Secure Joint Communication and Sensing

Onur Günlü[1], Matthieu Bloch[2], Rafael F. Schaefer[1], and Aylin Yener[3]
[1]Chair of Communications Engineering and Security, University of Siegen, {onur.guenlue, rafael.schaefer}@uni-siegen.de
[2]School of Electrical and Computer Engineering, Georgia Institute of Technology, matthieu.bloch@ece.gatech.edu
[3]Department of Electrical and Computer Engineering, The Ohio State University, yener@ece.osu.edu

***Abstract*—This work considers mitigation of information leakage between communication and sensing operations in joint communication and sensing systems. Specifically, a discrete memoryless state-dependent broadcast channel model is studied in which (i) the presence of feedback enables a transmitter to simultaneously achieve reliable communication and channel state estimation; (ii) one of the receivers is treated as an eavesdropper whose state should be estimated but which should remain oblivious to a part of the transmitted information. The model abstracts the challenges behind security for joint communication and sensing if one views the channel state as a characteristic of the receiver, e.g., its location. For independent and identically distributed (i.i.d.) states, perfect output feedback, and when part of the transmitted message should be kept secret, a partial characterization of the secrecy-distortion region is developed. The characterization is exact when the broadcast channel is either physically-degraded or reversely-physically-degraded. The characterization is also extended to the situation in which the entire transmitted message should be kept secret. The benefits of a joint approach compared to separation-based secure communication and state-sensing methods are illustrated with a binary joint communication and sensing model.**

I. Introduction

The vision for next generation mobile communication networks includes a seamless integration of the physical and digital world. Key to its success is the network's ability to automatically react to changing environments thanks to tight harmonization of communication and sensing [1]. For instance, a mmWave joint communication and radar system can be used to detect a target or to estimate crucial parameters relevant to communication and adapt the communication scheme accordingly [2]. Joint communication and sensing (JCAS) techniques are envisioned more broadly as key enablers for a wide range of applications, including connected vehicles and drones.

Several information-theoretic studies of JCAS have been initiated, drawing on existing results for joint communication and state estimation [3]–[6]. Motivated by the integration of communication and radar for mmWave vehicular applications, [7] considers a model in which messages are encoded and sent through a state-dependent channel with generalized feedback both to reliably communicate with a receiver and to estimate the channel state by using the feedback and transmitted codewords. The optimal trade-off between the communication rate and channel-state estimation distortion is then characterized for memoryless JCAS channels and i.i.d. channel states that are causally available at the receiver and estimated at the transmitter by using a strictly causal channel output. Follow up works have extended the model to multiple access channels [8] and broadcast channels [9].

The nature of JCAS mandates the use of a single modality for the communication and sensing functions so that sensing signals carry information, which then creates situations in which leakage of sensitive information can occur. For example, a target illuminated for sensing its range has the ability to gather potentially sensitive information about the transmitted message [10]. As the sensing performance and secrecy performance are both measured with respect to the signal received at the sensed target, there exists a trade-off between the two [2]. To capture and characterize this trade-off, we extend the JCAS model in [7] by introducing an eavesdropper in the network. The objective of the transmitter is then to simultaneously communicate reliably with the legitimate receiver, estimate the channel state, and hide a part of the message from the eavesdropper. The channel state is modeled as a two-component state capturing the characteristics of each individual receiver, the feedback is modeled as perfect output feedback for simplicity, and the transmitted message is divided into two parts, only one of which should be kept (strongly) secret (this is called partial secrecy in [11]). We develop inner and outer bounds for the secrecy-distortion region of this partial-secrecy scenario under a strong secrecy constraint when i.i.d. channel states are causally available at the corresponding receivers. The bounds match when the JCAS channel is physically- or reversely-physically-degraded and the outer bound also applies to the case of noisy generalized feedback. We also extend these characterizations to the case in which the entire transmitted message should be kept secret. The proposed secure JCAS models can be viewed as extensions of the wiretap channel with feedback models [12]–[19]. Our achievability proof leverages the output statistics of random binning (OSRB) method [20]–[22] to obtain strong secrecy. A binary JCAS channel example with multiplicative Bernoulli states illustrates how secure JCAS methods may outperform separation-based secure communication and state-sensing methods.

II. Problem Definition

We consider the secure JCAS model shown in Fig. 1, which includes a transmitter equipped with a state estimator, a legitimate receiver, and an eavesdropper (Eve). The transmitter attempts to reliably transmit a uniformly distributed message $M = (M_1, M_2) \in \mathcal{M} = \mathcal{M}_1 \times \mathcal{M}_2$ through a memoryless state-dependent JCAS channel with known statistics $P_{Y_1Y_2Z|S_1S_2X}$ and i.i.d. state sequence $(S_1^n, S_2^n) \in \mathcal{S}_1^n \times \mathcal{S}_2^n$

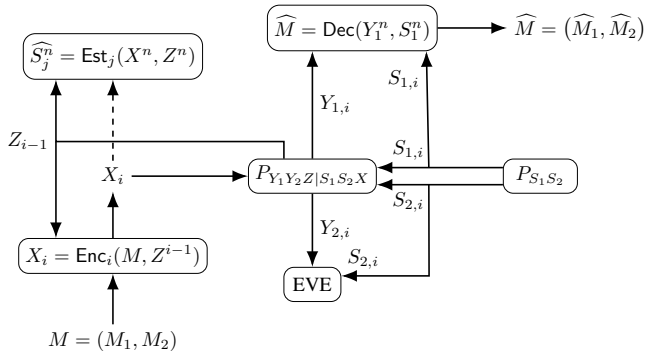

Fig. 1. JCAS model with partial secrecy, where only M_2 should be kept secret from Eve, for $j = 1, 2$ and $i = [1 : n]$. We mainly consider JCAS with perfect output feedback, where $Z_{i-1} = (Y_{1,i-1}, Y_{2,i-1})$.

generated according to a known joint probability distribution $P_{S_1S_2}$. The transmitter calculates the channel inputs X^n as $X_i = \mathsf{Enc}_i(M, Z^{i-1}) \in \mathcal{X}$ for all $i = [1 : n]$, where $\mathsf{Enc}_i(\cdot)$ is an encoding function and $Z^{i-1} \in \mathcal{Z}^{i-1}$ is the delayed channel output feedback. The legitimate receiver that observes $Y_{1,i} \in \mathcal{Y}_1$ and $S_{1,i}$ for all channel uses $i = [1 : n]$ should reliably decode both M_1 and M_2 by forming the estimate $\widehat{M} = \mathsf{Dec}(Y_1^n, S_1^n)$, where $\mathsf{Dec}(\cdot)$ is a decoding function. The eavesdropper that observes $Y_{2,i} \in \mathcal{Y}_2$ and $S_{2,i}$ should be kept ignorant of M_2. Finally, the transmitter estimates the state sequence (S_1^n, S_2^n) as $\widehat{S_j^n} = \mathsf{Est}_j(X^n, Z^n) \in \widehat{\mathcal{S}_j}^n$ for $j = 1, 2$, where $\mathsf{Est}_j(\cdot, \cdot)$ is an estimation function. Unless specified otherwise, all sets $\mathcal{S}_1$, $\mathcal{S}_2$, $\widehat{\mathcal{S}}_1$, $\widehat{\mathcal{S}}_2$, $\mathcal{X}$, $\mathcal{Y}_1$, $\mathcal{Y}_2$, and $\mathcal{Z}$ are finite.

For simplicity, we consider the perfect output feedback case in which for all $i = [2 : n]$ we have

$$Z_{i-1} = (Y_{1,i-1}, Y_{2,i-1}). \tag{1}$$

Although this is explicitly used in our achievability proofs, some of our converse results hold for generalized feedback. We next define the strong secrecy-distortion region for the problem of interest.

Definition 1. A secrecy-distortion tuple (R_1, R_2, D_1, D_2) is *achievable* if, for any $\delta > 0$, there exist $n \geq 1$, one encoder, one decoder, and two estimators $\mathsf{Est}_j(X^n, Y_1^n, Y_2^n) = \widehat{S_j^n}$ such that

$$\frac{1}{n} \log |\mathcal{M}_j| \geq R_j - \delta \qquad \text{for } j = 1, 2 \quad \text{(rates)} \tag{2}$$

$$\Pr\left[M \neq \widehat{M}\right] \leq \delta \qquad \text{(reliability)} \tag{3}$$

$$I(M_2; Y_2^n | S_2^n) \leq \delta \qquad \text{(strong secrecy)} \tag{4}$$

$$\mathbb{E}\left[d_j(S_j^n, \widehat{S_j^n})\right] \leq D_j + \delta \qquad \text{for } j = 1, 2 \quad \text{(distortions)} \tag{5}$$

where $d_j(s^n, \widehat{s^n}) = \frac{1}{n} \sum_{i=1}^n d_j(s_i, \widehat{s}_i)$ for $j = 1, 2$ are bounded per-letter distortion metrics.

The secrecy-distortion region $\mathcal{R}_{\text{PS,POF}}$ is the closure of the set of all achievable tuples with partial secrecy and perfect output feedback.

The use of per-letter distortion metrics $d_j(\cdot, \cdot)$ in conjunction with i.i.d. states simplifies the problem to a rate distortion region characterization [7]–[9]; in fact, past observations are independent of present and future ones, lending the transmitter no state prediction ability to adapt its transmission on the fly. Analyzing JCAS models with memory leads to conceptually different results, see, e.g., [23].

Remark 1. The strong secrecy condition (4) is equivalent to $I(M_2; Y_2^n, S_2^n) \leq \delta$ since the transmitted message is independent of the state sequence.

III. Bounds for JCAS with Partial-Secrecy

We next provide inner and outer bounds for the secrecy-distortion region $\mathcal{R}_{\text{PS,POF}}$; only a proof sketch is provided, full details are available in [24, Section VI].

Define $[a]^+ = \max\{a, 0\}$ for $a \in \mathbb{R}$.

Proposition 1 (Inner Bound)**.** $\mathcal{R}_{\text{PS,POF}}$ *includes the union over all joint distributions* $P_U P_{V|U} P_{X|V}$ *of the rate tuples* (R_1, R_2, D_1, D_2) *such that*

$$R_1 \leq I(U; Y_1 | S_1) \tag{6}$$

$$R_2 \leq \min\{R_2',\ (I(V; Y_1 | S_1) - R_1)\} \tag{7}$$

$$D_j \geq \mathbb{E}[d_j(S_j, \widehat{S}_j))] \qquad \textit{for } j = 1, 2 \tag{8}$$

where

$$P_{UVXY_1Y_2S_1S_2} = P_{U|V} P_{V|X} P_X P_{S_1S_2} P_{Y_1Y_2|S_1S_2X}, \tag{9}$$

$$R_2' = [I(V; Y_1 | S_1, U) - I(V; Y_2 | S_2, U)]^+ + H(Y_1 | Y_2, S_2, S_1, V) \tag{10}$$

and $\mathsf{Est}_j^*(x, y_1, y_2) = \hat{s}_j$ *for* $j = 1, 2$ *are per-letter state estimators such that* $d_j(x, y_1, y_2)$ *is equal to*

$$\underset{\tilde{s} \in \widehat{\mathcal{S}}_j}{\operatorname{argmin}} \sum_{s_j \in \mathcal{S}_j} P_{S_j|XY_1Y_2}(s_j | x, y_1, y_2)\, d_j(s_j, \tilde{s}). \tag{11}$$

One can limit $|\mathcal{U}|$ *to*

$$(\min\{|\mathcal{X}|, |\mathcal{Y}_1|, |\mathcal{Y}_2|\} + 2) \tag{12}$$

and $|\mathcal{V}|$ *to*

$$(\min\{|\mathcal{X}|, |\mathcal{Y}_1|, |\mathcal{Y}_2|\} + 2) \cdot (\min\{|\mathcal{X}|, |\mathcal{Y}_1|, |\mathcal{Y}_2|\} + 1). \tag{13}$$

Proposition 2 (Outer Bound)**.** $\mathcal{R}_{\text{PS,POF}}$ *is included in the union over all joint distributions* P_{VX} *of the rate tuples in (8) and*

$$R_1 \leq I(V; Y_1 | S_1) \tag{14}$$

$$R_2 \leq \min\Big\{\big(H(Y_1, S_1 | Y_2, S_2) - H(S_1 | Y_2, S_2, V)\big), \big(I(V; Y_1 | S_1) - R_1\big)\Big\} \tag{15}$$

where (9) with constant U *follows and we can apply the deterministic per-letter estimators* $\mathsf{Est}_j^*(x, y_1, y_2) = \hat{s}_j$ *for* $j = 1, 2$ *by using (11). One can limit the cardinality to*

$$|\mathcal{V}| \leq (\min\{|\mathcal{X}|, |\mathcal{Y}_1|, |\mathcal{Y}_2|\} + 1). \tag{16}$$

Proof Sketches for Propositions 1 and 2: We use the OSRB method [21], [22] for the achievability proofs, applying the steps in [25, Section 1.6], see also [26]. While the strictly causal observation of the i.i.d. state through feedback does not provide opportunities to improve reliability, feedback offers significant opportunities to improve secrecy. Hence, we apply a block Markov coding scheme that consists of $b \geq 2$ transmission blocks, each with n channel uses, to transmit $(b-1)$ independent messages $M(k) = (M_1(k), M_2(k))$. In every block, secret keys are distilled from the states and used to protect messages in the subsequent block. In the following, all n-letter random variables are i.i.d. according to (9) for all $k = [1:b]$, obtained by fixing $P_{U|V}$, $P_{V|X}$, and P_X so that there exist associated per-letter estimators $\mathsf{Est}_j(x, y_1, y_2) = \widehat{S_j}$ for $j = 1, 2$ that satisfy

$$\mathbb{E}[d_j(S_j^n, \mathsf{Est}_j^n(X^n, Y_1^n, Y_2^n))] \leq D_j + \epsilon_n \tag{17}$$

where $\epsilon_n > 0$ such that $\epsilon_n \to 0$ when $n \to \infty$. The block k under consideration is indicated by adding the argument (k) to the variables, e.g., $M(k)$ refers to the message in block k, etc.

For all blocks $k = [1:b]$ we construct codes as follows. For every sequence $u^n(k)$, independently and uniformly assign two random bin indices $(F_{\mathrm{u}}(k), W_{\mathrm{u}}(k))$ such that $F_{\mathrm{u}}(k) \in [1 : 2^{n\widetilde{R}_{\mathrm{u}}}]$ and $W_{\mathrm{u}}(k) \in [1 : 2^{nR_{\mathrm{u}}}]$. To each $v^n(k)$, independently and uniformly assign three random indices $(F_{\mathrm{v}}(k), W_{\mathrm{v}}(k), L_{\mathrm{v}}(k))$ such that $F_{\mathrm{v}}(k) \in [1 : 2^{n\widetilde{R}_{\mathrm{v}}}]$, $W_{\mathrm{v}}(k) \in [1 : 2^{nR_{\mathrm{v}}}]$, and $L_{\mathrm{v}}(k) \in [1 : 2^{n\overline{R}_{\mathrm{v}}}]$. Finally, to each $y_1^n(k-1)$, independently and uniformly assign a random index $L_{\mathrm{y}_1}(k-1) \in [1 : 2^{n\overline{R}_{\mathrm{y}_1}}]$. Conceptually, the indices $F(k) = (F_{\mathrm{u}}(k), F_{\mathrm{v}}(k))$ represent the public choice of an independent encoder-decoder pair in block $k \in [1:b]$, while the indices $W(k) = (W_{\mathrm{u}}(k), W_{\mathrm{v}}(k), L_v(k))$ represent the messages that should be reliably reconstructed at the decoder. Only $W_{\mathrm{v}}(k)$ should be directly kept secret from the eavesdropper. $L_{\mathrm{v}}(k)$ represents a non-secure additional message that should be reliably reconstructed at the decoder and can be kept secret by applying a one-time pad as used in the chosen-secret model [27]–[29]. The role of the index $L_{\mathrm{y}_1}(k-1)$, which is known at all legitimate parties thanks to the perfect output feedback, is to provide the required key for the one-time pad in block k. The messages of interest for our original problem, are obtained for all $k = [2:b]$ as $M_1(k) = W_{\mathrm{u}}(k)$ and $M_2(k) = (W_{\mathrm{v}}(k), L_{\mathrm{v}}(k))$, so that $R_1 = R_{\mathrm{u}}$ and $R_2 = R_{\mathrm{v}} + \overline{R}_{\mathrm{v}}$. The rate region is obtained by deriving sufficient conditions on the rates introduced above to ensure that all indices have the desired reliability or secrecy properties.

In slightly more details, the indices $F_{\mathrm{u}}(k)$ and $W_{\mathrm{u}}(k)$ are almost independent and uniformly distributed for all $k = [1:b]$ if [21, Theorem 1]

$$R_{\mathrm{u}} + \widetilde{R}_{\mathrm{u}} < H(U). \tag{18}$$

The indices $(F_{\mathrm{u}}(k), W_{\mathrm{u}}(k), F_{\mathrm{v}}(k), W_{\mathrm{v}}(k), L_{\mathrm{v}}(k))$ are almost mutually independent and uniformly distributed for all $k = [1:b]$ if [21, Theorem 1]

$$R_{\mathrm{u}} + \widetilde{R}_{\mathrm{u}} + R_{\mathrm{v}} + \widetilde{R}_{\mathrm{v}} + \overline{R}_{\mathrm{v}} < H(V, U). \tag{19}$$

The indices $F_{\mathrm{v}}(k)$ and $W_{\mathrm{v}}(k)$ are also almost independent of $(Y_2^n(k), S_2^n(k), U^n(k))$ and uniformly distributed for all $k = [1:b]$ if

$$R_{\mathrm{v}} + \widetilde{R}_{\mathrm{v}} < H(V|Y_2, S_2, U). \tag{20}$$

The random bin index $L_{\mathrm{y}_1}(k-1)$ is almost independent of $(Y_2^n(k-1), S_2^n(k-1), S_1^n(k-1), V^n(k-1), U^n(k-1))$ and uniformly distributed for all $k = [2:b]$ if

$$\overline{R}_{\mathrm{y}_1} = \overline{R}_{\mathrm{v}} < H(Y_1|Y_2, S_2, S_1, V). \tag{21}$$

Using a Slepian-Wolf [30] decoder, one can reliably reconstruct $U^n(k)$ from $(Y_1^n(k), S_1^n(k), F_{\mathrm{u}}(k))$ for all $k = [1:b]$ if [21, Lemma 1]

$$\widetilde{R}_{\mathrm{u}} > H(U|Y_1, S_1). \tag{22}$$

The decoder can then reliably reconstruct $V^n(k)$ from $(Y_1^n(k), S_1^n(k), F_{\mathrm{v}}(k), U^n(k))$ for all $k = [1:b]$ if

$$\widetilde{R}_{\mathrm{v}} > H(V|Y_1, S_1, U). \tag{23}$$

Applying Fourier-Motzkin elimination [31] to (18)-(23), and using the typical average lemma [32, pp. 26] and selection lemma [33, Lemma 2.2] the proof of the inner bound follows.

The proof of the outer bound follows by using standard properties of the Shannon entropy and by defining $V_i \triangleq (M_1, M_2, Y_1^{i-1}, S_1^{i-1})$ such that $V_i - X_i - (Y_{1,i}, Y_{2,i}, S_{1,i}, S_{2,i})$ form a Markov chain for all $i \in [1:n]$. ∎

Remark 2. Since we consider perfect feedback as in (1), the outer bound proposed in Proposition 2 is also valid for the general JCAS problem depicted in Fig. 1, in which the feedback Z_{i-1} can be any noisy version of $(Y_{1,i-1}, Y_{2,i-1})$.

We next characterize the exact strong secrecy-distortion regions for physically-degraded and reversely-physically-degraded JCAS channels with partial secrecy and perfect output feedback, defined below; see also [9, Definition 2].

Definition 2. A JCAS channel $P_{Y_1Y_2|S_1S_2X}$ is *physically-degraded* if we have

$$P_{Y_1Y_2|S_1S_2X} = P_{S_1} P_{Y_1|S_1X} P_{Y_2S_2|S_1Y_1}. \tag{24}$$

The channel is *reversely-physically-degraded* if the degradation order is changed and

$$P_{Y_1Y_2|S_1S_2X} = P_{S_2} P_{Y_2|S_2X} P_{Y_1S_1|S_2Y_2}. \tag{25}$$

The physically-degraded corresponds to a situation in which the observations (Y_2^n, S_2^n) of the eavesdropper are degraded versions of observations (Y_1^n, S_1^n) of the legitimate receiver with respect to the channel input X^n.

Theorem 1. *$\mathcal{R}_{\text{PS,POF}}$ for a physically-degraded JCAS problem with partial secrecy and perfect output feedback is the region defined in Proposition 2.*

Proof Sketch: Since the outer bound given in Proposition 2 does not assume any degradedness, the converse proof for Theorem 1 follows from the outer bound. Furthermore, the achievability proof for Theorem 1 follows by modifying the proof of the inner bound in Proposition 1. We next provide a sketch of the modifications for a physically-degraded JCAS.

First, U^n is eliminated from the achievability proof. Second, to each $v^n(k)$ we assign four random bin indices $(F_{\mathrm{v}}(k), W_{\mathrm{v}_1}(k), W_{\mathrm{v}_2}(k), L_{\mathrm{v}}(k))$ such that $F_{\mathrm{v}}(k) \in [1 : 2^{n\widetilde{R}_{\mathrm{v}}}]$, $W_{\mathrm{v}_1}(k) \in [1 : 2^{nR_{\mathrm{v}_1}}]$, $W_{\mathrm{v}_2}(k) \in [1 : 2^{nR_{\mathrm{v}_2}}]$, and $L_{\mathrm{v}}(k) \in [1 : 2^{n\overline{R}_{\mathrm{v}}}]$ for all $k = [1 : b]$ independently such that $M_1(k) = W_{\mathrm{v}_1}(k)$ and $M_2(k) = (W_{\mathrm{v}_2}(k), L_{\mathrm{v}}(k))$. As in (23), we impose the reliability constraint

$$\widetilde{R}_{\mathrm{v}} > H(V|Y_1, S_1) \tag{26}$$

as in (20) and (21) we impose the strong secrecy constraints

$$R_{\mathrm{v}_2} + \widetilde{R}_{\mathrm{v}} < H(V|Y_2, S_2) \tag{27}$$

$$\overline{R}_{\mathrm{v}} < H(Y_1|Y_2, S_2, S_1, V) \tag{28}$$

and as in (19) we impose the mutual independence and uniformity constraint

$$R_{\mathrm{v}_1} + R_{\mathrm{v}_2} + \widetilde{R}_{\mathrm{v}} + \overline{R}_{\mathrm{v}} < H(V). \tag{29}$$

We remark that we have $H(V|Y_2, S_2) \geq H(V|Y_1, S_1)$ for all physically-degraded JCAS channels, i.e., we obtain

$$\begin{aligned}[I(V;Y_1|S_1) - I(V;Y_2|S_2)]^+ &\\ \overset{(a)}{=} H(V|Y_2, S_2) - H(V|Y_1, S_1)& \end{aligned} \tag{30}$$

where (a) follows because V is independent of (S_1, S_2) and since

$$V - X - (Y_1, S_1) - (Y_2, S_2) \tag{31}$$

form a Markov chain for these JCAS scenarios. Define

$$\begin{aligned} R'_{2,\mathrm{deg}} &= [I(V;Y_1|S_1) - I(V;Y_2|S_2)]^+ + H(Y_1|Y_2, S_2, S_1, V) \\ &\overset{(a)}{=} H(V|Y_2, S_2) - H(V|Y_1, S_1) + H(Y_1|Y_2, S_2, S_1, V) \\ &= H(Y_1, V|Y_2, S_2, S_1) - H(V|Y_1, S_1) + H(S_1|Y_2, S_2) \\ &\quad - H(S_1|Y_2, S_2, V) \\ &= H(V|Y_2, S_2, S_1, Y_1) - H(V|Y_1, S_1) + H(Y_1|Y_2, S_2, S_1) \\ &\quad + H(S_1|Y_2, S_2) - H(S_1|Y_2, S_2, V) \\ &\overset{(b)}{=} H(Y_1, S_1|Y_2, S_2) - H(S_1|Y_2, S_2, V) \end{aligned} \tag{32}$$

where (a) follows by (30) and (b) follows from the Markov chain in (31).

Applying the Fourier-Motzkin elimination to (26)-(29) and using steps similar to those in the proof for Proposition 1, given in [24, Section VI], the achievability proof follows. ■

Lemma 1. *$\mathcal{R}_{\mathrm{PS,POF}}$ for a reversely-physically-degraded JCAS problem with partial secrecy and perfect output feedback is the union over all joint distributions P_{VX} of the rate tuples satisfying (8), (14), and*

$$R_2 \leq \min\left\{H(Y_1|Y_2, S_2, S_1),\ \left(I(V;Y_1|S_1) - R_1\right)\right\} \tag{33}$$

for joint distributions as in (9) but with constant U, and we can apply the deterministic per-letter estimators $\mathsf{Est}_j^*(x, y_1, y_2) = \hat{s}_j$ for $j = 1, 2$ by using (11). One can limit the cardinality to

$$|\mathcal{V}| \leq \min\{|\mathcal{X}|, |\mathcal{Y}_1|, |\mathcal{Y}_2|\}. \tag{34}$$

The proof for Lemma 1 is given in [24, Section III] and follows by showing that the inner and outer bounds in Propositions 1 and 2, respectively, match after elimination of U^n from the proof of achievability, as in the proof for Theorem 1 above.

IV. Bounds for JCAS with Single Secure Message

We next give inner and outer bounds for the JCAS problem with perfect output feedback, in which there is a single message $M = M_2$ that should be kept secret from an eavesdropper, i.e., $M_1 = \varnothing$ in Fig. 1. For this problem, the definitions of an achievable secrecy-distortion tuple (R, D_1, D_2) and corresponding strong secrecy-distortion region $\mathcal{R}_{\mathrm{POF}}$ follow similarly as in Definition 1 by eliminating (M_1, R_1) and by replacing $(M_2, R_2, \mathcal{R}_{\mathrm{PS,POF}})$ with $(M, R, \mathcal{R}_{\mathrm{POF}})$, respectively.

Proposition 3. (Inner Bound): *$\mathcal{R}_{POF}$ includes the union over all joint distributions P_{VX} of the rate tuples (R, D_1, D_2) satisfying (8) and*

$$R \leq \min\{R''_2,\ I(V;Y_1|S_1)\} \tag{35}$$

where

$$P_{VXY_1Y_2S_1S_2} = P_{V|X} P_X P_{S_1S_2} P_{Y_1Y_2|S_1S_2X}, \tag{36}$$

$$\begin{aligned} R''_2 = [I(V;Y_1|S_1) - I(V;Y_2|S_2)]^+ &\\ + H(Y_1|Y_2, S_2, S_1, V)& \end{aligned} \tag{37}$$

and apply the deterministic per-letter estimators $\mathsf{Est}_j^*(x, y_1, y_2) = \hat{s}_j$ for $j = 1, 2$ by using (11). One can limit the cardinality as in (16).

Proposition 4. (Outer Bound): *$\mathcal{R}_{POF}$ is included in the union over all P_X of the rate tuples satisfying (8) and*

$$\begin{aligned} R \leq \min\Big\{&\big(H(Y_1, S_1|Y_2, S_2) - H(S_1|Y_2, S_2, X)\big), \\ &I(X;Y_1|S_1)\Big\} \end{aligned} \tag{38}$$

where we can apply the deterministic per-letter estimators $\mathsf{Est}_j^*(x, y_1, y_2) = \hat{s}_j$ for $j = 1, 2$ by using (11).

The proof of the inner bound in Proposition 3 follows by eliminating U^n in the proof of the inner bound for Proposition 1 such that $R_1 = R_{\mathrm{v}_1} = 0$ and by imposing (26)-(29) after replacing R_{v_2} with R_{v} since for this case we have $M(k) = (W_{\mathrm{v}}(k), L_{\mathrm{v}}(k))$ for all $k = [1 : b]$. See [24, Section IV] for the proof of the outer bound.

Similar to Section III, we characterize the exact strong secrecy-distortion regions for the JCAS problem with perfect output feedback when the JCAS channel $P_{Y_1Y_2|S_1S_2X}$ is physically-degraded, as in (24), or reversely-physically-degraded, as in (25).

Theorem 2. *$\mathcal{R}_{POF}$ for a physically-degraded JCAS problem with perfect output feedback is the region defined in Proposition 4.*

Proof: Since the outer bound given in Proposition 4 is valid for any JCAS channel, the converse proof for Theorem 2 follows from Proposition 4. Furthermore, the achievability proof for Theorem 2 follows by modifying the proof of the inner bound for Theorem 1 such that we assign $V^n(k) = X^n(k)$ for all $k = [1 : b]$ and then apply the same OSRB steps for $X^n(k)$ rather than $V^n(k)$. ∎

Lemma 2. *$\mathcal{R}_{POF}$ for a reversely-physically-degraded JCAS problem with perfect output feedback is the union over all P_X of the rate tuples in (8) and*

$$R \leq \min\left\{H(Y_1|Y_2, S_2, S_1),\ I(X; Y_1|S_1)\right\} \tag{39}$$

where we can apply the deterministic per-letter estimators $\mathsf{Est}_j^*(x, y_1, y_2) = \hat{s}_j$ for $j = 1, 2$ by using (11).

The proof for Lemma 2 is given in [24, Section IV] and follows by showing that the inner and outer bounds in Propositions 3 and 4, respectively, match after assigning $V^n = X^n$ in the proof of achievability, i.e., we choose $V = X$ that is allowed by (36).

V. Binary JCAS Channel with Multiplicative Bernoulli States Example

We next consider a JCAS with perfect output feedback example, in which JCAS channel input and output alphabets are binary with multiplicative Bernoulli states, i.e., we have

$$Y_1 = S_1 \cdot X, \qquad Y_2 = S_2 \cdot X \tag{40}$$

where $P_{S_1S_2}(0,0) = (1-q)$, $P_{S_1S_2}(1,1) = q\alpha$, and $P_{S_1S_2}(1,0) = q(1-\alpha)$ for fixed $q, \alpha \in [0,1]$, so the JCAS channel satisfies (24) [9, Section IV-A].

Define the binary entropy function, for any $c \in [0,1]$, as

$$H_b(c) = -c\log c - (1-c)\log(1-c). \tag{41}$$

Lemma 3. *The strong secrecy-distortion region $\mathcal{R}_{POF}$ for a binary JCAS channel with multiplicative Bernoulli states characterized by parameters (q, α), and with Hamming distortion metrics is the union over all $p = \Pr[X = 1]$ of the rate tuples*

$$R \leq q(1-\alpha)H_b(p) \tag{42}$$
$$D_1 \geq (1-p)\cdot\min\{q, (1-q)\} \tag{43}$$
$$D_2 \geq (1-p)\cdot\min\{q\alpha, (1-q\alpha)\}. \tag{44}$$

Proof Sketch: The proof follows by evaluating the strong secrecy-distortion region $\mathcal{R}_{\text{POF}}$ defined in Theorem 2 since the JCAS channel considered is physically-degraded. Proofs for (43) and (44) follow by choosing $\mathsf{Est}_j^*(1, y_1, y_2) = y_j$ and $\mathsf{Est}_j^*(0, y_1, y_2) = \mathbb{1}\{\Pr[S_j = 1] > 0.5\}$ for $j = 1, 2$ that result from (11), which are equivalent to the proofs for [9, Eqs. (27c) and (27d)]. We next have $I(X; Y_1|S_1) = qH_b(p)$, which is equivalent to the proof for [9, Eq. (27a)] with $r = 1$. Furthermore, we obtain

$$\begin{aligned} &H(Y_1, S_1|Y_2, S_2) - H(S_1|Y_2, S_2, X) \\ &\overset{(a)}{=} H(S_1|S_2) + H(Y_1|S_1, Y_2, S_2) - H(S_1|S_2) \\ &\overset{(b)}{=} P_{S_1S_2}(1,0)H(Y_1|S_1 = 1, S_2 = 0) \\ &\overset{(c)}{=} P_{S_1S_2}(1,0)H(X) = q(1-\alpha)H_b(p) \end{aligned} \tag{45}$$

where (a) follows since $S_1 - S_2 - (Y_2, X)$ form a Markov chain for the considered JCAS channel, (b) follows since if $S_1 = 0$, then $Y_1 = 0$, and if $(S_1, S_2) = (1, 1)$, then $Y_1 = Y_2$, and (c) follows since $Y_1 = X$ that is because of $S_1 = 1$ and since X is independent of (S_1, S_2). Therefore, we have

$$\begin{aligned} R &\leq \min\Big\{\big(H(Y_1, S_1|Y_2, S_2) - H(S_1|Y_2, S_2, X)\big), \\ &\qquad\qquad I(X; Y_1|S_1)\Big\} \\ &= q(1-\alpha)H_b(p) \end{aligned} \tag{46}$$

which follows since $\alpha \leq 1$. ∎

We remark that for the considered example, the rate of the securely transmitted message is upper bounded by $\big(H(Y_1, S_1|Y_2, S_2)$–$H(S_1|Y_2, S_2, X)\big)$ rather than $I(X; Y_1|S_1)$, the latter of which is the upper bound for the rate for the same example when there is no secrecy constraint [9, Corollary 4]. Thus, the amount of rate loss due to the strong secrecy constraint is $q\alpha H_b(p)$ for this JCAS example. Furthermore, one can show that JCAS methods achieve significantly better performance than separation-based secure communication and state-sensing methods. First, one can show that the maximum secure communication rate in (42) is achieved with $p = 0.5$, whereas the minimum distortions in (43) and (44) are achieved with $p = 1$ that results in zero communication rate. Then, applying time sharing between the tuples achieved by the separation based methods to convexify and enlarge the region, we observe that the secrecy-distortion region that can be achieved by applying the JCAS methods is strictly larger than the region being achieved by the separation based methods. These analyses are analogous to the comparisons between joint and separation-based secrecy and reliability methods for the secret key agreement problem, as discussed in [34]–[36].

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