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Exploring students' learning support use in digital game-based math learning: A mixed-methods approach using machine learning and multi-cases study

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A R T I C L E I N F O

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A B S T R A C T

Digital game-based math learning environments (math DGBLE) are promising platforms that provide students with opportunities to master conceptual understanding and cultivate mathematical thinking, on which the contemporary mathematics education places an emphasis. Literature on learning support in digital game-based learning (DGBL) rarely investigate learners' support-use behaviors and interaction patterns in relation to math learning. We addressed this research gap in this exploratory mixed-methods study. We designed and developed a packet of learning supports (i.e., *Task Planner* and *Math Story*) in a math DGBLE. *Task Planner* is designed to assist learners' systematic and planned efforts for math problem solving whereas *Math Story* features historical stories and real-life applications of math concepts. With the data collected via mixed-methods approach, we extracted six clusters of learning-support-use behaviors via unsupervised machine learning technique (i.e., Gaussian Mixture Model), including 1) skills development and application of mathematical problem decomposition, 2) conceptual knowledge development, 3) metacognitive mathematical connections, 4) metacognitive regulation, 5) information selection using cognitive aids, and 6) sustained motivation for necessary aversive practices. Qualitative multi-cases study revealed nuanced details regarding learners' interactions with the learning supports in DGBLE. Results showed that the designed in-game learning supports facilitated individual meaningful and mindful math problem-solving experiences. The findings suggested that a mixed-methods research design integrating machine learning with multi-cases study could act as a tool for learner behavior and interaction pattern research that informs the design of adaptive and effective DGBL.

1. Introduction

Contemporary mathematics education emphasizes the importance of providing students with opportunities for mastering conceptual understanding and cultivating mathematical thinking, and one way to achieving this goal is using learning technologies (Lehtinen et al., 2017). For instance, digital game-based learning environments (DGBLE) can be designed as a discovery-oriented space that provides students with opportunities to actively cultivate discipline-focused, context-rich problem solving (Dai et al., 2022; Kiili,

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2007; Moyer-Packenham et al., 2019). It is expected that game-based learners will engage in self-initiated knowledge construction when solving ill-structured problems (Ke, 2016; Kiili, 2007).

However, game-based problem-solving processes can be aversive or cognitive-demanding. Learners may struggle and feel frustrated, resulting in unsuccessful learning experience or overloaded cognitive processing (De Jong & Van Joolingen, 1998). One way to overcome this barrier is to incorporate deliberate practice (Ericsson et al., 1993), by which learners are coached, goal-driven, and self-aware (Lehtinen et al., 2017), in a game-based, complex learning environment. In mathematics education, deliberate practice is essential for learners to actively develop intricate and integrative mental schemas and genuinely recognize why math works in certain patterns (Lehtinen et al., 2017). Deliberate practice can be achieved by the deployment of *scaffolding or learning support* for digital game-based learning (DGBL). In-game learning support is reported as an effective approach to assisting students to overcome cognitive overload in complex learning environments (Liu et al., 2020; De Jong & Van Joolingen, 1998; Lehtinen et al., 2017; Zumbach et al., 2020). Yet, the design of learning support to facilitate game-based math problem solving remains poorly understood. In-game learning support requires further investigation because it has unique dynamicity in DGBLE. To elaborate, in-game learning support should be designed seamlessly in the game world, be coherent with the in-game actions, and be governed by the game rules (Ke, 2016). All these design challenges for in-game learning support make purposeful explorations of learners' behaviors in using in-game learning support warranted.

The design of learning environments (including learning supports) can only become productive when learners' perspectives are understood (Confrey, 1990). Despite prolific research on learners' behavioral and problem-solving patterns in DGBLE (Pan & Dai, 2022), prior studies on *learning* support in DGBLE rarely investigate learners' support-use behavior and interaction patterns, particularly in the setting of game-based math learning. In the current study, we have designed a packet of learning supports in DGBLE that assist learners to practice mathematical problem solving and develop math conceptual understanding in context. We aim to investigate learners' learning support-use behaviors during math problem solving in a DGBLE. Specifically, for the exploratory purposes, we ask: *How do learners use in-game learning* support *in digital game-based math learning?*

1.1. Underpinning theories of learning support for math problem solving in DGBLE

Learning support, or instructional scaffolding, refers to instructions that experts provide to the learners, aim to help learners develop expertise, and ultimately, perform tasks independently (Puntambekar & Hübscher, 2005). It can be embedded in a learning game to enhance learning of the domain-specific knowledge (Ke, 2016). Using learning support in DGBLE shares the theoretical ground with *facilitated problem solving* in problem-based learning (e.g., Hmelo-Silver, 2004). Learners' problem solving processes were scaffolded for skill and knowledge development until they can perform the tasks independently in DGBLE (Belland et al., 2017; Hmelo-Silver, 2004; Wood et al., 1976). Other contemporary theories underlying the use of learning support in DGBLE include cognitive information processing for problem solving (Mayer, 2002; Mayer & Moreno, 2003) and self-regulated learning (e.g., Zumbach et al., 2020). Because DGBLE involves complex multimedia, learners need supports to select relevant information, coordinate the tasks, and be mindful of regulating their game-based learning activities.

Literature has dominantly focused on *cognitive* and *meta-cognitive learning* supports for learning performance in complex learning environments (Cai et al., 2022; Kim et al., 2018). For example, Zumbach et al. (2020) found that cognitive and metacognitive supports are effective in enhancing knowledge acquisition in social science in a serious game. A recent meta-analysis classified cognitive and metacognitive learning supports into seven predominant types——reflection, feedback, hints, exposition, collaboration, mixed, and

Table 1
The classification of learning support, its description, and corresponding learning supports used in this study.

Categorized functions of learning support in math DGBLE	Description	Forms of learning support designed in this study
Skills development and application of mathematical problem decomposition	An interactive stepwise solution guides for decomposition of math problem solving.	Faded worked example; modeling; problem decomposition guide • Task Planner
Conceptual knowledge development	Providing frames, cues, or conceptual support for learners to experiment with their mathematical understanding for problem solving.	Representations; explicit instruction • Task planner • Math story
Metacognitive mathematical connections	Facilitating learners to make connections among mathematical concepts in the game as well as with the real world to reinforce math problem solving.	Cues; representations • Task Planner • Math Story
Metacognitive regulation	Making learners to be aware of how to plan, think, and execute math problem solving.	Prompts for reflections; information on and cues for real-life applications • Task Planner • Math Story
Information selection using cognitive aids	Making learners to be cognizant of what to look at so that the learners are on the right track.	Explicit instruction; cues; hints • Task Planner • Math story
Sustained motivation for necessary aversive practices	Motivation is being sustained for deliberate learning and problem-solving practices.	Videos; prompts • Task Planner • Math Story

others (Cai et al., 2022). They reported that leaning support can effectively enhance learning in DGBLE ($g = 0.43$, 95% CI: [0.30, 0.56]), but there were no statistically significant differences among different types of learning supports. The insignificant difference among various learning supports may be explained by the design characteristics of and tensions among different in-game learning supports (Reiser, 2004). However, it is still unclear how a packet of learning supports is used. There is still a critical lack of research on learner behavior and interaction patterns in relation to learning support usage, especially for math problem solving.

1.2. Classification of learning supports for math problem solving in DGBLE

Based on prior research, the functions of learning supports for math problem solving in DGBLE can be categorized as facilitating: 1) skills of math problem decomposition (Mayer, 2002; ter Vrugte et al., 2017), 2) conceptual knowledge development (Belland et al., 2017; Confrey, 1990; diSessa, 2014), 3) metacognitive regulation (De Jong & Van Joolingen, 1998; Puntambekar & Hübscher, 2005; Zumbach et al., 2020), 4) metacognitive mathematics connections (García-García & Dolores-Flores, 2021; Moyer-Packenham et al., 2019), 5) information selection (Wouters & Van Oostendorp, 2013), and 6) sustained motivation for necessary aversive practices (Ke, 2008; Lehtinen et al., 2017). Table 1 summarized the categorized functions of the learning supports and forms of learning supports examined in this study.

Problem decomposition. It is essential as it helps learners to decompose the contextualized math problem into smaller and digestible steps for successful math problem solving. Worked example is a frequently used learning support for scaffolding problem decomposition. However, providing worked example could lead to passive learning because learners may just replicate the solutions without deep thinking (Shute et al., 2021). As a result, providing faded worked example is considered as a better way to fostering students' math knowledge acquisition (ter Vrugte et al., 2017). The faded worked examples allow learners to actively engage in systematic planning and solution reasoning rather than copying and pasting the problem solutions (De Jong & Van Joolingen, 1998). Specifically, a stepwise learning support for math problem decomposition may be useful to help students deliberately practice math problem-solving skills (i.e., identifying the problems, systematically planning the solutions, recognizing the resources to solve the problem, analyzing and applying the solutions, and evaluating the solutions) (Lehtinen et al., 2017; Mayer, 2002).

Conceptual knowledge development. It is suggested that learners' failure of solving a problem is attributed to the lack of knowledge that leads to different ways of thinking compared to experts (diSessa, 2014). The proponents (Confrey, 1990; diSessa, 2014) assume that students use preconceptions (from retrievable past experiences) to act on the task at hand and gradually gear toward the expert's thinking with the development of conceptual knowledge. As such, learning support in DGBLE should facilitate conceptual knowledge development by providing frames and cues while giving students sufficient time and freedom to experiment and test their mathematical understanding (Confrey, 1990).

Metacognitive regulation. Noting that the development of conceptual knowledge is situated in the actions in DGBLE, reflection about actions (Weinert, 1987) is therefore a critical metacognition for successful math problem solving. Learning support in DGBLE should address learners' needs during self-regulated mathematical problem solving, including the learning strategies for constantly monitoring and reflecting on their decisions of solution planning, thinking, and executing (Zumbach et al., 2020). Moreover, the design of supports for metacognitive regulation can help learners: "a) relate their new knowledge to their prior understanding, b) mindfully abstract knowledge, and c) understand how their learning and problem-solving strategies might be reapplied" (Hmelo-Silver, 2004, p. 247).

Metacognitive mathematics connections. Making connections is a metacognitive skill that is important for knowledge to be stabilized and internalized (Confrey, 1990). Sensing and discerning mathematical connections between the learning support and the tasks can result in deeper learning in DGBLE (Moyer-Packenham et al., 2019). Learning support can facilitate students' mathematical connections by providing learners with different mathematical representations (e.g., learners use symbolic math representations in the learning support to translate into enactive representations and execute in the game world), prompting mathematical problem decomposition, and making associations between part-whole relations of the mathematical concepts (e.g., García-García & Dolores-Flores, 2021; Moyer-Packenham et al., 2019).

Information selection. Given the complexity of DGBLE, novice learners can feel overwhelmed and focus on irrelevant information (Wouters & Van Oostendorp, 2013). Wouters and van Oostendorp's (2013) meta-analysis found that if a learning support facilitates information selection in DGBLE, it can result in a relatively larger learning effect size ($d = 0.46$, $p < .001$) than learning support for information organization/integration. Learning support for information selection can be presented in the form of cues/hints through open-ended questions and instructional videos (Cai et al., 2022; Moyer-Packenham et al., 2019). Some studies suggested that learning support for information selection can be designed in a packet (Ke, 2016; Reiser, 2004). For example, in a science learning game called *Crystal Island*, the learning support packet consists of information presentation, pedagogical agents, virtual books, and posters. Students can interact with this multifunctional learning-support packet to frame their investigation and exploration (Taub et al., 2018).

Sustained motivation for necessary aversive practices. Learning and solving problems in math DGBLE may be perceived as aversive practices (Lehtinen et al., 2017). Motivation support in problem-based learning aims at maintaining learners' intrinsic motivation towards learning, that is, students are motivated to learn because they have the needs to learn (Hmelo-Silver, 2004). Additionally, motivation support should facilitate learners' appreciation of the target discipline, integrating history in mathematical teaching has been suggested as a way to meeting such a purpose (Pan et al., 2020; Radford et al., 2002). For instance, Bellomo and Wertheimer (2010) reported that students in the history-integrated mathematics classroom appeared to recognize values of mathematics and connect mathematics with real world and various domains.

1.3. Natural language processing (NLP) for educational research

The exponential growth in computing power has brought possibilities for researchers and practitioners to create data-driven personalized learning experiences (Dessì et al., 2019). Using NLP to automatically track learners' performance improvements has been extensively applied in education, such as writing assessment, intelligent tutoring systems (ITS), computer-supported collaborative learning (CSCL), and Massive Open Online Courses (MOOCs) (McNamara et al., 2017). For instance, Moon et al. (2020) validated the feasibility of using NLP for automatic assessment to track learners' cognitive and emotional states to provide adaptive training for learners with autism spectrum disorder in a virtual-reality-supported learning environment. Chary et al.'s (2019) review suggested that learners' verbalization and speech data are indicative of their thinking processes and learning behaviors. Indeed, NLP is increasingly used to enhance teaching and learning by informing on the learners' behaviors and needs (Litman, 2016).

Although the applications of NLP are promising in assessing students' interactions and predicting their game-based task performance, they do not accommodate the mining of learners' utterances that reflects their learning behavior and interaction patterns (Dessì et al., 2019). Machine learning, "a method of artificial intelligence for supervised and unsupervised classification and profiling" (Zawacki-Richter et al., 2019, p.3), can be applied to classify learner interaction patterns through learners' speech utterances. In addition, Chary et al. (2019) emphasized that using NLP is "to translate, or map, words or phrases onto concepts" (p. 78); this entails (1) tokenization, (2) lemmatization, and (3) latent semantic meaning analysis for conceptual understanding. Therefore, in this study, we use NLP combined with cluster analysis to classify how learners use variant learning supports for game-based math problem solving. Due to the nature and complexity of speech data, we do not assume that a single utterance belongs to an exact cluster, instead, it can belong to more than one or no cluster (i.e., soft clustering) (Baker, 2010). Moreover, because speech data mining is an unsupervised machine-learning problem, Gaussian Mixture Model (GMM) is an ideal technique to identify clusters among the mixture components (Constantinopoulos et al., 2006). We provide detailed descriptions of how we use GMM in the Methods section.

2. Methods

2.1. Research design

A convergent mixed-methods research design (Creswell & Plano Clark, 2018) was endorsed in this study to examine students' learning support use in digital game-based math learning. Quantitative and qualitative analyses were conducted simultaneously to validate both results. Quantitative data analysis, via GMM, classified students' learning support use. Qualitative data analysis, based on a multi-cases study design, offered deeper insights regarding classified learning support usage behaviors. The cross validation of quantitative and qualitative data helped provide more accurate and nuanced interpretation of the results.

2.2. Participants

We collected data from 58 college students. Five of them never showed up thus the data included in this study were from 53 students (62% female) at a large public university in the southeastern U.S. Aligning with our exploratory study purposes, we used purposeful sampling (i.e., sampling college students) to mitigate the effect of prior knowledge on math context problem solving, to acquire illuminating information, and to seek the design-based knowledge of how the in-game learning support was used or processed in the math DGBLE. The sample consisted of 60% Caucasian, 24%, Hispanic/Latino, 7% African American, 7% Asian, and 2% Pacific Islander. Sixty percent of the participants self-reported that they had prior gaming experience (e.g., console games, video games, or computer games) whereas the other 40% had no gaming experience. Nine percent was freshman, 23% sophomore, 38% junior, and 30% was in their senior year. Sixty-two percent of the participants were Science, Technology, Engineering, and Mathematics (STEM) major (e.g., Chemistry, Biology, Exercise Physiology, or a pre-medical track) whereas 38% was non-STEM major (e.g., English, Finance, Education, or Teacher Education programs); but all participants were enrolled in at least one education course. All participation sessions were audio- and video-recorded. The recorded speech data were then transcribed verbatim by a professional speech-to-text service and reviewed by two trained human coders.

2.3. Procedures and data sources

Each student in the study participated in a 2-h session consisted of gameplay, think-aloud, and semi-structured interviewing. All sessions were conducted through a videoconferencing tool (i.e., Zoom) with one participant at a time.

The primary data came from multiple sources, including video and audio recordings, think-aloud, participatory observations, semi-structured interviews, and researcher's reflective logs. Trustworthiness was established by triangulating multiple data sources and methodologies, collecting rich and saturated data, and constantly reflecting on the evidence. Think-aloud, semi-structured interviews, and audio recordings were key data sources in this study as we conducted unsupervised machine learning with speech data. For think-aloud, learners were asked to externalize what they were doing and thinking when interacting with the learning supports, which helped researchers recognize learners' "intrinsic cognitive activities" (Chang & Johnson, 2021, p. 141). Vasalou et al. (2017) suggested that think-aloud protocols can be encouraged for DGBLE studies and think-aloud can occur naturally during gameplay of which they called game talk. Such game talk is useful to provide ideas and feedback and understand learner experience for game design (Vasalou et al., 2017). Semi-structured interviews and participatory observations were conducted throughout and at the end of the gameplay session, respectively. Given the exploratory nature of the study, the interview protocol consists of open-ended questions such as *How* *did you feel about the Task Planner and Math Story? How do you think other students can use them? How do you think Task Planner/Math Story may help or not help math learning in the game? What kind of knowledge, if any, is in Task Planner/Math Story?* (see section 2.4 for the detailed descriptions of *Task Planner* and *Math Story*).

2.4. Math DGBLE and the learning supports

We used *E-Rebuild* as the game-based math learning environment. It is a 3D architecture-themed game. The goal for the players in *E-Rebuild* is to reconstruct villages after a natural disaster (Ke et al., 2019). There are four main environment-themed episodes (i.e., island, desert, farm, and school) examined in this study, which are inclusive of around 50 game sub-episodes. In each sub-episode, learners are given math problems or tasks depicting the math topics of ratios and proportional reasoning, geometry, or arithmetic reasoning. There are two types of learning supports in the game—*Task Planner* and *Math Story*. Both support features are unobtrusive and ever-present in the game interface at learners' control (Ke, 2016, Ke et al., 2019), presented via a "Dr. Fox" panel (see Fig. 1).

Task Planner is interactive and aimed to prompt and guide learners with math problem representation and decomposition (see Fig. 2). Task Planner has been designed in alignment with Common Core State Standards (CCSS) of mathematics for middle school students and the core game mechanics, supporting seven main in-game actions underlying game tasks: allocating, building, collecting, covering, folding, placing, and trading.

Math Story is aimed to promote conceptual learning by providing epistemic and historical stories about the evolution of task-relevant mathematical concepts. Math Story also supports access, equity, and culturally relevant mathematics pedagogy (Ladson--Billings, 1995; NCTM, 2017) from diverse perspectives, including historical evolution of math in western, middle eastern, and eastern cultures. There are five major modules of historical stories in Math Story (see Fig. 3(a)).

These learning supports underwent iterative design experiments and refinements during the past three years. The design conjectures and philosophy governing the two learning supports are that they do not provide direct answers or problem solutions for the learners to replicate. Instead, they guide learners' mathematical thinking and scaffold them for purposeful learning (Ke, 2016).

2.5. Data analyses

2.5.1. Gaussian Mixture Model (GMM)

We collected natural speech data to understand how students used learning support in DGBLE. We did not assume one instance to be exclusive in one cluster; instead, the probability that one instance belongs to each cluster was calculated, which is considered as soft clustering. For example, general math concepts or descriptions in one instance can belong to multiple clusters. GMM, a type of unsupervised machine learning approach, assumes that data instances are generated from a fixed number of (multivariate) Gaussian distributions with their own means and covariances (McLachlan & Basford, 1988; McLachlan & Peel, 2000). Compared to K-means clustering, GMM is more versatile in the sense that they can better handle datasets that have non-spherical clusters.

Each data instance in a GMM has a density that can be mathematically represented by a linear combination of Gaussian densities

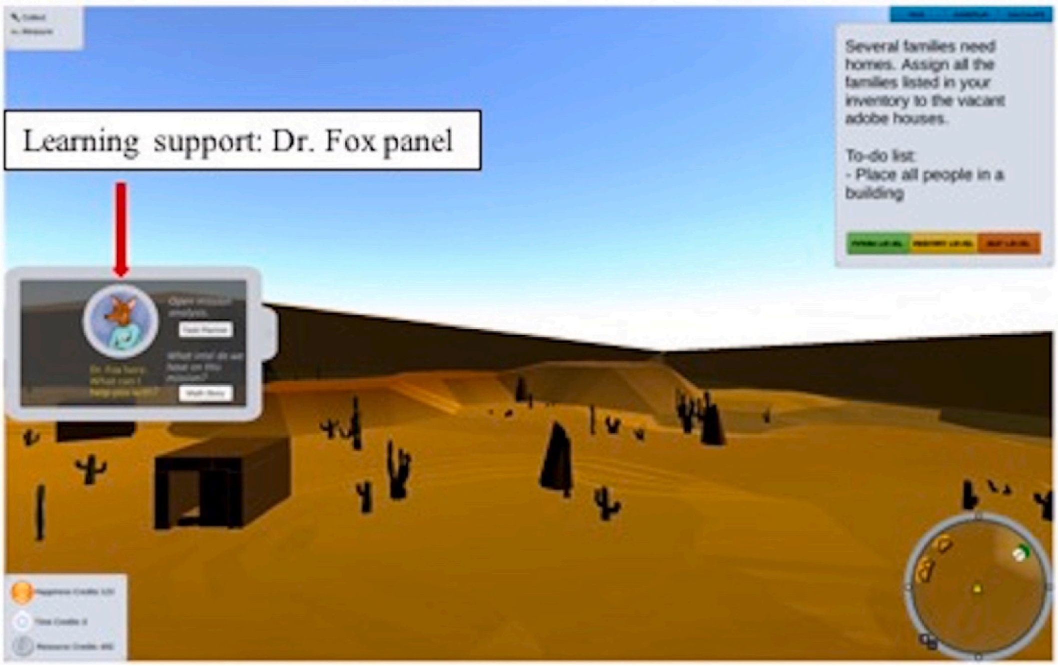

Fig. 1. Learning support features are unobtrusive and ever-present/on learners' demand in Dr. Fox panel.

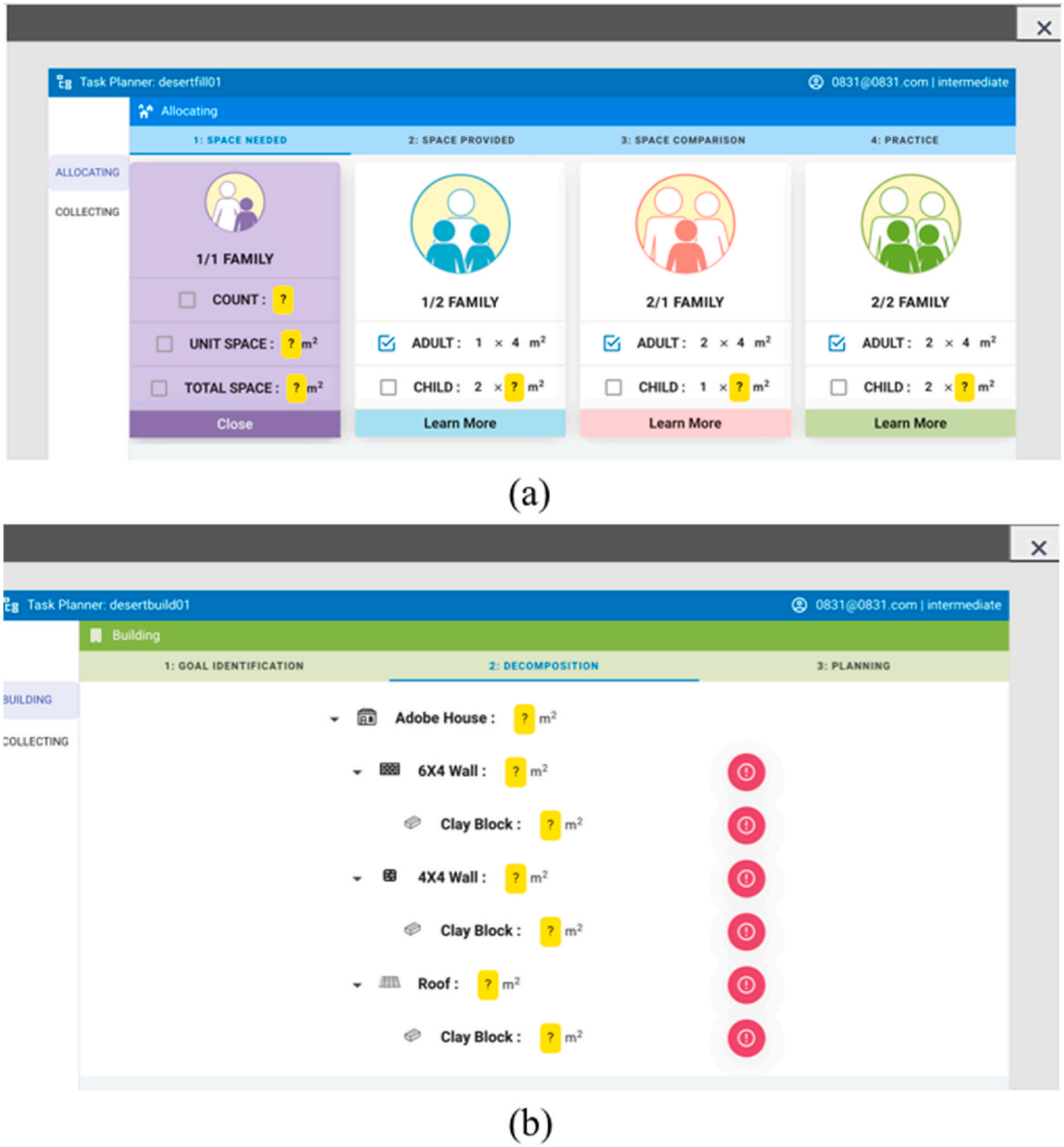

Fig. 2. Sample Task Planner demonstrating (a) problem structure and (b) task decomposition.

(McLachlan & Basford, 1988). The coefficients of the linear combination, called mixing coefficients (Peel & McLachlan, 2000), as well as the means and covariances of the Gaussian components, are then estimated by maximizing the likelihood function based on the expectation-maximization algorithm (Dempster et al., 1977; McLachlan & Krishnan, 1997). The probability that a data instance belongs to a cluster can be computed by evaluating the corresponding posterior density; and the most likely cluster a data instance belongs to, is the one that has the largest probability. We present the formula below, where $p_k(\boldsymbol{x})$ is the probability that a data instance $\boldsymbol{x}$ belongs to cluster k, π_k is the k-th mixing coefficient, and $N(\boldsymbol{x}|\mu_k, \Sigma_k)$ is the probability density for the data instance $\boldsymbol{x}$ corresponding to the k-th Gaussian component with mean μ_k and covariance Σ_k:

$$p_k(\boldsymbol{x}) = \frac{\pi_k N(\boldsymbol{x}|\mu_k, \Sigma_k)}{\sum_{j=1}^{K} \pi_j N(\boldsymbol{x}|\mu_j, \Sigma_j)}$$

2.5.1.1. Data processing for GMM. To apply GMM for the speech dataset, punctuations, numbers and stop words (uninformative words such as "and," "the," or "a") are first removed from each dialogue, followed by a vectorization process that turns the speech into numerical feature vectors. Specifically, the vectorization process involves tokenizing the words of each speech so that they are represented by integers, counting the occurrences of the tokens, and normalizing the counts by the term-frequency-inverse document

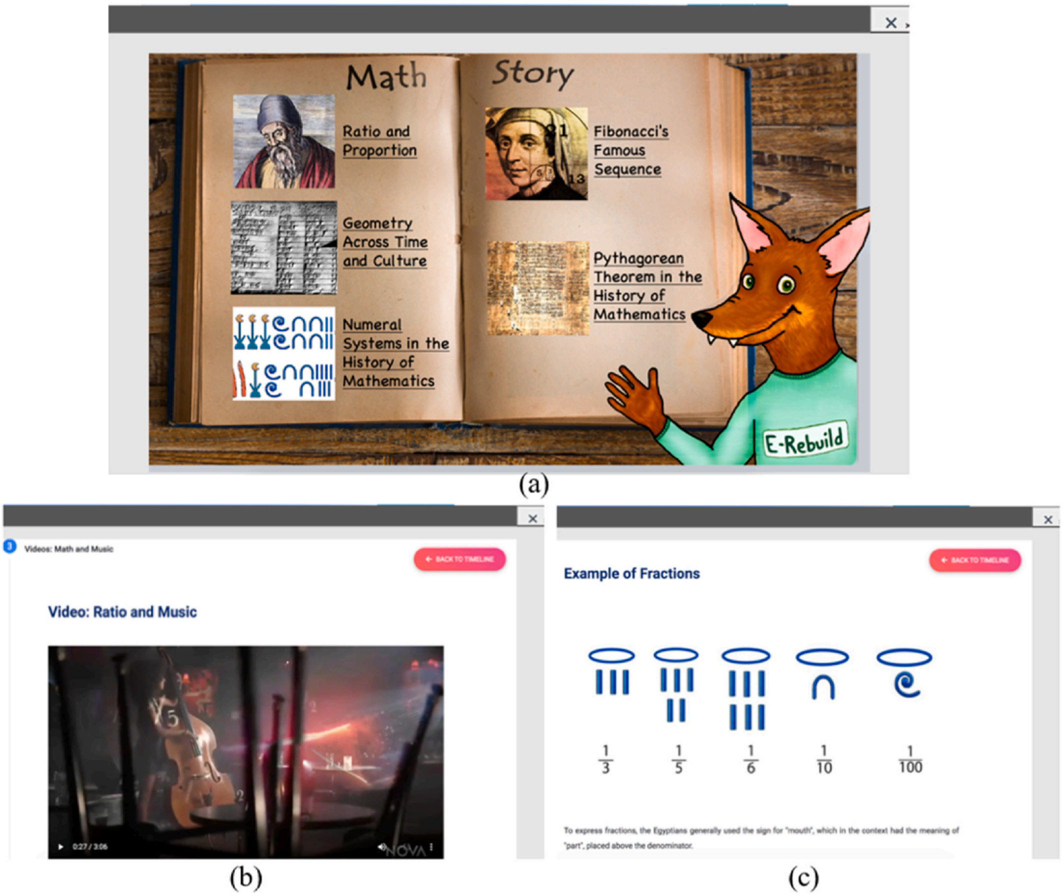

Fig. 3. Math Story overview page (a) and sample Math Story content demonstrating real-life applications of ratio (b) and historical content on fractions (c).

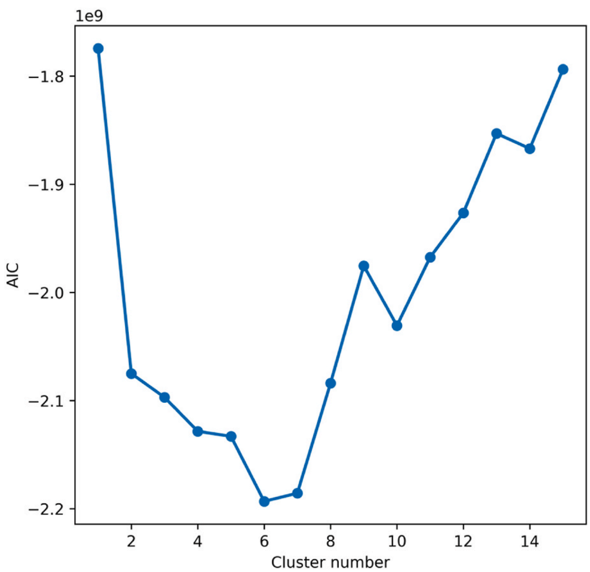

Fig. 4. Akaike information criterion (AIC) scores corresponding to cluster numbers ranging from 1 to 15 for the Gaussian mixture models.

frequency (tf-idf) weighting scheme, with the formula below, where $\mathrm{tf}_{t,d}$ is the number of times a term t occurs in a given document d, N is the total number of documents in the document set, and df_t is the number of documents that contain the term t:

$$\mathrm{tf}-\mathrm{idf}_{t,d} = \mathrm{tf}_{t,d} \times \left(\log\frac{1+N}{1+\mathrm{df}_t}+1\right)$$

The resulting extracted numerical features were then fed to the GMM algorithm for a variety of numbers of clusters. To determine the number of clusters (i.e., Gaussian distributions), we used the Akaike information criterion (AIC); the optimal number of clusters is the one that has the smallest information criterion (see Fig. 4).

2.5.2. Qualitative analysis of multi-cases

Based on a multi-cases study design, where each participant is a case, qualitative data were analyzed through open coding with a purpose of comparing with the clusters extracted and providing *how* and *why* learners used learning support in each cluster. Constant comparison technique (Glaser & Strauss, 1967) was used for qualitative data analysis, depicting the salient characteristics that define the vivid actions and behaviors in each cluster. Consequently, the first cycle of open coding was specifically *in vivo* coding (Miles et al., 2020). The second cycle coding was pattern coding where categories were analyzed and compared to identify surprising insights that were not aligning with our original working design and research conjectures. All names were pseudonym in this paper. For example, C1 is Case (or Participant) One. Finally, the categories were converged and validated with the clusters.

3. Results

3.1. Clustering results of GMM

With GMM, clusters ranging from 1 to 15 were calculated with 37,240 speech data instances. Fig. 4 shows the AICs corresponding to the different numbers of cluster. The AIC score initially dropped as the number of cluster increased, reaching the minimum when there were six clusters. As more clusters were considered, the AIC score started to increase, indicating that the optimal choice of the numbers of cluster is six. Therefore, the results hereafter were based on grouping the data instances into six clusters. The quantity of data instances belonging to each cluster was visualized in Fig. 5. Frequency and percentage of each cluster were shown in Table 2, with associated examples in Table 3. The clusters were named by examining the extracted features in each cluster in accordance with the

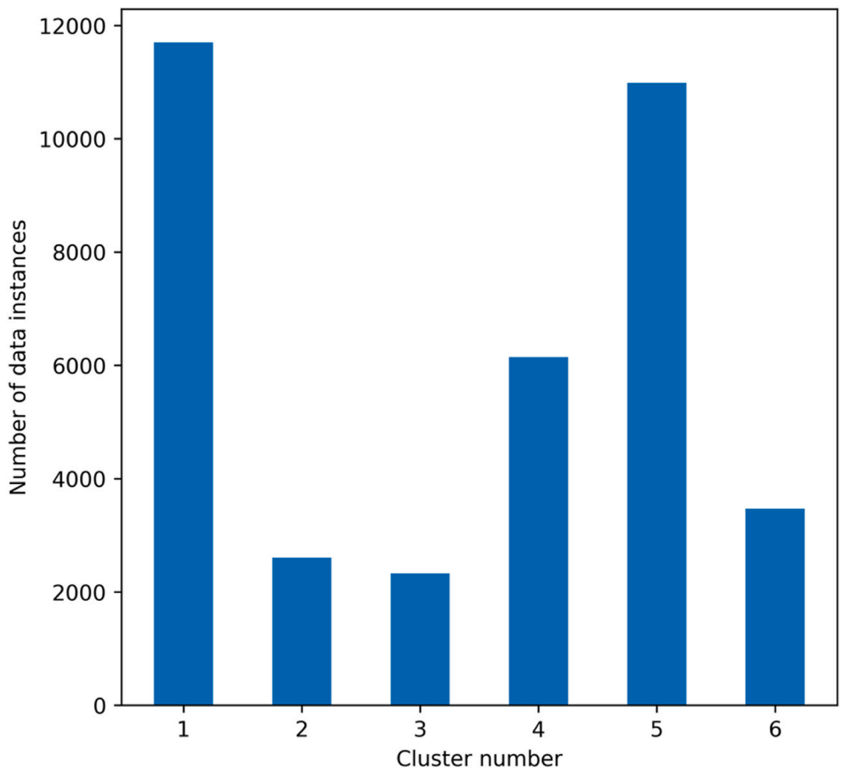

Fig. 5. Distribution of the cluster for learning support use (Cluster 1 = Skills development and application of mathematical problem decomposition, 2 = Conceptual knowledge development, 3 = Metacognitive mathematical connections, 4 = Metacognitive regulation, 5 = Information selection using cognitive aids, 6 = Sustained motivation for necessary aversive practices).

Table 2
Frequency and percentage of each cluster.

Clusters	Percentage	Frequency
1. Skills development and application of mathematical problem decomposition	31.40%	11,702
2. Conceptual knowledge development	7.00%	2607
3. Metacognitive mathematical connections	6.30%	2331
4. Metacognitive regulation	16.50%	6148
5. Information selection using cognitive aids	29.50%	10,985
6. Sustained motivation for necessary aversive practices	9.30%	3467

Table 3
The associated example of each cluster.

Clusters	Examples
1. Skills development and application of mathematical problem decomposition	• It's like the *operation* almost, like *breaks it down*, makes it simpler to what you actually doing … and you build the knowledge from what you already know. • So this one tells you … it *breaks down* the steps you need to solve this problem. • I like this, I like this a lot. This whole page [in the Task Planner] … it's like your *thought processes*, when solving the problem, you should write it down the steps, there are a lot of *steps* that you have to do intuitively, it's laid out here … it's straightforward.
2. Conceptual knowledge development	• I have to do it to *understand* it … I found like … that one, *I realized a pattern.* So I follow the pattern … I use it to fill out the rest … • It kind of explains the areas … like the whole home, and then we take a part of it so … it's kind of like *reverse thinking*, how do you get from the bigger thing to the smaller thing, and like how you build and all that … *it creates like a full understanding … like all aspects*. • Math Story makes me understand Task Planner … *it provides you like the background knowledge for doing math.*
3. Metacognitive mathematical connections	• This video shows *the connection* to the lesson [the in-game math task], because it directly shows how the family was a unit rate like one child is this many. • I think *it relates to* the game in the sense that it's showing you how ratio and math work in everyday life when it's putting tile down in a house and painting a basketball court, things like that. • Like Task Planner *helps you go through the game*, and gives you more analysis about that.
4. Metacognitive regulation	• It gives me more exposures to all kinds of questions, and angles, and it gives me a better understanding. And like more practice in *another way of thinking* because *it comes easier for me to look at like "goal identification" one but the planning one is harder.* • I think it *helps you organize the task before you do it.* • I would honestly use Math Story *after* I've completed the level as a more conceptual learning-the-history-of-it type. *Or even before it*, as informative before playing the game.
5. Information selection using cognitive aids	• I think *it's hinting. It's giving you better of a hint* as to how to work with the ratios. So, instead of giving you the ratios, instead of giving you everything, you just need *to find out* the numbers. • Well, when you first see the question … on the gameplay, if it's something that's … seems difficult for you, you would go to either the task planner or the math story, and *see if there is anything that you can find* to help you solve the answer in order to collect the people. • *I was way more focused on everything* else that was happening and like one of the first problems was like a harder problem, like a harder ratio. But with this [Math Story] like you can kind of build upon it instead of just jumping right into something and then you know what to *focus* on.
6. Sustained motivation for necessary aversive practices	• I feel good. *I feel confident.* If I am given more problems I can do it. • As I really looked at it, you can tell … kind of from the first thing, it's geometry across time and culture, so that … that should have *sparked my interest.* • I found the system of the symbol a little bit weird … but it's a bit *interesting* … cause it's like an exponential symbol system kind of thing … so it's like the Roman numbers, it's *interesting* to see. It's not boring, but like not something very *exciting.* I mean I am *curious* about it, it's something different.

literature reviewed in section 1.2.

3.2. Prominent characteristics of each cluster

To better understand the nuances of how learners used learning support in each cluster, we used qualitative data to supplement and enrich the clustering results of GMM, and to feature salient characteristics defining each cluster.

3.2.1. Skills development and application of mathematical problem decomposition

As indicated in the findings from machine learning clustering, math problem decomposition is the most frequently mentioned theme for learning support use, predominantly caused by using the *Task Planner*. For example, learners particularly suggested that Task Planner provided "step-by-step" problem walkthrough (C16), and then "it'll be easier to break down [math problems] *on your own*" (C46). Furthermore, the participants also noticed that the Task Planner provided mathematical problem representation and allowed a practice for mental abstraction instead of giving the answers to the problem/task (C36, C5, C11). It also enhanced the learning experience as a supplement learning resource to the game world (C36):

It worked me through the mental math aspect of the entire … math problem … and it allows a much faster process of just getting to the final answer with … like this step-by-step process … but it never explicitly told me what the answers were. I think there's like a fine line where you need to have kind of like being tossed out on your own (re: in the game world) to try to figure it out first and then at some point you're allowed to use these resources to try to help guide your processing.

In addition, learners thought that Task Planner helped students to form a way of thinking in math problem solving, as C32 put, "it shows students, how to solve a problem, kind of step-by-step. So the more they practice that then the more they'll be able to formulate their own ideas on how to solve a problem." Another participant found the learning support features "*interactive*" (C19), he explained how he would integrate both features to support math problem solving: "to me, I would rather practice it, step-by-step, then read through [Math Story], and do it, and then possibly try it. I mean there's no doubt that this requires a higher level of understanding."

3.2.2. Conceptual knowledge development

Developing conceptual knowledge to understand how and why math came about is another frequently mentioned theme. For example, C32 suggested that Math Story can build students' mathematical conceptual understanding. In particular, after reviewing Math Story, students can "get a better understanding and make sure … they really know *how to do it* … that way they're not just memorizing the steps for a certain problem, but kind of seeing *how to think behind it* … the thing with math … it's not … showing people how to do problems as much as *its showing people how to think in a way of solving a problem*. So that they can do that when different things are throwing up at them."

Besides, conceptual knowledge development appeared to involve a process of knowledge transformation. In the case of Camryn (C7), she had trouble in solving a ratio-relevant math problem. Specifically, we observed conceptual development and knowledge transformation process based on her interactions with the math problem and learning support features. Her successful math problem solving in the game level was supported by the Task Planner on one hand. On the other, it was the Math Story that supported her *to elucidate why* and understand meaning behind *ratio*. See Fig. 6 below.

3.2.3. Metacognitive mathematical connections

With the help of learning supports, learners were also found to engage in metacognitive learning by connecting mathematical concepts in three ways: 1) connecting the concepts in the *two* learning supports *with each other*, 2) connecting the learning support with the game intrinsically, and 3) connecting the learning support with the real world applications. For example, Case 9 (C9) discussed how the packet of learning supports framed, cued, and "opened her mind" when she was experiencing difficulty solving a ratio problem. After using both Math Story on *Egyptian numeral system* and the Task Planner for the problem, she connected the learning support with the gameplay intrinsically:

"(Analyzing the math problem) I was like, okay, we are *dividing* here [after reading Math Story] … and then it was asking what was a square of a child [in the Task Planner, the space a child occupies, as in x^2]. So I guess it allowed me to kind of (using two index fingers rolling besides her head) … utilize that [the content in learning supports] into the concept … that's how I was like (snapped her fingers) … I am going to go back in the game and check that [the game math task], to see if this is going to open a little bit of my light bulb, and it did. I did notice that *I made that reference connection* … so it kind of opened my mind … provided that little solid

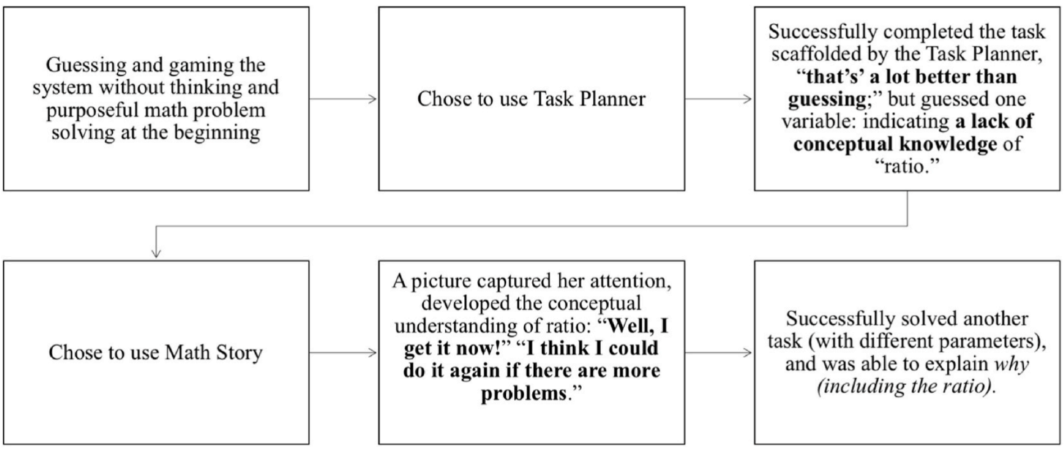

Fig. 6. The conceptual knowledge development of Camryn (C7) visualized by the gameplay behaviors progressed.

foundation."

Sometimes the connections did not occur immediately. For example, C29 did not see the connection until after several rounds of careful reading; he started to make connections between the math task (solving a volume problem) and the Math Story (in Ancient Middle East on Geometry): "Okay, I get it now. It's just like solving for a volume."

When using a Math Story, C19 was cognizant regarding why he used it in a math game-based learning experience: "I have a feeling it's about to come in, and in the next game we're gonna learn about *how math is applied in real life* and then *we're gonna test it out in the game*."

C51, with an experience of "tiling an entire swimming pool before," revealed that it was "*the combination of the game task (a tiling task) and the Task Planner*" that created a connection between problem solving and real-life application. Using Task Planner, she carefully calculated the tiles needed: "because you don't want to buy more materials than you need … if this was a real construction site, you're not trying to waste money. You're going at a price." The metacognitive mathematical connections between the game task, Task Planner, and real-life experience have made the learning personally meaningful.

3.2.4. Metacognitive regulation

Learning supports helped learners to regulate their math problem solving process, as C6 explained: "I think because the whole point was to get you to think what you're trying to do. And then like tells you like … 'identifying your goal' [showing a part of the Task Planner]. When you're given a big task like making a house, this really makes it less overwhelming. And also the interface looks really pretty," she further added that she would prioritize Dr. Fox panel (containing both learning supports) than external help from peers or experts: "Honestly I would do Dr. Fox before I ask someone around me so that I could have *a good plan*." C11 elaborated more on this, "you don't need someone there to help you. And I think it's also like they're definitely going to like once they know that the feature is there. They are sort of like … are *aware of it* and will *be able to use it when they need to*, so it's sort of like they are self-regulating because they're like *aware of their own* … like all the weaknesses, for when they do need that."

Task Planner also acted as a tool to facilitate metacognitive analysis for learners who had no trouble solving the math problem. For example, C30 reported: "I mean I had gotten through the task, and using this [Task Planner] is more like *how to think about it*, I think this was definitely more helpful, especially after finishing … it's like … *rationalize* it." Another observation is that participants metacognitively compared the math problem solving processes in the Task Planner with their own solutions after they solved the game task. C11 weighed in on this: "I think like for these kinds of students for them to see an alternate solution through the Task Planner is

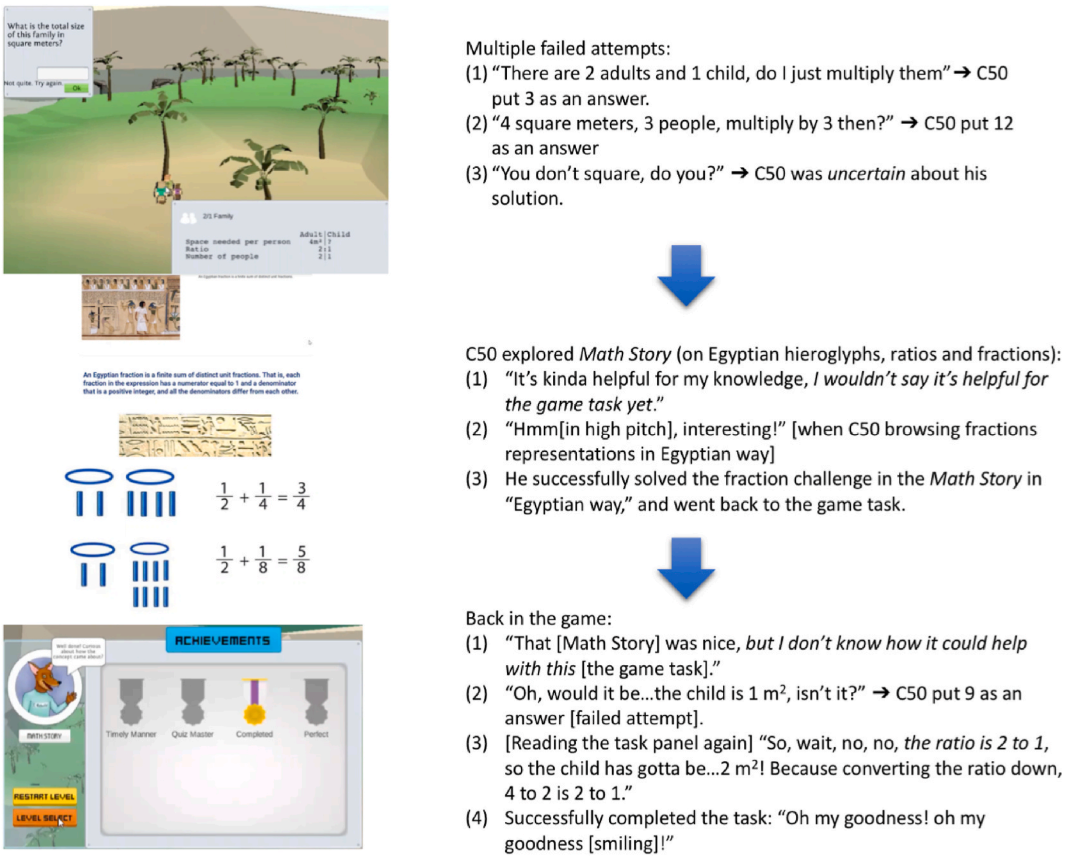

Fig. 7. How Math Story helped C50 select information in the game world, and successfully completed the game task.

still helpful just because every … like … math problem that they encounter is going to be slightly different. So *knowing all the different ways* to solve one can help you solve for, like, future problems."

In contrast to C30 who used learning support *after* experienced the game task for metacognition, other cases suggested using learning support *before* the game task. For example, C46 explained, "you should use it first because, like it tells you what … like … the math that you're going to be doing in the game so like now, I know that I'm going to be doing that the Pythagorean theorem." Similarly, C16 also suggested that "it's giving the game a purpose," and the learning support(s) can be used at the "beginning for knowledge."

3.2.5. Information selection using cognitive aids

The learning support features in DGBLE appeared to help learners focus on task-related problem solving. For example, C50 told that Task Planer helped him concentrate on relevant tasks when feeling confused: "*if I didn't understand the directions, or I was confused I would use Task Planner.*" Moreover, learning supports helped learners focus on the information pertaining key concepts to solve the math problem at hand. As shown in Fig. 7, after several failed attempts on the game task, C50 selected to use Math Story for help. Initially, he wasn't sure how Math Story would help him. But after completing the conceptual math exercises in Math Story, he went back to the game. At first, however, he still could not solve the game task. He then paid attention to the information of "ratio" in the game task panel, and successfully completed the game task (see Fig. 7). Being asked, "*how did you figure out the ratio?*" C50 explained: "*actually hieroglyphs helped me there, thinking back to that [when he went back at the game task].*" The activities in Math Story helped him to select the crucial information in the task panel, and identify the key concept for solving the problem.

Visual aids that represent the math problem in the learning supports were used to assist information selection and processing for math problem solving. Learners were able to focus on the parameters that are important for the task; some learners also combined the visual aids in the Task Planner with out-of-game embodiment to further structure the math problem (see Fig. 8).

3.2.6. Sustained motivation for necessary aversive practices

Both Task Planner and Math Story were able to sustain motivation, though in different ways. For Task Planner, learners suggested that it made the math tasks "less overwhelming; " while using Math Story made game-based math problem-solving personally meaningful for students who were interested in the intersections between the math concepts and the history (C16, C33, C11). For example, C16 elaborated: "I think this like 3D version of the game makes me more motivated and the parts of the lesson [Math Story] that I was most motivated with *stuff that I found to be interesting*, like the symbols and the Roman numerals stuff like that … um … that I was like always curious about. And so those portions were like *motivational*."

The motivational function of Math Story also involved helping learners appreciate math as it is: "It's just *fun* to … like it was just *interesting* to watch it; the Math Story is definitely *interesting* … *it adds to the beauty of math*."

For those who are not interested in history, Math Story provides a purposeful learning experience featuring the behind-the-scene of the math concepts of the task at hand. C11 elucidated, "I think that's also *interesting*. I think it's nice for them to sort of get a greater understanding of where math comes from and like sort of the people who helped shape it."

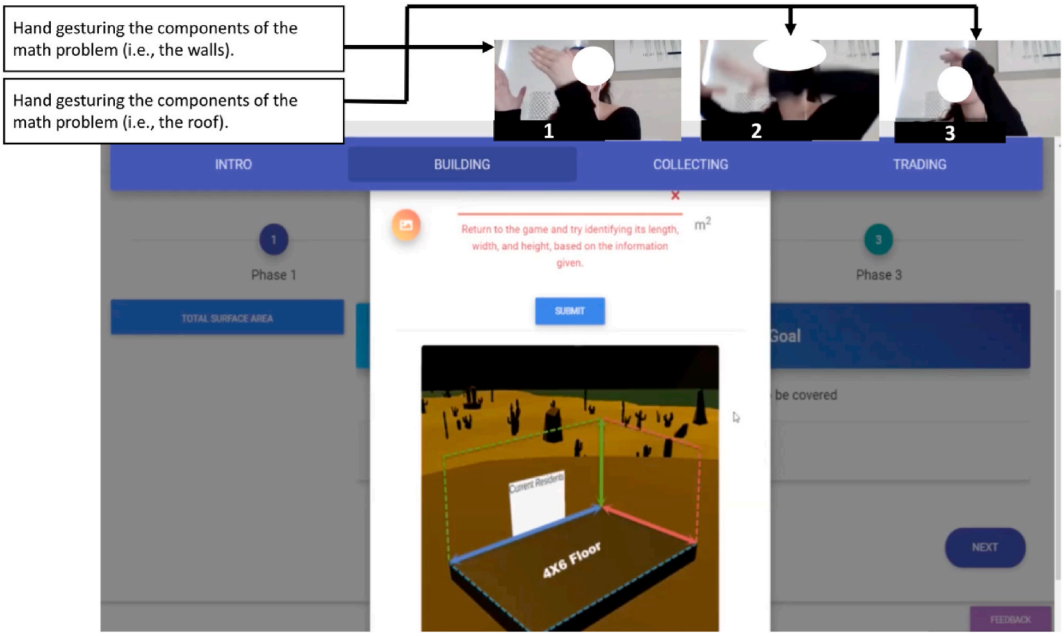

Fig. 8. An example of Task Planner as a cognitive aid visually guiding the learner to focus on the key information and structure of the math problem. The learners used out-of-game embodiment to assist learning.

4. Discussion

This study explored learners' behaviors of using learning supports in a game-based math learning environment. By using GMM, we identified six clusters of learners' learning support use during game-based math problem solving. Multi-cases analyses provided nuanced details of each cluster. With these evidence-based learner interaction patterns for learning support use, we discuss theoretical, design, and methodological implications for learning support research in math DGBL.

First, the two most frequently demonstrated learning-support-use behaviors were "skills development and application of mathematical problem decomposition" and "information selection using cognitive aids." This finding suggests that learners used learning supports to meet their imminent in-game math problem solving cognitive needs (Mayer, 2002; Mayer & Moreno, 2003), which is aligned with the designer's conjecture that in-game learning support should provide interactive prompts and cues for problem decomposition and structuralization. This is important for math problem solving based on deliberate practice (De Jong & Van Joolingen, 1998; Lehtinen et al., 2017). In addition, we found other four learning-support-use behaviors. These learning-support use behaviors indicate that math problem solving should involve systematic conceptual knowledge and skills development, metacognition, connections, and sustainable motivation for aversive practices (e.g., Confrey, 1990; diSessa, 2014; Lehtinen et al., 2017; Puntambekar & Hübscher, 2005). Among them, developing systematic conceptual knowledge and making connections are in line with designer's purpose for making math problem-solving personally meaningful by promoting the realization of *whys* and *hows* behind math concepts, and connecting real-life applications (Radford et al., 2002). Learners also used learning supports for metacognition. They suggested that the use of learning supports prior to the gameplay should be designed to create purposeful learning (Ke, 2016) because the learners will gauge what to expect for the upcoming game-based math task.

The other four learning support-use behaviors were less frequently observed in this study. This finding partially explains why in prior research on these functions of learning support were less studied (Belland et al., 2007). Possible reasons are that most learners did not have the motivation or cognitive resources to engage in game task-irrelevant knowledge and skills development.

The current study extended prior research by providing an empirical investigation of a mixed packet of learning supports (Cai et al., 2022). We found that learners can integrate different learning supports to scaffold their in-game math problem solving. Multi-cases analyses also showed that both types of learning supports in this study coherently scaffold a comprehensive math problem-based learning——solving the math task and explaining the underlying concept. We also found that these learning supports facilitated learners' in-game representational reasoning via visual aids and out-of-game embodiment for math learning. This unique phenomenon extended the embodied learning research (Nathan & Walkington, 2017) by highlighting an active role learning support could play in activating embodied cognition. Finally, our findings suggested that the use of the packet of learning supports can foster the development of metacognitive knowledge and skills. It is in alignment with Vasalou et al.'s (2017) suggestion that designing metacognitive support in DGBLE is critical.

Methodologically, we demonstrated the promising benefits of using a mixed-methods design. We corroborated our natural-language-based machine learning findings with qualitative, multimodal behavioral analysis in the rich context. Unsupervised machine learning technique helps to classify learner interaction patterns. Soft clustering (i.e., Gaussian Mixture Model) is especially useful in our study because it reflects the nature of human discourse via a flexible approach in clustering (Baker, 2010). For example, natural language such as *"how to think"* can be clustered into either conceptual knowledge development or metacognitive regulation. For educational data occurring in naturalistic interactions, soft clustering can be a reasonable and applicable approach. The mixed-methods design has also enabled us to explore the complex design characteristics and nuanced differences among different learning supports (Reiser, 2004).

5. Limitations and future research

Some limitations should be considered when interpreting the results of the current study. First, given the exploratory nature of this inquiry, we purposefully selected advanced learners to study the design of learning support in DGBLE. Future research should recruit school students to further investigate how they interact with a variety of learning supports. Second, we focused on individual online sessions using multiple user-testing techniques. Future studies can focus on classroom or after-school group learning settings. During the study, researchers should encourage and remind students who are not familiar with think-aloud practices (Cotton & Gresty, 2006) to externalize their thinking processes, or engage in game talk (Vasalou et al., 2017). The approach of "prompted think-aloud" (Cotton & Gresty, 2006, p. 50) with audio- and screen-recording is suggested. Finally, future research should consider an experimental design in investigating the effects of learning supports on cognitive, metacognitive, and motivational outcomes in DGBLE, the aspects of learning outcomes associated with the learning-support usage as we discovered in the current study.

6. Conclusion

To facilitate math problem solving in complex environments such as digital learning games, embedding learning support is a critical design task. In this study we sought to understand how learners use learning support to inform evidence-based design and game pedagogy. Our results support existing theories in learning support (Belland et al., 2007; Puntambekar & Hübscher, 2005; Reiser, 2004) while portraying the unique phenomenon of learning-support-use behaviors in digital learning games. We argue for adopting a mixed-methods design, by integrating machine learning with multi-cases study, to investigate learner interaction patterns and design personally meaningful learning experiences.

Credit statement

Chih-Pu Dai:Conceptualization, Methodology, Data Collection and Analysis, Writing – original draft; Writing – review & editing; **Fengfeng Ke**: Software, Resources, Writing – review & editing, Funding acquisition; **Yanjun Pan**: Conceptualization, Methodology, Writing – review & editing; **Yaning Liu**:Methodology, Data Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on reasonable request.

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