Model Discrimination of Switched Nonlinear Systems With Temporal Logic-Constrained Switching

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***Abstract*—This letter considers the model discrimination problem for switched nonlinear systems, where the switching sequence is constrained by metric/signal temporal logic specifications. Specifically, we propose an optimization-based algorithm for analyzing the detectability of the models from noisy, finite data as well as a model discrimination algorithm for nonlinear parameter-varying systems to rule out models that are inconsistent with observations at run time, by checking the feasibility of corresponding mixed-integer linear programs. Moreover, we apply the algorithms to nonlinear systems subject to (*m*,*k*)-firm data losses and explicitly provide the integer constraints corresponding to the (*m*,*k*)-firm constraints for lossy/missing data. Finally, we demonstrate the effectiveness of our approaches using several illustrative examples on fault detection, swarm consensus and intent identification problems.**

***Index Terms*—Switched systems, model validation, fault detection.**

I. Introduction

CYBER-PHYSICAL systems (CPS), which combine embedded computers and communication networks with physical processes, are often complex and involve logical/discrete elements, e.g., changing or switching between modes to satisfy some task and/or safety specifications, in addition to continuous states and dynamics. Moreover, large-scale deployments of sensors with shared communication channels may be plagued by missing data, which can be viewed as another type of switching constraints involving the measurements. At the same time, anomaly/fault detection and model identification are vital for safe and high-performance operation of these CPS so that potential faults and changes in the system behaviors can be quickly detected and dealt with, if needed.

Literature Review. The model discrimination problem is to identify/separate models given a finite sequence of measured input-output data and knowledge of the system dynamics [1], [2]. This can be considered using a modeling invalidation framework, which aims to check whether the observed input-output data is compatible/consistent with one member in the valid model set [3]. The model invalidation problem has recently been explored for various types of systems, i.e., linear parameter varying systems [4], [5], nonlinear systems [6], uncertain systems [7], switched auto-regressive models [8] and switched affine systems [2], [9], [10].

Moreover, to analyze the detectability of the models, the notion of T-distinguishability (or T-detectability) is introduced in [2], [10] to find upper bounds on the required time horizon T to distinguish one model from the other, if such a T exists. The concept of T-distinguishability is closely related to the notion of state/mode distinguishability of switched linear systems [11], [12], finite-state systems [13] and switched nonlinear systems [14]. Furthermore, discrete-time system properties are given as linear temporal logic (LTL) specifications and used for anomaly and fault detection in [15], while a monitor finite state machine is constructed for LTL formulas and further encoded as mixed-integer constraints in the T-distinguishability problem in [16].

Additionally, in the context of control and estimation with lossy measurements, an (m,k)-firm policy/specification requires that at least m out of any k sequential data or measurements are successfully delivered/transmitted [17], and the overall system behavior governed by the (m, k)-firmness property can be represented as a constrained switched linear system [18]. The effect of the (m,k)-firm specification on control performance and stability is analyzed in [19] and strategies to determine the parameters k and m that conserve the stability of the system is researched in [20]. To our knowledge, the model discrimination problem with data losses that satisfy (m,k)-firmness and more generally, for switched nonlinear systems with temporal logic-constrained switching has not been studied in the literature.

Contribution: In this letter, we consider an extension of the model discrimination problem for switched affine systems in [16] to switched nonlinear systems, where the switching/mode sequence is constrained by metric/signal temporal logic (MTL/STL) specifications. We first propose an

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optimization-based algorithm for analyzing the detectability of the models via finding a finite time T for which any pair of models cannot be identical for any initial state, noise signals and admissible switching sequences. Specifically, we show that a sufficient detectability test for any pair of models, also known as the T-distinguishability (or T-detectability) problem, e.g., [2], [10], can be achieved for switched nonlinear dynamics by leveraging piecewise affine abstraction tools in [7], [21], [22], [23]. This test then reduces to a feasibility check of a mixed-integer linear program (MILP), which is an alternative to [16] for the special case of switched affine systems that need not explicitly construct a monitor finite state machine and is shown in simulations to result in shorter computation times. Moreover, given noisy input-mode-output data at run time, we present a model discrimination algorithm for nonlinear parameter-varying systems by similarly building upon the abstraction tools in [7], [21], [22], [23], which also boils down to an MILP.

In addition, we describe how the proposed algorithms apply for solving the problems of detectability analysis and model discrimination for constrained nonlinear systems with (m,k)-firm data losses and explicitly provide the integer constraints corresponding to the (m,k)-firm property. Finally, we demonstrate and compare the effectiveness of our proposed approaches using several illustrative examples on fault detection, swarm consensus and intent identification, including the comparison of various mixed-integer encodings of switched dynamics in the model discrimination problems.

II. Preliminaries

A. Notations

$\|v\|_i$ for $i = \{1, \infty\}$ denotes the i-norm of a vector $v \in \mathbb{R}^n$. The set of integers from a through b is denoted by $\mathbb{Z}_a^b$ and $[\underline{m}, \overline{m}] = \{m \in \mathbb{R}^p : \underline{m} \leq m \leq \overline{m}\}$ is an interval in $\mathbb{R}^p$.

B. Metric/Signal Temporal Logic (MTL/STL)

Definition 1 (Atomic Proposition): An atomic proposition is a statement on the system variables/signals that is either *True* (1 *or* $\top$) or *False* (0 *or* $\perp$).

Let Σ be a finite set of modes. The syntax of MTL/STL formulas over Σ is given by:

$$\varphi ::= p \mid \neg\varphi \mid \varphi_1 \vee \varphi_2 \mid \varphi_1 \mathcal{U}_{[t_1,t_2]} \varphi_2, \tag{1}$$

where $p \in \Sigma$, while $\neg$, $\vee$, and $\mathcal{U}_{[t_1,t_2]}$ are the negation, disjunction, and time-constrained until operators, respectively, and $[t_1, t_2] \subset [0, \infty)$ is an interval of reals. Applying the grammar given in (1), we can also define next ($\bigcirc$; for discrete-time systems), conjunction ($\wedge$), implication ($\Rightarrow$), eventually in $[t_1, t_2]$ ($\Diamond_{[t_1,t_2]}$), and always in $[t_1, t_2]$ ($\Box_{[t_1,t_2]}$) as $\bigcirc\varphi = \top\mathcal{U}_{[0,1]}\varphi$, $\varphi_1 \wedge \varphi_2 = \neg(\neg\varphi_1 \vee \neg\varphi_2)$, $\varphi_1 \Rightarrow \varphi_2 = \neg\varphi_1 \vee \varphi_2$, $\Diamond_{[t_1,t_2]}\varphi = \top\mathcal{U}_{[t_1,t_2]}\varphi = \bigvee_{\tau=t_1}^{t_2} \bigcirc^\tau \varphi$, and $\Box_{[t_1,t_2]}\varphi = \neg\Diamond_{[t_1,t_2]}\neg\varphi = \bigwedge_{\tau=t_1}^{t_2} \bigcirc^\tau\varphi$, respectively. Further, we abbreviate $\mathcal{U}_{[0,\infty)}$, $\Diamond_{[0,\infty)}$, $\Box_{[0,\infty)}$ as $\mathcal{U}$, $\Diamond$, $\Box$.

Definition 2 (MTL/STL Semantics): Let $\boldsymbol{\sigma}$ be an ω-word over Σ, i.e., $\boldsymbol{\sigma} \in \Sigma^\omega$, and let σ_t be t^{th} element of $\boldsymbol{\sigma}$. The MTL/STL semantics is defined as follows:

1) $(\boldsymbol{\sigma}, t) \models p \Leftrightarrow \sigma_t = p$,
2) $(\boldsymbol{\sigma}, t) \models \neg\varphi \Leftrightarrow (\boldsymbol{\sigma}, t) \not\models \varphi$,
3) $(\boldsymbol{\sigma}, t) \models \varphi_1 \vee \varphi_2 \Leftrightarrow (\boldsymbol{\sigma}, t) \models \varphi_1$ or $(\boldsymbol{\sigma}, t) \models \varphi_2$,
4) $(\boldsymbol{\sigma}, t) \models \varphi_1 \wedge \varphi_2 \Leftrightarrow (\boldsymbol{\sigma}, t) \models \varphi_1$ and $(\boldsymbol{\sigma}, t) \models \varphi_2$,
5) $(\boldsymbol{\sigma}, t) \models \varphi_1\mathcal{U}_{[t_1,t_2]}\varphi_2 \Leftrightarrow \exists t' \in [t+t_1, t+t_2] : (\boldsymbol{\sigma}, t') \models \varphi_2$ and $\forall t'' \in [t, t'] : (\boldsymbol{\sigma}, t'') \models \varphi_1$,
6) $(\boldsymbol{\sigma}, t) \models \Diamond_{[t_1,t_2]}\varphi \Leftrightarrow \exists t' \in [t + t_1, t + t_2], (\boldsymbol{\sigma}, t') \models \varphi$,
7) $(\boldsymbol{\sigma}, t) \models \Box_{[t_1,t_2]}\varphi \Leftrightarrow \forall t' \in [t + t_1, t + t_2], (\boldsymbol{\sigma}, t') \models \varphi$.

We write $\boldsymbol{\sigma} \models \varphi$ if $(\boldsymbol{\sigma}, 0) \models \varphi$.

Moreover, we introduce several definitions for (infinite-length) MTL/STL specifications, which will be useful later.

Definition 3 (Valid Subtrace of an MTL/STL): A length-T word $\boldsymbol{q} \in \Sigma^T$ is called a *valid subtrace* from t_0 of an MTL/STL formula φ, if there exist a t_0-length prefix $\boldsymbol{p} \in \Sigma^{t_0}$ and a suffix $\boldsymbol{r} \in \Sigma^\omega$ such that their ω-word concatenation $\boldsymbol{pqr}$ satisfies φ, i.e., $\boldsymbol{pqr} \models \varphi$. Further, the set of length-T valid subtraces from t_0 is denoted as $V_{t_0}^T(\varphi)$.

Definition 4 (MTL/STL Bound of an MTL/STL [24]): The bound of an MTL/STL formula φ, denoted by b^φ, is the time length required to evaluate the satisfaction of φ and is recursively computed as follows: 1) $b^{\neg\varphi} = b^\varphi$; 2) $b^{\varphi_1 \wedge \varphi_2} = \max(b^{\varphi_1}, b^{\varphi_2})$; 3) $b^{\Diamond_{[t_1,t_2]}\varphi} = b^{\Box_{[t_1,t_2]}\varphi} = t_2 + b^\varphi$; and 4) $b^{\varphi_1\mathcal{U}_{[t_1,t_2]}\varphi_2} = t_2 + \max(b^{\varphi_1}, b^{\varphi_2})$.

Next, we present the integer encoding of MTL/STL formulas, as an extension of the LTL encoding found in [25], which we will use later for directly encoding MTL/STL formulas in the model discrimination problems, without first constructing a monitor automaton as in [16]. For brevity, we will now only present constraints for the satisfaction of each operator of the MTL/STL semantics, i.e., $(\boldsymbol{\sigma}, t) \models \varphi$ for the following operators, where p, q and p_i are atomic propositions, and P_φ^t is the truth value of formula φ at time t, as defined in [25]:

Negation: The formula $\varphi = \neg p$ can be modeled as:

$$P_\varphi^t = (1 - P_p^t). \tag{2}$$

Disjunction: The formula $\varphi = \bigvee_{i=1}^k p_i$ can be modeled as:

$$P_\varphi^t \leq \Sigma_{i=1}^k P_{p_i}^t; \quad P_\varphi^t \geq P_{p_i}^t, i \in \mathbb{Z}_1^k. \tag{3}$$

Conjunction: The formula $\varphi = \bigwedge_{i=1}^k p_i$ can be modeled as:

$$P_\varphi^t \geq \Sigma_{i=1}^k P_{p_i}^t - (k-1); \quad P_\varphi^t \leq P_{p_i}^t, i \in \mathbb{Z}_1^k. \tag{4}$$

Next: The formula $\varphi = \bigcirc p$ can be modeled as:

$$P_\varphi^t = P_p^{t+1}. \tag{5}$$

Until: The formula $\varphi = p\mathcal{U}_{[t_1,t_2]}q$ can be modeled as:

$$\begin{aligned} &\alpha_{tj} \geq P_q^j + \Sigma_{\tau=t}^{j-1} P_p^\tau - (j-t), j \in \mathbb{Z}_{t+t_1}^{t+t_2}; \\ &\alpha_{tj} \leq P_q^j, \ \alpha_{tj} \leq P_p^\tau, j \in \mathbb{Z}_{t+t_1}^{t+t_2}, \tau \in \mathbb{Z}_t^{j-1}; \\ &P_\varphi^t \leq \Sigma_{j=t+t_1}^{t+t_2} \alpha_{tj}, P_\varphi^t \geq \alpha_{tj}, j \in \mathbb{Z}_{t+t_1}^{t+t_2}. \end{aligned} \tag{6}$$

Eventually: The formula $\varphi = \Diamond_{[t_1,t_2]}p$ can be modeled as:

$$P_\varphi^t \leq \Sigma_{\tau=t+t_1}^{t+t_2} P_p^\tau; \quad P_\varphi^t \geq P_p^\tau, \tau \in \mathbb{Z}_{t+t_1}^{t+t_2}. \tag{7}$$

Always: The formula $\varphi = \Box_{[t_1,t_2]}p$ can be modeled as:

$$P_\varphi^t \geq \Sigma_{\tau=t+t_1}^{t+t_2} P_p^\tau - (t_2 - t_1); \quad P_\varphi^t \leq P_p^\tau, \tau \in \mathbb{Z}_{t+t_1}^{t+t_2}. \tag{8}$$

III. Problem Formulation

A. Modeling Framework

Consider a set of N_m discrete-time switched nonlinear system models $\{\mathcal{G}^l\}_{l=1}^{N_m}$, with each model $\mathcal{G}^l$ given by:

$$x_{t+1} = f_{\sigma_t}^l(x_t, u_t, w_t), y_t = g_{\sigma_t}^l(x_t, u_t, v_t), \tag{9}$$

where $x_t \in \mathcal{X}$ denotes the system state at time instant t with a closed interval domain $\mathcal{X} = [\underline{x}, \overline{x}] \subset \mathbb{R}^n$, $u_t \in \mathcal{U}$ is the control input with a closed interval domain $\mathcal{U} = [\underline{u}, \overline{u}] \subset \mathbb{R}^m$, $w_t \in \mathcal{W}$ and $v_t \in \mathcal{V}$ are process noise and measurement noise with a closed interval domain $\mathcal{W} = [\underline{w}, \overline{w}] \subset \mathbb{R}^{n_w}$ and $\mathcal{V} = [\underline{v}, \overline{v}] \subset \mathbb{R}^{n_v}$, respectively, σ_t is the (controlled or uncontrollable) discrete switching signal/mode, from a finite set Σ with cardinality $|\Sigma| = K$, and y_t is the system output at time instant t, while $f_\sigma : \mathcal{X} \times \mathcal{U} \times \mathcal{W} \to \mathbb{R}^n$ and

$g_\sigma : \mathcal{X}\times\mathcal{U}\times\mathcal{V} \to \mathbb{R}^{n_y}$ are vector fields describing the nonlinear dynamics of each mode $\sigma \in \Sigma$ of the system. Each model $\mathcal{G}^l$ also includes an MTL/STL formula φ^l that constrains the set of admissible switching signals $\boldsymbol{\sigma} \triangleq \{\sigma_t\}_0^\infty \in \Sigma^\omega$, i.e., $\boldsymbol{\sigma} \models \varphi^l$ must hold. Moreover, to obtain algorithms with a finite number of variables, we assume that formulas φ^l are of the (unbounded global/safety) form:

$$\varphi^l = \phi_b^l \wedge \Box\phi_g^l, \tag{10}$$

where ϕ_b^l and ϕ_g^l are bounded negation-free MTL/STL formulas with bounds $b^{\phi_b^l}$ and $b^{\phi_g^l}$ (see Definition 4).

Further, we will also consider the special case of nonlinear models with lossy/missing data, where the models $\mathcal{G}^l$ in (9) reduce to ones where each f^l is independent of σ_t and

$$g_{\sigma_t}^l(x_t, u_t, v_t) = \begin{cases} g^l(x_t, u_t, v_t), & \text{if } \sigma_t = 1,\\ \emptyset, & \text{if } \sigma_t = 0,\end{cases} \tag{11}$$

and consider a lossy data model where the switching signal σ_t satisfies the (m,k)-firm property [17], [18], defined as:

Definition 5 ((m,k)-Firm Specification): Given two integers m and k, $m \le k$, at least m steps system outputs are measured or observed for any k sequential steps.

1) Abstraction of Nonlinear Functions: To deal with nonlinearities in $\mathcal{G}^l$, i.e., the functions f_σ^l and g_σ^l for each $\sigma \in \Sigma$, we propose to leverage a result in [21], [23] to over-approximate the nonlinearities with piecewise affine inclusions. The precision of the abstraction can be improved with more and better chosen *partitions*, as defined below, but may result in longer computation times due to more integer variables in our solutions in Section IV (see [23] for details).

Definition 6 (Partition): For each function f_σ^l, a *partition* $\mathcal{I}_\sigma^{f,l}$ of the closed bounded region $\mathcal{X}\times\mathcal{U}\times\mathcal{W} \subseteq \mathbb{R}^{n+m+n_w}$ is a collection of $q_\sigma^{f,l}$ subregions $\mathcal{I}_\sigma^{f,l} = \{I_{\sigma,i}^{f,l} | i \in \mathbb{Z}_1^{q_\sigma^{f,l}}\}$ such that $\mathcal{X}\times\mathcal{U}\times\mathcal{W} \subseteq \bigcup_{i=1}^{q_\sigma^{f,l}} I_{\sigma,i}^{f,l}$ and $I_{\sigma,i}^{f,l} \cap I_{\sigma,j}^{f,l} = \partial I_{\sigma,i}^{f,l} \cap \partial I_{\sigma,j}^{f,l}$, $\forall i \ne j \in \mathbb{Z}_1^q$, where $\partial I_{\sigma,\ell}^{f,l}$ is the boundary of set $I_{\sigma,\ell}^{f,l}$. Similarly, a *partition* $\mathcal{I}_{g,\sigma}^l$ of the closed bounded region $\mathcal{X}\times\mathcal{U}\times\mathcal{V} \subseteq \mathbb{R}^{n+m+n_v}$ for each function g_σ^l can be defined.

We assume the partitions to be polytopic. Then, for each polytopic subregion $I_{\sigma,i}^{f,l} \in \mathcal{I}_\sigma^{f,l}$ (or $I_{\sigma,j}^{g,l} \in \mathcal{I}_\sigma^{g,l}$) that partitions the domain of interest, the nonlinear function f_σ^l (or g_σ^l) can be over-approximated/abstracted by a pair of affine functions $\underline{f}_{\sigma,i}^l$, $\overline{f}_{\sigma,i}^l$ (or $\underline{g}_{\sigma,j}^l$, $\overline{g}_{\sigma,j}^l$) by solving a linear programming (LP) problem [21], [23]. As a result, for all $(x,u,w) \in I_{\sigma,i}^{f,l}$ (or $(x,u,v) \in I_{\sigma,j}^{g,l}$), the function $f_\sigma^l(x,u,w)$ (or $g_\sigma^l(x,u,v)$) is sandwiched/framed by a pair of affine functions, i.e., $\underline{f}_{\sigma,i}^l(x,u,w) \le f_\sigma^l(x,u,w) \le \overline{f}_{\sigma,i}^l(x,u,w)$ (or $\underline{g}_{\sigma,j}^l(x,u,v) \le g_\sigma^l(x,u,v) \le \overline{g}_{\sigma,j}^l(x,u,v)$) with

$$\begin{aligned}
\underline{f}_{\sigma,i}^l(x,u,w) &= \underline{A}_{\sigma,i}^l x + \underline{B}_{\sigma,i}^l u + \underline{W}_{\sigma,i}^l w + \underline{h}_{\sigma,i}^{f,l},\\
\overline{f}_{\sigma,i}^l(x,u,w) &= \overline{A}_{\sigma,i}^l x + \overline{B}_{\sigma,i}^l u + \overline{W}_{\sigma,i}^l w + \overline{h}_{\sigma,i}^{f,l},\\
\underline{g}_{\sigma,j}^l(x,u,v) &= \underline{C}_{\sigma,j}^l x + \underline{D}_{\sigma,j}^l u + \underline{V}_{\sigma,j}^l v + \underline{h}_{\sigma,j}^{g,l},\\
\overline{g}_{\sigma,j}^l(x,u,v) &= \overline{C}_{\sigma,j}^l x + \overline{D}_{\sigma,j}^l u + \overline{V}_{\sigma,j}^l v + \overline{h}_{\sigma,j}^{g,l},
\end{aligned}$$

where $\underline{A}_{\sigma,i}^l$, $\overline{A}_{\sigma,i}^l$, $\underline{B}_{\sigma,i}^l$, $\overline{B}_{\sigma,i}^l$, $\underline{C}_{\sigma_t,j}^l$, $\overline{C}_{\sigma,j}^l$, $\underline{D}_{\sigma,j}^l$, $\overline{D}_{\sigma,j}^l$, $\underline{W}_{\sigma,i}^l$, $\overline{W}_{\sigma,i}^l$, $\underline{V}_{\sigma,j}^l$, $\overline{V}_{\sigma,j}^l$, $\underline{h}_{\sigma,i}^{f,l}$, $\overline{h}_{\sigma,i}^{f,l}$, $\underline{h}_{\sigma,j}^{g,l}$ and $\overline{h}_{\sigma,j}^{g,l}$ are of appropriate dimensions and are constants that are determined by the abstraction algorithm in [21], [23]. The abstracted piecewise affine interval models $\mathcal{H}^l$ is then given by:

$$\begin{aligned}
\begin{pmatrix} \underline{A}_{\sigma_t,i}^l x_t + \underline{B}_{\sigma_t,i}^l u_t \\ +\underline{h}_{\sigma_t,i}^{f,l} + \underline{W}_{\sigma_t,i}^l w_t \end{pmatrix} &\le x_{t+1} \le \begin{pmatrix} \overline{A}_{\sigma_t,i}^l x_t + \overline{B}_{\sigma_t,i}^l u_t \\ +\overline{h}_{\sigma_t,i}^{f,l} + \overline{W}_{\sigma_t,i}^l w_t \end{pmatrix},\\
\begin{pmatrix} \underline{C}_{\sigma_t,j}^l x_t + \underline{D}_{\sigma_t,j}^l u_t \\ +\underline{h}_{\sigma_t,j}^{g,l} + \underline{V}_{\sigma_t,j}^l v_t \end{pmatrix} &\le y_t \le \begin{pmatrix} \overline{C}_{\sigma_t,j}^l x_t + \overline{D}_{\sigma_t,j}^l u_t \\ +\overline{h}_{\sigma_t,j}^{g,l} + \overline{V}_{\sigma_t,j}^l v_t \end{pmatrix},
\end{aligned} \tag{12}$$

where their corresponding polytopic subregions $I_{\sigma_t,i}^{f,l}$ and $I_{\sigma_t,j}^{g,l}$ are given by the following linear constraints:

$$\begin{aligned}
S_{\sigma_t,i}^{x,l} x_t + S_{\sigma_t,i}^{u,l} u_t + S_{\sigma_t,i}^{w,l} w_t &\le \beta_{\sigma_t,i}^l,\\
M_{\sigma_t,j}^{x,l} x_t + M_{\sigma_t,j}^{u,l} u_t + M_{\sigma_t,j}^{v,l} v_t &\le \alpha_{\sigma_t,j}^l,
\end{aligned} \tag{13}$$

respectively, with $S_{\sigma_t,i}^{x,l}$, $S_{\sigma_t,i}^{u,l}$, $S_{\sigma_t,i}^{w,l}$, $M_{\sigma_t,j}^{x,l}$, $M_{\sigma_t,j}^{u,l}$, $M_{\sigma_t,j}^{v,l}$, $\beta_{\sigma_t,i}^l$ and $\alpha_{\sigma_t,j}^l$ of appropriate dimensions. Moreover, the MTL/STL formula φ^l for each model remains unchanged.

Next, to solve the model discrimination problem via model invalidation, we extend the definition in [2] of the length-T behavior of original constrained switched nonlinear and abstracted piecewise affine inclusion models, $\mathcal{G}^l$ and $\mathcal{H}^l$:

Definition 7 (Length-T Behavior of Original Model $\mathcal{G}^l$): The length-T behavior of the constrained switched nonlinear model $\mathcal{G}^l$ at time t_0 is the set of all length-T input-mode-output trajectories compatible with $\mathcal{G}^l$, given by:

$$\begin{aligned}
\mathcal{B}_{t_0}^T(\mathcal{G}^l) := \{&\{u_t,\sigma_t,y_t\}_{t=t_0}^{t_0+T-1} \mid u_t \in \mathcal{U}, \{\sigma_t\}_{t=t_0}^{t_0+T-1} \in V_{t_0}^T(\varphi^l),\\
&x_t \in \mathcal{X}, w_t \in \mathcal{W}, v_t \in \mathcal{V} \quad \forall t \in \mathbb{Z}_{t_0}^{t_0+T-1} \text{ s.t. (9) holds}\}.
\end{aligned}$$

Definition 8 (Length-T Behavior of Abstracted Model $\mathcal{H}^l$): The length-T behavior of the constrained abstracted model $\mathcal{H}^l$ at time t_0 is the set of all length-T input-mode-output trajectories compatible with $\mathcal{H}^l$, given by:

$$\begin{aligned}
\mathcal{B}_{t_0}^T(\mathcal{H}^l) := \{&\{u_t,\sigma_t,y_t\}_{t=t_0}^{t_0+T-1} | u_t \in \mathcal{U}, \{\sigma_t\}_{t_0}^{t_0+T-1} \in V_{t_0}^T(\varphi^l),\\
&x_t \in \mathcal{X}, w_t \in \mathcal{W}, v_t \in \mathcal{V} \quad \forall t \in \mathbb{Z}_{t=t_0}^{t_0+T-1} \text{s.t. (12)–(13) hold}\}.
\end{aligned}$$

Using the above definitions of system behaviors as well as the fact that $\mathcal{H}^l$ is a piecewise affine abstraction of $\mathcal{G}^l$ (by construction) with the same MTL/STL specification φ^l, we can conclude that $\mathcal{B}^T(\mathcal{G}^l) \subseteq \mathcal{B}^T(\mathcal{H}^l)$ for all t_0 and T.

B. Problem Statement

Next, we describe the model discrimination problems we consider. Specifically, we want to analyze the detectability of the models, i.e., to determine if the models in a model set can be distinguished from each other from finite input-mode-output data, as well as design a model discrimination algorithm to identify the true model at run time.

Problem 1 (Detectability Analysis for a Set of Constrained Switched Nonlinear Models $\{\mathcal{G}^l\}_{l=1}^{N_m}$): Given a set of constrained switched nonlinear models, $\{\mathcal{G}^l\}_{l=l}^{N_m}$, and a time horizon T, determine whether the set of models are T-detectable/-distinguishable, i.e., whether $\exists t_0$ such that:

$$\bigcap_{l=1}^{N_m} \mathcal{B}_{t_0}^T(\mathcal{G}^l) = \emptyset. \tag{14}$$

Problem 2 (Model Discrimination for $\{\mathcal{G}^l\}_{l=1}^{N_m}$): Given an input-mode-output trajectory $\{u_t,\sigma_t,y_t\}_{t=t_0}^{t_0+T-1}$, a set of constrained switched nonlinear models $\{\mathcal{G}^l\}_{l=l}^{N_m}$ and a finite horizon T, determine which model the trajectory belongs to. That is, to find an i that for some t_0 satisfies

$$\mathcal{B}_{t_0}^T(\mathcal{G}^i) \ne \emptyset \wedge (\mathcal{B}_{t_0}^T(\mathcal{G}^j) = \emptyset, \forall j \in \mathbb{Z}_1^{N_m}, j \ne i). \tag{15}$$

However, since the original models $\mathcal{G}^l$ are nonlinear and thus, "hard" to directly compute with, we propose to solve a "simpler" version of the above problems by leveraging the affine abstraction tools described in the previous section and the property that $\mathcal{B}^T_{t_0}(\mathcal{G}^l) \subseteq \mathcal{B}^T_{t_0}(\mathcal{H}^l)$, which is *sufficient* for solving the original problems, i.e., in lieu of solving Problems 1 and 2, we will solve the following.

Problem 1 (Detectability Analysis for a Set of Constrained Piecewise Affine Models $\{\mathcal{H}^l\}_{l=1}^{N_m}$): Given a set of constrained abstracted piecewise affine models, $\{\mathcal{H}^l\}_{l=1}^{N_m}$ and a time horizon T, determine whether the set of models are T-distinguishable/detectable, i.e., whether $\exists t_0$ such that:

$$\bigcap_{l=1}^{N_m} \mathcal{B}^T_{t_0}(\mathcal{H}^l) = \emptyset. \tag{16}$$

Problem 2 (Model Discrimination for $\{\mathcal{H}^l\}_{l=1}^{N_m}$): Given an input-mode-output trajectory $\{u_t, \sigma_t, y_t\}_{t=t_0}^{t_0+T-1}$, a set of constrained abstracted piecewise affine models $\{\mathcal{H}^l\}_{l=1}^{N_m}$, and a finite horizon T, determine which model the trajectory belongs to. That is, to find an i that for some t_0 satisfies

$$\mathcal{B}^T_{t_0}(\mathcal{H}^i) \neq \emptyset \wedge (\mathcal{B}^T_{t_0}(\mathcal{H}^j) = \emptyset, \forall j \in \mathbb{Z}_1^{N_m}, j \neq i). \tag{17}$$

Further, for the lossy data case, we aim to explicitly provide the constraints on $\{\sigma_t\}$ for the (m,k)-firm property.

Problem 3 [(m,k)-Firm Constraints]: Given the (m,k)-firm property and a finite horizon T, find the integer constraints on the switching signal $\{\sigma_t\}_{t=t_0}^{t_0+T-1}$ that encode the property.

IV. Main Result

In this section, we present optimization-based approaches to solve Problem 1.1 (and, in turn, Problem 1) and Problem 2.2 (and, in turn, Problem 2), as well as Problem 3.

First, we propose a detectability analysis algorithm for T-distinguishability (i.e., to solve Problem 1.1) with MTL/STL formulas of the form in (10), where only a finite number of variables are necessary, and the (guaranteed) detection time T is found by solving the problem below with increasing T:

Theorem 1 (T-Distinguishability): A pair of constrained abstracted piecewise affine inclusion models $\mathcal{H}^i$ and $\mathcal{H}^j$, $i \neq j$, with MTL/STL formulas φ^i and φ^j of the form in (10) is T distinguishable if the following is infeasible for any $t_0 \in \mathbb{Z}_0^{\max(\phi_b^i,\phi_b^j)+\max(\phi_g^i,\phi_g^j)+1}$ (with a search over t_0):

$$\text{Find } x_t^\star, w_t^\star, v_t^\star, u_t, y_t, \sigma_t, s_{t,*}^\star, \tilde{s}_{t,\dagger}^\star, a_{t,*}^\star, \tilde{a}_{t,\dagger}^\star, c_t^\sigma, z_t^\sigma$$

$$\text{s.t. } \forall \star \in \{i,j\}, \sigma \in \Sigma, * \in \mathbb{Z}_1^{q_\sigma^{f,\star}}, \dagger \in \mathbb{Z}_1^{q_\sigma^{g,\star}}, t \in \mathbb{Z}_{t_0}^{t_0+T-1}:$$

$$x_{t+1}^\star \leq \overline{A}_{\sigma,*}^\star x_t^\star + \overline{B}_{\sigma,*}^\star u_t + \overline{W}_{\sigma,*}^\star w_t^\star + \overline{h}_{\sigma,*}^{f,\star} + (s_{t,*}^\star + c_t^\sigma)\mathbb{1}, \tag{18a}$$

$$x_{t+1}^\star \geq \underline{A}_{\sigma,*}^\star x_t^\star + \underline{B}_{\sigma,*}^\star u_t + \underline{W}_{\sigma,*}^\star w_t^\star + \overline{h}_{\sigma,*}^{f,\star} - (s_{t,*}^\star + c_t^\sigma)\mathbb{1}, \tag{18b}$$

$$S_{\sigma,*}^{x,\star} x_t^\star + S_{\sigma,*}^{u,\star} u_t + S_{\sigma,*}^{w,\star} w_t^\star \leq \beta_{\sigma,*}^\star + (s_{t,*}^\star + c_t^\sigma)\mathbb{1}, \tag{18c}$$

$$y_t \leq \overline{C}_{\sigma,\dagger}^\star x_t^\star + \overline{D}_{\sigma,\dagger}^\star u_t + \overline{V}_{\sigma,\dagger}^\star v_t^\star + \overline{h}_{\sigma,\dagger}^{g,\star} + (\tilde{s}_{t,\dagger}^\star + c_t^\sigma)\mathbb{1}, \tag{18d}$$

$$y_t \geq \underline{C}_{\sigma,\dagger}^\star x_t^\star + \underline{D}_{\sigma,\dagger}^\star u_t + \underline{V}_{\sigma,\dagger}^\star v_t^\star + \underline{h}_{\sigma,\dagger}^{g,\star} - (\tilde{s}_{t,\dagger}^\star + c_t^\sigma)\mathbb{1}, \tag{18e}$$

$$M_{\sigma,\dagger}^{x,\star} x_t^\star + M_{\sigma,\dagger}^{u,\star} u_t + M_{\sigma,\dagger}^{v,\star} v_t^\star \leq \alpha_{\sigma,\dagger}^\star + (\tilde{s}_{t,\dagger}^\star + c_t^\sigma)\mathbb{1}, \tag{18f}$$

$$a_{t,*}^\star \in \{0,1\}, \tilde{a}_{t,\dagger}^\star \in \{0,1\}, z_t^\sigma \in \{0,1\}, \tag{18g}$$

$$\text{SOS}-1: (a_{t,*}^\star, s_{t,*}^\star), \text{SOS}-1: (\tilde{a}_{t,\dagger}^\star, \tilde{s}_{t,\dagger}^\star), \text{SOS}-1: (z_t^\sigma, c_t^\sigma), \tag{18h}$$

$$\sum_{\xi=1}^{q_\sigma^{f,\star}} a_{t,\xi}^\star = 1, \sum_{\xi=1}^{q_\sigma^{g,\star}} \tilde{a}_{t,\xi}^\star = 1, \sum_{\xi=1}^{|\Sigma|} z_t^\xi = 1, \tag{18i}$$

$$w_t^\star \in \mathcal{W}, v_t^\star \in \mathcal{V}, u_t \in \mathcal{U}, x_t^\star \in \mathcal{X}, \tag{18j}$$

$$\{\sigma_t\}_{t=\underline{t}}^{\overline{t}} \in V_{\underline{t}}^{\overline{t}-\underline{t}+1}(\varphi^\star), \tag{18k}$$

where $s_{t,*}^\star$, $\tilde{s}_{t,\dagger}^\star$ and c_t^σ are slack variables, $z_t^\sigma = 1$ corresponds to $\sigma_t = \sigma$, the set of valid subtraces $V_{\underline{t}}^{\overline{t}-\underline{t}+1}(\varphi^\star)$ is constructed recursively using (2)–(8) and SOS-1 refers to Special Ordered Set of type 1 (i.e., at most one member of the set can be nonzero [26]), with $\underline{t} = t_0 - \max(\phi_g^i, \phi_g^j)$ and $\overline{t} = t_0 + T - 1 + \max(\phi_g^i, \phi_g^j)$ if $t_0 > \max(\phi_b^i, \phi_b^j)$, and otherwise, with $\underline{t} = 0$ and $\overline{t} = \max(t_0 + T - 1 + \max(\phi_g^i, \phi_g^j), \phi_b^i, \phi_b^j)$.

Proof: $a_{t,*}^\star = 1$ and $z_t^\sigma = 1$ imply that (18a)–(18c) hold, since the SOS-1 constraints in (18h) ensure that $s_{t,*}^\star = 0$ and $c_t^\sigma = 0$ correspondingly. On the contrary, if $a_{t,*}^\star = 0$ and/or $z_t^\sigma = 0$, it means that $s_{t,*}^\star$ and/or c_t^σ, are free and thus (18a)–(18c) hold trivially. Similarly, $\tilde{a}_{t,\dagger}^\star = 1$ and $z_t^\sigma = 1$, or $\tilde{a}_{t,\dagger}^\star = 0$ and/or $z_t^\sigma = 0$ imply that (18d)–(18f) hold. In addition, (18i) ensures that, at each time step t, only one partition is valid for each of the state and output equations, and only one switching signal/mode is valid. Thus, if the above problem is infeasible, it means that there exists no common behavior that is satisfied by both models, i.e., $\mathcal{B}^T_{t_0}(\mathcal{H}^i) \bigcap \mathcal{B}^T_{t_0}(\mathcal{H}^j) = \emptyset$; hence, the pair of models is distinguishable from each other. Note that horizon for σ_t and z_t^σ is extended backwards and forwards as determined by $\underline{t}, \overline{t}$ such that the constraints induced by $\varphi^\star$ that affects switching signal/mode sequence within the horizon from t_0 and t_0+T-1 can be fully expressed using (2)–(8), and thanks to the assumed form for $\varphi^\star$ in (10) with bounded $\phi_b^\star$ and $\phi_g^\star$, $\overline{t}$ is bounded. Finally, from (10), it can also be deduced that the MILP in (1) for all $t_0 \geq \max(\phi_b^i, \phi_b^j)+\max(\phi_g^i, \phi_g^j)+1$ remains the same. ■

Next, we present a model invalidation algorithm (see Algorithm 1) that enables us to discriminate among all models, i.e., to solve Problem 2.2, if all model pairs are T-distinguishable according to Theorem 1. If not all pairs are T-distinguishable, Algorithm 1 will instead return the set of all models that are consistent with the given input-mode-output data up to the current time step t_c. Note that since the mode sequence is observed, the models in (9) reduces to nonlinear parameter-varying (NPV) systems, and thus, the model invalidation algorithm derived below applies to NPV models:

Theorem 2: Given a constrained abstracted piecewise affine inclusion model $\mathcal{H}^l$ and a length-T input-mode-output sequence $\{u_t, \sigma_t, y_t\}_{t=t_c-T+1}^{t_c}$ at time t_c, the model is invalidated if the following problem is infeasible for any $t_c \in \mathbb{Z}_0^\infty$:

$$\text{Find } x_t, w_t, v_t, s_{t,*}, a_{t,*}, s_{t,\dagger}, a_{t,\dagger}$$

$$\text{s.t. } \forall t \in \mathbb{Z}_{t_c}^{t_c+T-1}, * \in \mathbb{Z}_1^{q_{\sigma_t}^{f,l}}, \dagger \in \mathbb{Z}_1^{q_{\sigma_t}^{f,l}}:$$

$$x_{t+1} \leq \overline{A}_{\sigma_t,*}^l x_t + \overline{B}_{\sigma_t,*}^l u_t + \overline{W}_{\sigma_t,*}^l w_t + \overline{h}_{\sigma_t,*}^{f,l} + s_{t,*}\mathbb{1}, \tag{19a}$$

$$x_{t+1} \geq \underline{A}_{\sigma_t,*}^l x_t + \underline{B}_{\sigma_t,*}^l u_t + \underline{W}_{\sigma_t,*}^l w_t + \underline{h}_{\sigma_t,*}^{f,l} - s_{t,*}\mathbb{1}, \tag{19b}$$

$$S_{\sigma_t,*}^{x,l} x_t + S_{\sigma,*}^{u,l} u_t + S_{\sigma,*}^{w,l} w_t \leq \beta_{\sigma_t,*}^l + s_{t,*}\mathbb{1}, \tag{19c}$$

$$y_t \leq \overline{C}_{\sigma_t,\dagger}^l x_t + \overline{D}_{\sigma_t,\dagger}^l u_t + \overline{V}_{\sigma_t,\dagger}^l v_t + \overline{h}_{\sigma_t,\dagger}^{g,l} + \tilde{s}_{t,\dagger}\mathbb{1}, \tag{19d}$$

$$y_t \geq \underline{C}_{\sigma_t,\dagger}^l x_t + \underline{D}_{\sigma_t,\dagger}^l u_t + \underline{V}_{\sigma_t,\dagger}^l v_t + \underline{h}_{\sigma_t,\dagger}^{g,l} - \tilde{s}_{t,\dagger}\mathbb{1}, \tag{19e}$$

$$M_{\sigma_t,\dagger}^{x,l} x_t + M_{\sigma_t,\dagger}^{u,l} u_t + M_{\sigma_t,\dagger}^{v,l} v_t \leq \alpha_{\sigma_t,\dagger}^l + \tilde{s}_{t,\dagger}\mathbb{1}, \tag{19f}$$

$$a_{t,*} \in \{0,1\}, \text{ SOS}-1: (a_{t,*}, s_{t,*}), \sum_{\xi=1}^{q_{\sigma_t}^{f,l}} a_{t,\xi} = 1, \tag{19g}$$

$$\tilde{a}_{t,\dagger} \in \{0,1\}, \text{ SOS}-1: (\tilde{a}_{t,\dagger}, \tilde{s}_{t,\dagger}), \sum_{\xi=1}^{q_{\sigma_t}^{g,l}} \tilde{a}_{t,\xi} = 1, \tag{19h}$$

Algorithm 1: Model Discrimination With Length T

```
Data: Models {G^l}_{l=1}^{N_m},
      Input-Output Sequence = {u_t, σ_t, y_t}_{t=t_0-T+1}^{t_0}
function findModel({G^l}_{l=1}^{N_m}, {u_t, σ_t, y_t}_{t=t_0-T+1}^{t_0})
    valid ← {G^l}_{l=1}^{N_m};
    for l = 1:N_m do
        Check Feasibility of Theorem 2;
        if infeasible then
            Remove l from valid;
        end
    end
    return valid
```

$$w_t \in \mathcal{W},\ v_t \in \mathcal{V},\ x_t \in \mathcal{X}. \tag{19i}$$

Proof: The construction follows similar steps as in Theorem 1, but with only one model and a given mode sequence. ■

Further, we consider the special case of nonlinear system models with lossy/missing data in (11), where the lossy data pattern satisfies the (m,k)-firm property (see Definition 5). In this case, the model discrimination algorithm based on Algorithm 1 and Theorem 2 remains the same, while the mode sequence constraint corresponding to the (m,k)-firm property for the T-distinguishability test in (18k) of Theorem 1 can be explicitly given (Problem 3) by the following:

Proposition 1 ((m,k)-Firm Constraints): The (m,k)-firm property for (11) corresponds to the MTL/STL formula:

$$\varphi = \Box \bigvee_{i_1=0}^{k-m} \cdots \bigvee_{i_m=i_{m-1}+1}^{k-1} \left(\bigwedge_{j\in\{i_\ell\}_{\ell=1}^{m}} \bigcirc^j(\sigma=1) \right)$$

with $\phi_b = 0$ and $\phi_g = k$, and the associated constraint for $\{\sigma_t\}_{t=\underline{t}}^{\bar{t}} \in V_{\underline{t}}^{\bar{t}-\underline{t}+1}(\varphi)$ in (18k) can be explicitly written as:

$$\forall t \in \mathbb{Z}_{\underline{t}}^{\bar{t}-k+1} : \sum_{\xi=t}^{t+k-1} z_\xi^\sigma \geq m. \tag{20}$$

Proof: This follows directly from Definition 5. ■

V. Simulation Examples

The simulations in Sections V-A and V-C are implemented in MATLAB 2019b with Gurobi v8.1 [26] on a 1.3 GHz dual-core machine with 16 GB RAM, while the simulations in Section V-B are performed on Arizona State University's Agave Cluster on a single thread of one of the cores of Intel Xeon E5-2680 v4 CPU processor running at 2.40GHz with 16GB RAM using MATLAB 2017a with Gurobi v8.0.0 [26].

A. 2D Numerical Fault Detection Example

First, we consider the fault detection problem of two 2D switched affine systems $\{\mathcal{G}^l\}_{l\in\{\mathtt{h},\mathtt{f}\}}$. Here $l = \mathtt{h}$ represents the healthy operation model and $l = \mathtt{f}$ the faulty model. A fault (i.e., a permanent switching from model h to model f) is guaranteed to be detected with a T-delay if $\{\mathcal{G}^l\}_{l\in\{\mathtt{h},\mathtt{f}\}}$ is T-distinguishable. System $\mathcal{G}^l$ has the form $x_{t+1} = A^l_{\sigma_t} x_t + h^l_{\sigma_t} + w_t$, $y_t = x_t + v_t$, where $\sigma_t \in \{1, 2\}$ is the discrete control input (controlled switching signal), with disturbance $w_t \in [-0.01, 0.01] \times [-0.01, 0.01]$, and noise $v_t \in [-0.1, 0.1] \times [-0.1, 0.1]$. The states x_t are restricted within $\mathcal{X} = [-5, 5] \times [-5, 5]$ and the initial state set $\mathcal{X}_0 = [1.5, 2.5] \times [0.5, 1.5]$. The healthy system matrices are:

$$A_1^{\mathrm{h}} = \begin{bmatrix} 0.794 & 0.723 \\ -0.260 & 0.794 \end{bmatrix}, h_1^{\mathrm{h}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$
$$A_2^{\mathrm{h}} = \begin{bmatrix} 0.794 & 0.434 \\ -0.434 & 0.794 \end{bmatrix}, h_1^{\mathrm{h}} = \begin{bmatrix} 0.115 \\ 0.457 \end{bmatrix},$$

while for the faulty system, $A_1^{\mathrm{f}} = A_2^{\mathrm{f}} = A_1^{\mathrm{h}}$, $h_1^{\mathrm{f}} = h_2^{\mathrm{f}} = h_1^{\mathrm{h}}$ (i.e., a linear system, not a switched system). We also assume that the switching signal $\boldsymbol{\sigma} = \{\sigma_t\}_{t=0}^{\infty}$ for the healthy model must satisfy an MTL/STL formula $\varphi^{\mathrm{h}} = \phi_1^{\mathrm{h}} \wedge \phi_2^{\mathrm{h}}$, where

$$\phi_1^{\mathrm{h}} = \Box\Big(\Box_{[0,4]}(\sigma = 1) \Rightarrow \bigcirc^5 \neg(\sigma = 1)\Big),$$
$$\phi_2^{\mathrm{h}} = \Box\big((\neg(\sigma = 2) \wedge \bigcirc(\sigma = 2)) \Rightarrow \Box_{[2,3]}(\sigma = 2)\big), \tag{21}$$

whereas there is no switching signal for the faulty model. The detectability analysis for $\{\mathcal{G}^l\}_{l\in\{\mathtt{h},\mathtt{f}\}}$ can be formulated as Problem 1. Note that the system is not detectable within any finite steps unless the MTL/STL constraint φ^{h} is considered. This is because, without φ^{h}, the system is allowed to stay at mode 1 for all time (i.e., $\sigma_t = 1, \forall t$), in which case the healthy model can behave identically as the faulty one. Our proposed approach shows that the system $\{\mathcal{G}^l\}_{l\in\{\mathtt{h},\mathtt{f}\}}$ is 11-detectable, which is consistent with that obtained using the automaton-based approach proposed in [16]. Moreover, we compared several encoding techniques for the switched affine dynamics in the literature. Table I shows the computation time for these methods, where the column "Big-M" corresponds to using the `implies` function in YALMIP [27], the "SOS-1" column to the encoding in Theorem 1, and the AuxVar column to the encoding of the switched dynamics as a summation using auxiliary variables similar to [16]. It can be seen that the SOS-1 encoding (in our approach) is the most efficient in this fault detection example.

TABLE I
Comparison of Computation Times of Different Encoding Techniques in the Fault Detection Example

Parameter	Big-M	SOS-1	AuxVar
Solving Time (s)	1.273	0.634	0.848
Total Time (s)	3.164	2.361	2.993

B. Swarm Consensus Example

We further apply our approach to determine the distinguishability of models of a swarm of drones that implements a leader-follower altitude consensus protocol. The problem setup here is similar to that in the previous example, but with more states and more realistic scenarios. The system $\{\mathcal{G}^l\}_{l\in\{\mathtt{c},\mathtt{r}\}}$ consists of a *complying* mode and a *rogue* mode, both governed by 8D switched affine dynamics, where the discrete control input σ_t represents the controlled switching among different communication network topologies and different leading drone's altitude set point. We impose MTL/STL constraints similar to (21) on the sequence $\boldsymbol{\sigma} = \{\sigma_t\}_{t=0}^{\infty}$ to capture the dwell-time restriction for each set point and the communication network's joint connectedness condition that must hold to achieve consensus. The detailed system equations and MTL/STL formulas can be found in [16], where an MILP is also used to analyze detectability, but the MTL/STL constraints are encoded with the binary representation of a monitor automaton constructed from the MTL/STL formula. Although the results using our method, shown in Table II, match those obtained using the approach and implementation in [16], our method is able to reach the conclusion much faster than the method in [16], which took more than 2 weeks to determine the distinguishability of the models.

TABLE II
DETECTION TIMES AND TIME TAKEN FOR VARIOUS NOISE LEVELS IN THE SWARM CONSENSUS EXAMPLE

Noise Bound $\|w\|$	Detection Time T	Time Taken (s)
0	26	12806
0.025	28	9562
0.05	29	18738

TABLE III
DETECTION TIMES FOR INTENT IDENTIFICATION UNDER (m,k)-FIRM DATA LOSS WITH DIFFERENT (m,k) PARAMETERS

$m=1, k=1$	$m=4, k=5$	$m=3, k=5$	$m=5, k=6$
$T=7$	$T=9$	$T=11$	$T=9$

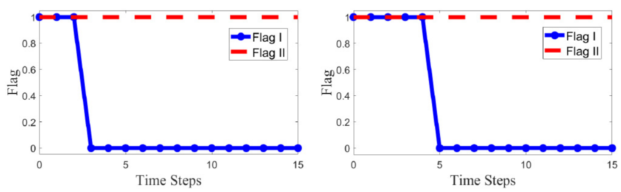

Fig. 1. Model discrimination results with two different intents, $i \in \{\text{I}, \text{II}\}$ under (m,k)-firm data losses: (Left) $m = k = 1$; (Right) $m = 3$, $k = 5$. Flag i is 1 when model i is not (yet) invalidated and is 0, if invalidated.

C. Swarm Intent Identification Example With Data Loss

In this example, we consider the swarm intent identification scenario in [7] with two swarm intent models (see [7] for details): the swarm intends to move towards the centroid of the swarm (Model I), or the swarm moves away from the centroid (Model II). An affine abstraction algorithm as described in [21] is applied on the system dynamics to obtain each abstracted swarm model represented by a piece-wise affine inclusion model as in (12). We first applied the T-distinguishability algorithm (Theorem 1) without (m,k)-firm constraints and found that the two intent models are distinguishable when $T = 7$. This can be viewed as a special case of (m,k)-firm specification with $m = k = 1$. In addition, we considered the identification problem with (m,k)-firm data loss with various m and k and the results are shown in Table III. Further, we applied the model discrimination algorithm in Theorem 2 for the first and third cases and the results are depicted in Fig. 1. As expected, the detection time for both T-distinguishability and model discrimination grows with increasing data loss.

VI. CONCLUSION

In this letter, we considered the model discrimination problem for switched nonlinear systems with switching sequences that satisfy given metric/signal temporal logic specifications. In particular, we leveraged tools for abstracting/over-approximating nonlinear dynamics with simpler inclusion models and proposed detectability analysis and model discrimination algorithms to identify the true model using finite, noisy input-mode-output data at run time, which consist of feasibility checks of the corresponding mixed-integer linear programs. In addition, we explicitly provided the integer constraints for the special case of (m,k)-firm data loss models. The effectiveness of our proposed approaches are illustrated and compared with existing approaches using several examples of fault detection, swarm consensus and swarm intent identification.

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REFERENCES

[1] V. Venkatasubramanian, R. Rengaswamy, K. Yin, and S. N. Kavuri, "A review of process fault detection and diagnosis: Part I: Quantitative model-based methods," *Comput. Chem. Eng.*, vol. 27, no. 3, pp. 293–311, 2003.
[2] F. Harirchi, S. Z. Yong, and N. Ozay, "Guaranteed fault detection and isolation for switched affine models," in *Proc. IEEE Conf. Decis. Control (CDC)*, 2017, pp. 5161–5167.
[3] R. S. Smith and J. C. Doyle, "Model invalidation: A connection between robust control and identification," in *Proc. Amer. Control Conf.*, 1989, pp. 1435–1440.
[4] F. D. Bianchi and R. S. Sánchez-Peña, "Robust identification/invalidation in an LPV framework," *Int. J. Robust Nonlinear Control*, vol. 20, no. 3, pp. 301–312, 2010.
[5] M. Sznaier and M. C. Mazzaro, "An LMI approach to control-oriented identification and model (in) validation of LPV systems," *IEEE Trans. Autom. Control*, vol. 48, no. 9, pp. 1619–1624, Sep. 2003.
[6] S. Prajna, "Barrier certificates for nonlinear model validation," *Automatica*, vol. 42, no. 1, pp. 117–126, 2006.
[7] Z. Jin, Q. Shen, and S. Z. Yong, "Optimization-based approaches for affine abstraction and model discrimination of uncertain nonlinear systems," in *Proc. IEEE Conf. Decis. Control*, 2019, pp. 7976–7981.
[8] N. Ozay, M. Sznaier, and C. Lagoa, "Convex certificates for model (in)validation of switched affine systems with unknown switches," *IEEE Trans. Autom. Control*, vol. 59, no. 11, pp. 2921–2932, Nov. 2014.
[9] F. Harirchi and N. Ozay, "Guaranteed model-based fault detection in cyber–physical systems: A model invalidation approach," *Automatica*, vol. 93, pp. 476–488, Jul. 2018.
[10] F. Harirchi, S. Z. Yong, and N. Ozay, "Passive diagnosis of hidden-mode switched affine models with detection guarantees via model invalidation," in *Diagnosability, Security and Safety of Hybrid Dynamic and Cyber-Physical Systems*. Cham, Switzerland: Springer, 2018, pp. 227–251.
[11] H. Lou and R. Yang, "Conditions for distinguishability and observability of switched linear systems," *Nonlinear Anal. Hybrid Syst.*, vol. 5, no. 3, pp. 427–445, 2011.
[12] M. Babaali and G. J. Pappas, "Observability of switched linear systems in continuous time," in *Proc. HSCC*, 2005, pp. 103–117.
[13] E. De Santis and M. D. Di Benedetto, "Observability and diagnosability of finite state systems: A unifying framework," *Automatica*, vol. 81, pp. 115–122, Jul. 2017.
[14] N. Ramdani, L. Travé-Massuyès, and C. Jauberthie, "Mode discernibility and bounded-error state estimation for nonlinear hybrid systems," *Automatica*, vol. 91, pp. 118–125, May 2018.
[15] E. Asarin, A. Donzé, O. Maler, and D. Nickovic, "Parametric identification of temporal properties," in *Proc. Int. Conf. Runtime Verif.*, 2011, pp. 147–160.
[16] L. Yang and N. Ozay, "Fault detectability analysis of switched affine systems with linear temporal logic constraints," in *Proc. IEEE Conf. Decis. Control (CDC)*, 2019, pp. 5779–5786.
[17] G. Quan and X. Hu, "Enhanced fixed-priority scheduling with (m,k)-firm guarantee," in *Proc. IEEE Real Time Syst. Symp.*, 2000, pp. 79–88.
[18] E. P. van Horssen, A. R. B. Behrouzian, D. Goswami, D. Antunes, T. Basten, and W. P. M. H. Heemels, "Performance analysis and controller improvement for linear systems with (m, k)-firm data losses," in *Proc. Eur. Control Conf. (ECC)*, 2016, pp. 2571–2577.
[19] N. Jia, Y.-Q. Song, and R.-Z. Lin, "Analysis of networked control system with packet drops governed by (m,k)-firm constraint," in *Proc. Fieldbus Syst. Appl.*, 2006, pp. 63–70.
[20] F. Felicioni, N. Jia, Y.-Q. Song, and F. Simonot-Lion, "Impact of a (m,k)-firm data dropouts policy on the quality of control," in *Proc. IEEE Int. Workshop Fact. Commun. Syst.*, 2006, pp. 353–359.
[21] K. R. Singh, Q. Shen, and S. Z. Yong, "Mesh-based affine abstraction of nonlinear systems with tighter bounds," in *Proc. IEEE Conf. Decis. Control*, 2018, pp. 3056–3061.
[22] S. M. Hassaan, M. Khajenejad, S. Jensen, Q. Shen, and S. Z. Yong, "Incremental affine abstraction of nonlinear systems," *IEEE Control Syst. Lett.*, vol. 5, no. 2, pp. 653–658, Apr. 2021.
[23] Z. Jin, Q. Shen, and S. Z. Yong, "Mesh-based piecewise affine abstraction with polytopic partitions for nonlinear systems," *IEEE Control Syst. Lett.*, vol. 5, no. 5, pp. 1543–1548, Nov. 2021, doi: 10.1109/LCSYS.2020.3041312.
[24] A. Dokhanchi, B. Hoxha, and G. Fainekos, "On-line monitoring for temporal logic robustness," in *Proc. Int. Conf. Runtime Verif.*, 2014, pp. 231–246.
[25] S. Karaman, R. G. Sanfelice, and E. Frazzoli, "Optimal control of mixed logical dynamical systems with linear temporal logic specifications," in *Proc. IEEE Conf. Decis. Control*, 2008, pp. 2117–2122.
[26] *Gurobi Optimizer Reference Manual*, Gurobi Optimization, Inc., Houston, TX, USA, 2015. [Online]. Available: http://www.gurobi.com
[27] J. Lofberg, "YALMIP: A toolbox for modeling and optimization in MATLAB," in *Proc. CACSD*, Taipei, Taiwan, 2004, pp. 284–289.