

# Strong increase in ultrasound attenuation below $T_c$ in $\text{Sr}_2\text{RuO}_4$ : Possible evidence for domains

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Recent experiments suggest that  $\text{Sr}_2\text{RuO}_4$  has a two-component superconducting order parameter (OP). A two-component OP has multiple degrees of freedom in the superconducting state that can result in low-energy collective modes or the formation of domain walls—a possibility that would explain a number of experimental observations including the smallness of the signature of time reversal symmetry breaking at  $T_c$  and telegraph noise in critical current experiments. We use resonant ultrasound spectroscopy to perform ultrasound attenuation measurements across the superconducting  $T_c$  of  $\text{Sr}_2\text{RuO}_4$ . We find that compressional sound attenuation increases by a factor of 7 immediately below  $T_c$ , in sharp contrast with what is found in both conventional ( $s$ -wave) and high- $T_c$  ( $d$ -wave) superconductors. Our observations are most consistent with the presence of domain walls that separate different configurations of the superconducting OP. The fact that we only observe an increase in sound attenuation for compressional strains, and not for shear strains, suggests an inhomogeneous superconducting state formed of two distinct, accidentally degenerate superconducting OPs that are not related to each other by symmetry. Whatever the mechanism, a factor of 7 increase in sound attenuation is a singular characteristic that must be reconciled with any potential theory of superconductivity in  $\text{Sr}_2\text{RuO}_4$ .

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## I. INTRODUCTION

One firm, if perhaps counterintuitive, prediction of Bardeen, Cooper, and Schrieffer (BCS) theory is the contrasting behavior of the nuclear spin-lattice relaxation rate,  $1/T_1$ , and the ultrasonic attenuation,  $\alpha$  [1]. One might expect both  $1/T_1$  and  $\alpha$  to decrease upon cooling from the normal state to the superconducting (SC) state as both processes involve the scattering of normal quasiparticles. In the SC state, however, Cooper pairing produces quantum coherence between quasiparticles of opposite spin and momentum. These correlations produce “coherence factors” that add constructively for nuclear relaxation and produce a peak—the Hebel-Slichter peak—in  $1/T_1$  immediately below  $T_c$  [2]. In contrast, the coherence factors add destructively for sound attenuation and there is an immediate drop in  $\alpha$  below  $T_c$  [3]. These experiments provided some of the strongest early evidence for the validity of BCS theory [1], and the drop in sound attenuation below  $T_c$  was subsequently confirmed in many elemental superconductors [4–7].

It came as a surprise, then, when peaks in the sound attenuation were discovered below  $T_c$  in two heavy-fermion superconductors:  $\text{UPt}_3$  and  $\text{UBe}_{13}$  [8–10]. Specifically, peaks were observed in the longitudinal sound attenuation—when the sound propagation vector  $\mathbf{q}$  is parallel to the sound po-

larization  $\mathbf{u}$ : ( $\mathbf{q} \parallel \mathbf{u}$ ). Transverse sound attenuation ( $\mathbf{q} \perp \mathbf{u}$ ), on the other hand, showed no peak below  $T_c$  but instead decreased with power-law dependencies on  $T$  that were ultimately understood in terms of the presence of nodes in the SC gap [11]. Various theoretical proposals were put forward to understand the peaks in the longitudinal sound attenuation, including collective modes, domain-wall friction, and coherence factors [12–15], but the particular mechanisms for  $\text{UPt}_3$  and  $\text{UBe}_{13}$  were never pinned down (see Sigrist and Ueda [16] for a review). What is clear, however, is that a peak in sound attenuation below  $T_c$  is not a prediction of BCS theory and surely indicates unconventional superconductivity.

The superconductivity of  $\text{Sr}_2\text{RuO}_4$  has many unconventional aspects, including time reversal symmetry (TRS) breaking [17–19], the presence of nodal quasiparticles [20–22], and a two-component SC order parameter (OP) [23,24]. These observations have led to various recent theoretical proposals for the SC OP [25–31], requiring further experimental inputs to differentiate between them. Not only should the coherence factors differ for  $\text{Sr}_2\text{RuO}_4$  compared to the  $s$ -wave BCS case, but there is the possibility of low-energy collective modes [32,33] and domain-wall motion [34], all of which could be observable in the ultrasonic attenuation when measured at appropriate frequencies.

Prior ultrasonic attenuation measurements on  $\text{Sr}_2\text{RuO}_4$  reported a power-law temperature dependence of the transverse sound attenuation, interpreted as evidence for nodes in the gap [20], but found no other unconventional behavior. The

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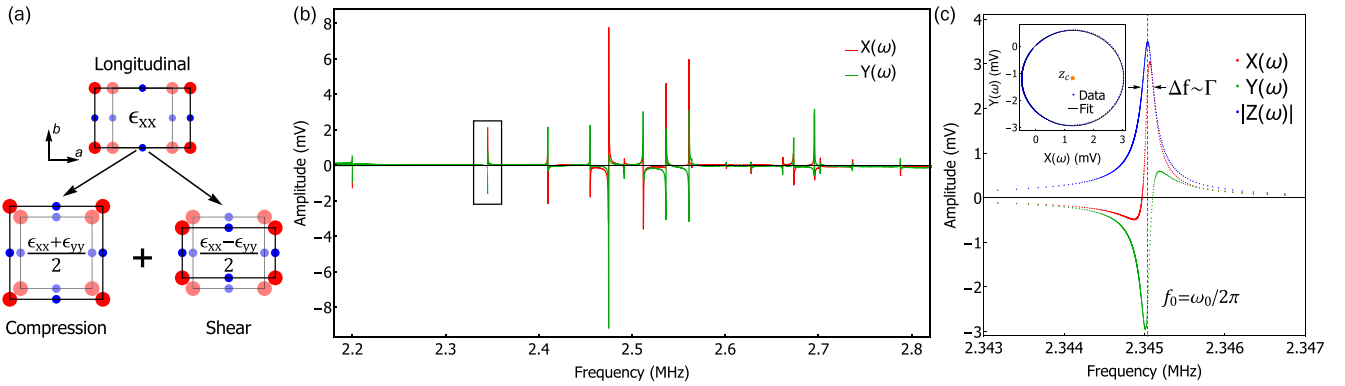


FIG. 1. Measuring ultrasonic attenuation with resonant ultrasound spectroscopy. (a) The  $\text{Sr}_2\text{RuO}_4$  unit cell under a deformation corresponding to the longitudinal strain  $\epsilon_{xx}$ , associated with the elastic constant  $c_{11}$ . This mode is a superposition of pure compression  $\epsilon_{xx} + \epsilon_{yy}$  and pure shear  $\epsilon_{xx} - \epsilon_{yy}$ , associated with the elastic constants  $(c_{11} + c_{12})/2$  and  $(c_{11} - c_{12})/2$ , respectively. (b) Resonant ultrasound spectrum of  $\text{Sr}_2\text{RuO}_4$  between 2.2 and 2.8 MHz.  $X(\omega)$  and  $Y(\omega)$  are the real and the imaginary parts of the response. The boxed resonance is shown in detail in (c). (c) Zoom-in on the resonance near 2.34 MHz. The center of the resonance and the linewidth are indicated. Inset shows the same resonance plotted in a complex plane and fit to a circle— $z_c$  denotes the center of the circle.

ultrasound technique employed in these previous measurements, pulse-echo ultrasound, can measure a pure shear response in the transverse configuration but measures a combination of compression and shear response in the longitudinal configuration in a tetragonal crystal like  $\text{Sr}_2\text{RuO}_4$  [35]. In particular, the L100 mode measures the elastic constant  $c_{11}$ , which is a mixture of pure compression,  $(c_{11} + c_{12})/2$ , and pure shear,  $(c_{11} - c_{12})/2$  [see Fig. 1(a)]. Shear and compression strains couple to physical processes in fundamentally different ways and thus effects that couple exclusively to compressional sound may have been missed in previous measurements. In addition, pulse echo operates at frequencies of order 100 MHz and higher, which may be too high in frequency—or too short in wavelength—to observe certain dynamical processes associated with large-scale correlations in the system. Thus, attenuation measurements that can separate the compression and shear responses, as well as measure at lower frequencies, may reveal features of the superconducting state in  $\text{Sr}_2\text{RuO}_4$  not observed in previous experiments.

## II. EXPERIMENT

We have measured the ultrasound attenuation of  $\text{Sr}_2\text{RuO}_4$  across  $T_c$  using resonant ultrasound spectroscopy (RUS). RUS allows us to obtain the attenuation in all the independent symmetry channels in a single experiment (i.e., for all five symmetry components of strain in  $\text{Sr}_2\text{RuO}_4$ ), and operates at frequencies of order 1 MHz. The sample space in our RUS apparatus requires exchange gas in order to thermalize the sample, preventing us from measuring below 1.25 K (see Ghosh *et al.* [23] for details of our custom-built, low-temperature RUS apparatus and the Supplemental Material [36] for details on the lock-in technique [37] used to measure the spectra.

The high-quality  $\text{Sr}_2\text{RuO}_4$  crystal used in this experiment was grown by the floating zone method—more details about the sample growth can be found in Bobowski *et al.* [38]. A single crystal was precision-cut along the  $[110]$ ,  $[\bar{1}\bar{1}0]$ , and  $[001]$  directions and polished to the dimensions  $1.50 \text{ mm} \times$

$1.60 \text{ mm} \times 1.44 \text{ mm}$ , with  $1.44 \text{ mm}$  along the tetragonal  $c$  axis. The sample quality was characterized by heat capacity and AC susceptibility measurements, as reported in Ghosh *et al.* [23]. The SC  $T_c$  measured by these techniques—approximately 1.43 K—agrees well with the  $T_c$  seen in our RUS experiment, indicating that the sample underwent uniform cooling during the experiment.

RUS measures the mechanical resonances of a three-dimensional solid. The frequencies of these resonances depend on the elastic moduli, density, and geometry of the sample, while the widths of these resonances are determined by the ultrasonic attenuation [39,40]. Because each resonance mode is a superposition of multiple kinds of strain, the attenuation in all strain channels can be extracted by measuring a sufficient number of resonances—typically two or three times the number of unique strains (of which there are five for  $\text{Sr}_2\text{RuO}_4$ ).

A segment of a typical RUS spectrum from our experiment is shown in Fig. 1(b) (see the Supplemental Material [36] for the full spectrum). Each resonance can be modeled as the response  $Z(\omega)$  of a damped harmonic oscillator driven at frequency  $\omega$  [see Fig. 1(c)],

$$Z(\omega) = X(\omega) + iY(\omega) = Ae^{i\phi}/[(\omega - \omega_0) + i\Gamma/2], \quad (1)$$

where  $X$  and  $Y$  are the real and imaginary parts of the response, and  $A$ ,  $\Gamma$ , and  $\phi$  are the amplitude, linewidth, and phase, respectively. The real and imaginary parts of the response form a circle in the complex plane. The response is measured at a set of frequencies that space the data points evenly around this circle: this is the most efficient way to precisely determine the resonant frequency  $\omega_0$  and the linewidth  $\Gamma$  in a finite time (see Shekhter *et al.* [41] for details of the fitting procedure). We plot the temperature dependence of the linewidth for all our resonances measured through  $T_c$  in the Supplemental Material [36]. For comparison, the attenuation  $\alpha$  measured in conventional pulse-echo ultrasound is related to the resonance linewidth via  $\alpha = \Gamma/v$ , where  $v$  is the sound velocity (see the Supplemental Material [36] for a simple derivation).

### III. RESULTS

When the sound wavelength,  $\lambda = \frac{2\pi}{q}$ , is much longer than the electronic mean free path  $l$ , i.e., when  $ql \ll 1$ , the electron-phonon system is said to be in the “hydrodynamic” limit [42] (this is different than the hydrodynamic limit of electron transport). Given that the best  $\text{Sr}_2\text{RuO}_4$  has a mean free path that is at most of the order of a couple of microns, and that our experimental wavelengths are of the order of 1 mm, we are well within the hydrodynamic limit. In this regime, we can express the linewidth  $\Gamma$  of a resonance  $\omega_0$  as

$$\frac{\Gamma}{\omega_0^2} = \sum_j \alpha_j \frac{\eta_j}{c_j}, \quad (2)$$

where  $c_j$  and  $\eta_j$  are the independent components of the elastic and viscosity tensors, respectively [see the Supplemental Material [36] for a derivation of Eq. (2)]. Note that  $c_j$  and  $\eta_j$  can also be understood as the real and imaginary parts, respectively, of the full, dynamic elastic tensor. The coefficients  $\alpha_j$  define the composition of a resonance, with  $\alpha_j = \partial(\ln \omega_0^2)/\partial(\ln c_j)$  and  $\sum_j \alpha_j = 1$  [39].

We measured the linewidths of 18 resonances and resolved them into the independent components of the viscosity tensor. Because viscosity depends only weakly on frequency in a Fermi liquid, and because  $\text{Sr}_2\text{RuO}_4$  is a good Fermi liquid at low temperatures (just above  $T_c$ ) [43], we can directly compare our measured viscosities to those made at much higher frequencies by pulse-echo ultrasound. These comparisons are made below, with further discussion in the Supplemental Material [36]. The tetragonal symmetry of  $\text{Sr}_2\text{RuO}_4$  dictates that there are only six independent viscosity components, arising from the five irreducible representations of strain in  $D_{4h}$  plus one component arising from coupling between the two distinct compression strains [23]. The six symmetry-resolved components of viscosity in  $\text{Sr}_2\text{RuO}_4$  are plotted in Fig. 2.

The shear viscosity  $(\eta_{11} - \eta_{12})/2$  decreases below  $T_c$  in a manner similar to what is observed in conventional superconductors [3,4]. We find that  $(\eta_{11} - \eta_{12})/2$  is much larger than the other two shear viscosities, which is consistent with previous pulse-echo ultrasound experiments [20,44]. On converting attenuation to viscosity, we find very good agreement between the resonant ultrasound and pulse-echo measurements of  $(\eta_{11} - \eta_{12})/2$ . This is nontrivial because the bare sound attenuation—before conversion to viscosity—is two orders of magnitude larger in the pulse-echo experiments than in the RUS experiments. The much larger magnitude of  $(\eta_{11} - \eta_{12})/2$ , in comparison to  $\eta_{66}$ , may be due to the fact that the  $\epsilon_{xx} - \epsilon_{yy}$  strain is associated with pushing the  $\gamma$  Fermi surface pocket toward the Van Hove singularity [45]. The small values of  $\eta_{44}$  and  $\eta_{66}$  are comparable to the experimental background and any changes at  $T_c$  are too small to resolve at these low frequencies (see the Supplemental Material [36] for a discussion of the intrinsic and extrinsic contributions [46] to this background).

In contrast to the rather conventional shear viscosities, the three compressional viscosities each exhibit a strong increase below  $T_c$ . For in-plane compression—the strain that should couple strongest to the largely two-dimensional superconductivity of  $\text{Sr}_2\text{RuO}_4$ —this increase is more than a factor of 7. The viscosity slowly decreases as the temperature is lowered

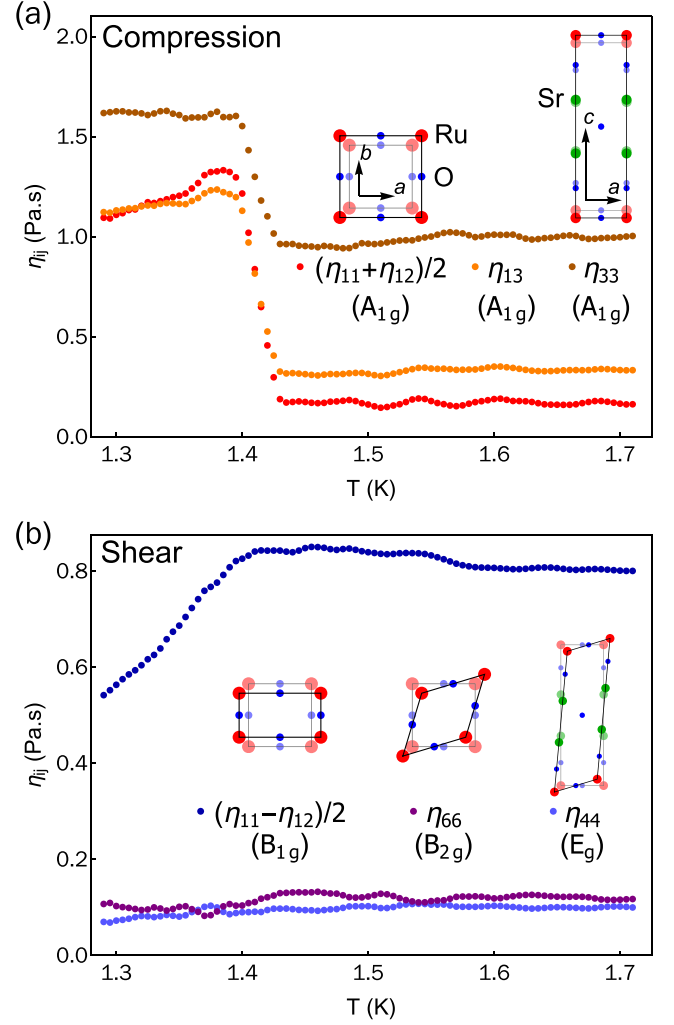


FIG. 2. Symmetry-resolved sound viscosity in  $\text{Sr}_2\text{RuO}_4$ . (a) Compressional and (b) shear viscosities through  $T_c$ . The irreducible strain corresponding to each viscosity is shown— $\eta_{13}$  arises due to coupling between the two  $A_{1g}$  strains. The compressional viscosities increase immediately below  $T_c$ , whereas no such features are observed in the shear viscosities.

after peaking just below  $T_c$ . The large increase in compression viscosity below  $T_c$  was not observed in previous longitudinal sound attenuation measurements made by pulse-echo ultrasound [20,44]. There are two likely explanations for this. First, the L100 mode measured in pulse-echo experiments measures  $\eta_{11}$ , which should be thought of as a mixture of the shear viscosity  $(\eta_{11} - \eta_{12})/2$  and the compression viscosity  $(\eta_{11} + \eta_{12})/2$  [Fig. 1(a)]. Because  $(\eta_{11} - \eta_{12})/2$  is an order of magnitude larger than  $(\eta_{11} + \eta_{12})/2$ , the shear viscosity completely dominates the signal (see the Supplemental Material [36] for a comparison of  $\eta_{11}$  from RUS and pulse-echo experiments). Second, the pulse-echo experiments are conducted at frequencies that are two orders of magnitude higher than in the RUS experiments. The difference in timescales between the ultrasound and the dynamics system is critical because sound attenuation is intrinsically a dynamical quantity, thus the two techniques operating at different frequencies can observe

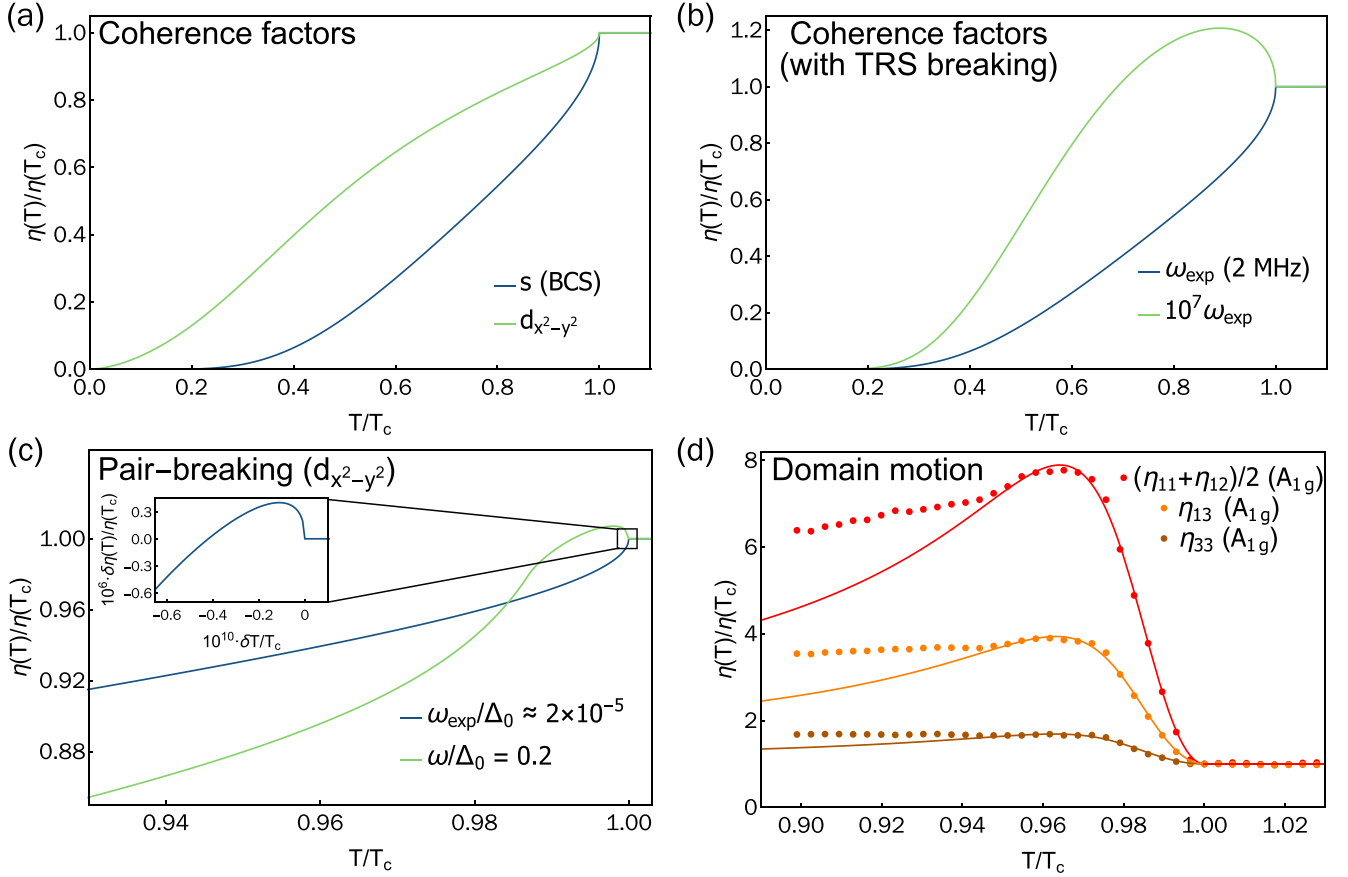


FIG. 3. Comparison of different mechanisms for sound attenuation in the superconducting state. (a) Normalized viscosity  $[\eta(T)/\eta(T_c)]$  for an isotropic  $s$ -wave gap and a  $d_{x^2-y^2}$  gap, calculated within the BCS framework. (b)  $\eta(T)/\eta(T_c)$  for a time reversal symmetry breaking gap below  $T_c$ . A peak is seen at high enough frequencies (approximately terahertz) but not at our experimental frequencies (approximately megahertz). (c) Attenuation peak at different frequencies due to pair-breaking effects in a  $d_{x^2-y^2}$  gap. The inset shows the plot at our experimental frequency in detail—a tiny peak is seen about 0.01 nK below  $T_c$  [ $\delta\eta(T) = \eta(T) - \eta(T_c)$  and  $\delta T = T - T_c$ ]. (d) Normalized viscosity in the  $A_{1g}$  channels of  $\text{Sr}_2\text{RuO}_4$  through  $T_c$ , fit to the viscosity expected from domain-wall motion below  $T_c$ .

different phenomena—we will return to this idea later on in the discussion.

#### IV. ANALYSIS

We consider three possible mechanisms that could give rise to such an increase in sound attenuation below  $T_c$ . First, we calculate sound attenuation within a BCS-like framework that accounts for the differences in coherence factors that occur for various unconventional SC OPs. We find that a peak can indeed arise under certain circumstances but not under our experimental conditions. Second, we consider phonon-induced Cooper pair breaking in the SC state. This mechanism does lead to a sound attenuation peak just below  $T_c$  but it is inaccessibly narrow in our experiment. Finally, we show that a simple model of sound attenuation due to the formation of SC domains best matches the experimental data.

First, we calculate the change in sound attenuation due to coherent quasiparticle scattering in the SC state. The coherent scattering of Bogoliubov quasiparticles off of phonons results in suppressed sound attenuation below  $T_c$  in an  $s$ -wave superconductor [1]. In general, however, the coherence factors depend on the structure of the superconducting OP, motivating

the idea that an unconventional superconducting OP might produce a peak in the sound attenuation below  $T_c$ . We find that a  $d_{x^2-y^2}$  gap cannot not produce a peak in sound attenuation below  $T_c$  [Fig. 3(a); see the Supplemental Material [36] for details of the calculation]. For a TRS breaking gap, such as  $p_x + ip_y$  or  $d_{xz} + id_{yz}$ , a Hebel-Slichter-like peak can appear below  $T_c$  if sufficiently large-angle scattering is allowed, but this scattering is only accessible at frequencies that are orders of magnitude higher than what is used in our experiment [Fig. 3(b)]. Hence we rule out coherent scattering as the mechanism of increased attenuation below  $T_c$ .

Next we consider phonon-induced Cooper pair breaking as a mechanism for increased attenuation, similar to what is found below  $T_c$  in superfluid  $^3\text{He-B}$  [47]. Pair breaking in BCS superconductors requires a minimum energy of  $2\Delta_0$ , where  $\Delta_0$  is the gap magnitude. While superconducting gaps are typically much larger than ultrasound frequencies—the maximum gap magnitude in  $\text{Sr}_2\text{RuO}_4$ , for example, is  $2\Delta \sim 0.65$  meV or approximately 1 THz [48]—the gap does go to zero at  $T_c$  and at the nodes of certain OPs. Our calculations show that ultrasound frequencies of order  $\sim 10$  GHz are required to produce an experimentally discernible peak with a  $d_{x^2-y^2}$  gap [Fig. 3(c)]. At our experimental frequencies, the



peak is only visible within 0.01 nK of  $T_c$ . For a fully gapped superconductor, like the TRS breaking state  $p_x + ip_y$ , the peak is suppressed even further. This clearly rules out pair breaking as the origin of the increased sound attenuation.

Finally, we consider the formation of domain walls in the superconducting state. Domain walls separate regions of degenerate OP configurations, such as  $p_x + ip_y$  and  $p_x - ip_y$ , and can extract energy from sound waves by oscillating about their equilibrium positions [14]. Sigrist and Ueda [16] derive an expression for how domain-wall motion leads to enhanced sound attenuation, which we write in the form

$$\eta(\omega, T) = A \frac{\rho_s^2}{\omega^2 + \omega_{DW}^2}, \quad (3)$$

where  $\rho_s$  is the superfluid density (proportional to the square of the superconducting gap),  $\omega$  is the angular frequency of the sound wave,  $\omega_{DW}$  is the lowest vibrational frequency of the domain wall, and all microscopic parameters have been subsumed into the coefficient  $A$  (see the Supplemental Material [36] for details of the parameters included in  $A$ ). Near  $T_c$ ,  $\rho_s$  and  $\omega_{DW}$  can be expanded within a Ginzburg-Landau (GL) formalism as  $\rho_s \propto |T - T_c|$  and  $\omega_{DW} \propto |T - T_c|^{3/2}$ . This gives an explicit temperature dependence to Eq. (3):

$$\eta(\omega, T) = A \frac{|T/T_c - 1|^2}{\omega^2 + \omega_1^2 |T/T_c - 1|^3}, \quad (4)$$

where  $\omega_1$  is  $\omega_{DW}$  in the limit  $T \rightarrow 0$ .

We fit all three measured viscosities to Eq. (4) and extract  $\omega_1 = 500 \pm 25$  MHz [Fig. 3(d)]. As the temperature approaches  $T_c$  from below, the domain-wall frequency decreases to zero, producing a peak in the attenuation when the ultrasound frequency is approximately equal to the domain-wall frequency. Note that  $\eta(\omega)$  becomes frequency dependent in the presence of domain walls, in contrast to the frequency-independent viscosity of the Fermi liquid state above  $T_c$ . We use the average experimental frequency  $\omega = 2.5$  MHz to extract  $\omega_1$ . Although our analysis uses resonance frequencies spanning 1.7–3.2 MHz, the position of the peak in  $\eta$  changes by only about 14 mK over this frequency range, justifying our use of a single frequency for the fit (see the Supplemental Material [36] for plots at different frequencies).

The fit of Eq. (4) deviates from the data for  $T/T_c \lesssim 0.95$ . This may be because of additional temperature dependencies, such as the temperature dependence of the domain-wall frequency, that are not captured by the GL expansion, which is only valid near  $T_c$  [16]. Nevertheless, Eq. (4) captures the correct shape of the rapid increase in attenuation below  $T_c$  in all three compression channels, using the same value of  $\omega_1$  for all three fits. The extracted frequency scale of  $\omega_1 \approx 500$  MHz is also reasonable: studies of sound attenuation in nickel at MHz frequencies show similar magnitudes of increase in the magnetically ordered state when domains are present [49]. We note that the results of Josephson interferometry measurements have previously been interpreted as evidence for SC domains in  $\text{Sr}_2\text{RuO}_4$  [19].

Previous pulse-echo ultrasound measurements, performed at 83 MHz, did not identify any peak in  $\eta_{11}$  below  $T_c$  [20]. As we show in the Supplemental Material [36], the peak produced by Eq. (3) becomes very broad at 83 MHz. Coupled with the

fact that the temperature dependence of  $\eta_{11}$  is dominated by the strong temperature dependence of  $(\eta_{11} - \eta_{12})/2$ , it would be impossible to identify a peak below  $T_c$  at typical pulse-echo frequencies.

## V. DISCUSSION

The factor of 7 increase we find in the in-plane compressional viscosity is without precedent in a superconductor. For comparison, longitudinal attenuation increases by 50% below  $T_c$  in  $\text{UPt}_3$  [10], and by a bit more than a factor of 2 in  $\text{UBe}_{13}$  [9]. There is also a qualitative difference between the increase in  $\text{Sr}_2\text{RuO}_4$  and the increase seen in the heavy-fermion superconductors: the attenuation peaks sharply below  $T_c$  in both  $\text{UPt}_3$  and  $\text{UBe}_{13}$ , with a peak width of approximately 10% of  $T_c$ . The compressional attenuation in  $\text{Sr}_2\text{RuO}_4$ , by contrast, decreases by only about 10% over the same relative temperature range. This suggests that something highly unconventional occurs in the SC state of  $\text{Sr}_2\text{RuO}_4$ , leading to a large increase in sound attenuation that is not confined to temperatures near  $T_c$ . The mechanism we find most consistent with the data is domain-wall motion.

Assuming that we have established the likely origin of the increase in sound attenuation, we consider its implications for the superconductivity of  $\text{Sr}_2\text{RuO}_4$ . The formation of domains requires a two-component OP, either symmetry enforced or accidental, reaffirming the conclusions of recent ultrasound studies of the elastic moduli and the sound velocity [23,24].

We can learn more about which particular OPs are consistent with our experiment by considering which symmetry channels show an increase in attenuation. Domains attenuate ultrasound when the application of strain raises or lowers the condensation energy of one domain in comparison to a neighboring domain. A simple example is the “nematic” superconducting state proposed by Benhabib *et al.* [24], which is a  $d$ -wave OP of the  $E_g$  representation, transforming as  $\{d_{xz}, d_{yz}\}$ . Under  $(\epsilon_{xx} - \epsilon_{yy})$  strain, domains of the  $d_{xz}$  configuration will be favored over the  $d_{yz}$  configuration (depending on the sign of the strain). This will cause some domains to grow and others to shrink, attenuating sound through the mechanism proposed by Sigrist and Ueda [16]. We find no increase in  $(\eta_{11} - \eta_{12})/2$  below  $T_c$ , suggesting that a  $\{d_{xz}, d_{yz}\}$  OP cannot explain the increase in compressional sound attenuation.

More generally, the lack of increase in attenuation in any of the shear channels implies that the SC state of  $\text{Sr}_2\text{RuO}_4$  does not break rotational symmetry. Domains that are related to each other by time reversal symmetry can also be ruled out: there is no strain that can lift the degeneracy between, for example, a  $p_x + ip_y$  domain and a  $p_x - ip_y$  domain. The observed increase in sound attenuation under compressional strain is therefore quite unusual: as Sigrist and Ueda [16] point out, compressional strains can never lift the degeneracy between domains that are related by *any* symmetry, since compressional strains do not break the point group symmetry of the lattice. Instead, attenuation in the compressional channel requires domains that couple differently to compressional strain, which in turn requires domains that are accidentally degenerate. Examples that are consistent with both NMR [50] and ultrasound [23,24] include  $\{d_{x^2-y^2}, g_{xy}(x^2-y^2)\}$  [30,31,51]

and  $\{s, d_{xy}\}$  [52]. Then, for example, domains of  $d_{x^2-y^2}$  will couple differently to compressional strain than domains of  $g_{xy(x^2-y^2)}$ , leading to the growth of one domain type and an increase in compressional sound attenuation below  $T_c$ . Shear strain, meanwhile, does not change the condensation energy of any single-component OP (e.g.,  $s$ ,  $d_{xy}$ ,  $d_{x^2-y^2}$ , or  $g_{xy(x^2-y^2)}$ ) to first order in strain, which means that the lack of increase in shear attenuation below  $T_c$  is also consistent with an accidentally degenerate OP. This is also consistent with the lack of a cusp in  $T_c$  under applied shear strain [53,54].

Recent theoretical work [34] has shown that domain walls between  $d_{x^2-y^2}$  and  $g_{xy(x^2-y^2)}$  OPs may provide an explanation of the observation of half-quantum vortices in  $\text{Sr}_2\text{RuO}_4$  without a spin-triplet OP [55]—a result that is otherwise inconsistent with the singlet pairing suggested by NMR [50]. Willa *et al.* [31], followed by Yuan *et al.* [34], have shown that domains between such states stabilize a TRS breaking  $d_{x^2-y^2} \pm ig_{xy(x^2-y^2)}$  state near the domain wall. This would naturally explain why probes of TRS breaking, such as the Kerr effect and  $\mu\text{SR}$  [18,56], see such a small effect at  $T_c$  in  $\text{Sr}_2\text{RuO}_4$ .

One significant challenge for the two-component OP scenario is that, whether accidentally degenerate or not, a two-component OP should generically produce two superconducting  $T_c$ 's. The lack of a heat capacity signature from an expected second transition under uniaxial strain [57] can only be explained if the second, TRS-breaking transition is particularly weak—a result that might be consistent with the

TRS-breaking state appearing only along domain walls. Finally, it is worth noting that there are other mechanisms of ultrasonic attenuation that we have not explored here, including collective modes and gapless excitations such as edge currents that might appear along domain walls even if the domains are related by symmetry. Future ultrasound experiments under applied static strain and magnetic fields are warranted as certain types of domain walls can couple to these fields, thereby affecting the sound attenuation through  $T_c$ .

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