A Pattern-Recognition-Based Mesh Refinement Method for the Moment Method Analysis of Electromagnetic Scattering Problems

Christian Díaz-Cáez and Su Yan
Department of Electrical Engineering and Computer Science, Howard University, Washington, DC 20059-1015, USA
christian.diaz@bison.howard.edu; su.yan@howard.edu

***Abstract*—Basis functions that describe both the amplitude distribution and the traveling wave phase variation of the induced surface current have been successfully applied over large patches. Although such basis functions generate accurate results for smooth and convex objects, they cannot describe accurately the current distribution on objects with non-smooth and/or concave surfaces. In this work, a nonuniform mesh refinement method is developed based on a current pattern recognition technique to describe standing wave distributions more accurately. The simulation results on the nonuniform mesh grids achieve a much better accuracy and a lower overall computational cost.**

I. Introduction

The phase extracted (PE) basis functions have been developed to solve efficiently electromagnetic scattering problems from electrically large complex objects [1]. By incorporating a traveling wave phase factor in the definition, the PE basis functions can be defined on patches as large as half a wavelength, which reduce the number of unknowns and alleviate the computational and memory requirements significantly. Unfortunately, although the PE basis functions have been applied successfully on the scattering analysis of smooth and convex objects, they cannot describe accurately the current singularity at geometrical edges, corners, or tips, or standing wave patterns on non-smooth or concave surfaces such as cavities. This difficulty can be partly alleviated by combining low-order PE basis functions with high-order hierarchical basis functions [2]. While such a combination can be regarded as the *p*-refinement, another approach is the local *h*-refinement where only the mesh elements associated with geometrical discontinuities or standing wave patterns are refined to better capture the local current variation.

In this paper, a set of rules are proposed based on geometrical and physical features of the problem to identify elements that need to be refined. It is shown that with the proposed rules, the mesh refinement can be automated, which eliminates the need of any human intervention. A numerical example from an electrically large non-smooth object is presented to demonstrate the performance of the method.

II. Pattern-Recognition-Based Mesh Refinement

It has been proven in [1] that the induced surface current in a scattering problem is dominated by traveling wave components if the scatterer is smooth and convex. The phase variation of the traveling wave is determined by the incident plane wave. As a result, the plane wave phase factor can be introduced to the traditional basis functions, such as the curvilinear Rao-Wilton-Glisson (CRWG) basis functions [3], so that the unknown surface current $\boldsymbol{J}$ can be expanded with the PE basis functions $\boldsymbol{j}_n(\boldsymbol{r})\,\mathrm{e}^{\mathrm{i}\boldsymbol{k}^{\mathrm{inc}}\cdot\boldsymbol{r}}$ as

$$\boldsymbol{J}(\boldsymbol{r}) = \sum_{n=1}^{N} a_n\,\boldsymbol{j}_n(\boldsymbol{r})\,\mathrm{e}^{\mathrm{i}\boldsymbol{k}^{\mathrm{inc}}\cdot\boldsymbol{r}} \qquad (1)$$

where a_n, $\boldsymbol{j}_n$, $\boldsymbol{k}^{\mathrm{inc}}$, and N stand for the unknown expansion coefficients, the CRWG basis functions, the wave vector of the incident plane wave, and the total number of unknowns, respectively.

When solving scattering problems with smooth and convex surfaces, the use of PE basis functions defined on triangular patches up to half a wavelength ($\lambda/2$) can generate numerical solutions with a good accuracy [1], [2]. However, when the scatterer has geometrical discontinuities or encounters strong multiple reflections and mutual couplings between different parts, the induced standing wave components can dominate over the traveling wave components. To describe such standing wave components properly, a denser mesh, for example, $\lambda/10$, can be used in standing wave regions. Unfortunately, without an adequate physical understanding of the scattering and coupling mechanism, it is difficult to determine the location of the standing wave regions. As a result, it usually requires human intervention to manually specify the regions that need a denser geometrical discretization (referred to as the target regions hereafter).

In this work, a two-step approach is proposed to automatically identify the target regions. In the first step, a coarse mesh is employed in the method of moments (MoM) solution of a scattering problem using the PE basis functions. Due to the use of a coarse mesh, the total number of unknowns is very small and the solution can be performed very fast. Based on such a solution, the standing wave ratio (SWR) of the surface current can be estimated in triangular patch k as

$$\mathrm{SWR} = \max_{\forall i\in\mathcal{N}_k} \|\boldsymbol{J}(\boldsymbol{r}_i)\| / \min_{\forall i\in\mathcal{N}_k} \|\boldsymbol{J}(\boldsymbol{r}_i)\| \qquad (2)$$

where $\mathcal{N}_k$ denotes triangle k and its direct neighbors. A large SWR indicates a stronger standing wave pattern, which requires a finer mesh to resolve the induced current variation. In addition, geometrical discontinuities usually induce strong current reflections and edge singularities, and hence, need to be resolved with a finer mesh. In sum, the following set of rules is

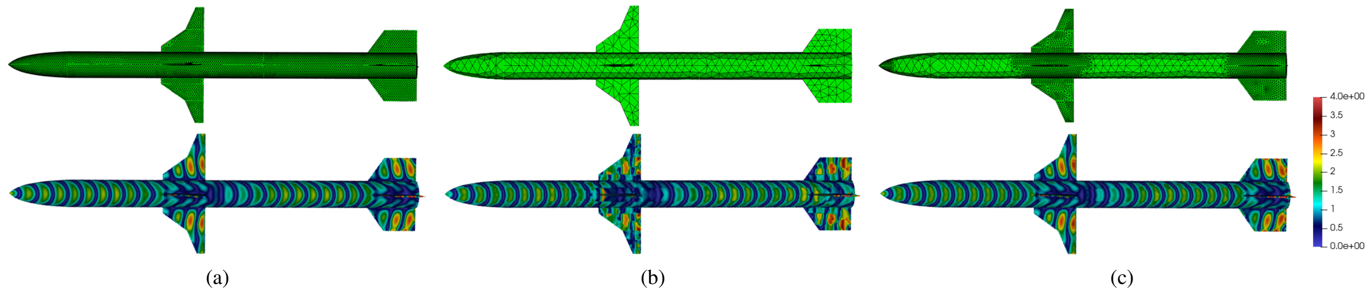

Fig. 1. Geometrical discretizations and induced current distributions over the surface of a missile-like object using (a) CRWG basis functions defined on a dense mesh with an average patch size of $\lambda/10$; (b) PE basis functions defined on a coarse mesh with an average patch size of $\lambda/2.5$; and (c) PE basis functions defined on a nonuniform mesh.

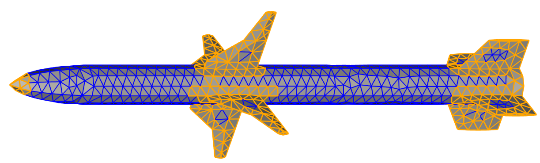

Fig. 2. The missile-like object with target regions identified and highlighted.

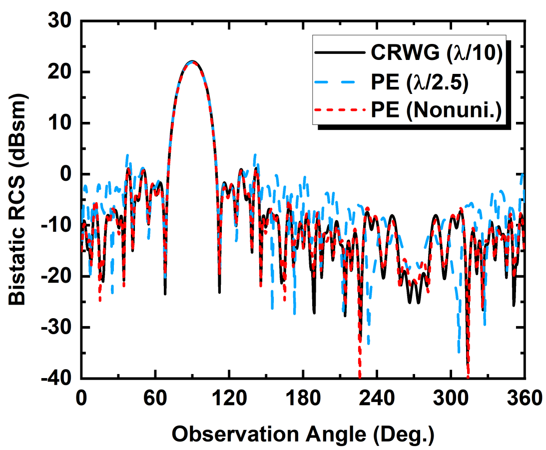

Fig. 3. Bistatic RCS of the missile-like object. The result from CRWG basis functions is used as the reference.

proposed to identify the target regions in a mesh. Specifically, when a patch

1) is associated with an open geometrical boundary;
2) is located at a geometrical discontinuity (e.g., edges, corners, and tips); or
3) has a SWR higher than a preset threshold;

it is identified as a target region and needs to be refined.

In the second step, the scattering problem is solved again using the refined, nonuniform mesh with PE basis functions. Due to the presence of traveling wave regions, PE basis functions are still required in this step to properly describe the traveling wave components of the induced surface current.

III. Numerical Example

Electromagnetic scattering from a PEC missile-like object is presented in this section. The total length of the target is 4.7 m with its electrical size being 23.5λ at the frequency of 1.5 GHz. Under the illumination of the incident plane wave coming from the nose direction, the induced surface currents are solved using the combined-field integral equation (CFIE). Figure 1(a) presents the reference solution obtained by CRWG basis functions defined on a dense mesh with an average patch size of $\lambda/10$. Figures 1(b) and (c) show the numerical solutions from PE basis functions defined on a coarse mesh of $\lambda/2.5$ and a nonuniform mesh, respectively. It is clear that the solution from the coarse mesh cannot resolve the strong induced current on the wings and tail fins. By refining the mesh over the nose and all four wings and tail fins, the current distribution on the nonuniform mesh matches that of the reference solution very well. Shown in Fig. 2 is the highlighted target regions that are refined. These regions are automatically identified using the rules described in the preceding section with the SWR threshold chosen as 10. Apparently, the proposed method identifies both geometrical discontinuities and strong mutual coupling regions successfully. The bistatic radar cross section (RCS) is shown in Fig. 3, from which it can be seen that the nonuniform mesh results in a much more accurate solution than the coarse mesh. Finally, the computational data are reported in Tab. I. It is seen that the memory requirement and the total CPU time of the two PE solutions (59.6 min.) are significantly less than those of the CRWG solution.

Table I. Comparison of Computational Data.

Basis Function (Mesh Size)	No. of Unknowns	Memory (Mb)	Total CPU Time (min.)
CRWG ($\lambda/10$)	$44,736$	2072.11	99.5
PE ($\lambda/2.5$)	$2,817$	61.07	2.2
PE (Nonuni.)	$26,607$	1309.87	57.4

References

[1] Z.-P. Nie, S. Yan, S. He, and J. Hu, “On the basis functions with traveling wave phase factor for efficient analysis of scattering from electrically large targets,” *Prog. Electromagn. Res.*, vol. 85, pp. 83–114, 2008.

[2] Z.-P. Nie, S. Ren, S. Yan, S. He, and J. Hu, “Modified phase-extracted basis functions for efficient analysis of scattering from electrically large targets,” *Proc. IEEE*, vol. 101, no. 2, pp. 401–413, 2012.

[3] R. D. Graglia, D. R. Wilton, and A. F. Peterson, “Higher order interpolatory vector bases for computational electromagnetics,” *IEEE Trans. Antennas Propag.*, vol. 45, no. 3, pp. 329–342, Mar. 1997.