},t)d\boldsymbol{\theta}. \tag{15}$$

The implementation of PDEM requires to feed sampled dynamic responses to Equation (14) and to use it numerically. It usually adopts the number theoretical net (NT-net) developed by Hua & Wang (1981), which is a QMC sequence, to improve computation efficiency.

Although PDEM is a general framework for stochastic dynamic analysis of engineering systems, its application to dynamic reliability problems is somewhat limited. Theoretically, we can use PDEM to obtain the temporal PDF $p_{\boldsymbol{Z}}(z,t)$ and thus get the temporal reliability $R(t)$ given limit state function $g(\boldsymbol{Z})$ of a system. As far as the authors' knowledge, existing PDEM's applications to reliability analysis of stochastic dynamic systems mainly focus on the equivalent static reliability over a time period. It transfers the reliability evaluation for stochastic dynamic systems to characterizing distribution of the *extreme value* (Chen & Li, 2007). To get the PDF of the *extreme value* using the PDEM, a virtual stochastic process needs to be constructed in a way that the *extreme value* is a temporal value of the response at a certain instant of time (Chen & Li, 2007, Liu et al., 2018, Miao et al., 2020).

The numerical issue associated with solving Equation (13) is one potential problem that may hinder the PDEM's application. Equation (13) contains the first order deviation of the stochastic response $\boldsymbol{Z}(t)$, which implies that it can only deal with differentiable stochastic dynamic responses. Additionally, there are stability issues when solving Equation (13) with numerical techniques.

4 COMPUTATIONAL EXPERIMENTS

In this section, we implement the Quasi Monte Carlo method (QMC), the principled Monte Carlo method (PMC) equipped with a Probably Approximately Correct (PAC) algorithm, the Gamma Bernoulli approximation scheme (GBAS), and the Probability Density Evolution Method (PDEM) to evaluate the dynamic source-terminal (S-T) connectivity reliability of a hypothetical infrastructure network (see Figure 1). We will compare their performance with the results by an exact method being the baseline.

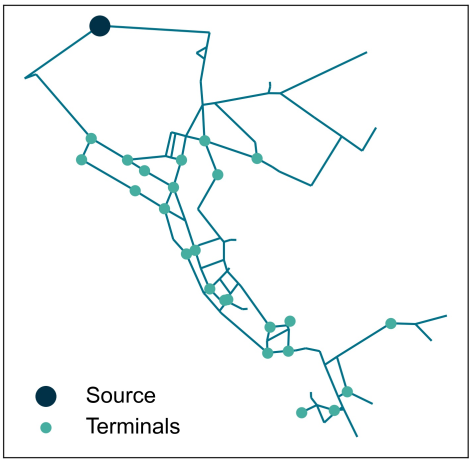

Figure 1. Layout of the case network.

The hypothetical network is built based on a benchmark water distribution network (WDN), *Net 3* (USEPA 2016). It consists of 1 source, 97 nodes, among which 24 are terminals, and 117 links. Each link is assigned with a hypothetical linear oscillator with artificial characteristic parameters to model its dynamic response $u(t)$ to stochastic excitation $w(t)$. Each link has two states, 'safe'('failure'), when the response $u(t)$ is less than (is equal to or greater than) its capacity u_c.

In our computational experiments, we adopt the widely studied linear oscillator with the stochastic dynamic equation (SDE):

$$\frac{du}{dt} + \beta u = \lambda w(t) \tag{16}$$

where u is the performance measure, and $w(t)$ is a truncated Winer process. The Wiener process is a Gaussian process with the increment $W(t+\delta t) - W(t)$ over any time interval $[t, t+\delta t], \delta t > 0$ as a Gaussian variable with $E(W(t+\delta t) - W(t)) = 0, V(W(t+\delta t) - W(t)) = \delta t \sigma^2$. Equation (16) originally describes the velocity of a free particle in Brownian motion in physics, where u is the velocity of the particle, β is the coefficient of viscous friction, and $w(t)$ is the acceleration induced by the fluctuating force. Uhlenbeck & Ornstein (1930) obtained the analytical solution of this equation.

We choose the Wiener process as the stochastic excitation process because we can project it to a random space consisting of a denumerable set of orthogonal random variables using Karhunen-Loeve expansion (Ghanem & Spanos, 1991; Masri & Miller, 1982). The expansion has an explicit closed form over $(0, T)$:

$$W(t) = \sum_{i=1}^{\infty} v_i \frac{sin(\pi t(i-1/2)/T)}{\pi t(i-1/2)/T} \tag{17}$$

where $\{v_i\}$ are independent normal variables. In our experiments, we adopt a truncated Wiener process over $(0, 1)$ with $\sigma^2 = 1$:

$$W_1(t) = \sum_{i=1}^{3} v_i \frac{sin\pi t(i-1/2)}{\pi(i-1/2)} \tag{18}$$

We obtain an closed form solution for Equation (16) with the truncated Winer process applying the strategy in Sarkka & Solin (2019)

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(pp. 49 – 52):

$$u(t) = e^{-\beta t} u_0 + \sqrt{2}\lambda \sum_{i=1}^{3} h(i^*, \beta, t) v_i \quad (19)$$

with $h(i^*, \beta, t) = \frac{sin\pi ti^* - \pi i^*(e^{-\beta t} - cos\pi ti^*)}{(\beta^2 + \pi^2 i^{*2})\pi i^*}$, $i^* = i - 1/2$, u_0 being the initial value. The solution $u(t)$ is also a Gaussian process with mean $E(u(t)) = e^{-\beta t} u_0$ and variance $\sigma^2(u(t)) = 2\lambda^2 \sum_{i=1}^{3} h^2(i^*, \beta, t)$.

4.1 *Dynamic S-T connectivity reliability evaluation*

We evaluate the dynamic S-T connectivity reliability of the case network when it is under truncated Winer excitation. Denote the S-T dynamic reliability for a source terminal pair st_j at time t as $R_j(t)$. It is the probability that at time t the system is at a connected source terminal pair st_j state.

Suitable parameters for each link are predetermined by sampling from uniform distributions, with $u_0 \sim U(0.8, 1.2), \beta \sim U(0.08, 0.5), \lambda \sim U(1.4, 3.3)$. We assume that each link's capacity u_c equals to its initial performance measure u_0. Also, each link is assigned a linear oscillator (Equation (16)), so that each link is resilient and its state can return back to 'safe' from 'failure' at anytime. The dynamic S-T reliability over $[0, 1)$ for each terminal is evaluated at every instant $\{t_m = 0.01m\}$, where $m = 0, 1, 2, \ldots, 99$.

The *Exact* method evaluates the dynamic S-T connectivity reliability from the reliability of each link l using a recursive decomposition algorithm (Li & He 2002). All the three numerical methods, PMC, QMC, and PDEM have to rely on simulations to obtain realizations of each link's stochastic response. We use the classic fourth order Runge-Kutta scheme for dynamic link response computation.

At each time instant t_m, the PMC and QMC are direct methods that can evaluate the dynamic S-T reliability $R_j(t_m)$ by characterizing the distribution of the S-T connectivity state $Z_j(t_m)$, which is a Bernoulli random variable. We have the following relationship:

$$R_j(t_m) = P(Z_j(t_m) = 1) = \mu_{Z_j(t_m)} \quad (20)$$

$$\hat{R}_j(t_m) = \frac{1}{N} \sum_{i=1}^{N} Z_j(t_m) \quad (21)$$

To compare the QMC and the PDEM in the same context, we adopt the framework for calculating $R_j(t_m)$ in Liu et al. (2018) for both methods. The dynamic $R_j(t_m)$ is estimated from the distribution of the S-T connectivity index $CI_j(t_m)$

$$R_j(t_m) = P(CI_j(t_m) \geq 1) \quad (22)$$

The difference between the QMC and the PDEM is that QMC approximates the numerical PDF of $CI_j(t_m)$ directly from the simulation realizations $\{CI_{j,i}(t_m)\}$ using a smoothing technique, kernel density estimation, while the PDEM obtains the numerical PDF of $CI_j(t_m)$ by solving Equation (13) with $\boldsymbol{Z} = CI_j$.

We use the number theoretical net (NT-net) scheme by Hua & Wang (1981) to generate QMC samples for $\mathbf{v} = (v_1, v_2, v_3)$ in Equation (18). The basic idea of the NT-net is to employ an integer generator vector $(N, Q_1, Q_2, \ldots, Q_d)$ to generate a set of points over the unit cube I^d, where N is the number of samples. The points are taken using the following relationship:

$$P_{NT} = \{v_i = (v_{i,1}, v_{i,2}, \ldots, v_{i,d}),\ i = 1, 2, \ldots, N\}$$
$$v_{i,q} = \frac{2iQ_q - 1}{2N} - int(\frac{2iQ_q - 1}{2N}), q = 1, 2, \ldots, d \quad (23)$$

We first generate N samples of $\boldsymbol{v}$ using NT-net and substitute them to Equation (18) obtaining N excitation samples. Then, we feed them to Equation (16) and solve response realizations using the fourth order Runge-Kutta scheme. Finally, we calculate the $CI_{j,i}(t_m)$ for each response i using the connectivity index algorithm in Liu et al. (2018).

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4.2 *Results*

Our primary computational experiments results include: dynamic S-T reliability evaluation for a typical S-T pair st_{20} calculated by three methods; relative errors ε_0 of $R_{20}(t_m)$, defined as $\varepsilon_0(j,t_m) = (\hat{R}_{20}(t_m) - R_{20}(t_m))/R_{20}(t_m)$; and performance comparison between the QMC and the PDEM.

Figure 2 illustrates the temporal evolution trajectories of the dynamic S-T reliability of S-T pair st_{20}. In the whole time domain, $\hat{R}_{20}$ sharply decreases in the beginning and stays relatively stable afterwards. The sample size for QMC and PDEM is the same, $N = 418$. The sample size of the PMC with GBAS varies from around three times to five times of the QMC sample size.

Figure 3 presents the trajectories of ε_0 that reflect quantitative measures of quality. The ε_0 for all approximation schemes is stable and is within 0.1 in the time range $(0.2, 1)$. The PAC with GBAS yields its expected confidence interval with the fraction that the absolute value of ε_0 is larger than 0.05 less than 0.05. With the QMC and the PDEM, ε_0 has a clear trend of decreasing in the time domain $(0, 0.2)$, and yield higher relative accuracy than the PMC with $\varepsilon = 0.05, \delta = 0.05$.

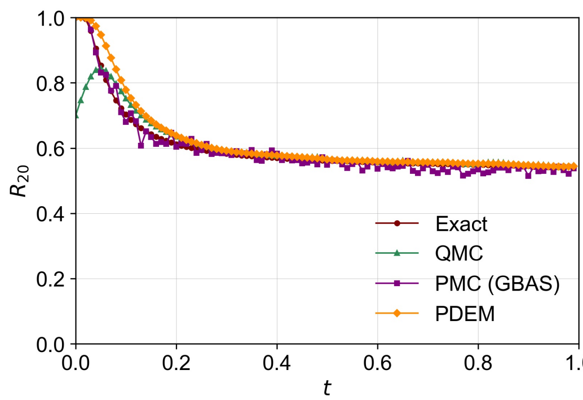

Figure 2. Dynamic S-T reliability approximations for pair st_{20} (GBAS: $\varepsilon = 0.05, \delta = 0.05$).

The early distributions of CI_{20} in Figure 4 can provide insights about the early large ε_0 with the QMC method and the PDEM. At $t = 0.05$, CI_{20} is close to its initial value 1.0, and it is narrowly distributed with a small variance. The QMC method and the PDEM approximate a PDF with scattered probability density values over discretized intervals. KDE is a smoothing technique, so when its band width is coarse relative to the spreading of the PDF, over smoothness will occur, as shown by the CI_{20} at $t = 0.05$ in Figure 4. As for the PDEM, the interval containing the initial value $CI_{20} = 1$ possesses the concentrated probability density at $t = 0$. The probability density evolves to disperse to adjacent intervals with time. In the time range $(0, 0.2)$, the discretized concentration of probability density leads to under smoothness of the PDF.

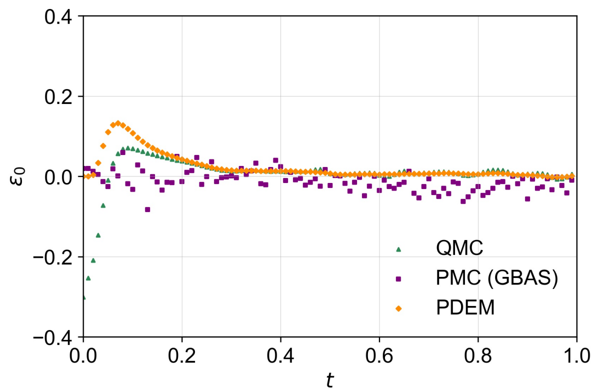

Figure 3. Relative errors ε_0 of $\hat{R}_{20}$ (GBAS: $\varepsilon = 0.05, \delta = 0.05$).

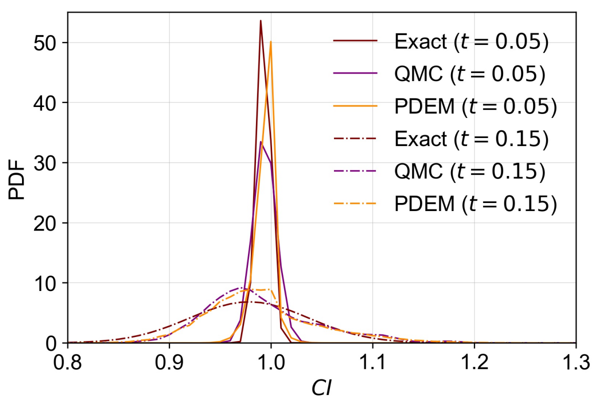

Figure 4. Temporal PDF of CI_{20}

5 CONCLUSIONS

We demonstrate that simulation based methods are most suitable for dynamic reliability analysis of infrastructure networks relative to methods leveraging on statistical descriptions considering the highly non-linear behaviors from complex interactions. As a consequence, we introduce QMC sampling techniques to improve the computational efficiency of the simulation based approximation. We also propose to embed the expectation estimation from MCS into PAC algorithms to deliver guaranteed accuracy of reliability. We demonstrate our proposed methods on a hypothetical dynamic S-T connectivity reliability problem, and compare it with an emerging approach, the PDEM.

The QMC sampling technique turns out to be very effective for improving computation efficiency. QMC also achieves similar accuracy to that of the PDEM, while the former is more versatile and efficient for practical applications. The QMC method directly uses sample trajectories to approximate the marginal distribution of a random variable with KDE, while PDEM constructs a probability density evolution equation using the sample trajectories and solves it for the marginal PDF. However, one should be careful with small numbers of QMC samples to approximate the PDF of a random variable. For QMC and PDEM, either over smoothness or under smoothness can occur depending on the fineness of their numerical discretization. The Cumulative density function (CDF) may be a better alternative for reliability evaluation using QMC samples as it does not require extra smoothness.

The MCS with PAC algorithms is promising to provide accuracy guarantee for reliability estimation. The PMC method should replace the crude MCS when rigorous estimation accuracy is desired. Also, the PMC method is superior than the crude MCS when serving as a fair baseline comparison. The PAC algorithm introduced in this paper can be infeasible for rare-event estimations, new schemes that combine the efficiency of QMC and the rigorous accuracy guarantees of PAC algorithms are desired.

In addition to the stochastic dynamic reliability problems, the QMC and the PMC methods are applicable to other reliability problems as general techniques for uncertainty propagation quantification. In future work, the authors will test the robustness of them with different benchmark problems. We will develop portfolios of reliability analysis schemes applying advanced uncertainty quantification and accuracy guarantee techniques for mission-critical applications.

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