# An Automated Adaptive $h$-Refinement Technique for Solving SIEs With Nonconformal Meshes

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***Abstract*—An adaptive mesh refinement ($h$-refinement) technique is proposed to solve surface integral equations (SIEs) for electromagnetic scattering analysis. Based on the physics of the induced surface current and the geometry of the scatterer, rules are developed to identify regions for mesh refinement. Basis function with a phase factor are employed to better describe the phase variation of the current. In the proposed technique, the problem is first solved on a coarse mesh. Based on the coarse solution, regions are identified for refinement, which produces a nonconformal mesh. The discontinuous Galerkin integral equation method is then employed to solve the problem again on the nonconformal mesh for an improved accuracy. A numerical example is presented to demonstrate the performance of the proposed method.**

## I. Introduction

Electromagnetic scattering problems are commonly solved by numerical methods such as the finite element method and the method of moments (MoM). In a numerical simulation, the solution domain needs to be discretized into small mesh elements. The electrical size of the mesh is critical for the accuracy of the numerical solution. While a denser mesh with smaller elements results in a better numerical accuracy, it also leads to higher computational and storage costs, making it expensive to solve scattering problems from electrically large objects. For objects with smooth and convex surfaces, it has been shown that describing the phase variation of the induced surface currents can reduce the total number of unknowns effectively [1]. By incorporating a traveling wave phase factor into the traditionally used low-order RWG basis functions, the resulting phase extracted basis functions (PEBFs) can be defined on mesh elements as large as half a wavelength.

While the use of the PEBFs in smooth and convex objects demonstrated their modeling capability, when it comes to the modeling of objects with geometrical discontinuities, such as edges, corners, and tips, and those with strong physical mutual couplings, such as reflectors and cavities, the modeling efficacy of the PEBFs deteriorates. In our previous efforts, we have presented a set of rules based on the geometrical and physical features of the problem to identify regions that need to be refined [2]. Based on those proposed rules, a mesh refinement technique has been developed such that an algorithm can be automated [3]. In [2], the mesh refinement was implemented in a nonuniform but conformal fashion. However, a conformal discretization requires a considerably larger number of unknowns, and hence, more memory and CPU time in the numerical solution, when compared to a nonconformal meshing strategy.

In this paper, the automated mesh refinement method presented in [2] and [3] is extended to operate with a nonconformal mesh. It is shown that the method presented here can solve electrically extra-large problems and reduce the overall computational cost.

## II. Automated Nonconformal Mesh Refinement

An automated mesh refinement technique is proposed to identify areas that need finer mesh elements by exploring the unique traveling-wave-describing nature of the PEBFs. Because the traveling wave is determined by the incident plane wave, a phase factor can be introduced into a traditional basis function, such as the RWG basis functions. The PEBF [1] is defined as $\boldsymbol{j}_n(\boldsymbol{r})\,\mathrm{e}^{\mathrm{i}\boldsymbol{k}^{\mathrm{inc}}\cdot\boldsymbol{r}}$, where $\boldsymbol{j}_n$ stands for the curvilinear RWG basis functions and $\boldsymbol{k}^{\mathrm{inc}} = k_0\hat{\boldsymbol{k}}^{\mathrm{inc}}$ stands for the vector wavenumber of the incident plane wave.

In the proposed method, an initial coarse mesh is first employed, on which the PEBFs are defined and utilized to obtain a fast but "rough" solution of the scattering problem. Clearly, the total number of unknowns in the initial mesh is very small, and the solution can be performed very fast. Based on such a rough solution, an automated mesh refinement technique is applied to automatically identify areas that need a finer mesh. A nonuniform and nonconformal mesh is then generated by refining the identified mesh elements to smaller sizes while keeping the rest of the mesh untouched. Elements in regions with strong standing waves are expected to present a large standing wave ratio (SWR) of the surface current as defined in [2]. In summary, an element need refinement if it

1) is associated with an open geometrical boundary;
2) is located at a geometrical discontinuity (e.g., edges, corners, or tips); or
3) has an SWR higher than a preset threshold.

After the refinement, for the MoM to process the nonconformal mesh, the discontinuous Galerkin integral equation (DGIE) method [4] is employed with the PEBFs to solve the scattering problem again. The DGIE method permits the PEBFs to be discontinuous across a nonconformal interface of two neighboring elements. Therefore, it is highly suitable for the MoM simulation of electrically extra-large problems with the PEBFs and the automated mesh refinement technique.

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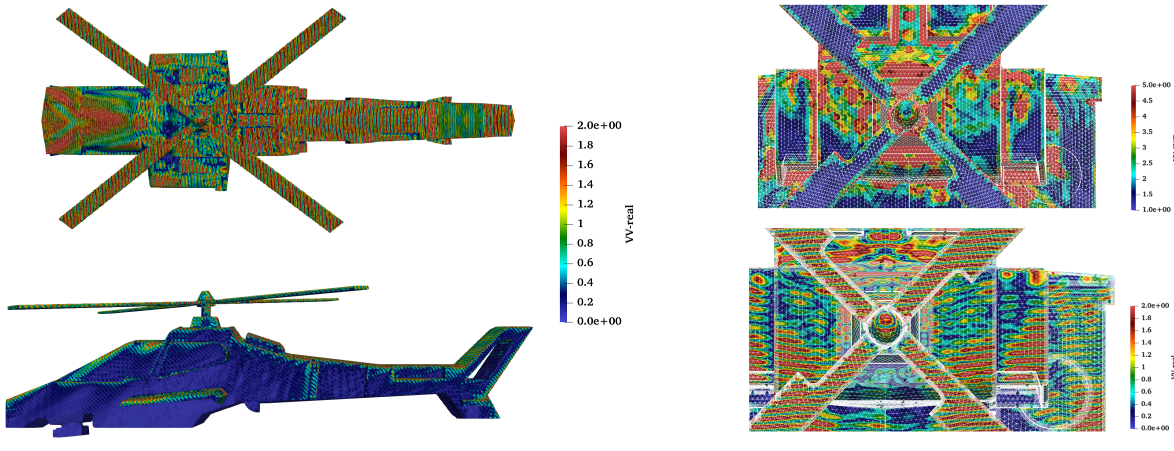


(a) (b)

Fig. 1. Modeling and simulation of electromagnetic scattering from a helicopter-like object. (a) Top and side views of the induced surface current; (b) SWR distribution over a uniform and conformal coarse mesh (top); and the induced surface current over a nonuniform and nonconformal mesh (bottom).

Using the contour/boundary interior penalty presented in [4]. A general expression to calculate the MoM system with the DGIE method is given by

$$\begin{aligned}
&\sum_{n=1}^{N} a_n \Big\{ \frac{\mathrm{i}k_0}{2} \int_S \boldsymbol{v}_m \cdot \int_{S'} \boldsymbol{j}_n G_0 \, \mathrm{d}\boldsymbol{r}' \, \mathrm{d}\boldsymbol{r} \\
&+ \frac{1}{2\mathrm{i}k_0} \int_S \boldsymbol{\nabla} \cdot \boldsymbol{v}_m \int_{S'} \boldsymbol{\nabla}' \cdot \boldsymbol{j}_n G_0 \, \mathrm{d}\boldsymbol{r}' \, \mathrm{d}\boldsymbol{r} \\
&- \frac{1}{2\mathrm{i}k_0} \oint_C \hat{\boldsymbol{t}}_m \cdot \boldsymbol{v}_m \int_{S'} \boldsymbol{\nabla}' \cdot \boldsymbol{j}_n G_0 \, \mathrm{d}\boldsymbol{r}' \, \mathrm{d}\ell \\
&- \frac{1}{2\mathrm{i}k_0} \int_S \boldsymbol{\nabla} \cdot \boldsymbol{v}_m \oint_{C'} \hat{\boldsymbol{t}}_n \cdot \boldsymbol{j}_n G_0 \, \mathrm{d}\ell' \, \mathrm{d}\boldsymbol{r} \\
&+ \frac{1}{4} \int_S \boldsymbol{v}_m \cdot \boldsymbol{j}_n \, \mathrm{d}\boldsymbol{r} + \frac{1}{2} \mathrm{P.V.} \int_S \boldsymbol{v}_m \cdot \hat{\boldsymbol{n}} \times \int_{S'} \boldsymbol{\nabla} G_0 \boldsymbol{j}_n \, \mathrm{d}\boldsymbol{r}' \, \mathrm{d}\boldsymbol{r} \\
&+ \beta \int_{C_{mn}} \hat{\boldsymbol{t}}_m \cdot \boldsymbol{v}_m \cdot \frac{1}{\mathrm{i}k_0} \left[ \hat{\boldsymbol{t}}_{mn} \cdot \boldsymbol{j}_m(\boldsymbol{r}) + \hat{\boldsymbol{t}}_{nm} \cdot \boldsymbol{j}_n(\boldsymbol{r}) \right] \mathrm{d}\ell \Big\} \\
&= \frac{1}{2} \int_S \boldsymbol{v}_m \cdot \boldsymbol{E}^{\mathrm{inc}} \, \mathrm{d}\boldsymbol{r} + \frac{1}{2} \int_S \boldsymbol{v}_m \cdot \hat{\boldsymbol{n}} \times Z_0 \boldsymbol{H}^{\mathrm{inc}}(\boldsymbol{r}) \, \mathrm{d}\boldsymbol{r} \qquad (1)
\end{aligned}$$

where $\boldsymbol{v}_m$, $\boldsymbol{j}_n$, $\hat{\boldsymbol{t}}_m$, $\hat{\boldsymbol{t}}_n$, $Z_0$, $\beta$ and $G_0$ denote the testing function, basis function, unit normal vector on contours $C$ and $C'$, wave impedance in free space, penalty/stabilization term coefficient [4], and the scalar Green's function in free space, respectively.

## III. Numerical Example

In this section, the electromagnetic scattering from a helicopter-like object is presented. The object has a physical length of $4.86$ m and an electrical size of $97.2\lambda$, illuminated by an incident wave at 6 GHz. The incident angle is $\theta = 90°$ and $\phi = 45°$. For simplicity, the entire object, including the window glass, is treated as a perfect electric conductor. The snapshot of the surface current distribution over the object based on the solution of PE-DGIE is presented in Fig. 1 in different viewing angles. The SWR distribution and the nonconformal mesh after the automated mesh refinement are as well presented. From the top view in Fig. 1 (a), a large number of wave oscillations is observed, indicating the large electrical size of the object. The complicated surface current distribution is resulted from the multiple current reflection and mutual couplings. The numerical solution for this object using CRWG-CFIE does not converge, which manifests the unique descriptive capabilities of the PEBFs. In Table I, the number of unknowns and the computational resources are summarized. In this table, NC stands for not converged.

Table I. Comparison of computational data at 6 GHz and VV-polarization.

| Basis Function (Mesh Size) | No. of Unknowns | Memory (Mb) | Total CPU Time (min.) |
|---|---|---|---|
| CRWG ($\lambda/10$) | $3,857,210$ | $28,966.10$ | NC |
| PE ($\lambda/2.5$) | $185,097$ | $5,776.51$ | 42.0 |
| PE (Nonuni.) | $796,147$ | $11,786.80$ | 94.0 |

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