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Janus and RG interfaces in three-dimensional gauged supergravity. Part II. General α

Michael Gutperle and Charlie Hultgreen-Mena

Mani L. Bhaumik Institute for Theoretical Physics,
Department of Physics and Astronomy, University of California,
475 Portola Plaza, Los Angeles, CA 90095, U.S.A.

E-mail: gutperle@physics.ucla.edu, charlie.hultgreen@physics.ucla.edu

Abstract: In this paper, we continue the study of Janus and RG-flow interfaces in three dimensional supergravity continuing the work presented in [1]. We consider $\mathcal{N} = 8$ gauged supergravity theories which have a $\mathcal{N} = (4, 4)$ AdS_3 vacuum with $D^1(2, 1; \alpha) \times D^1(2, 1; \alpha)$ symmetry for general α. We derive the BPS flow equations and find numerical solutions. Some holographic quantities such as the entanglement entropy are calculated.

Keywords: AdS-CFT Correspondence, Supergravity Models

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1 Introduction

Janus solutions provide a holographic description of interface conformal field theories. Generally, the solutions are constructed by considering an AdS_d slicing of a higher dimensional space where the other fields depend non-trivially on the slicing coordinate(s). For example the original Janus solution [2], deforms the $AdS_5 \times S^5$ vacuum of type IIB and is given by an AdS_4 slicing where the dilation depends non-trivially on a single slicing coordinate and approaches two different values on the two boundary components. The solution is dual to an interface of $N = 4$ super Yang-Mills theory where the coupling g_{YM} jumps across a co-dimension one interface [3]. More general Janus solutions preserving supersymmetry were constructed as $AdS_4 \times S^2 \times S^2$ space warped over a Riemann surface [4]. These solutions are dual to supersymmetric interface theories in $N = 4$ SYM [5–7]. For other Janus solutions in ten and eleven dimensions, see e.g. [8–11]. In general, constructing such solutions

is quite difficult due to the fact that the supersymmetry variations, as well the equations of motion, depend on more than one warping coordinate and the resulting equations are nonlinear partial differential equations. A useful approach is to construct Janus solutions in lower dimensional gauged supergravities (see for example [12–22]). Such solutions are often easier to obtain, can be uplifted to ten or eleven dimensions or can be used to explore qualitative features of Janus solution in a bottom-up approach.

In lower dimensional gauged supergravities it is often the case that in addition to a maximally supersymmetric AdS vacuum there are extrema with a reduced amount of supersymmetry. One of the aims of the present paper is to construct holographic Janus solutions which correspond to RG interfaces [23], between different AdS vacua.[1] This paper is a continuation of the work presented [1], which considered three-dimensional $\mathcal{N} = 8$ gauged supergravity with $n = 4$ vector multiplets, first discussed in [27]. This theory has an AdS_3 vacuum with maximal $\mathcal{N} = (4, 4)$ supersymmetry as well as two families of AdS_3 vacua with $\mathcal{N} = (1, 1)$ supersymmetry [28]. The gauged supergravity has a parameter α on which the embedding tensor for the gauged supergravity depends. For this theory the dual superconformal algebra of the $\mathcal{N} = (4, 4)$ vacuum is given by the "large" superconformal algebra $D^1(2, 1; \alpha) \times D^1(2, 1; \alpha)$, and the three-dimensional supergravity is believed to be a truncation of M-theory on $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ [29–32]. In the previous paper we considered the special case of $\alpha = 1$ for which the explicit expressions become simpler. Here we will analyze the case for general α, using both analytical and numerical methods.

The structure of this paper is as follows: in section 2 we review the three dimensional gauged supergravity with $n = 4$ vector multiplets used here. We consider three truncations where the gauge fields as well as some scalars can consistently be set to zero and fix the $\mathcal{N} = (1, 1)$ vacua for general α. In section 3 we derive the BPS flow equations for an AdS_2 sliced Janus ansatz, this generalizes and streamlines the discussion of [1]. In section 4 we present the flow equations for the three truncations and integrate them numerically for the three truncations. For the second and third truncations where $\mathcal{N} = (1, 1)$ AdS vacua exists we present examples of RG-flow interfaces. In section 5 we use the solutions to calculate some holographic observables. In particular we determine the masses of the fluctuating scalars around the $\mathcal{N} = (1, 1)$ vacua. The mass squared of the scalar fluctuations is positive and quite large, which means that the scalar fluctuations around the fixed point are repulsive in the UV. This implies that the initial conditions have to be fine tuned in order to reach the fixed point. We discuss our results and possible directions for future research in section 6.

2 Three-dimensional $\mathcal{N} = 8$ gauged supergravity

In this section, we review the $\mathcal{N} = 8$ gauged supergravity first constructed in [27] mainly following the conventions of [1]. The bosonic field content consists of a graviton $g_{\mu\nu}$, Chern-Simons gauge fields $B_\mu^{\mathcal{M}}$, and scalars fields living in a $G/H = \mathrm{SO}(8, n)/\mathrm{SO}(8) \times \mathrm{SO}(n)$ coset, which has $8n$ degrees of freedom before gauging. The scalar fields are parametrized by a G-valued matrix $L(x)$ in the vector representation, which transforms under H and

[1] See [24–26] for other examples of holographic RG-flow interfaces.

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the gauge group $G_0 \subseteq G$ by right and left multiplication of group elements respectively.

$$L(x) \to g_0(x) L(x) h^{-1}(x) \tag{2.1}$$

for $g_0 \in G_0$ and $h \in H$. The Lagrangian is invariant under such transformations. In this paper we use the following index conventions:

- $I, J, \ldots = 1, 2, \ldots, 8$ for SO(8).
- $r, s, \ldots = 9, 10, \ldots, n+8$ for SO(n).
- $\bar{I}, \bar{J}, \ldots = 1, 2, \ldots, n+8$ for SO($8, n$).
- $\mathcal{M}, \mathcal{N}, \ldots$ for generators of SO($8, n$).

Let the generators of G be $\{t^{\mathcal{M}}\} = \{t^{\bar{I}\bar{J}}\} = \{X^{IJ}, X^{rs}, Y^{Ir}\}$, where Y^{Ir} are the noncompact generators. Explicitly, the generators of the vector representation are given by

$$(t^{\bar{I}\bar{J}})^{\bar{K}}{}_{\bar{L}} = \eta^{\bar{I}\bar{K}} \delta^{\bar{J}}_{\bar{L}} - \eta^{\bar{J}\bar{K}} \delta^{\bar{I}}_{\bar{L}} \tag{2.2}$$

where $\eta^{\bar{I}\bar{J}} = \mathrm{diag}(++++++++--\cdots)$ is an SO($8, n$)-invariant tensor. These generators satisfy the typical SO($8, n$) commutation relations,

$$[t^{\bar{I}\bar{J}}, t^{\bar{K}\bar{L}}] = 2\Big(\eta^{\bar{I}[\bar{K}} t^{\bar{L}]\bar{J}} - \eta^{\bar{J}[\bar{K}} t^{\bar{L}]\bar{I}}\Big) \tag{2.3}$$

The gauging of the supergravity is characterized by an embedding tensor $\Theta_{\mathcal{M}\mathcal{N}}$ (which has to satisfy various identities [33] in order to define a consistent theory) that determines which isometries are gauged, the coupling to the Chern-Simons fields, and additional terms in the supersymmetry transformations and action depending on the gauge coupling g. We will look at the particular case in [28] where $n \geq 4$ and the gauged subgroup is the $G_0 = \mathrm{SO}(4) \times \mathrm{SO}(4)$ subset of the SO(8) $\subset$ SO($8, n$). The embedding tensor has the non vanishing entries,[2]

$$\Theta_{\bar{I}\bar{J},\bar{K}\bar{L}} = \begin{cases} \alpha \varepsilon_{\bar{I}\bar{J}\bar{K}\bar{L}} & \text{if } \bar{I}, \bar{J}, \bar{K}, \bar{L} \in \{1,2,3,4\} \\ \varepsilon_{\bar{I}\bar{J}\bar{K}\bar{L}} & \text{if } \bar{I}, \bar{J}, \bar{K}, \bar{L} \in \{5,6,7,8\} \\ 0 & \text{otherwise} \end{cases} \tag{2.4}$$

Note that the gauging depends on a real parameter α. As discussed in [28], the maximally supersymmetric AdS_3 vacuum has an isometry group,

$$D^1(2,1;\alpha) \times D^1(2,1;\alpha) \tag{2.5}$$

which corresponds to the family of "large" superconformal algebras of the dual SCFT. In this paper we generalize the analysis of [1] where the case $\alpha = 1$ was considered to the case of general α. Note that in the special case $\alpha = 1$ the super algebra becomes more familiar $D^1(2,1;1) = \mathrm{OSp}(4|2)$.

[2] We use the conventions $\varepsilon_{1234} = \varepsilon_{5678} = 1$.

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From the embedding tensor, the G_0-covariant currents can be obtained,

$$L^{-1}(\partial_\mu + g\Theta_{\mathcal{MN}}B_\mu^{\mathcal{M}}t^{\mathcal{N}})L = \frac{1}{2}\mathcal{Q}_\mu^{IJ}X^{IJ} + \frac{1}{2}\mathcal{Q}_\mu^{rs}X^{rs} + \mathcal{P}_\mu^{Ir}Y^{Ir} \tag{2.6}$$

It is convenient to define the $\mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}$ tensors,

$$L^{-1}t^{\mathcal{M}}L = \mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}t^{\mathcal{A}} = \frac{1}{2}\mathcal{V}^{\mathcal{M}}{}_{IJ}X^{IJ} + \frac{1}{2}\mathcal{V}^{\mathcal{M}}{}_{rs}X^{rs} + \mathcal{V}^{\mathcal{M}}{}_{Ir}Y^{Ir} \tag{2.7}$$

and the T-tensor,

$$T_{\mathcal{A}|\mathcal{B}} = \Theta_{\mathcal{MN}}\mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}\mathcal{V}^{\mathcal{N}}{}_{\mathcal{B}} \tag{2.8}$$

The T-tensor is used to construct the tensors $A_{1,2,3}$ which will appear in the scalar potential and the supersymmetry transformations,

$$\begin{aligned} A_1^{AB} &= -\frac{1}{48}\Gamma^{IJKL}_{AB}T_{IJ|KL} \\ A_2^{A\dot{A}r} &= -\frac{1}{12}\Gamma^{IJK}_{A\dot{A}}T_{IJ|Kr} \\ A_3^{\dot{A}r\dot{B}s} &= \frac{1}{48}\delta^{rs}\Gamma^{IJKL}_{\dot{A}\dot{B}}T_{IJ|KL} + \frac{1}{2}\Gamma^{IJ}_{\dot{A}\dot{B}}T_{IJ|rs} \end{aligned} \tag{2.9}$$

where A, B and $\dot{A}, \dot{B}$ are SO(8)-spinor indices. Our conventions for the SO(8) Gamma matrices are presented in appendix A.

Here we choose the spacetime signature $\eta^{ab} = \mathrm{diag}(+---)$ as mostly negative. The bosonic Lagrangian and scalar potential are given by

$$\begin{aligned} e^{-1}\mathcal{L}_{\mathrm{bos}} &= -\frac{1}{4}R + \frac{1}{4}\mathcal{P}_\mu^{Ir}\mathcal{P}^{Ir\,\mu} + V - \frac{1}{4}e^{-1}\varepsilon^{\mu\nu\rho}g\Theta_{\mathcal{MN}}B_\mu^{\mathcal{M}}\Big(\partial_\nu B_\rho^{\mathcal{N}} + \frac{1}{3}g\Theta_{\mathcal{KL}}f^{\mathcal{NK}}{}_{\mathcal{P}}B_\nu^{\mathcal{L}}B_\rho^{\mathcal{P}}\Big) \\ V &= \frac{1}{4}g^2\Big(A_1^{AB}A_1^{AB} - \frac{1}{2}A_2^{A\dot{A}r}A_2^{A\dot{A}r}\Big) \end{aligned} \tag{2.10}$$

The supersymmetry variations take the following form

$$\begin{aligned} \delta\chi^{\dot{A}r} &= \frac{1}{2}i\Gamma^I_{A\dot{A}}\gamma^\mu\varepsilon^A\mathcal{P}_\mu^{Ir} + gA_2^{A\dot{A}r}\varepsilon^A \\ \delta\psi_\mu^A &= \Big(\partial_\mu\varepsilon^A + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\varepsilon^A + \frac{1}{4}\mathcal{Q}_\mu^{IJ}\Gamma^{IJ}_{AB}\varepsilon^B\Big) + igA_1^{AB}\gamma_\mu\varepsilon^B \end{aligned} \tag{2.11}$$

The Einstein equations of motion are

$$R_{\mu\nu} - \mathcal{P}_\mu^{Ir}\mathcal{P}_\nu^{Ir} - 4Vg_{\mu\nu} = 0 \tag{2.12}$$

and the gauge field equations of motion are

$$e\mathcal{P}^{Ir\,\lambda}\Theta_{\mathcal{QM}}\mathcal{V}^{\mathcal{M}}{}_{Ir} = \varepsilon^{\lambda\mu\nu}\Big(\Theta_{\mathcal{QM}}\partial_\mu B_\nu^{\mathcal{M}} + \frac{1}{6}gB_\mu^{\mathcal{M}}B_\nu^{\mathcal{K}}\Big(\Theta_{\mathcal{MN}}\Theta_{\mathcal{KL}}f^{\mathcal{NL}}{}_{\mathcal{Q}} + 2\Theta_{\mathcal{MN}}f^{\mathcal{LN}}{}_{\mathcal{K}}\Theta_{\mathcal{LQ}}\Big)\Big) \tag{2.13}$$

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2.1 The $n = 4$ case

The smallest number of matter multiplets where multiple supersymmetric vacua exist is $n = 4$. The symmetries of the theory are a local $G_0 = \mathrm{SO}(4) \times \mathrm{SO}(4)$ and a global $\mathrm{SO}(n)$ with $n = 4$. Consequently, the scalar potential only depends on $8 \cdot 4 - 3 \cdot 6 = 14$ fields out of the original 32. Moreover, a further consistent truncation outlined in [28] is performed where the coset representative depends only on eight of the fourteen scalars.

$$L = \begin{pmatrix} \cos A & \sin A \cosh B & \sin A \sinh B \\ -\sin A & \cos A \cosh B & \cos A \sinh B \\ 0 & \sinh B & \cosh B \end{pmatrix}$$
$$A = \mathrm{diag}(p_1, p_2, p_3, p_4)\,, \qquad B = \mathrm{diag}(q_1, q_2, q_3, q_4) \tag{2.14}$$

We will not display the general form of the tensors A_1 and A_2 defined in (2.9) here. The scalar potential has terms up to order α^2.

$$g^{-2}V = \frac{1}{2} + \frac{1}{4}\sum_i x_i^2 - \frac{1}{4}\sum_{i<j<k} x_i^2 x_j^2 x_k^2 - \frac{1}{2}\prod_i x_i^2 + \alpha\left(-\prod_i x_i y_i + \prod_i \sqrt{1 + x_i^2 + y_i^2}\right)$$
$$+ \alpha^2\left(\frac{1}{2} + \frac{1}{4}\sum_i y_i^2 - \frac{1}{4}\sum_{i<j<k} y_i^2 y_j^2 y_k^2 - \frac{1}{2}\prod_i y_i^2\right) \tag{2.15}$$

where all indices run form 1 to 4 unless otherwise indicated and we used the following definition of scalar fields

$$x_i = \cos p_i \sinh q_i\,, \qquad y_i = \sin p_i \sinh q_i \tag{2.16}$$

The $\mathcal{Q}_\mu$ and $\mathcal{P}_\mu$ currents do not depend on α, excluding the $g\Theta_{\mathcal{MN}} B_\mu^{\mathcal{M}} \mathcal{V}^{\mathcal{N}}{}_{\mathcal{A}}$ term, they are given by

$$\mathcal{Q}_\mu^{IJ} = \begin{pmatrix} 0 & 0 & 0 & 0 & \cosh q_1 \partial_\mu p_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cosh q_2 \partial_\mu p_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cosh q_3 \partial_\mu p_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cosh q_4 \partial_\mu p_4 \\ -\cosh q_1 \partial_\mu p_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\cosh q_2 \partial_\mu p_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\cosh q_3 \partial_\mu p_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\cosh q_4 \partial_\mu p_4 & 0 & 0 & 0 & 0 \end{pmatrix}_{IJ}$$
$$\mathcal{Q}_\mu^{rs} = 0$$
$$\mathcal{P}_\mu^{Ir} = \begin{pmatrix} \sinh q_1 \partial_\mu p_1 & 0 & 0 & 0 \\ 0 & \sinh q_2 \partial_\mu p_2 & 0 & 0 \\ 0 & 0 & \sinh q_3 \partial_\mu p_3 & 0 \\ 0 & 0 & 0 & \sinh q_4 \partial_\mu p_4 \\ \partial_\mu q_1 & 0 & 0 & 0 \\ 0 & \partial_\mu q_2 & 0 & 0 \\ 0 & 0 & \partial_\mu q_3 & 0 \\ 0 & 0 & 0 & \partial_\mu q_4 \end{pmatrix}_{Ir} \tag{2.17}$$

Using these matrices, we can check that the combination $\mathcal{P}_\mu^{Ir} \mathcal{V}^{JK}{}_{Ir}$ vanishes whenever the indices $J, K \in \{1, 2, 3, 4\}$ or $J, K \in \{5, 6, 7, 8\}$. This implies that there is no source for $B_\mu^{\mathcal{M}}$ in the gauge field equation of motion (2.13), so it is consistent to set $B_\mu^{\mathcal{M}} = 0$. We will make this choice from now on.

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The kinetic term for the scalars in the action (2.10) can be expressed in terms of the x_i and y_i using the relations (2.16) and takes the form

$$\frac{1}{4}\mathcal{P}^{Ir}_{\mu}\mathcal{P}^{Ir\,\mu} = -\frac{1}{4}\sum_{i=1}^{4}\frac{1}{1+x_i^2+y_i^2}\Big((1+y_i^2)(\partial_\mu\partial^\mu x_i - 2x_i y_i \partial_\mu x_i \partial^\mu y_i' + (1+x_i^2)\partial y_i \partial^\mu y_i\Big) \quad (2.18)$$

This expression will be needed for determining masses of the fluctuations of the scalar fields around the supersymmetric vacua.

2.2 Truncations and supersymmetric AdS_3 vacua

In order to make our analysis more tractable, we make further truncations to reduce the number of independent scalar fields. Below we consider three consistent truncations, which together explore the AdS_3 vacua with $\mathcal{N} = (4,4)$ and $\mathcal{N} = (1,1)$ supersymmetry. All of the results are generalizations of the $\alpha = 1$ case discussed in [1].

2.2.1 Truncation 1

The first truncation is given by denoting $q_1 = q$, $p_1 = p$ and setting all remaining $q_i = p_i = 0$ for $i = 2,3,4$. The scalar potential is

$$V = \frac{g^2}{4}\Big(2(1+\alpha^2) + 4\alpha\cosh q + (\cos^2 p + \alpha^2 \sin^2 p)\sinh^2 q\Big) \quad (2.19)$$

The $\mathcal{N} = (4,4)$ vacuum is given by setting $q = 0$ and the vacuum potential is $V_0 = \frac{1}{2}g^2(1+\alpha)^2$. In the x, y coordinates the $\mathcal{N} = (4,4)$ vacuum is given by $x_i = y_i = 0$. This is the only supersymmetric vacuum for this truncation. We note that for the choice $\alpha = 1$ the potential is independent of the scalar field p. We note we will chose $g = 1/(1+\alpha)$ in order to set the potential at the $\mathcal{N} = (4,4)$ vacuum to be $V_0 = \frac{1}{2}$, which corresponds to a unit radius AdS_3.

2.2.2 Truncation 2

The second truncation is given by setting all the q and p equal, i.e. $q_i = q$, $p_i = p$ for $i = 1,2,3,4$. The scalar potential becomes

$$\begin{aligned} V = \frac{g^2}{2}\Big\{&(1-\cos^2 p\sinh^2 q)(1+\cos^2 p\sinh^2 q)^3 + \alpha^2(1-\sin^2 p\sinh^2 q)(1+\sin^2 p\sinh^2 q)^3 \\ &+ \alpha(2+4\sinh^2 q + 2\sinh^4 q - 2\sin^4 p\cos^4 p\sinh^8 q)\Big\} \end{aligned} \quad (2.20)$$

or in terms of the x, y fields, the potential will take the following form

$$V = \frac{g^2}{2}\Big\{(x^2-1)(x^2+1)^3 + \alpha^2(y^2-1)(y^2+1)^3 + 2\alpha(2x^2(1+y^2) + (1+y^2)^2 - x^4(y^4-1))\Big\} \quad (2.21)$$

As before the $\mathcal{N} = (4,4)$ vacuum is given by $q = 0$ or $x = y = 0$. There are $\mathcal{N} = (1,1)$ vacua which are located at

$$\begin{aligned} x &= \pm\frac{1}{6\sqrt{3}\alpha}\left(-12\alpha^2 - \frac{2^{\frac{2}{3}}Y^{\frac{1}{3}}(Y + 2\alpha^2(-3+8\alpha))}{(3+2\alpha)} - \frac{2^{\frac{1}{3}}y^{\frac{2}{3}}(Y + 2\alpha(-18+\alpha(-15+8\alpha)))}{(3+2\alpha)^2}\right)^{\frac{1}{2}} \\ y &= \pm\frac{1}{3}\left(-1 + \frac{2^{\frac{2}{3}}Y^{\frac{1}{3}}}{\alpha} + \frac{4\,2^{\frac{1}{3}}(3+2\alpha)}{Y^{\frac{1}{3}}}\right)^{\frac{1}{2}} \end{aligned} \quad (2.22)$$

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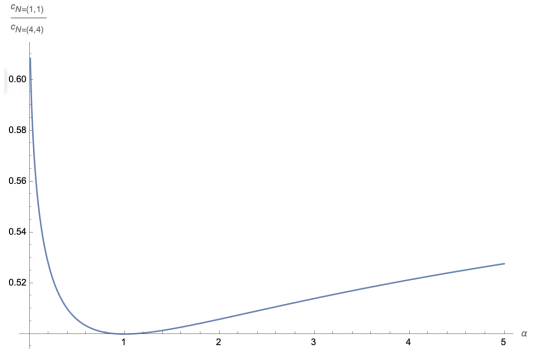

Figure 1. Ratios of central charges for the $\mathcal{N}=(1,1)$ and $\mathcal{N}=(4,4)$ vacua.

where Y is given by

$$Y = 3i\alpha^{\frac{3}{2}}\sqrt{96+3\alpha(61+32\alpha)} - \alpha^2(9+16\alpha) \tag{2.23}$$

The central charge of the dual CFT is related to the AdS radius and the value of the potential V_0 at its minimum

$$c = \frac{3R_{\text{AdS}}}{2G_N} = \frac{3}{\sqrt{2V_0}}\frac{1}{2G_N} \tag{2.24}$$

Choosing $g = 1/(1+\alpha)$ sets the AdS radius of the $\mathcal{N}=(4,4)$ vacuum to one and the ratio of the central charge of the $\mathcal{N}=(4,4)$ to the $\mathcal{N}=(1,1)$ vacuum as a function of α becomes

$$\frac{c_{\mathcal{N}=(1,1)}}{c_{\mathcal{N}=(4,4)}} = \frac{1}{\sqrt{2V_0^{N=(1,1)}(\alpha)}} \tag{2.25}$$

The expressions derived in (2.22) are not very illuminating and we present a plot of the ratio of the central charges for the two vacua in the figure 1. It is interesting to note that the ratio of central charges is minimized for the special value $\alpha = 1$.

2.2.3 Truncation 3

The third truncation is given by setting the first three q and p equal, i.e. $q_i = q$, $p_i = p$ for $i = 1,2,3$, and setting the remaining $q_4 = p_4 = 0$. The scalar potential is

$$V = \frac{g^2}{4}\Big(2{+}3\cos^2 p\sinh^2 q{-}\cos^6 p\sinh^6 q{+}4\alpha\cosh^3 q{+}\alpha^2(2{+}3\sin^2 p\sinh^2 q{-}\sin^6 p\sinh^6 q)\Big) \tag{2.26}$$

or in the x, y variables

$$V = \frac{g^2}{4}\Big((2+3x^2-x^6) + 4\alpha(1+x^2+y^2)^{\frac{3}{2}} + \alpha^2(2+3y^2-y^6)\Big) \tag{2.27}$$

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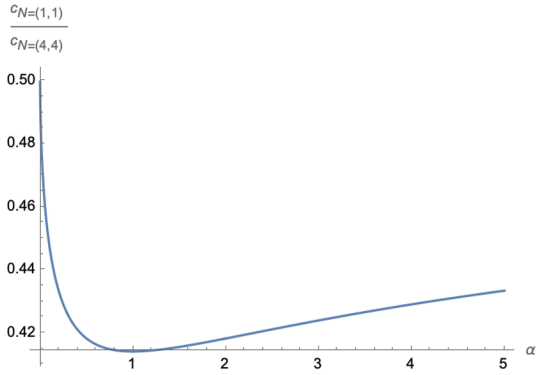

Figure 2. Ration of central charges for the $\mathcal{N}=(1,1)$ and $\mathcal{N}=(4,4)$ vacua.

The $\mathcal{N}=(4,4)$ vacuum is given by $q=0$ or $x=y=0$ as before, and $\mathcal{N}=(1,1)$ vacua can be determined by finding the extrema for the potential (2.27) away from the origin.

$$y=\pm\sqrt{\frac{1}{2}}\left(1+\varepsilon(\alpha)\sqrt{1-\frac{4}{X^{\frac{1}{3}}}+\frac{2X^{\frac{1}{3}}}{3\alpha^2}}+\sqrt{2+\frac{4}{X^{\frac{1}{3}}}-\frac{2X^{\frac{1}{3}}}{3\alpha^2}+\frac{2(\alpha^2-4)}{\alpha^2\sqrt{1-\frac{4}{X^{\frac{1}{3}}}+\frac{2X^{\frac{1}{3}}}{3\alpha^2}}}}\right)^{\frac{1}{2}}$$
$$x=\pm\left(\frac{\alpha^2}{4}(y^4-1)^4-1-y^2\right)^{\frac{1}{2}} \tag{2.28}$$

where we used the abbreviation

$$X=3\alpha^2\left(9-9\alpha^2+\sqrt{81-138\alpha^2+81\alpha^4}\right) \tag{2.29}$$

The $\varepsilon(\alpha)$ is a sign which selects a branch of the solutions which gives real x,y depending on α and we have $\varepsilon(\alpha)=+1$ for $\alpha<2$ and $\varepsilon(\alpha)=-1$ for $\alpha>2$. We can plot the ratio of the central charges which is given by (2.25), determined from the potential (2.27). We note that the qualitative behavior of the ratio for truncation 2 and 3 is very similar, in particular the central charge for the $\mathcal{N}=(1,1)$ vacuum is minimized at $\alpha=1$.

3 Janus flow equations

In this section we will derive the BPS flow equations, expanding on the construction in our previous paper [1]. The Janus ansatz for the bosonic fields is give by

$$\mathrm{d}s^2=e^{2B(u)}\left(\frac{\mathrm{d}t^2-\mathrm{d}z^2}{z^2}\right)-\mathrm{d}u^2\,,\qquad q_i=q_i(u)\,,\qquad p_i=p_i(u) \tag{3.1}$$

The Chern-Simons gauge fields is set to zero $B_\mu^{\mathcal{M}}=0$. We will check that the source term on the right hand side of the gauge field equation of motion (2.13) is zero for the solutions considered in this paper.

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The gravitino supersymmetry variation $\delta\psi_\mu^A = 0$ is

$$
\begin{aligned}
0 &= \partial_t\varepsilon + \frac{1}{2z}\gamma_0\Big(\gamma_1 - B' e^B \gamma_2 + 2ige^B A_1\Big)\varepsilon \\
0 &= \partial_z\varepsilon + \frac{1}{2z}\gamma_1\Big(-B' e^B \gamma_2 + 2ige^B A_1\Big)\varepsilon \\
0 &= \partial_u\varepsilon + \frac{1}{4}\mathcal{Q}_u^{IJ}\Gamma^{IJ}\varepsilon + ig\gamma_2 A_1\varepsilon
\end{aligned}
\tag{3.2}
$$

where we have suppressed the SO(8)-spinor indices of ε^A and A_1^{AB}. The spin-$\frac{1}{2}$ variation $\delta\chi^{\dot{A}r} = 0$ is

$$
\left(-\frac{i}{2}\Gamma^I \mathcal{P}_u^{Ir}\gamma_2 + gA_2^r\right)_{A\dot{A}}\varepsilon^A = 0\,, \qquad r = 9, 10, \ldots, 8+n \tag{3.3}
$$

The matrix A_1 defined in (2.9) has eigenvectors

$$
A_1^{AB} n_\pm^{(i)B} = \pm w_i n_\pm^{(i)A}\,, \qquad i = 1, 2, 3, 4 \tag{3.4}
$$

For a supersymmetric AdS_3 vacuum the eigenvalue w_i is related to the value of the potential evaluated at the vacuum via

$$
w_{\text{vac}}^2 = \frac{V_{\text{vac}}}{2g^2} \tag{3.5}
$$

and the associated eigenvectors $n_\pm^{(i)}$ determine the supersymmetries of the vacuum. For the $\mathcal{N} = (4,4)$ vacuum the $w_i, i = 1, \cdots 4$ all satisfy (3.5) and hence the vacuum preserves eight supersymmetries. For the $\mathcal{N} = (1,1)$ vacuum only one of the four $n_\pm^{(i)}$ and w_i satisfies (3.5). In the following we drop the index $^{(i)}$ to denote the supersymmetric eigenvalue w and the eigenvector $n_\pm^A$.

The general ansatz for unbroken supersymmetry ε^A for the Janus solution is given by

$$
\varepsilon^A = (f_+ n_+^A + f_- n_-^A)\zeta_+ + (g_+ n_+^A + g_- n_-^A)\zeta_- \tag{3.6}
$$

where $\zeta_\pm$ are Killing spinors for a unit radius AdS_2

$$
D_\mu\zeta_\eta = i\frac{\eta}{2}\gamma_\mu\zeta_\eta\,, \qquad \mu = t, z, \quad \eta = \pm 1 \tag{3.7}
$$

3.1 Gravitino variation

The t, z components of the gravitino variation can be expressed as follows by using the properties of the AdS_2 Killing spinors,

$$
\begin{aligned}
0 = {} & i\Big\{(f_+ n_+^A + f_- n_-^A)\zeta_+ - (g_+ n_+^A + g_- n_-^A)\zeta_-\Big\} && (3.8) \\
& + iB' e^B i\gamma_2\Big\{(f_+ n_+^A + f_- n_-^A)\zeta_+ + (g_+ n_+^A + g_- n_-^A)\zeta_-\Big\} \\
& + 2igwe^B\Big\{(f_+ n_+^A - f_- n_-^A)\zeta_+ + (g_+ n_+^A - g_- n_-^A)\zeta_-\Big\} && (3.9)
\end{aligned}
$$

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Using $i\gamma_2\zeta_\eta = \zeta_{-\eta}$ and the linear independence of the $n_\pm$ and $\zeta_\pm$, one obtains a set of equations,

$$\begin{aligned} f_+ + B'e^B g_+ + 2gwe^B f_+ &= 0 \\ -g_+ + B'e^B f_+ + 2gwe^B g_+ &= 0 \\ f_- + B'e^B g_- - 2gwe^B f_- &= 0 \\ -g_- + B'e^B f_- - 2gwe^B g_- &= 0 \end{aligned} \tag{3.10}$$

It is convenient to define the following expressions

$$\gamma(u) = \sqrt{1 - \frac{e^{-2B}}{4g^2w^2}}\,, \qquad \sqrt{1-\gamma^2(u)} = \frac{e^{-B}}{2gw} \tag{3.11}$$

The equations (3.10) can then be solved by

$$f_+ = \frac{\sqrt{1-\gamma^2}-1}{\gamma} g_+\,, \qquad f_- = \frac{\sqrt{1-\gamma^2}+1}{\gamma} g_- \tag{3.12}$$

if the integrability condition

$$\begin{aligned} B' &= \pm\sqrt{4g^2w^2 - e^{-2B}} \\ &= \pm 2gw\gamma \end{aligned} \tag{3.13}$$

is satisfied. This equation provides us with a differential equation for the metric factor B.

3.2 Spin $\frac{1}{2}$ variation

The spin-$\frac{1}{2}$ variation (3.3) takes the following form of a projector

$$\left(M^{(r)AB} i\gamma_2 + \delta^{AB}\right)\varepsilon^B = 0 \tag{3.14}$$

where

$$M^{(r)}_{AB} = -\frac{1}{2g}\left(\Gamma^I \mathcal{P}^{Ir}_u (A_2^r)^{-1}\right)^T_{AB} \tag{3.15}$$

Note that there is a projector for each r, which all have to be satisfied and the resulting flow equations are mutually consistent for a supersymmetric Janus solution to exist. This analysis will be performed for the particular truncations presented in section 2.2.

Inserting ε^A given by (3.6) into the spin $\frac{1}{2}$ projector gives

$$\begin{aligned} 0 = {} & (f_+ n_+^A + f_- n_-^A)\zeta_+ + (g_+ n_+^A + g_- n_-^A)\zeta_- \\ & + M^{AB} i\gamma_2 \Big\{ (f_+ n_+^B + f_- n_-^B)\zeta_+ + (g_+ n_+^B + g_- n_-^B)\zeta_- \Big\} \end{aligned} \tag{3.16}$$

We have dropped the index r for notational convenience. Using the fact that the two dimensional Killing spinors are orthogonal we can project (3.16) onto the $n_\pm$ and $\zeta_\pm$ components.

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This produces four equations

$$\begin{aligned} f_+ n_+^2 + M_{++} g_+ + M_{+-} g_- &= 0 \\ g_+ n_+^2 + M_{++} f_+ + M_{+-} f_- &= 0 \\ f_- n_-^2 + M_{+-} g_+ + M_{--} g_- &= 0 \\ g_+ n_-^2 + M_{+-} f_+ + M_{--} f_- &= 0 \end{aligned} \tag{3.17}$$

where we denoted $n_\pm^2 = n_\pm^A n_\pm^A$ and we define

$$M_{++} = n_+^A M^{AB} n_+^B \,, \qquad M_{--} = n_-^A M^{AB} n_-^B \,, \qquad M_{+-} = M_{-+} = n_+^A M^{AB} n_-^B \tag{3.18}$$

If there is more than one $n_\pm$ (as in truncation 1) one has to choose linear combinations for which $M_{\pm\pm}, M_{\pm\mp}$ take the same form for all $n_\pm^{(i)}$, which is a consistency condition. Using (3.12) it can be shown that equations (3.17) can only be satisfied if we have

$$M_{++} = \gamma n_+^2 \,, \qquad M_{--} = -\gamma n_-^2 \,, \qquad M_{+-} = M_{-+} = \sqrt{1-\gamma^2}\sqrt{n_+^2 n_-^2} \tag{3.19}$$

In all cases we consider, the M_{--} equation is automatically satisfied if the M_{++} equation is satisfied. Hence (3.19) provides two independent equations. It follows from (3.15) that these equations are linear in the first derivatives of the scalar fields and provide the BPS flow equations for the scalars. The complete set of flow equations is given by these equations and the flow equation for the metric factor (3.13), coming from the gravitino variation.

4 Janus and RG-flow solutions

In this section we obtain the flow equations and solve them numerically for the three truncations considered in this paper. Since the first truncation does not have $\mathcal{N} = (1,1)$ vacua the BPS flows will correspond to Janus solutions interpolating between $\mathcal{N} = (4,4)$ vacua. For the two other truncations we find Janus as well as RG-flow interface solutions.

4.1 Truncation 1

The matrix A_1 for this truncation is given by

$$A_1 = \begin{pmatrix} 0 & 0 & 0 & a & 0 & 0 & b & 0 \\ 0 & 0 & -a & 0 & 0 & 0 & 0 & b \\ 0 & -a & 0 & 0 & -b & 0 & 0 & 0 \\ a & 0 & 0 & 0 & 0 & -b & 0 & 0 \\ 0 & 0 & -b & 0 & 0 & 0 & 0 & -a \\ 0 & 0 & 0 & -b & 0 & 0 & a & 0 \\ b & 0 & 0 & 0 & 0 & a & 0 & 0 \\ 0 & b & 0 & 0 & -a & 0 & 0 & 0 \end{pmatrix} \tag{4.1}$$

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where

$$a = \frac{1}{2}\cos p(\alpha + \cosh q), \quad b = \frac{1}{2}\sin p(1 + \alpha\cosh q) \tag{4.2}$$

The eigenvalue of A_1 are $\pm w_0$ which is given by

$$w_0 = \sqrt{a^2 + b^2} = \frac{1}{2}\sqrt{\cos^2 p(\alpha + \cosh q)^2 + \sin^2 p(1 + \alpha\cosh q)^2} \tag{4.3}$$

The eigenvectors are given by

$$n_{\pm}^{(1)} = \begin{pmatrix} 0 \\ \frac{a \pm w_0}{b} \\ \frac{-a \mp w_0}{b} \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad n_{\pm}^{(2)} = \begin{pmatrix} 0 \\ \frac{-a \pm w_0}{b} \\ \frac{-a \pm w_0}{b} \\ 0 \\ -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad n_{\pm}^{(3)} = \begin{pmatrix} \frac{-a \pm w_0}{b} \\ 0 \\ 0 \\ \frac{a \mp w_0}{b} \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \quad , n_{\pm}^{(4)} = \begin{pmatrix} \frac{a \pm w_0}{b} \\ 0 \\ 0 \\ \frac{a \pm w_0}{b} \\ 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \tag{4.4}$$

The matrix M^{AB} defined in (3.15) takes the following form for the truncation 1

$$M = \begin{pmatrix} 0 & 0 & 0 & m_1 & 0 & 0 & -m_2 & 0 \\ 0 & 0 & -m_1 & 0 & 0 & 0 & 0 & -m_2 \\ 0 & -m_1 & 0 & 0 & m_2 & 0 & 0 & 0 \\ m_1 & 0 & 0 & 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & m_2 & 0 & 0 & 0 & 0 & -m_1 \\ 0 & 0 & 0 & m_2 & 0 & 0 & m_1 & 0 \\ -m_2 & 0 & 0 & 0 & 0 & m_1 & 0 & 0 \\ 0 & -m_2 & 0 & 0 & -m_1 & 0 & 0 & 0 \end{pmatrix} \tag{4.5}$$

with

$$m_1 = \frac{\alpha \sin p\, p' + \cos p\, \mathrm{csch}\, q\, q'}{g(\cos^2 p + \alpha^2 \sin^2 p)}, \qquad m_2 = \frac{\cos p\, p' - \alpha \sin p\, \mathrm{csch}\, q\, q'}{g(\cos^2 p + \alpha^2 \sin^2 p)}, \tag{4.6}$$

Using the definitions (4.2) and (4.3) the flow equations (3.19) for the scalars p, q and the metric function B (3.13) can be written relatively compactly

$$\begin{aligned} p' &= \frac{g}{w_0}\left(\alpha(a\gamma - b\sqrt{1-\gamma^2})\sin p - (b\gamma + a\sqrt{1-\gamma^2})\cos p\right) \\ q' &= \frac{g\sinh q}{w_0}\left(\alpha(b\gamma + a\sqrt{1-\gamma^2})\sin p + (a\gamma - b\sqrt{1-\gamma^2})\cos p\right) \\ B' &= \pm\sqrt{4g^2 w_0^2 - e^{-2B}} \end{aligned} \tag{4.7}$$

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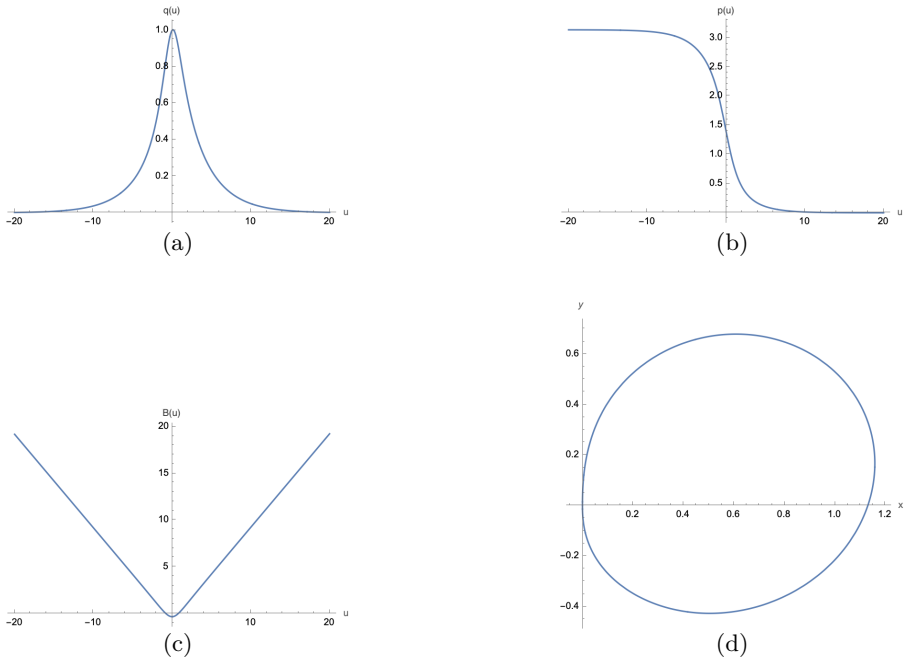

Figure 3. (a)-(c) plots of q, p, B respectively, (d) parametric plot of the Janus flow in the x, y variables. The initial conditions are $q(0) = 1.0$ and $p(0) = 1.5$ and $\alpha = 2.3$.

This system of ordinary differential equations can only be integrated numerically. We will choose the coordinate u such that the turning point of the metric function where $B'(u) = 0$ is located at $u = 0$. We then use the BPS equations (4.7) to determine $p'(0)$, $q'(0)$ and $B(0)$ for a given $q(0)$ and $p(0)$. We then integrate the equations of motion following from the variation of the Lagrangian (2.10). This means that all our solutions depend on two initial conditions $q(0)$ and $p(0)$. We have given an illustrative example of the flows we can obtain in figure 3.

4.2 Truncation 2

The matrix A_1 for this truncation is given by

$$A_1 = \begin{pmatrix} 0 & 0 & 0 & \frac{a+c}{4} & 0 & 0 & 0 & 0 \\ 0 & -\frac{c}{2} & \frac{-a+c}{4} & 0 & b & 0 & 0 & b \\ 0 & \frac{-a+c}{4} & -\frac{c}{2} & 0 & -b & 0 & 0 & -b \\ \frac{a+c}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b & -b & 0 & \frac{c}{2} & 0 & 0 & \frac{-a+c}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{a+c}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{a+c}{4} & 0 & 0 \\ 0 & b & -b & 0 & \frac{-a+c}{4} & 0 & 0 & \frac{c}{2} \end{pmatrix} \tag{4.8}$$

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where

$$
\begin{aligned}
a &= 2\cos^4 p(\alpha\cosh^4 q) + 2\sin^4 p(1+\alpha\cosh^4 q)\\
b &= \sin p\cos p\cosh q(\cos^2 p(\alpha+\cosh^2 q) - \sin^2 p(1+\alpha\cosh^2 q))\\
c &= (1+\alpha)\cosh^2 q\sin^2 2p
\end{aligned}
\tag{4.9}
$$

The eigenvectors $n^{(1)}_{\pm}$ of A_1 with eigenvalues $\pm w_0$ corresponding to the unbroken $\mathcal{N}=(1,1)$ supersymmetries are given by

$$
n^{(1)}_{\pm} = \begin{pmatrix} 0 \\ \frac{a-3c\pm 4w_0}{8b} \\ -\frac{a-3c\pm 4w_0}{8b} \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \qquad w_0 = \frac{1}{4}\sqrt{64b^2+(3c-a)^2}
\tag{4.10}
$$

We have checked that the extremum (2.22) does satisfy the supersymmetry condition (3.5) for the w_0 defined above and hence corresponds to an AdS vacuum with $\mathcal{N}=(1,1)$ supersymmetry. The rest of the eigenvectors of A_1 do not have eigenvalues which satisfy the supersymmetry condition (3.5) for the $\mathcal{N}=(1,1)$ vacuum. We chose the initial conditions the same way as in section 4.1.

In figure 4 we display examples of solutions to the flow equations representing Janus flows between $\mathcal{N}=(4,4)$ vacua, $\mathcal{N}=(1,1)$ vacua and RG-flow Janus solutions between $\mathcal{N}=(4,4)$ and $\mathcal{N}=(1,1)$ vacua. We note that the flows involving the $\mathcal{N}=(1,1)$ vacua are a new feature of the truncation. As discussed in section 5.1 the $\mathcal{N}=(1,1)$ is a repulsive fixed point of the flow and to obtain the numerical solutions one has to fine tune the initial conditions at the turning point to approach the $\mathcal{N}=(1,1)$ vacuum. This implies that choosing an initial $p(0)$, the initial $q(0)$ for which an RG-flow solution exists, is fixed. A third kind of flow solution corresponds to a Janus solution interpolating between $\mathcal{N}=(1,1)$ vacua, since both vacua are repulsive such solutions only exist for a discrete set of initial conditions. Note that the asymptotic value of p when the $\mathcal{N}=(4,4)$ vacuum is obtained can take any value and determines the angle with which the point $(x,y)=(0,0)$ is approached in the parametric x,y plot.

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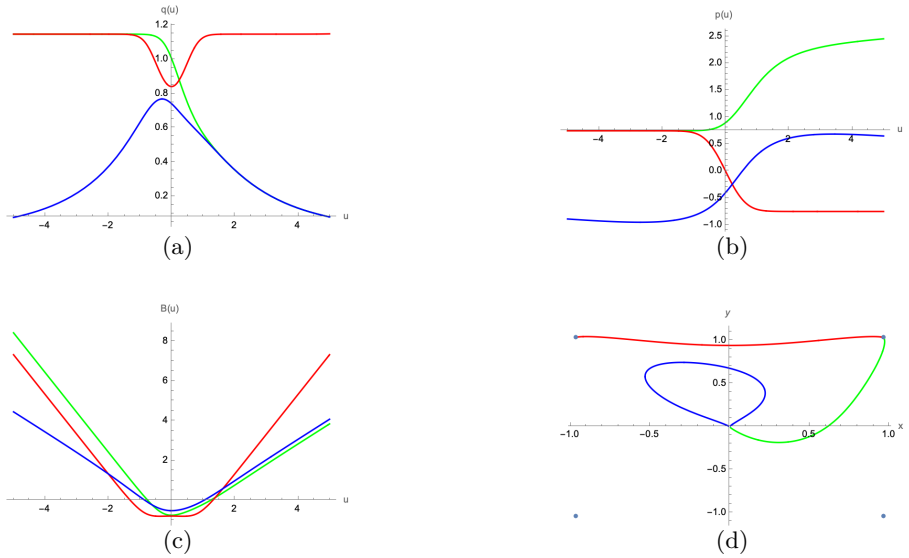

Figure 4. Truncation 2: (a)-(c) plots of p, q, B respectively, (d) parametric plot of the Janus flow in the x, y variables, the $\mathcal{N} = (4, 4)$ vacuum is at the origin and the dots denote the locations of the $\mathcal{N} = (1, 1)$ vacua. Blue: Janus between $\mathcal{N} = (4, 4)$ vacua, red: Janus between $\mathcal{N} = (1, 1)$ vacua, green: RG-Janus between $\mathcal{N} = (4, 4)$ and $\mathcal{N} = (1, 1)$. We have set $\alpha = 1.2$ for these examples.

4.3 Truncation 3

The matrix A_1 for this truncation is given by

$$A_1 = \begin{pmatrix} 0 & 0 & 0 & \frac{b+c}{2} & 0 & \frac{a+d}{2} & 0 & 0 \\ 0 & -c & \frac{-b+c}{2} & 0 & \frac{-a+d}{2} & 0 & 0 & d \\ 0 & \frac{-b+c}{4} & -c & 0 & -d & 0 & 0 & \frac{a-d}{2} \\ \frac{b+c}{2} & 0 & 0 & 0 & 0 & 0 & \frac{-a-d}{2} & 0 \\ 0 & \frac{-a+d}{2} & -d & 0 & c & 0 & 0 & \frac{-b+c}{2} \\ \frac{a+d}{2} & 0 & 0 & 0 & 0 & 0 & \frac{b+c}{2} & 0 \\ 0 & 0 & 0 & \frac{-a-d}{2} & 0 & \frac{b+c}{2} & 0 & 0 \\ 0 & d & \frac{a-d}{2} & 0 & \frac{-b+c}{2} & 0 & 0 & c \end{pmatrix} \tag{4.11}$$

where

$$\begin{aligned} a &= \sin^3 p(1 + \alpha \cosh^3 q) \\ b &= \cos^3 p(\alpha + \cosh^3 q) \\ c &= \sin^2 p \cos p \cosh q(1 + \alpha \cosh q) \\ d &= \cos^2 p \sin p \cosh q(\alpha + \cosh q) \end{aligned} \tag{4.12}$$

The eigenvectors $n_{\pm}^{(1)}$ of A_1 with eigenvalues $\pm w_0$ corresponding to the unbroken $\mathcal{N} =$

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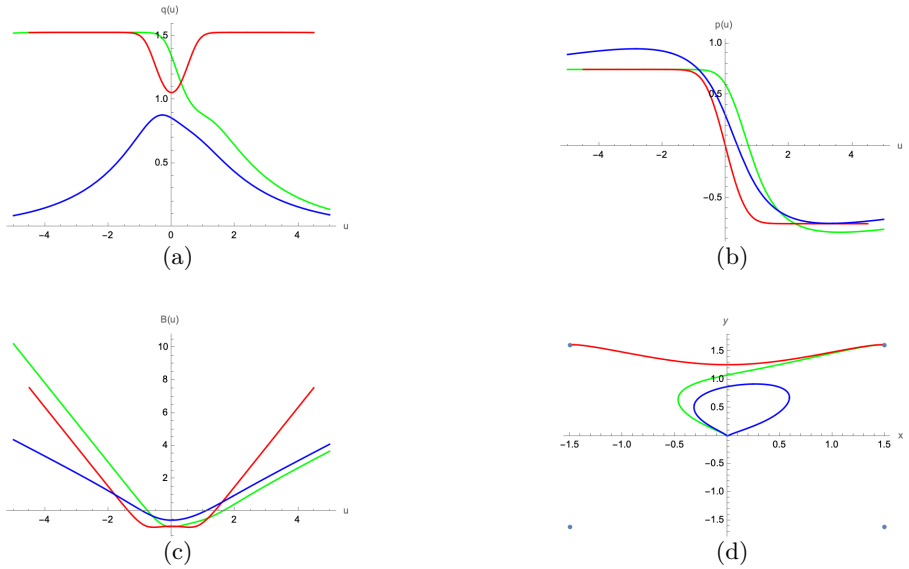

(a) (b) (c) (d)

Figure 5. Truncation 3: (a)-(c) plots of p, q, B respectively, (d) parametric plot of the Janus flow in the x, y variables, the $\mathcal{N} = (4, 4)$ vacuum is at the origin and the dots denote the locations of the $\mathcal{N} = (1, 1)$ vacua. Blue: Janus between $\mathcal{N} = (4, 4)$ vacua, red: Janus between $\mathcal{N} = (1, 1)$ vacua, green: RG-Janus between $\mathcal{N} = (4, 4)$ and $\mathcal{N} = (1, 1)$. We have set $\alpha = 1.2$ for these examples.

$(1, 1)$ supersymmetries are given by

$$n_{\pm}^{(1)} = \begin{pmatrix} 0 \\ -\frac{b-3c\pm 2w_0}{a-3d} \\ \frac{b-3c\pm 2w_0}{a-3d} \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad w_0 = \frac{1}{2}\sqrt{(a-3d)^2 + (b-3c)^2} \tag{4.13}$$

The rest of the eigenvectors of A_1 do not have eigenvalues which satisfy the supersymmetry condition (3.5) for the $\mathcal{N} = (1, 1)$ vacuum. Note that all of them reduce to the ones of truncation 1 for the $\mathcal{N} = (4, 4)$ vacuum.

In figure 5 we display a sample of solutions to the flow equations representing Janus flows between $\mathcal{N} = (4, 4)$ vacua, $\mathcal{N} = (1, 1)$ vacua and RG-flow Janus solutions between an $\mathcal{N} = (4, 4)$ and $\mathcal{N} = (1, 1)$ vacuum. We note that the solutions behave qualitatively similar to the ones displayed for truncation 2.

5 Holographic calculations

In this section we will perform some holographic calculations for the solutions obtained in the section 4. In particular we will calculate the masses for the fluctuations of the scalar

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$\mathcal{N}=(4,4)$	m^2	Δ_+	Δ_-
δx_i	$-\frac{1+2\alpha}{(1+\alpha)^2}$	$\frac{1+2\alpha}{1+\alpha}$	$\frac{1}{1+\alpha}$
δy_i	$-\frac{\alpha(2+\alpha)}{(1+\alpha)^2}$	$\frac{2+\alpha}{1+\alpha}$	$\frac{\alpha}{1+\alpha}$

Table 1. Mass and conformal dimensions of scalar fluctuations for the $\mathcal{N}=(4,4)$ vacuum.

fields around the $\mathcal{N}=(4,4)$ and $\mathcal{N}=(1,1)$ vacua. This will allow us to identify the dimensions of the dual operators which are turned on in the flows. One of the results is that for truncation 2 and 3 the mass squared of the fluctuations are positive, corresponding to operators with scaling dimensions $\Delta > 2$. Since the behavior near the AdS vacuum is given by

$$\lim_{\varepsilon\to 0}\phi \sim \bar{\phi} + c_1\varepsilon^{\Delta} + c_2\varepsilon^{2-\Delta} + \cdots \tag{5.1}$$

where $\varepsilon \to 0$ corresponds to approaching the AdS boundary, the initial conditions have to be fine tuned in order to make the repulsive term $c_2\varepsilon^{2-\Delta}$ very small. In addition we consider the entanglement entropy of a symmetric region around the defect [34–38] and give a prescription to obtain the defect entropy (or g-factor) [39].

5.1 Operator spectrum

The $\mathcal{N}=(4,4)$ vacuum has $q_i = 0, i = 1,2,3,4$. Since the kinetic terms for p_i are vanishing the x_i, y_i defined in (2.16) are better suited to analyze the fluctuations. Expanding around the $x_i = y_i = 0$ vacuum one finds for the quadratic term of the fluctuations,

$$\frac{1}{e}\mathcal{L}_{(2)} = \frac{1}{4}\sum_i \left(\partial_\mu\delta x_i\partial^\mu\delta x_i + \partial_\mu\delta y_i\partial^\mu\delta y_i\right) + \frac{g_c^2}{4}\sum_i\left((1+2\alpha)\delta x_i^2 + \alpha(\alpha+2)\delta y_i^2\right) \tag{5.2}$$

from which we can read off the masses of the scalar fluctuations. Then the masses determine the conformal scaling dimensions

$$\Delta_\pm = 1 \pm \sqrt{1+m^2R^2} \tag{5.3}$$

where R is the AdS radius of the vacuum. Setting $g_c = 1/(1+\alpha)$ to obtain a unit radius AdS_3 for the $\mathcal{N}=(4,4)$ vacuum and the standard AdS/CFT relation the conformal dimensions of the dual operators are displayed in table 1. Note that Δ_+ gives the scaling dimension of the dual operator in the standard quantization which takes values between $1 < \Delta_+ < 2$ for $\alpha > 0$, whereas Δ_- corresponds to the alternative quantization and $0 < \Delta_- < 1$ for $\alpha > 0$. Supersymmetric flows are related to the standard quantization which we will adapt in the following [40]. We note that the $\mathcal{N}=(4,4)$ vacuum is attractive since both x and y are dual to operators with $\Delta < 2$ and the initial conditions do not have to be fine tuned for (5.1) to approach the vacuum value.

For truncations 2 and 3 we can determine the scaling dimensions of the operators at the $\mathcal{N}=(1,1)$ vacuum by expanding the scalar action around the vacuum to second order and diagonalizing the resulting scalar Lagrangian. The resulting expressions are quite unwieldy and we present the plots of the scaling dimensions of the two modes as a function of α in figure 6. We note that the scaling dimensions are larger than 2 and hence the $\mathcal{N}=(1,1)$ corresponds to a repulsive fixed point and the initial conditions have to be fine-tuned.

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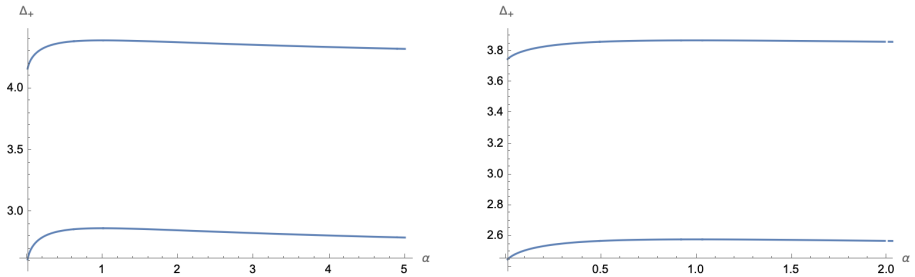

(a) (b)

Figure 6. (a) Conformal dimension of operator dual to scalar fluctuations around the $\mathcal{N} = (1, 1)$ vacuum for truncation 2, (b) same for truncation 3.

5.2 Holographic entanglement entropy

The Ryu-Takayanagi prescription [41] relates holographic entanglement entropy to the area of a minimal surface in the bulk which when approaching the AdS boundary ends at the border of the entangling surface. For a three dimensional static bulk spacetime this corresponds to a geodesic in the bulk which terminates at the ends of the entangling interval on the boundary. For the AdS_2 sliced metric (3.1) and an entangling surface which is symmetric about the defect and of length $2L$, such a geodesic is simply parameterized by u and constant $z = L$. The entanglement entropy is then given by

$$S_{EE}(L) = \frac{1}{4G_N}\int_{u_{-\infty}}^{u_{+\infty}} du = \frac{1}{4G_N}(u_{-\infty} - u_{+\infty}) \tag{5.4}$$

where $u_{\pm\infty}$ will be related to an UV Fefferman-Graham cutoff in the following. We will generalize the derivation of [34, 35] to the case of an RG-flow interface where the AdS radius and hence the central charge take different values on both sides of the interface. The asymptotic behavior of the metric is determined by the metric function $B(u)$ as $u \to \pm\infty$

$$\lim_{u\to\pm\infty} B(u) = \pm\frac{u}{R_\pm} + \ln\lambda_\pm - \ln 2 + o\left(\frac{1}{u}\right) \tag{5.5}$$

In the two asymptotic regions we can define a Fefferman-Graham coordinate system by defining a new coordinates $\hat{u}_\pm$

$$u \to \pm\infty : \quad u = R_\pm\hat{u}_\pm \mp R_\pm \ln\lambda_\pm + o\left(\frac{1}{u}\right) \tag{5.6}$$

and then the coordinates $\zeta_\pm, \eta$

$$\begin{aligned} u \to +\infty : \qquad & e^{-2\hat{u}_+} = \frac{1}{4}\frac{\zeta_+^2}{\eta^2} + o(\zeta_+^4), \qquad & z = \eta\left(1 + \frac{1}{2}\frac{\zeta_+^2}{\eta^2}\right) + o(\zeta_+^4) \\ u \to -\infty : \qquad & e^{2\hat{u}_-} = \frac{1}{4}\frac{\zeta_-^2}{\eta^2} + o(\zeta_-^4), \qquad & z = -\eta\left(1 + \frac{1}{2}\frac{\zeta_-^2}{\eta^2}\right) + o(\zeta_-^4) \end{aligned} \tag{5.7}$$

This expansion is valid for $\eta \gg \zeta_\pm$, i.e. if we consider an entanglement interval which is far away from the interface. In this limit the metric becomes

$$\zeta_\pm \to 0 \qquad ds^2 = R_\pm^2\left(\frac{-d\zeta_\pm^2 - d\eta^2 + dt^2}{\zeta_\pm^2}\right) + o(1) \tag{5.8}$$

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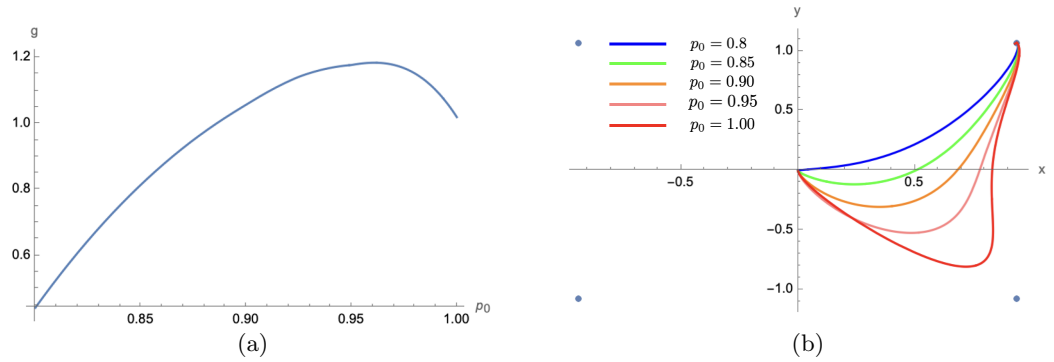

(a) (b)

Figure 7. (a) Plot of boundary entropy for RG-flow interface for truncation 2, as a function of initial condition p_0 at the turning point for $\alpha = 1.4$. (b) Illustration of RG-flows for some initial values of p_0.

It follows that $R_\pm$ defined in (5.5), corresponds to the asymptotic AdS radius and the left and right side of the interface respectively and a Fefferman-Graham cutoff is given by setting $\zeta_\pm = \varepsilon$. For the entanglement region located at $z = L$ in follows from (5.8) that the FG cutoff is related to the $u_\pm$ cutoff as follows

$$u_\pm = \mp R_{\pm\infty} \ln\left(\frac{1}{2}\frac{\varepsilon}{L}\right) \mp R_\pm \ln \lambda_\pm \tag{5.9}$$

Plugging this into (5.4) gives the entanglement entropy

$$\begin{aligned} S_{EE} &= \frac{1}{4G_N}\left((R_+ + R_-)\ln\frac{L}{2\varepsilon} - R_+ \ln\lambda_+ - R_- \ln\lambda_-\right) \\ &= \frac{c_+ + c_-}{6}\ln\frac{L}{2\varepsilon} - \frac{c_+}{6}\ln\lambda_+ - \frac{c_-}{6}\ln\lambda_- + o(\varepsilon) \end{aligned} \tag{5.10}$$

the constant term gives the boundary entropy

$$g = -\frac{c_+}{6}\ln\lambda_+ - \frac{c_-}{6}\ln\lambda_- \tag{5.11}$$

Where $c_\pm$ is the central charge for the two CFTs on either side of the RG interface. The g-factor is given by the second and third term in (5.10). For a Janus interface we have $c_+ = c_- = c$, whereas the central charges differ on both sides of the interface for a RG-flow interface. It is straightforward to determine the $R_\pm$ and $\ln\lambda_\pm$ by numerically fitting the metric functions (see plot (c) in figures 4 and 5) to determine the slope and the intercept (5.5) in the limit of large $|u|$. We will give an example of numerical results by presenting the g-factors as for the RG-interface between the $\mathcal{N} = (4,4)$ vacuum and a $\mathcal{N} = (1,1)$ vacuum in truncation 2. As discussed in section 4.2 there exists a unique RG-flow interface for a choice on initial condition p_0. In figure 7 we present the g-factor as a function of the initial condition for a particular value of $\alpha = 1.4$.

6 Discussion

In this paper we found holographic interface solutions in three dimensional gauged supergravity theories. An important feature of these theories is that they have AdS vacua which

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preserve $\mathcal{N} = (1, 1)$ supersymmetry in addition to the $\mathcal{N} = (4, 4)$ AdS vacuum. This feature allows us to find solutions which correspond to interfaces between two $\mathcal{N} = (4, 4)$ vacua on both side, $\mathcal{N} = (1, 1)$ on both sides, as well as RG-flow interfaces which have a $\mathcal{N} = (4, 4)$ on one side and $\mathcal{N} = (1, 1)$ vacuum on the other. We derived BPS flow equations which are three first order nonlinear differential equations for the two scalars p, q which are non zero in the truncations as well as the warp factor B of the AdS_2 slicing. By using the freedom to shift the warping coordinate u by a constant we can choose the initial conditions for the flow as the value of p and q at the turning point of the warp factor, where $B' = 0$. In fact we use the BPS equations to determine the initial conditions for the second order equation motion following from the variation of the action. The numerical accuracy of the solution is tested by checking the BPS equations away from the point where the initial conditions were fixed.

The $\mathcal{N} = (1, 1)$ extrema are repulsive fixed points of the flow and hence the initial condition have to be fine tuned using a shooting method. This is possible by fixing one scalar initial condition and varying the other in order to come closer and closer to the $\mathcal{N} = (1, 1)$ vacuum in the flow. Our results indicate that the qualitative behavior of the solutions for general α is quite similar to the behavior of the $\alpha = 1$ solutions obtained in [1]. In addition we have considered the entanglement entropy for the Janus and RG-flow solutions. Since for the RG-flow solutions the central charges and hence AdS radii are different on both sides of the interface one has to carefully consider the UV cut-off. It is possible to determine the g-function or interface entropy from the numerical solution by a linear fit of the warp factor B.

We have considered truncations of the scalars to two nonzero scalars q and p (or x and y), it would be interesting to generalize this since it would then be possible to consider more complicated flows between different $\mathcal{N} = (1, 1)$ vacua. It would also be interesting to investigate the solutions we have found can be lifted and have a representation in $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ holography. It would also be interesting to see whether the prescription for the interface entropy can be applied to other examples of RG-flow interfaces. We leave these interesting questions for future work.

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A SO(8) Gamma matrices

The formulation of the $N = 8$ three dimensional gauged supergravity utilizes with 8×8 Gamma matrices $\Gamma^I_{A\dot{A}}$ and their transposes $\Gamma^I_{\dot{A}A}$, they satisfy the Clifford algebra,

$$\Gamma^I_{A\dot{A}}\Gamma^J_{\dot{A}B} + \Gamma^J_{A\dot{A}}\Gamma^I_{\dot{A}B} = 2\delta^{IJ}\delta_{AB} \tag{A.1}$$

Explicitly, we use the basis as given in Green-Schwarz-Witten [42],

$$\begin{aligned}
\Gamma^8_{A\dot{A}} &= 1 \otimes 1 \otimes 1\,, & \Gamma^1_{A\dot{A}} &= i\sigma_2 \otimes i\sigma_2 \otimes i\sigma_2 \\
\Gamma^2_{A\dot{A}} &= 1 \otimes \sigma_1 \otimes i\sigma_2\,, & \Gamma^3_{A\dot{A}} &= 1 \otimes \sigma_3 \otimes i\sigma_2 \\
\Gamma^4_{A\dot{A}} &= \sigma_1 \otimes i\sigma_2 \otimes 1\,, & \Gamma^5_{A\dot{A}} &= \sigma_3 \otimes i\sigma_2 \otimes 1 \\
\Gamma^6_{A\dot{A}} &= i\sigma_2 \otimes 1 \otimes \sigma_1\,, & \Gamma^7_{A\dot{A}} &= i\sigma_2 \otimes 1 \otimes \sigma_3
\end{aligned} \tag{A.2}$$

The matrices Γ^{IJ}_{AB}, $\Gamma^{IJ}_{\dot{A}\dot{B}}$ and similar are defined as antisymmetrized products of Γs with the appropriate indices contracted. For example,

$$\Gamma^{IJ}_{AB} \equiv \frac{1}{2}(\Gamma^I_{A\dot{A}}\Gamma^J_{\dot{A}B} - \Gamma^J_{A\dot{A}}\Gamma^I_{\dot{A}B}) \tag{A.3}$$

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