REVIEW SUMMARY

PHYSICS

Stranger than metals

Philip W. Phillips*, Nigel E. Hussey, Peter Abbamonte

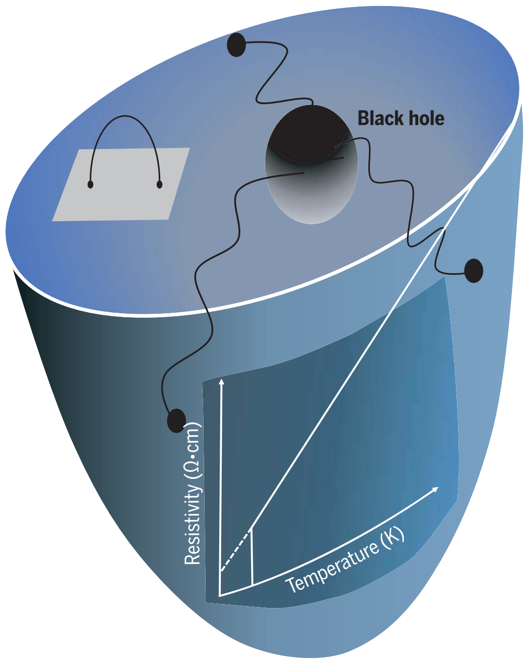
BACKGROUND: Landau's Fermi liquid theory provides the bedrock on which our understanding of metals has developed over the past 65 years. Its basic premise is that the electrons transporting a current can be treated as "quasiparticles"—electron-like particles whose effective mass has been modified, typically through interactions with the atomic lattice and/or other electrons. For a long time, it seemed as though Landau's theory could account for all the many-body interactions that exist inside a metal, even in the so-called heavy fermion systems whose quasiparticle mass can be up to three orders of magnitude heavier than the electron's mass. Fermi liquid theory also lay the foundation for the first successful microscopic theory of superconductivity.

In the past few decades, a number of new metallic systems have been discovered that violate this paradigm. The violation is most evident in the way that the electrical resistivity changes with temperature or magnetic field. In normal metals in which electrons are the charge carriers, the resistivity increases with increasing temperature but saturates, both at low temperatures (because the quantized lattice vibrations are frozen out) and at high temperatures (because the electron mean free path dips below the smallest scattering pathway defined by the lattice spacing). In "strange metals," by contrast, no saturation occurs, implying that the quasiparticle description breaks down and electrons are no longer the primary charge carriers. When the particle picture breaks down, no local entity carries the current.

ADVANCES: A new classification of metallicity is not a purely academic exercise, however, as strange metals tend to be the high-temperature phase of some of the best superconductors available. Understanding high-temperature superconductivity stands as a grand challenge because its resolution is fundamentally rooted in the physics of strong interactions, a regime where electrons no longer move independently. Precisely what new emergent phenomena one obtains from the interactions that drive the electron dynamics above the temperature where they superconduct is one of the most urgent problems in physics, attracting the attention of condensed matter physicists as well as string theorists. One thing is clear in this regime: The particle picture breaks down. As particles and locality are typically related, the strange metal raises the distinct possibility that its resolution must abandon the basic building blocks of quantum theory.

We review the experimental and theoretical studies that have shaped our current understanding of the emergent strongly interacting physics realized in a host of strange metals, with a special focus on their poster-child: the copper oxide high-temperature superconductors. Experiments are highlighted that attempt to link the phenomenon of nonsaturating resistivity to parameter-free universal physics. A key experimental observation in such materials is that removing a single electron affects the spectrum at all energy scales, not just the low-energy sector as in a Fermi liquid. It is observations of this sort that reinforce the breakdown of the single-particle concept. On the theoretical side, the modern accounts that borrow from the conjecture that strongly interacting physics is really about gravity are discussed extensively, as they have been the most successful thus far in describing the range of physics displayed by strange metals. The foray into gravity models is not just a pipe dream because in such constructions, no particle interpretation is given to the charge density. As the breakdown of the independent-particle picture is central to the strange metal, the gravity constructions are a natural tool to make progress on this problem. Possible experimental tests of this conjecture are also outlined.

OUTLOOK: As more strange metals emerge and their physical properties come under the scrutiny of the vast array of experimental probes now at our disposal, their mysteries will be revealed and their commonalities and differences cataloged. In so doing, we should be able to understand the universality of strange metal physics. At the same time, the anomalous nature of their superconducting state will become apparent, offering us hope that a new paradigm of pairing of nonquasiparticles will also be formalized. The correlation between the strength of the linear-in-temperature resistivity in cuprate strange metals and their corresponding superfluid density, as revealed here, certainly hints at a fundamental link between the nature of strange metallicity and superconductivity in the cuprates. And as the gravity-inspired theories mature and overcome the challenge of projecting their powerful mathematical machinery onto the appropriate crystallographic lattice, so too will we hope to build with confidence a complete theory of strange metals as they emerge from the horizon of a black hole. ■



Curved spacetime with a black hole in its interior and the strange metal arising on the boundary. This picture is based on the string theory gauge-gravity duality conjecture by J. Maldacena, which states that some strongly interacting quantum mechanical systems can be studied by replacing them with classical gravity in a spacetime in one higher dimension. The conjecture was made possible by thinking about some of the fundamental components of string theory, namely D-branes (the horseshoe-shaped object terminating on a flat surface in the interior of the spacetime). A key surprise of this conjecture is that aspects of condensed matter systems in which the electrons interact strongly—such as strange metals—can be studied using gravity.

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REVIEW

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Stranger than metals

Philip W. Phillips[1]*, Nigel E. Hussey[2,3], Peter Abbamonte[4]

In traditional metals, the temperature (*T*) dependence of electrical resistivity vanishes at low or high *T*, albeit for different reasons. Here, we review a class of materials, known as "strange" metals, that can violate both of these principles. In strange metals, the change in slope of the resistivity as the mean free path drops below the lattice constant, or as $T \to 0$, can be imperceptible, suggesting continuity between the charge carriers at low and high *T*. We focus on transport and spectroscopic data on candidate strange metals in an effort to isolate and identify a unifying physical principle. Special attention is paid to quantum criticality, Planckian dissipation, Mottness, and whether a new gauge principle is needed to account for the nonlocal transport seen in these materials.

To understand the essential tension between quantum mechanics and gravity, simply imagine two electrons impinging on the event horizon of a black hole. Whereas classical gravity predicts that they meet at the center, quantum mechanics forbids this if the electrons have the same spin. In essence, classical gravity has no way of preserving Pauli exclusion. Replacing classical general relativity with a quantum theory of gravity at small enough scales resolves the problem, but what is this scale?

In 1899, Planck formulated a universal length now regarded as the scale below which a quantum theory of gravity supplants its classical counterpart. The Planck scale,

$$\ell_\mathrm{P} = \sqrt{\frac{\hbar G}{c^3}} \qquad (1)$$

obtains by pure dimensional analysis on three fundamental constants: the speed of light, c, Newton's gravitational constant, G, and the quantum of uncertainty, $\hbar$ (Planck's constant, h, divided by 2π). This leads naturally to a Planck time as the ratio of the Planck length to the speed of light, ℓ_P/c. Such a Planckian analysis can be extended to many-body systems in contact with a heat bath. All that is necessary is to include the temperature T. A similar dimensional analysis then leads to

$$\tau_\mathrm{P} = \frac{\hbar}{k_\mathrm{B} T} \qquad (2)$$

as the shortest time for heat loss in a many-body system obeying quantum mechanics, where k_B is Boltzmann's constant. Because no system parameters enter τ_P, this quantity occupies a similar fundamental role in analogy to the Planck length and is referred to as the Planckian dissipation time. Equation 2 has had previous incarnations (*1*, *2*); in the realm of charge transport, relevant to this article, it defines the time scale for scale-invariant or Planckian dissipation (*3*). Scale invariance follows because there is no scale other than temperature appearing in τ_P. Achieving such scale invariance necessitates a highly entangled many-body state. Such a state would lead to the breakdown of a local single-particle framework and the advent of new collective nonlocal entities as the charge carriers. Identifying the new propagating degrees of freedom constitutes the key mystery of strange metals.

Whereas the Planck scale ℓ_P requires high-energy accelerators much beyond anything now in use, such is not the case with physics at the Planckian dissipation limit. Early tabletop experiments on cuprate superconductors, for example, revealed a "strange metal" regime defined by a robust T-linear resistivity extending to the highest temperatures measured (*4*–*6*) (Fig. 1), a possible harbinger of Planckian dissipation. Recall that in a Fermi liquid, the conductivity can be well described by a Drude formula,

$$\sigma = \frac{n_e e^2}{m} \tau_\mathrm{tr} \qquad (3)$$

where n_e is the charge carrier density, e is the charge of an electron, m is its mass, and τ_tr is the transport lifetime, defined as

$$\tau_\mathrm{tr} = \frac{\hbar E_\mathrm{F}}{(k_\mathrm{B} T)^2} = \frac{E_\mathrm{F}}{k_\mathrm{B} T} \tau_\mathrm{P} \qquad (4)$$

which contains the Fermi energy E_F of the quasiparticles. No such energy scale appears in Eq. 2. If the scattering rate in cuprates is directly proportional to the resistivity, as it is in simple metals, T-linear resistivity is equivalent to scale-invariant Planckian dissipation only if $\tau_\mathrm{tr} = \alpha_1 \tau_\mathrm{P}$ with α_1 (the coefficient of T-linear resistivity) ~ 1. Although this state of affairs seems to be realized in a host of correlated metals, including the cuprates (*7*–*10*), there is no consensus concerning how accurately α_1 can be known and the assumptions that go into its determination. Regardless of the possible relationship with Planckian dissipation, what makes T-linear resistivity in the cuprates truly novel is its persistence—from millikelvin temperatures (in both the electron- and hole-doped cuprates) (*11*, *12*) up to 1000 K (in the hole-doped cuprates) (*4*, *6*)—and its omnipresence, the strange metal regime dominating large swathes of the temperature versus doping phase diagram (*13*). In normal metals (*14*, *15*) as well as some heavy fermion materials (*16*), the resistivity asymptotically approaches a saturation value at which the mean free path ℓ becomes comparable with the interatomic spacing a or more generally the Fermi wavelength λ_F—the minimum length over which a Bloch wave and its associated Fermi velocity and wave vector can be defined. In many correlated metals, collectively referred to as "bad metals," $\ell < a$ at high T, which violates the so-called Mott-Ioffe-Regel (MIR) limit (*5*, *6*, *1*, *16*–*18*). Remarkably, no saturation occurs in these bad metals across the MIR threshold, implying that the whole notion of a Fermi velocity of quasiparticles breaks down at high T. In certain cases, an example of which is shown in Fig. 1, there is no discernible change in slope as the MIR limit is exceeded. Although this circumstance occurs only in a narrow doping window (in cuprates) (*18*), such continuity does suggest that, even at low T, quasiparticles (*19*) cannot be the

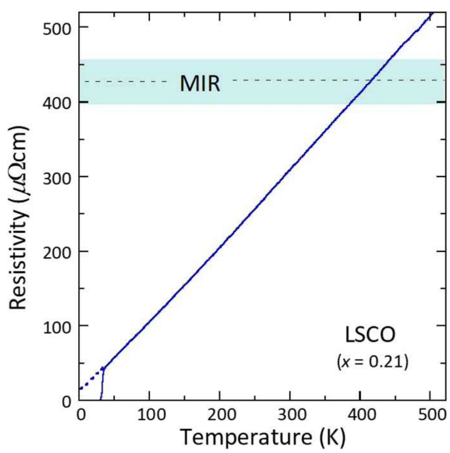

Fig. 1. *T*-linear resistivity in strange metals. Shown is the in-plane resistivity of $La_{2-x}Sr_xCuO_4$ (x = 0.21). The dotted points are extrapolated from high-field magnetoresistance data (*8*). The shaded area shows the Mott-Ioffe-Regel (MIR) boundary defined here as when the mean free path becomes comparable to the Fermi wavelength λ_F. [Adapted from (*8*, *18*)]

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effective propagating degrees of freedom. Evidently, in strongly correlated electron matter, the current-carrying degrees of freedom in the infrared (IR) need not have a particle interpretation.

Over time, the label "strange metal" has seemingly become ubiquitous, used to describe any metallic system whose transport properties display behavior that is irreconcilable with conventional Fermi liquid or Boltzmann transport theory. This catch-all phraseology, however, is unhelpful, as it fails to differentiate between the various types of non–Fermi liquid behavior observed, some of which deserve special deliberation on their own. We attempt to bring strange metal phenomenology into sharper focus by addressing a number of pertinent questions. Does the term refer to the resistive behavior of correlated electron systems at high or low temperatures, or both? Does it describe any T-linear resistivity associated with the Planckian time scale, or something unique? Does it describe the physics of a doped Mott insulator or the physics associated with quantum criticality (whose underlying origins may or may not include Mottness as a key ingredient)? Finally, does anything local carry the current, and if not, does explicating the propagating degrees of freedom in the strange metal require a theory as novel as quantum gravity?

Is strange metallicity ubiquitous?

A survey of the DC transport properties of several strange metal candidates is presented in Table 1 (*4*, *6*, *8*, *10*, *20*–*61*). In addressing the above question, we must first acknowledge the many definitions of strange metallic behavior that exist, the simplest being a material hosting a metallic-like resistivity in the absence of quasiparticles. A more precise, if empirical, definition centers on the T-linear resistivity, specifically one that is distinguishable from the resistivity manifested by simple metals (which is attributed to electron-phonon scattering). For a metal to be classified as strange, the T-linearity must extend far beyond the typical bounds associated with phonon-mediated resistivity. At low T, this is typically one-third of the Debye temperature, whereas at high T, it is the temperature at which the magnitude of the resistivity is roughly half the value corresponding to the MIR limit. A subset of correlated metals, such as $SrRuO_3$ (*62*) and Sr_2RuO_4 (*36*), exhibit T-linear resistivity at high T with a magnitude that clearly violates the MIR limit, but as the system cools down, conventional Fermi-liquid behavior is restored (*37*, *63*). Hence, although they are bona fide bad metals—exhibiting metallic resistivity beyond the MIR limit—they do not classify as strange (*16*, *64*).

Another subset, identified here as quantum critical metals, exhibit T-linear resistivity down to the lowest temperatures studied, but only at a singular quantum critical point (QCP) in their T versus tuning parameter g phase diagram at which a continuous quantum phase transition to a symmetry-broken phase is suppressed to $T = 0$. Here, g can be pressure, magnetic field, doping level, or even strain. In most cases, the phase transition in question is associated with finite-momentum antiferromagnetism [as in pure $YbRh_2Si_2$ (*49*), $CeCoIn_5$ (*55*), and $BaFe_2(As_{1-x}P_x)_2$ (*45*)], although similar behavior has recently been reported in systems exhibiting zero-momentum order, such as nematic $FeSe_{1-x}S_x$ (*41*) or ferromagnetic $CeRh_6Ge_4$ (*57*). Away from the QCP, the low-T resistivity recovers the canonical T^2 Fermi liquid form, albeit with a coefficient that is enhanced as the

Table 1. Summary of the DC transport properties of various strange metal candidates. First column: Candidate compound or family of compounds. For the hole-doped cuprates, underdoped (UD), optimally doped (OP) and overdoped (OD) compounds are treated separately; the transport properties of individual compounds within each subset are generic. For the electron-doped cuprates, only $La_{2-x}Ce_xCuO_4$ is listed; Pr- and Nd-based compounds show similar behavior. Second column: Bad metallic behavior; a check mark indicates T-linear resistivity beyond the Mott-Ioffe-Regel (MIR) limit. A cross indicates either a tendency toward saturation or a marked reduction in slope near the MIR limit. Third column: A check mark identifies systems that at any point in their respective phase diagram(s) exhibit T-linear resistivity down to the lowest temperatures studied. Fourth column: "Extended criticality" refers to systems where a predominant T-linear resistivity at low T extends over a finite region of the phase diagram. Fifth column: T^2 dependence of the inverse Hall angle cot Θ_H in the same temperature range where $\rho(T)$ is T-linear. Sixth column: Compounds satisfying the "Modified Kohler's" label have a low-field magnetoresistance (MR), defined as $[\rho(H, T) - \rho(0, T)]/\rho(0, T)$, that exhibits a T dependence similar to that of $\tan^2 \Theta_H$. Seventh and eighth columns: High-field MR behavior of strange metal candidates. The observation of an H-linear MR at high fields does not imply that the MR exhibits quadrature scaling over all fields and temperatures. *$FeSe_{1-x}S_x$ H-linear/quadrature MR seen in this family coexists with a more conventional MR contribution, indicating the presence of both strange metal and Fermi liquid–like components in the DC transport. **In $YbBAl_4$, although T-linear resistivity is observed over a wide pressure range, its limiting low-T dependence is $T^{1.5}$. A dash indicates no reports confirming or disproving the considered behavior.

	$\rho \propto T$ as $T \to \infty$	$\rho \propto T$ as $T \to 0$	Extended criticality	cot $\Theta_H \propto T^2$ (at low H)	Modified Kohler's (at low H)	H-linear MR (at high H)	Quadrature MR
UD p-cuprates	√ (*6*)	× (*20*)	× (*21*)	√ (*22*)	√ (*23*)	–	–
OP p-cuprates	√ (*4*)	–	–	√ (*24*)	√ (*25*)	√ (*26*)	× (*27*)
OD p-cuprates	√ (*6*)	√ (*8*)	√ (*8*)	√ (*28*)	× (*29*)	√ (*29*)	√ (*29*)
$La_{2-x}Ce_xCuO_4$	× (*30*)	√ (*31*, *32*)	√ (*31*, *32*)	× (*33*)	× (*34*)	√ (*35*)	× (*35*)
Sr_2RuO_4	√ (*36*)	× (*37*)	× (*38*)	× (*39*)	× (*37*)	× (*37*)	× (*37*)
$Sr_3Ru_2O_7$	√ (*10*)	√ (*10*)	× (*10*)	×	–	–	–
$FeSe_{1-x}S_x$	× (*40*)	√ (*41*)	× (*41*)	√ (*42*)	√ (*42*)	√* (*43*)	√* (*43*)
$BaFe_2(As_{1-x}P_x)_2$	× (*44*)	√ (*45*)	× (*45*)	–	√ (*46*)	√ (*47*)	√ (*47*)
$Ba(Fe_{1/3}Co_{1/3}Ni_{1/3})_2As_2$	–	√ (*48*)	× (*48*)	–	–	√ (*48*)	√ (*48*)
$YbRh_2Si_2$	× (*49*)	√ (*50*)	√ (*51*)	√ (*52*)	–	–	–
$YbBAl_4$	× (*53*)	√** (*53*)	√** (*53*)	–	–	–	–
$CeCoIn_5$	× (*54*)	√ (*55*, *56*)	× (*55*, *56*)	√ (*54*)	√ (*54*)	–	–
$CeRh_6Ge_4$	× (*57*)	√ (*57*)	× (*57*)	–	–	–	–
$(TMTSF)_2PF_6$	–	√ (*58*)	√ (*58*)	–	–	–	–
MATBG	√ (*59*)	√ (*60*)	√ (*60*)	√ (*61*)	–	–	–

QCP is approached and the order parameter fluctuations soften.

By contrast, in overdoped cuprates [both hole- (8, 9) and electron-doped (31, 32)], Ge-doped $YbRh_2Si_2$ (51), $YbBAl_4$ (53), and the organic Bechgaard salts (58), $\rho(T)$ is predominantly T-linear down to low T not at a singular point in these materials' respective phase diagrams, but over an extended range of the relevant tuning parameter. At first sight, this "extended criticality" is difficult to reconcile with current theories of quantum criticality, which predict a crossover to a purely T^2 resistivity and thus a recovery of Fermi liquid behavior at low T everywhere except at the (singular) QCP. Arguably, it is this feature—incompatibility with both standard Fermi liquid and quantum critical scenarios—that distinguishes a genuine strange metal. Intriguingly, in many of these systems, α_1 is found to scale with the superconducting transition temperature T_c. Moreover, for $La_{2-x}Ce_xCuO_4$ (31, 32) and bis-(tetramethyltetraselenafulvalene) hexafluorophosphate [$(TMTSF)_2PF_6$] (58), extended criticality emerges once the spin density wave transition has been fully suppressed, suggesting an intimate link between the strange metal transport, superconductivity, and the presence of critical or long-wavelength spin fluctuations. In hole-doped cuprates, however, the strange metal regime looks different, in the sense that the extended criticality emerges beyond the end of the pseudogap regime that does not coincide with a magnetic quantum phase transition (65). Furthermore, although the pseudogap plays host to a multitude of broken-symmetry states, the jury is still out as to whether any of these are responsible for pseudogap formation or are merely instabilities of it.

Besides T-linear resistivity, strange metals also exhibit anomalous behavior in their magnetotransport, including (i) a quadratic temperature dependence of the inverse Hall angle $\cot \Theta_H = \sigma_{xy}/\sigma_{xx}$ (24), (ii) a transverse magnetoresistance (MR) that at low field exhibits modified Kohler's scaling [$\Delta\rho/\rho(0) \propto \tan^2 \Theta_H \propto (1/T^2)^2$ or $1/(A + BT^2)^2$ (25)], and/or (iii) an H-linear MR at high fields that may or may not follow quadrature scaling [whereby $\Delta\rho/T \propto \sqrt{1+\gamma(H/T)^2}$] (43, 47). The combination of a modified Kohler's rule and T^2 Hall angle has been interpreted to indicate the presence of distinct relaxation times, either for different loci in momentum space (22) or for relaxation processes normal and tangential to the underlying Fermi surface (24). The H-linear MR, on the other hand, is inextricably tied to the T-linear zero-field resistivity via its H/T scaling relation, a relation that can also extend over a broad range of the relevant tuning parameter (29). In some cases, this link can be obscured, either because $\rho(T)$ itself is not strictly T-linear (29) or because the quadrature-scaling MR coexists with a more conventional orbital MR (43). Both sets of behavior highlight once again the possible coexistence of two relaxation times or two distinct charge-carrying sectors in real materials. Curiously, in single-band materials, quadrature scaling breaks down inside the pseudogap regime (26, 27), whereas modified Kohler's scaling is recovered (23, 25), suggesting that the two phenomena may be mutually exclusive in such materials. In multiband materials such as $FeSe_{1-x}S_x$, on the other hand, these different manifestations of strange metallic transport appear side by side (42, 43). Irrespective of these caveats and complexities, what is striking about the quadrature-scaling MR is that it occurs in systems with varied Fermi surface topologies, dominant interactions, and energy scales, hinting at some universal but as yet unidentified organizing principle.

Restricting the strange metal moniker, as done here, to materials that exhibit low-T T-linear resistivity over an extended region of phase space likewise restricts strange metallicity to a select "club." The following sections explore various possible attributes that they have in common.

Is it quantum criticality?

Scale-free T-linear resistivity is highly suggestive of some form of underlying quantum criticality in which the only relevant scale is the temperature governing collisions between excitations of the order parameter (66). In fact, following the advent of marginal Fermi liquid (MFL) phenomenology and its associated (T, ω)-linear self energies (67), the common interpretation of such T-linear resistivity was and still remains the nucleus of ideas centered on quantum criticality. The strict definition of quantum criticality requires the divergence of a thermodynamic quantity. In heavy fermion metals, the electronic heat capacity ratio C_{el}/T indeed grows as $\ln(1/T)$ as the antiferromagnetic correlations diverge (49, 55, 68). In certain hole-doped cuprates, C_{el}/T also scales as $\ln(1/T)$ at doping levels close to the end of the pseudogap regime (69), although here, evidence for a divergent length scale of an associated order parameter is currently lacking (70). Moreover, photoemission suggests that at a T-independent critical doping $p_c \approx 0.19$, all signatures of incoherent spectral features that define the strange metal cease, giving way to a more conventional coherent response (71). The abruptness of the transition suggests that it is first-order, posing a challenge to interpretations based solely on criticality.

As mentioned above, another major hurdle for the standard criticality scenario is that the T-linear resistivity persists over a wide range of the relevant tuneable parameter, whether doping [as is the case for cuprates (8, 9, 31, 32, 72) and magic-angle twisted bilayer graphene (MATBG) (60)], pressure [for $YbBAl_4$ (53) and the organics (58)], or magnetic field [for Ge-doped $YbRh_2Si_2$ (51)]. If quantum criticality is the cause, then it is difficult to fathom how a thermodynamic quantity can be fashioned to diverge over an entire phase.

Despite these difficulties, it is worth exploring the connection T-linear resistivity has with continuous quantum critical phenomena, which for the sake of argument we presume to be tied to a singular point in the phase diagram. Regardless of the origin of the QCP, universality allows us to answer a simple question: What constraints does quantum criticality place on the T dependence of the resistivity? The answer to this question should be governed only by the fundamental length scale for the correlations. The simplest formulation of quantum criticality is single-parameter scaling in which the spatial and temporal correlations are governed by the same diverging length (see Fig. 2). Making the additional assumption that the relevant charge carriers are formed from the quantum critical fluctuations, we find that a simple scaling analysis on the singular part of the free energy results in the scaling law

$$\sigma(\omega = 0, T) = \frac{q^2}{\hbar} f(\omega = 0) \left(\frac{k_B T}{\hbar c}\right)^{(d-2)/z} \quad (5)$$

(73) for the T dependence of the conductivity, where $f(\omega = 0)$ is a nonzero constant, q is the

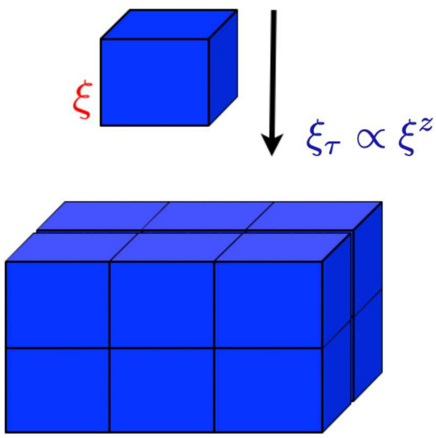

Fig. 2. Single-parameter scaling hypothesis. Depicted here is the collective scaling of a physical system near a critical point. The essential idea is that the correlations within each of the blocks shown are independent of one another. That is, spatial correlations in a volume smaller than the correlation volume, ξ^d, and temporal correlations on a time scale shorter than ξ_τ are small, and space-time regions of size $\xi^d\xi_\tau$ behave as independent blocks. At the critical point, the correlation length diverges. The single-scaling parameter hypothesis assumes that temporal correlations diverge also as a simple power of the spatial correlation length, namely $\xi_\tau \propto \xi^z$, where z is known as the dynamical critical exponent and by causality must exceed unity.

charge, and z is the dynamical exponent, which from causality must obey the inequality $z \geq 1$. Absent from this expression is any dependence on an ancillary energy scale—for example, E_F or the plasma frequency ω_p—as the only assumption is scale-invariant transport irrespective of the details of the system. The analogous expression for the optical conductivity is

$$\sigma(\omega, T = 0) \propto \omega^{(d-2)/z} \quad (6)$$

(74). In pure $YbRh_2Si_2$, for example, $\sigma^{-1}(\omega)$ follows an ω-linear dependence at low frequencies in the same region of the (T, H) phase diagram—the quantum critical "fan"—where $\rho(T)$ is also linear, consistent with this notion of single-parameter scaling (75). In cuprates, on the other hand, the situation is more nuanced. At intermediate frequencies—sometimes referred to as the mid-IR response—$\sigma(\omega)$ exhibits a ubiquitous $\omega^{-2/3}$ dependence (7). Although this feature in $\sigma(\omega)$ has been interpreted in terms of quantum critical scaling (7), it is inconsistent with the single-parameter scaling described above. At any doping level, $\sigma(\omega)$ in the cuprates exhibits a minimum at roughly the charge transfer scale of 1 eV. This is traditionally (76, 77) used as the energy scale demarcating the separation between intraband and interband transitions and hence serves to separate the low-energy from the high-energy continua. It has long been debated whether the broad sub-eV $\sigma(\omega)$ response in cuprates is best analyzed in terms of one or two components (77, 78). In the former case, the $\omega^{-2/3}$ tail is simply a consequence of the strong ω-linear dependence in $1/\tau_{tr}(\omega)$—as in MFL—whereas in the latter, it forms part of an incoherent response that is distinct from the coherent Drude weight centered at $\omega = 0$, which itself is described with either a constant or ω-dependent scattering rate.

Returning to the DC resistivity, we find that in cuprates, where $d = 3$, an exponent $z = -1$ is required to account for the T-linear dependence; this value is strictly forbidden by causality (73). For $d = 2$, as in the case of MATBG, the T dependence vanishes. This can be fixed with the replacement of $d \rightarrow d^* = 1$ for both materials. Although d^* can be construed as the number of dimensions (79) transverse to the Fermi surface, it is difficult to justify such a procedure here, as the persistence of T-linearity with no change in slope above and below the MIR requires a theory that does not rely on FL concepts such as a Fermi velocity or energy. Furthermore, it is well known that introducing d^* yields a power law for the heat capacity, $C \propto T^{3/2}$, which is not seen experimentally (80). On dimensional grounds, the $z = -1$ result in the context of the Drude formula is a consequence of compensating the square power of the plasma frequency with powers of T so that the scaling form of Eq. 5 is maintained. A distinct possibility is that perhaps some other form of quantum criticality beyond single-parameter scaling, such as a noncritical form of the entropy suggested recently (81), is at work here (see below).

Another critical feature of the conductivity is its behavior at finite wave vector k, which may be quantified by the dynamic charge susceptibility,

$$\chi''(k, \omega) = -\frac{k^2}{\omega e^2}\Re\sigma(k, \omega) \quad (7)$$

determined from electron energy-loss spectroscopy (EELS). A restriction on EELS is that it measures the longitudinal charge response, whereas optics yields the transverse. At vanishing momentum they are expected to be equal. Because optics has no momentum resolution, comparison with EELS can only be made as $k \rightarrow 0$. The primary charge excitation in strange metals is a 1-eV plasmon that was long believed to exhibit the same behavior as in a normal Fermi liquid (82, 83). Recent high-resolution momentum-resolved EELS (M-EELS) measurements have called this belief into question, showing that the large-k response is dominated by a continuum that remains flat to high energies, roughly 2 eV (84–86). Such behavior is reminiscent of the MFL (67) scenario except in that picture, the continuum persists up to a cutoff scale determined by the temperature, and not the Mott scale of 2 eV. In addition, the continuum exhibits scale-invariant features but with a dynamical critical exponent, $z \sim \infty$, not possible from a simple QCP.

We conclude, then, that no form of traditional quantum criticality can account easily for the power laws seen in strange metallic transport (although we recognize that T-linear resistivity is observed above what appear to be genuine singular QCPs). The photoemission experiments (71) indicating a first-order transition pose an additional problem exacerbated by the possibility that the criticality might be relevant to a whole region (8, 9, 51, 53, 58, 60, 65, 87, 88) rather than a point. Such criticality over an extended region is reminiscent of critical charged matter (89, 90) arising from dilatonic models in gauge-gravity duality. These ideas have been the most successful thus far in reproducing the various characteristics of strange metal physics (see below and Table 2).

Is it Planckian dissipation?

Whereas the electrical resistivity in metals can be measured directly, the scattering rate is entirely an inferred quantity. Herein lies the catch with Planckian dissipation. Angle-resolved photoemission (ARPES) experiments on cuprates as early as 1999 reported that the width of the momentum distribution curves (MDCs) at optimal doping along the nodal direction [(0, 0) to (π, π)] scale as a linear function of temperature and $a_0 + 0.75\omega$ for frequencies that exceed $2.5k_BT$ (91). The momentum linewidth, which in photoemission enters as Im Σ—the imaginary part of the self energy—can be used to define a lifetime through

$$\hbar v_k \Delta k = \mathrm{Im}\,\Sigma(k, \omega) = 2\frac{\hbar}{\tau} \quad (8)$$

where v_k is the group velocity for momentum state k. Extracting the slope from the data in Fig. 2 of (92) and using the experimentally reported Fermi velocity v_F = 1.1 eV/Å, we find that the single-particle scattering rate $\hbar/\tau \sim 1.7k_BT$ (i.e., on the order of the Planckian limit). Similar results were obtained in subsequent ARPES studies (92–94) with a key extension introduced in (88) whereby the width of nodal states was observed to obey a quadrature form described by the expression $[(\hbar\omega)^2 + (\beta k_BT)^2]^\lambda$, where λ is a doping-dependent exponent equal to ½ at optimal doping.

This extraction of the scattering rate from ARPES, however, is not entirely problem-free, as v_F is hard to define in ARPES experiments at energies close to the Fermi level and where, for the most part, the width of the state exceeds its energy. Indeed, the integral of the density of states using as input the v_F extracted from APRES measurements is found to account for only half of the as-measured electronic specific heat coefficient (95). Furthermore, this reliance on Fermiology (fermion phenomenology), present also in (10), leaves open the precise meaning of figure 2 of (10), in which the magnitude of the T-linear transport scattering rate extracted from the DC resistivity is found to scale as $1/v_F$ for a series of materials that violate the MIR limit at intermediate to high temperatures. Despite this, a similar extraction in (9), again using Fermiology but focusing on the low-T resistivity, also found a transport scattering rate close to the Planckian bound. This consistency between the two analyses reflects the curious fact that the T-linear slope of the DC resistivity does not vary markedly as the MIR threshold is crossed. It does not, however, necessarily justify either approach in validating T-linear scattering at the Planckian limit. Finally, although T-linearity and Planckian dissipation appear synonymous in the cuprates, this is not universally the case. In $YbRh_2Si_2$ (75), for example, the T-linear scattering rate is found to deviate strongly from the Planckian limit with $\tau_{tr} \sim 0.1\tau_P$ (52), and in the electron-doped cuprates, the notion of a Planckian limit to the scattering rate has recently been challenged (96). This certainly adds to the intrigue regarding quantum criticality as the underlying cause of Planckian dissipation, for which several alternative proposals have recently emerged (97, 98).

In principle, the optical conductivity permits an extraction of τ without recourse to Fermiology. Within a Drude model, the optical conductivity

$$\sigma(\omega) = \frac{1}{4\pi}\frac{\omega_p^2\tau_{tr}}{1 + i\omega\tau_{tr}} \quad (9)$$

Table 2. Snapshot of current theoretical modeling of the strange metal regime. Indicated are consistency with T-linear resistivity, $\omega^{-2/3}$ scaling of the mid-IR optical conductivity, quadrature-scaling magnetoresistance, extended quantum criticality, and what predictions are made in terms of experimental observables. Scenarios: MFL, marginal Fermi liquid; EFL, ersatz Fermi liquid; SYK, Sachdev-Ye-Kitaev; AdS/CFT, anti–de Sitter space/conformal field theory conjecture; AD/EMD, anomalous dimensions/Einstein-Maxwell-dilaton; HM, Hubbard model; QMC, quantum Monte Carlo; ED, exact diagonalization; CA, cold atoms; DMFT/EDMFT, dynamical mean-field theory/embedded dynamical mean-field theory; A-B, Aharonov-Bohm effect; ECFL, extremely correlated Fermi liquid; QCP, quantum critical point. *T-linear resistivity is an input. **A slope change occurs through the MIR. ***Quadrature scaling obtained only for a bivalued random resistor model (121) with equal weights (27). ****Although this scaling was thought to arise in pure AdS with an inhomogeneous charge density (123), later studies (124, 125) found otherwise.

	$\rho \propto T$ as $T \to 0$	$\rho \propto T$ as $T \to \infty$	$\sigma \propto \omega^{-2/3}$	Quadrature MR	Extended criticality	Experimental prediction
			Phenomenological			
MFL	√ (67)	× (67)	×	×	×	Loop currents (107)
EFL	— *	—	—	×	×	Loop currents (108)
			Numerical			
ECFL	×	(109)	—	—	×	×
HM (QMC/ED/CA)	— (110)	√ (110–114)	×	—	—	—
DMFT/EDMFT	√ (115)	√ (116, 117)	×	—	√ (117)	—
QCP	(118)	—	—	—	×	—
			Gravity-based			
SYK	√ (119, 120)	√** (120)	×	√*** (121)	—	×
AdS/CFT	√ (122)	√ (122)	√**** (90, 126)	×	×	×
AD/EMD	√ (127–129)	√ (90, 126, 127, 129, 130)	√ (90, 126, 130)	×	√ (126)	Fractional A-B (129)

contains only τ_{tr} and $\omega_p = \sqrt{4\pi n_e e^2/m}$. At zero frequency, the Drude formula naturally yields the DC conductivity σ_{DC} and an estimate for the relaxation rate can be extracted from the width at half maximum of the full Drude response. However, there is an important caveat: τ_{tr} is frequency-dependent in the cuprates, a condition that is consistent with various physical models including both the Fermi liquid and MFL scenarios as well as the large body (88, 91) of MDC analysis performed on the cuprates. Although this prevents a clean separation of the conductivity into coherent and incoherent parts, it was shown in (7) that in the low-frequency limit, $\omega < 1.5k_BT/\hbar$, $\tau_{tr} \sim 0.8\tau_P$, in agreement with the DC analysis of (9).

A second key issue remains: How can such Drude analysis be justified for those strange metals in which the MIR limit is violated and the Drude peak shifts to finite frequencies (16)? Indeed, in the high-T limit, "bad metallicity" can be ascribed to a transfer of spectral weight from low to high ω, rather than to an ever-increasing scattering rate (that within a Drude picture results in a continuous broadening of the Lorentzian fixed at zero frequency). Given the marked crossover in the form of $\sigma(\omega)$ at low frequencies, it is indeed mysterious that the slope of the T-linear resistivity continues unabated with no discernible change.

Is it Mottness?

Table 1 encompasses a series of ground states from which T-linear resistivity emerges. In some of these materials, such as the heavy fermions, the high- and low-energy features of the spectrum are relatively distinct in the sense that spectral weight transfer from the UV to the IR is absent. On the other hand, hole or electron doping of the parent cuprate induces a marked transfer of spectral weight of roughly 1 to 2 eV. As a result, the low-energy spectral weight grows (77, 99–102) at the expense of the degrees of freedom at high energy, a trend that persists (76) even inside the superconducting state. This is an intrinsic feature of Mott systems, namely that the number of low-energy degrees of freedom is derived from the high-energy spectral weight. As this physics is distinct from that of a Fermi liquid and intrinsic to Mott physics, it is termed "Mottness" (102). Notably, the mid-IR response with its characteristic $\omega^{-2/3}$ scaling is absent from the parent Mott insulating state. Hence, it must reflect the doping-induced spectral weight transfer across the Mott gap. It is perhaps not a surprise, then, that no low-T_c material exhibits such a mid-IR feature. In fact, some theories of cuprate superconductivity (103) credit its origin to the mid-IR scaling. We can quantify the total number of low-energy degrees of freedom that arise from the UV-IR mixing across the Mott gap by integrating the optical conductivity,

$$N_{\text{eff}}(\Omega) = \frac{2mV_{\text{cell}}}{\pi e^2}\int_0^{\Omega} \sigma(\omega)d\omega \qquad (10)$$

up to the optical gap $\Omega \approx 1.2$ eV, where V_{cell} is the unit cell volume. The energy scale of 1.2 eV corresponds to the minimum of the optical conductivity, as mentioned above. In a rigid-band semiconductor model in which such spectral weight transfer is absent, $N_{\text{eff}} = x$, where x is the number of holes. In the cuprates, however, N_{eff} exceeds x (Fig. 3). This is the defining feature of Mottness (99–102), as it is ubiquitous in Mott systems and strictly absent in weakly correlated metals. Even in many of the strange or quantum critical metals described in Table 1, there is little or no evidence that Mottness is playing a notable role. Such a distinction may thus offer a hint to the source of the uniqueness of the cuprate strange metal. In bad metals, on the other hand, a gradual transfer of low-frequency spectral weight out to energies on the order of the Mott scale is almost universally observed with increasing temperature (16), suggesting that Mottness is one of the key components of bad metallic transport.

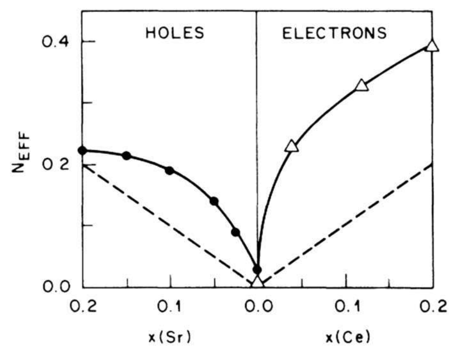

Fig. 3. Integrated optical conductivity N_{eff} exceeds dopant concentration. Shown is N_{eff} for electron-doped $Pr_{2-x}Ce_xCuO_{4-\delta}$ (triangles) and hole-doped $La_{2-x}Sr_xCuO_{4-\delta}$ (circles) The dashed line indicates what is expected for doping a semiconductor. The expectation is that each Ce or Sr atom contributes just a single charge carrier. [Reprinted with permission from (77)]

The optical response in cuprates tells us that there are degrees of freedom that couple to electromagnetism that have no interpretation in terms of doped holes. That is, they are not local entities, as they arise from the mixing of UV and IR degrees of freedom. It is such mixing that could account for the lack of any distinctive energy scale (*102*)—that is, scale invariance—underlying the strange metal. Additionally, optical conductivity studies showed (*104*) that throughout the underdoped regime of the cuprate phase diagram, the effective mass remains constant. As a result, the Mott transition proceeds by a vanishing of the carrier number rather than the mass divergence of the Brinkman-Rice scenario (*105*). [Note that although quantum oscillation experiments on underdoped cuprates show evidence for mass enhancement (*106*), this is thought to be tied to the charge order around 1/8 doping.] Such dynamical mixing between the UV and IR scales in Mott systems is well known to give rise to spectral weight in the lower Hubbard band (*100*–*102*) that exceeds the number of electrons, strictly $1 + x$, that the band can hold. Consequently, part of the electromagnetic response of strange metals at low energies has no interpretation in terms of electron quasiparticles, as it arises strictly from UV-IR mixing. Precisely how such mixing leads to scale-invariant T-linear resistivity remains unresolved.

Is it about gravity?

To frame the theoretical modeling of strange metallicity, we group the work into three principal categories: phenomenological, numerical, and gravity-related. Table 2 lists some representative contributions (*67*, *90*, *107*–*130*); because of space limitations, it is not possible to cite all relevant work. Both phenomenological models considered here require (such as the ersatz Fermi liquid or EFL) or predict (MFL) loop currents, but they do so for fundamentally different reasons. For EFL (*108*), such current order is needed to obtain a finite resistivity in the absence of momentum relaxation (certainly not a natural choice given the Drude fit to the optical conductivity discussed previously), whereas in MFL, loop currents (*107*) are thought to underpin the local fluctuation spectrum (*67*). The extremely correlated Fermi liquid theory (ECFL) (*109*) predicts a resistivity that interpolates between Fermi liquid–like T^2 at low T to T-linear for $T >> T_{FL}$. Quantum Monte Carlo (QMC) simulations (*110*, *111*, *113*, *114*) as well as cold atom experiments (*112*) on the Hubbard model have established that at high temperatures, the resistivity is indeed T-linear. The fermion-sign problem, however, prevents any definitive statement about the low-T behavior in the presence of Mott physics. Non–Fermi liquid transport in Sachdev-Ye-Kitaev (SYK)–based models (*131*–*133*) is achieved by an all-to-all random interaction. Although such interactions might seem initially unphysical, SYK models are nonetheless natural candidates to destroy Fermi liquids, which, by their nature, permit a purely local description in momentum space. As a result, Fermi liquids are impervious to generic repulsive local-in-space interactions (*134*). Coupling a Fermi liquid to an array of disordered SYK islands, however, leads (*120*, *121*) to a nontrivial change in the electron Green function across the MIR, and hence a change in slope of the resistivity is unavoidable (*121*), although it can be minimized through fine tuning (*120*).

An added feature of these disordered models is that in certain limits, they have a gravity dual (*132*, *133*, *135*, *136*). This state of affairs arises because the basic propagator (*132*, *133*, *135*) in the SYK model in imaginary time describes the motion of fermions, with appropriate boundary conditions, between two points of the asymptotic boundary of a hyperbolic plane. In real time, simply replacing the hyperbolic plane with the spacetime equivalent, namely two-dimensional anti–de Sitter (AdS) space (a maximally symmetric Lorentzian manifold with constant negative curvature), accurately describes all the correlators. It is from this realization that the dual description between a random spin model and gravity in AdS_2 lies (*132*, *133*, *137*). Hence, although the origins of SYK were independent of gravity, its correlators can be deduced from the asymptotics of the corresponding spacetime. At the asymptote, only the time coordinate survives and hence, ultimately, SYK dynamics is ultralocal in space with correlations diverging only in time, an instantiation of local quantum criticality.

Such local quantum criticality is not a new concept in condensed matter systems and indeed lies at the heart of MFL phenomenology (*67*) as well as dynamical mean field theory (DMFT) (*116*), and is consistent with the momentum-independent continuum found in the M-EELS data discussed earlier (*85*). The deeper question is why gravity has anything to do with a spin problem with nonlocal interactions. The issue comes down to criticality and to the structure of general relativity. The second equivalence principle on which general relativity is based states that no local measurement can detect a uniform gravitational field. A global measurement is required. The same is true for a critical system because no local measurement can discern criticality. Observables tied to the diverging correlation length are required. Hence, at least conceptually, it is not unreasonable to expect a link between critical matter and gravity. The modern mathematical machinery that makes it possible to relate the two is the gauge-gravity duality or the AdS/CFT (anti–de Sitter/conformal field theory) conjecture. The key claim of this duality (*138*–*140*) is that some strongly interacting quantum theories, namely ones that are at least conformally invariant in d dimensions, are dual to a theory of gravity in a $d + 1$ spacetime that is asymptotically AdS. The radial direction represents the energy with the quantum theory residing at the UV boundary, whereas the IR limit is deep in the interior at the black hole horizon. Hence, intrinsic to this construction is a separation between bulk (gravitational) and boundary (quantum mechanical) degrees of freedom. That the boundary of a gravitational object has features distinct from the bulk dates back to the observations of Beckenstein (*141*) and Hawking (*142*, *143*) that the information content of a black hole scales with the area, not the volume. The requirement that the boundary theory be strongly coupled then arises by maintaining that the AdS radius of curvature exceeds the Planck length ℓ_P. More explicitly, because the AdS radius and the coupling constant of the boundary theory are proportional, the requirement $R >> \ell_P$ translates into a boundary theory that is strongly coupled.

The first incarnation (*122*, *144*, *145*) of this duality in the context of fermion correlators involved modeling fermions at finite density in 2 + 1 dimensions. From the duality, the conformally invariant vacuum of such a system corresponds to gravity in AdS_4, the extra dimension representing the radial direction along which identical copies of the boundary CFT lie (albeit with differing energy scales). Surprisingly, what was shown (*122*) is that the low-energy (IR) properties of such a system in the presence of a charge density are determined by an emergent $AdS_2 \times R^2$ (with R^2 representing a plane) spacetime at the black hole horizon. The actual symmetry includes scale invariance and is denoted by $SL(2, R)$ (a special Lie group of real 2×2 matrices with a unit determinant). Once again, the criticality of boundary fermions is determined entirely by the fluctuations in time, that is, local quantum criticality as seen in SYK. The temperature and frequency dependence of the conductivity are then determined by the same exponent (*122*) as expected from Eqs. 5 and 6 and, as a result, a simultaneous description of T-linearity and $\omega^{-2/3}$ dependence is not possible, as noted in Table 2.

This particular hurdle is overcome by the anomalous dimensions/Einstein-Maxwell-dilaton (AD/EMD) theories (*89*, *90*, *126*, *146*–*149*), which, as indicated in Table 2, have been the most successful to date in describing the range of physics observed in strange metals. What is new here is the introduction of extra fields, dilatons for example, that permit hyperscaling violation (*79*) and anomalous dimensions (*89*, *90*, *126*, *146*–*149*) for all operators. Consequently, under a scale change of the coordinates, the metric is no longer unscathed. That is, the manifold is not fixed and it is the matter fields that determine the geometry. Such systems have a covariance, rather than scale invariance indicative of pure AdS metrics. A consequence of this covariance is that even the currents acquire anomalous dimensions. But how is this possible given that a tenet of field theory is that no amount of renormalization can change the dimension of the current (*150*) from $d - 1$? What makes this possible is that in EMD theories, the extra radial dimension allows physics beyond Maxwellian electromagnetism. For example, the standard Maxwell action, $S = \int dV_d F^2$ where $F = dA$, requires that the dimension of the gauge field be fixed to unity, $[A] = 1$ (*151*). EMD theories use instead an action of the form $S = \int dV_d dy y^a F^2$ where y is the radial coordinate of the $d + 1$ AdS spacetime. Comparing these two actions leads immediately to the conclusion that the dimension of A now acquires the value $[A] = 1 - (a/2)$. Hence, even in the bulk of the geometry, the dimension of the gauge field is not unity. Depending on the value of a, $a < 0$ at the UV conformal boundary or $a > 0$ at the IR at the black hole horizon, the equations of motion are nonstandard and obey fractional electromagnetism (*128*, *152*) consistent with a nontraditional dimension for the gauge field. In EMD theories, it is precisely the anomalous dimension (*89*, *90*, *126*, *146*–*148*) for conserved quantities that gives rise to the added freedom for extended quantum criticality to occur, enabling the simultaneous fitting (*130*) of T-linearity and $\omega^{-2/3}$ of the optical conductivity, and establishing the basis for a proposal for the strange metal based on $[A] = 5/3$ (*127*).

Within these holographic systems, a Drude-like peak in the optical conductivity can emerge both from the coherent (quasiparticle-like) sector (*153*) as well as from the incoherent ["unparticle" (*154*)] sector (*155*–*158*). Application of EMD theory has also provided fresh insights into the phenomenon of "lifetime separation" seen in the DC and Hall conductivities of hole-doped cuprates (*22*, *24*, *28*) as well as in other candidate strange metals (*52*, *61*). For a system with broken translational invariance, the finite density conductivity comprises two distinct components (*159*). The DC resistivity is dominated by the particle-hole symmetric term—whose Hall conductivity is consequently zero—whereas the T dependence of the Hall angle is set by the more conventional (umklapp) momentum relaxation that governs the response of the coherent charge density.

The success of EMD theories in the context of strange metal physics raises a philosophical question: Is all of this just a game? That is, is the construction of bulk theories with funky electromagnetism fundamental? The answer lies in Nöther's Second Theorem (NST) (*102*, *128*, *152*), a theorem far less known than her ubiquitous first theorem but ultimately of more importance as it identifies a shortcoming. To illustrate her first theorem, consider Maxwellian electromagnetism, which is invariant under the transformation $A_\mu \rightarrow A_\mu + \partial_\mu \Lambda$. This theorem states that there must be a conservation law with the same number of derivatives as in the gauge principle. Hence, the conservation law only involves a single derivative, namely $\partial_\mu J_\mu = 0$. This is Nöther's First Theorem (*160*) in practice.

What Nöther (*160*) spent the second half of her famous paper trying to rectify is that the form of the gauge transformation is not unique; hence, the conservation law is arbitrary. It is for this reason that in the second half (*160*) of her foundational paper, she retained all possible higher-order integer derivatives in the gauge principle. These higher-order derivatives both add constraints to and change the dimension of the current. Stated succinctly, NST (*160*) dictates that the full family of generators of U(1) invariance determines the dimension of the current. It is easy to see how this works. Suppose we can find a quantity $\hat{Y}$ that commutes with ∂_μ. That is, $\partial_\mu \hat{Y} = \hat{Y} \partial_\mu$. If this is so, then we can insert this into the conservation law with impunity. What this does is redefine the current: $\partial_\mu \hat{Y} J^\mu = \partial_\mu \tilde{J}^\mu$. The new current $\tilde{J}^\mu$ acquires whatever dimensions $\hat{Y}$ has, such that $[\tilde{J}^\mu] = d - 1 - d_Y$. But because of the first theorem, $\hat{Y}$ must have come from the gauge transformation and hence must ultimately be a differential operator itself. That is, there is an equally valid class of electromagnetisms with gauge transformations of the form $A_\mu \rightarrow A_\mu + \partial_\mu \hat{Y} \Lambda$. For EMD theories (*102*, *128*, *152*), $\hat{Y}$ is given by the fractional Laplacian, $\Delta^{(\gamma-1)/2}$ with $[A_\mu] = \gamma$ [with $\gamma = 1 - (a/2)$ to make contact with the EMD theories introduced earlier]. For most matter as we know it, $\gamma = 1$. The success of EMD theories raises the possibility that the strangeness of the strange metal hinges on the fact that $\gamma \neq 1$. This can be tested experimentally using the standard Aharonov-Bohm geometry (*128*, *129*) in which a hole of radius r is punched into a cuprate strange metal. Because $[A]$ is no longer unity, the integral of $A \cdot d\ell$ is no longer the dimensionless flux. For physically realizable gauges, this ultimately provides an obstruction to charge quantization. As a result, deviations (*128*, *129*) from the standard $\pi r^2 \times B$ dependence for the flux would be the key experimental feature that a nonlocal gauge principle is operative in the strange metal. An alternative would be, as Anderson (*161*) advocated, the use of fractional or unparticle propagators with the standard gauge principle. However, in the end, it all comes down to gauge invariance. The standard gauge-invariant condition prevents the power laws in unparticle stuff from influencing the algebraic fall-off of the optical conductivity (*130*, *162*), as they offer just a prefactor to the polarizations (*163*). The escape route, an anomalous dimension for the underlying gauge field, offers a viable solution, but the price is abandoning locality (*164*) of the action.

Is it important?

Given the immense difficulty in constructing a theory of strange metals, one might ask why bother? To gauge the importance of strange metals, look no further than Fig. 4. This figure shows that the coefficient α_1 of the T-linear resistivity component in the strange metal regime of overdoped hole-doped cuprates tracks the doping dependence of the $T = 0$ superfluid density $n_s(0)$. As mentioned earlier, a similar correlation exists between α_1 and T_c in electron-doped cuprates (*31*, *32*), the Bechgaard salts (*58*), and the iron pnictides (*58*), establishing a fundamental link between high-temperature superconductivity and strange metals.

For a long time, the drop in $n_s(0)$ with doping in cuprates was attributed to pair breaking, a symptom of the demise of the dominant pairing interaction within a disordered lattice. Recent mutual inductance measurements, however, have challenged this view, arguing that the limiting low-T behavior of $n_s(T)$ was incompatible with conventional pair-breaking scenarios (*165*). Certainly, the correlation between α_1 and $n_s(0)$ is unforeseen in such models. Moreover, if the strange metal regime is indeed populated with non-quasiparticle states, then Fig. 4 indicates a pivotal role for these states in the pairing condensate (*166*, *167*). On more general grounds, this result informs us that the door to unlocking cuprate superconductivity is through the strange metal regime, and any theory that divorces superconductivity from strange metals is unlikely to be a promising avenue. To conclude, solving the strange metal regime kills two birds with one stone. Perhaps there is some justice here. After all, we know from Pippard's (*168*) work, which can be reformulated (*128*, *152*) in terms of fractional

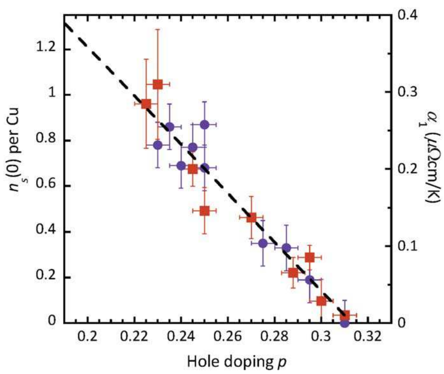

Fig. 4. Correlation between the superfluid density $n_s(0)$ and the coefficient a_1 of the T-linear resistivity. The data shown are for $Tl_2Ba_2CuO_{6+\delta}$ (Tl2201) across the strange metal regime. [Adapted from (*166*, *167*)]

Laplacians, that explaining superconductivity even in elemental metals necessitates a nonlocal relationship between the current and the gauge field. What seems to be potentially new about the cuprates is that now the normal state, as a result of the strange metallicity, also requires nonlocality.

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Stranger than metals

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Understanding an exotic phase
The nature of strange metals, a metallic phase with unconventional transport properties that appears in phase diagrams of materials such as cuprate superconductors, remains one of the major puzzles in condensed matter physics. Phillips *et al*. reviewed experimental and theoretical progress toward understanding this phase. The authors examined its relationship to quantum criticality, Planckian dissipation, and quantum gravity. —JS