

1 Dependence of Ice Crystal Size Distributions in High Ice
2 Water Content Conditions on Environmental Conditions:
3 Results from the HAIC-HIWC Cayenne Campaign

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Abstract

A new method that automatically determines the modality of an observed particle size distribution (*PSD*) and the representation of each mode as a gamma function was used to characterize data obtained during the High Altitude Ice Crystals and High Ice Water Content (HAIC-HIWC) project based out of Cayenne, French Guiana in 2015. *PSDs* measured by a 2-D stereo probe and a precipitation imaging probe for particles with maximum dimension (D_{max}) $> 55 \mu\text{m}$ were used to show how the gamma parameters varied with environmental conditions, including temperature (T) and convective properties such as cloud type, mesoscale convective system (MCS) age, distance away from the nearest convective peak, and underlying surface characteristics. Four kinds of modality *PSDs* were observed, unimodal *PSDs* and three types of multimodal *PSDs* (Bimodal1 with breakpoints $100 \pm 20 \mu\text{m}$ between modes, Bimodal2 with breakpoints $1000 \pm 300 \mu\text{m}$ and Trimodal *PSDs* with two breakpoints). The T and Ice Water Content (*IWC*) are the most important factors influencing the modality of *PSDs*, with the frequency of multimodal *PSDs* increasing with increasing T and *IWC*. An ellipsoid of equally plausible solutions in (N_o - λ - μ) phase space is defined for each mode of the observed *PSDs* for different environmental conditions. The percentage overlap between ellipsoids was used to quantify the differences between overlapping ellipsoids for varying conditions. The volumes of the ellipsoid decrease with increasing *IWC* for most cases, and (N_o - λ - μ) vary with environmental conditions related to distribution of *IWC*. HIWC regions are dominated by small irregular ice crystals and columns. The parameters (N_o - λ - μ) in each mode exhibit mutual dependence.

46 **1. Introduction**

47 Cirrus covers 20–30% of the Earth (Heymsfield and McFarquhar, 2002; Wylie et
48 al., 2005), and affects the vertical profile of radiative heating (e.g., Ackerman et al., 1988).
49 Anvil cirrus shields produced when convective ice detrains into the upper troposphere can
50 significantly impact the atmosphere's radiation budget (e.g., Machado and Rossow, 1993;
51 Fu et al., 1995; Zender and Kiehl, 1997; Del Genio and Kovari, 2002; Stephens, 2005).
52 The size, shape, and concentration of ice particles in cirrus have a large impact on cloud
53 radiative forcing, and hence determine cloud feedbacks that modify estimates of global
54 climate change (Ackerman et al., 1988; Mitchell et al., 2008; Sanderson et al., 2008;
55 Lawson et al., 2010). In addition, ice microphysical properties are of particular importance
56 due to the strong sensitivity of simulated deep convective systems to parameterizations of
57 microphysical process rates (Chen and Cotton, 1988; McCumber et al., 1991; Gilmore et
58 al., 2004; Milbrandt and Yau, 2005; McFarquhar et al., 2006; Morrison and Grabowski,
59 2008; Huang et al., 2020) that depend on the habits and size distributions of ice crystals.

60 The number distribution function [$N(D)$] of ice crystals determines many
61 microphysical properties (e.g., ice water content IWC , total number concentration N_t , mass-
62 weighted terminal velocity v_m , effective radius R_e , bulk extinction β , and single scattering
63 properties), parameterizations of which are important for the Earth system models (e.g.,
64 Stephens, 2005; Jakob and Klein, 1999; Sanderson et al., 2008; Jackson et al., 2015) and
65 weather models (e.g., Morrison et al., 2009; Bryan and Morrison, 2012; Stanford et al.,
66 2019) because they control distributions of latent heating and cooling, condensate loading
67 and radiant fluxes (Schlimme et al., 2005) that are directly coupled with model dynamics
68 (Morrison et al., 2015). Mass-weighted terminal velocity determined by particle size
69 distributions ($PSDs$) controls the cloud coverage and lifetime simulated by climate models
70 (Sanderson et al., 2008), and impacts spatial distributions of latent heating by affecting
71 microphysical process rates (i.e., riming aggregation, melting, evaporation, etc.). Further,
72 the accuracy of cirrus remote sensing retrievals depends on accurate representations of
73 $PSDs$ (Wolf et al., 2019). The effective diameter (D_e) depends on $PSDs$ and is commonly
74 used to parameterize single scattering properties needed for calculation of shortwave
75 radiative transfer (e.g., Fu, 1996; McFarquhar and Heymsfield, 1998; Mitchell, 2002). The
76 radar reflectivity is a higher-order moment of $PSDs$ in the Rayleigh scattering regime

77 (Smith, 1984). Thus, correct interpretation of ice crystal *PSDs* is critically needed for
78 development and evaluation of model parameterization schemes and remote sensing
79 retrievals (Gu et al., 2011).

80 Many microphysical parameterization schemes, such as the Milbrandt 2-moment
81 scheme (Milbrandt and Yau, 2005), the Thompson scheme (Thompson et al., 2008), the
82 State University of New York at Stony Brook scheme by Yanluan Lin (SBU-YLIN; Lin
83 and Colle, 2011), and the predicted particle properties (P3) scheme (Morrison and
84 Milbrandt, 2015) make assumptions about the number and type of categories of the
85 hydrometeors present and the shape of the *PSDs* in each category. A plethora of studies
86 found considerable sensitivity of high-resolution numerical weather prediction forecasts to
87 the selection of the microphysics schemes and the use of parameters in such schemes (e.g.,
88 Wang, 2002; McFarquhar et al., 2006; 2012; Zhu and Zhang, 2006; Li and Pu, 2008; Van
89 Weverberg et al., 2011; Clark et al., 2012; Huang et al., 2020).

90 *PSDs* have been fit using exponential or gamma functions in many parameterization
91 schemes (e.g., Walko et al., 1995; Meyers et al., 1997; Straka and Mansell, 2005; Milbrandt
92 and Yau, 2005). The gamma function used to represent the number distribution function
93 $N(D_{max})$ is typically represented by

$$94 \quad N(D_{max}) = N_o D_{max}^{\mu} e^{-\lambda D_{max}}. \quad (1)$$

95 with N_o the intercept, D_{max} the maximum crystal dimension, λ the slope, and μ the
96 dispersion parameter (e.g., Gilmore et al., 2004; Straka and Mansell, 2005; Milbrandt and
97 Yau, 2005; Moisseev and Chandrasekar, 2007; Wolf et al., 2019; Chen et al., 2020).
98 However, the units of N_o vary with μ in Eq. (1). Thus, the gamma function can alternately
99 be written

$$100 \quad N(D_{max}) = N_o \left(\frac{D_{max}}{D_o}\right)^{\mu} e^{-\lambda D_{max}}. \quad (2)$$

101 where $D_o = 1$ cm is frequently assumed so that N_o has identical units to $N(D_{max})$ ($\text{cm}^{-3} \mu\text{m}^{-1}$ or m^{-4} ; McFarquhar et al., 2015; Mascio et al., 2020).

103 Knowledge about the dependence of N_o , λ and μ on environmental conditions is
104 important, because such parameters affect the simulation of cloud properties (e.g.,
105 McCumber et al., 1991; McFarquhar et al., 2006). Many in-situ observations have been
106 used to fit the measured *PSDs* (e.g., Gunn and Marshall, 1958; Wong et al., 1988;
107 Heymsfield et al., 2002, 2009; McFarquhar and Black, 2004; McFarquhar et al., 2007) by

108 different techniques to minimize the difference between the observed *PSDs* and fitted $N(D)$.
109 For example, the least squares method (e.g., McFarquhar and Heymsfield, 1997), the
110 method of moments (e.g., Field, 2005; Smith and Kliche, 2005; Smith et al., 2009; Tian et
111 al., 2010; Handwerker and Straub, 2011), and the maximum-likelihood approach (e.g.,
112 Haddad et al., 1996) have all been used. Sometimes uncertainty has been taken into account
113 when estimating the fit parameters for observed *PSDs* to improve the fitting techniques
114 (e.g., Wong and Chidambaram, 1985; Chandrasekar and Bringi, 1987; Moisseev and
115 Chandrasekar, 2007; Smith et al., 2009; Handwerker and Straub, 2011). McFarquhar et al.
116 (2015) developed an Incomplete Gamma Fitting (IGF) Technique that uses statistical
117 uncertainty and variability in microphysical properties within a family of distributions to
118 represent the fit parameters as an ellipsoid of equally realizable solutions in $(N_o, \lambda$ and $\mu)$
119 phase space. The results showed that N_o , λ and μ exhibited some mutual dependence, which
120 must be accounted for when applying the results of the fits.

121 This technique has been used to study how the ellipsoids describing gamma
122 distributions vary with environmental conditions (e.g., Mascio et al., 2020) and
123 complement other studies showing how the most likely fit parameters vary with
124 environmental conditions (e.g., Thompson et al., 2008; Tian et al., 2010). However, these
125 studies did not consider the multimodal properties of *PSDs* even though previous studies
126 showed two peaks in measured *PSDs* (e.g., Varley, 1978; Mitchell et al., 1996; McFarquhar
127 and Heymsfield, 1997; Jensen et al., 2009; Zhao et al., 2010; Lawson et al., 2010).
128 Breakpoints between these peaks or between the two modes in the *PSDs* sometimes
129 occurred at a size near the cutoff between two instruments used to measure the *PSDs* (e.g.,
130 McFarquhar and Heymsfield, 1997; Zhao et al., 2010), but also frequently occurred at sizes
131 not near the cutoff between instruments (e.g., Jackson et al., 2015). Further, peaks and
132 breakpoints were also found in *PSDs* measured by only one probe (e.g., Mitchell et al.,
133 1996; Jensen et al., 2009). Attempts to parameterize multimodal size distributions include
134 those by Welch et al. (1980) who used two modified gamma distribution and Jackson et al.
135 (2015) who extended the McFarquhar et al. (2015) IGF technique to allow for a bimodal
136 gamma distribution function. Although these studies improved the understanding on the
137 distribution of ice crystals, sufficient data were not yet available to characterize the *PSDs*
138 from tropical zones and high *IWC* regions.

139 The High Altitude Ice Crystals (HAIC; Dezitter et al., 2013) and High Ice Water
140 Content (HIWC; Strapp et al., 2016a) projects collected airborne in-situ and remote sensing
141 data from large oceanic convective clouds, with the primary objective of collecting a
142 dataset of high IWC measurements for assessment of a new aircraft certification envelope
143 for the ice crystal environment (Strapp et al., 2021). Campaigns were performed using the
144 French Falcon-20 out of Darwin, Australia in 2014 (Leroy et al., 2017) and Cayenne,
145 French Guiana in 2015 (Dezitter et al., 2013; Strapp et al., 2016a), whereas the HIWC-
146 RADAR flight campaign out of Florida in 2015 (Yost et al., 2018, Ratvasky et al., 2019)
147 used the NASA DC-8. The HAIC-HIWC data provide a wealth of in-cloud microphysical
148 measurements that have been used to investigate HIWC regions (e.g., Protat et al., 2016;
149 Wolde et al., 2016; Fontaine et al., 2017; Leroy et al., 2016a, 2016b, 2017; Qu et al., 2018;
150 Yost et al., 2018; Huang et al., 2021). For example, a companion study (Hu et al., 2021)
151 investigated the dependence of *IWC* and median mass diameter (*MMD*) on environmental
152 conditions (e.g., temperature, vertical velocity, underlying surface characteristics defined
153 as whether the surface was land, ocean or coastline, mesoscale convective system (MCS)
154 age, the distance away from the convective core and the local strength of convection) for
155 data obtained from the Cayenne field campaign.

156 Brechner (2021) developed a new method that automatically identifies the
157 existence of up to three peaks in *PSDs*, integrating each mode over all crystal sizes rather
158 than having an artificial breakpoint between modes for *PSDs* as done by Jackson et al.
159 (2015). Brechner's (2021) method was used to investigate the nature of *PSDs* measured
160 during the first phase of the HAIC-HIWC field campaign collected in Darwin, Australia in
161 2014. The analysis of data collected during the second HAIC-HIWC campaign field
162 conducted from 9-29 May 2015 out of Cayenne, French Guiana presented in Hu et al. (2021)
163 is extended here to quantitatively describe the dependence of *PSDs* on *IWC* and
164 environmental conditions.

165 The remainder of this paper is organized as follows. Section 2 describes the HAIC-
166 HIWC dataset and methodology used to process the data. Section 3 presents a case study
167 on the nature of different modality *PSDs*. Investigations of how statistical distributions of
168 *PSDs* parameters vary with environmental conditions are shown in section 4. Section 5
169 summarizes the significance of this study and offers directions for future research.

170 **2. Data and Methodology**

171 **a. Overview of campaign**

172 The second HAIC-HIWC flight campaign based out of Cayenne (French Guiana)
173 9-29 May 2015 collected in-situ and remotely sensed data of oceanic convective clouds to
174 characterize and improve the understanding of the high altitude and high concentration ice
175 crystal environment. A total of 17 French Falcon-20 flights and 10 Canadian National
176 Research Council (NRC) Convair-580 flights are used here. The Convair-580 did almost
177 all of its sampling at levels corresponding to T of approximately -10°C due to its limited
178 ceiling. The French Falcon-20, equipped with in-situ cloud probes, conducted 17 flights in
179 oceanic, coastal and continental MCSs at varying levels centered at temperatures of mostly
180 -10° , -30° , and -45°C (Strapp et al., 2016a). The temperature refers to the location where
181 the ice particles were measured, but this may not necessarily be the level where they grew.
182 Hu et al. (2021) discussed the flights conducted, the probes used and the procedures by
183 which data were processed. Thus, only a brief summary of the most salient aspects is
184 offered here.

185 **b. In situ probes**

186 Hu et al. (2021) summarized the cloud microphysics probes installed on the Falcon-
187 20. These probes include a Stratton Park Engineering (SPEC, 2011) two-dimension stereo
188 probe (2D-S; nominally sizing diameter (D) between $10\text{--}1280\text{ }\mu\text{m}$), a Droplet
189 Measurement Technologies (DMT, 2009) precipitation image probe (PIP; $100\text{--}6400\text{ }\mu\text{m}$),
190 a DMT Cloud Droplet Probe (CDP-2; $2\text{--}49\text{ }\mu\text{m}$), an isokinetic evaporator probe (IKP2; \sim
191 $0.1\text{--}10\text{ g m}^{-3}$ at 200 m s^{-1}) for the primary measurement of bulk total water content TWC
192 (Strapp et al., 2016b), a Science Engineering Associates (SEA) Robust hot wire TWC probe,
193 a Rosemount Icing Detector, and a multi-beam 95 GHz Doppler cloud radar (RAdar
194 SysTem Airborne, RASTA). The Convair-580 was equipped with polarization radar (X
195 and W-band, 9.41 GHz and 94.05 GHz) and a suite of in-situ cloud particle probes,
196 including a 2D-S, a DMT PIP, a DMT CDP-2, an IKP2, where the parameters of these
197 probes are the same as those mounted on Falcon-20. In addition, a Particle Measuring
198 Systems (PMS) Forward Scattering Spectrometer Probe (FSSP-100; $2\text{--}47\text{ }\mu\text{m}$), SPEC
199 Cloud Particle Imager (CPI) with $2.3\text{ }\mu\text{m}$ pixel resolution, were used for further PSD
200 characterization, and an Aventech Aircraft Integrated Meteorological Measurements

201 System (AIMMS-20) measured temperature, static pressure and wind at the flying levels.
202 Licor hygrometer probes model 6262 and model 840A were used to measure water vapor
203 mixing ratio and calculate relative humidity and ice supersaturation following Korolev and
204 Isaac (2006). More information on the use of these probes for these flight campaigns can
205 be found in Strapp et al. (2020).

206 **c. In situ observations of PSDs**

207 Different definitions of particle size have been used to characterize *PSDs*. These
208 include the maximum dimension (D_{max}) (e.g., Mitchell and Arnott, 1994; McFarquhar and
209 Heymsfield, 1996; 1998; McFarquhar and Black 2004; Heymsfield et al., 2013; Jackson et
210 al., 2014; Korolev et al., 2014; Korolev and Field, 2015), the area-equivalent diameter
211 (Locatelli and Hobbs, 1974; Korolev et al., 2014; Waitz et al., 2021), and the mass-
212 equivalent diameter (Seifert and Beheng, 2006). D_{max} is used to characterize particle size
213 because this parameter has been used in most previous studies that examined observed ice
214 particle size distributions. Further parameterizations for mass and terminal fall speed that
215 are used in models are usually formulated in terms of maximum-dimension. Thus, the use
216 of maximum-dimension as opposed to the use of an area-equivalent diameter ensures self
217 consistency in model schemes (McFarquhar and Black, 2004). There are several different
218 ways that D_{max} has been calculated for a two-dimensional image (Locatelli and Hobbs,
219 1974; Brown and Francis, 1995; McFarquhar and Heymsfield, 1996; Mitchell and Arnott,
220 1994; Korolev and Field, 2015; Heymsfield et al., 2013; Wu and McFarquhar, 2016). The
221 definition of D_{max} as the diameter of a minimum enclosing circle for a two-dimensional
222 particle image is used here following Wu and McFarquhar (2016).

223 A composite *PSD* was derived from the 2D-S and PIP covering the size range D_{max}
224 from 55 to 12,845 μm at 10 μm resolution and is used to calculate cloud microphysical
225 parameters (e.g., mass concentration distribution, *MMD*, N_t) following the techniques
226 described by Hu et al. (2021). The 2D-S was used in the composite distribution for sizes
227 smaller than 800 μm . Likewise, the PIP was used for sizes larger than 1200 μm , and the
228 composite distribution between 800 and 1200 μm is a linear weighted mean of the 2D-S
229 and PIP distributions (Leroy et al., 2016).

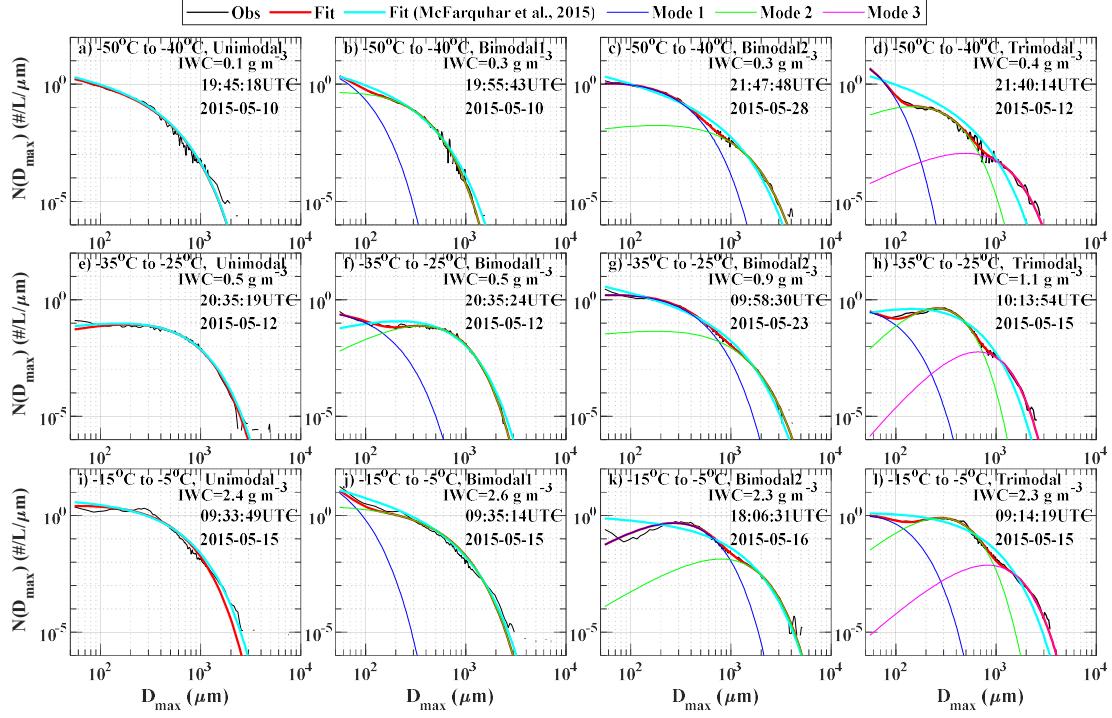
230 **d. Data processing and classifications**

Only time periods in ice-phase clouds are examined in this study. Time segments, when the Rosemount Icing Detector frequency was decreasing and was lower than 40 kHz (Mazin et al., 2001) or when N_t measured by the CDP-2 was larger than 10 cm^{-3} (Lance et al., 2010; Ding et al., 2020) identified the presence of liquid water content (*LWC*) and were removed. This represents only 0.81% of the data obtained by Falcon-20 (Hu et al., 2021). A frequency threshold of 39.7 kHz was applied to the different model of the Rosemount Icing Detector installed on the Convair-580 and showed that 24.7% of the data obtained by the Convair-580 was in liquid conditions. To remove tenuous clouds from analysis, only data with $IWC \geq 0.1 \text{ g m}^{-3}$ were used for both aircraft. The HIWC regions were defined to occur when the *IWC* was larger than 1.5 g m^{-3} without any threshold of *MMD* to use a definition consistent with previous studies (e.g., Leroy et al., 2016; Hu et al., 2021). The *IWC* and vertical velocity data were averaged over 5-second intervals to match the integration period used for the *PSD* data. For Falcon-20 flight observations, a total of 12,339 data points representing about 17.2 hours of data within organized MCSs were available for analysis, of which 1,931 data points were in HIWC regions. A total of 6,088 data points representing about 8.5 hours of data within organized MCSs were observed by Convair-580, of which 1,493 data points were in HIWC region.

To investigate the effects of environmental conditions and other MCS characteristics (e.g., temperature, vertical velocity, underlying surface characteristics, MCS age, distance away from the convective core) on *PSDs*, as well as the differences of *PSDs* between HIWC and other *IWC* regions, each measurement was classified according to the several environmental characteristics following the methodology of Hu et al. (2021).

253 **e. Methodology**

254 **1) Improved IGF technique**



257 Figure 1. Example of different modality PSDs and fits to those PSDs as a function of T using data
258 obtained during the HAIC-HIWC Cayenne campaign. (a) Unimodal, (b) Bimodal1, (c) Bimodal2, and
259 (d) Trimodal PSDs at $-50^{\circ}\text{C} \leq T \leq -40^{\circ}\text{C}$. (e–h) As Figures 1a–1d but for $-35^{\circ}\text{C} \leq T \leq -25^{\circ}\text{C}$. (i–l)
260 As Figures 1a–1d but for $-15^{\circ}\text{C} \leq T \leq -5^{\circ}\text{C}$. Black lines indicate observed PSDs, red lines fits to
261 observed PSD using method of Brechner (2021) that automatically determines number of modes in
262 PSDs, cyan lines represent fit assuming unimodal distribution using the algorithm of McFarquhar et al.
263 (2015), blue, green and pink lines indicate fits to first, second, and third mode of PSD respectively.

265 The algorithm in Brechner (2021) was used to fit the observed PSDs to a unimodal,
266 bimodal or trimodal gamma distribution. The algorithm automatically identifies the
267 number of modes and the location of each mode for a PSD. The IGF technique developed
268 by McFarquhar et al. (2015) was modified to determine the fit parameters for each mode.
269 To implement the technique, it is first assumed that all PSDs are trimodal, with $N(D_{max})$
270 hence expressed as

$$271 N(D_{max}) = N_{o1} \left(\frac{D_{max}}{D_o} \right)^{\mu_1} e^{-\lambda_1 D_{max}} + N_{o2} \left(\frac{D_{max}}{D_o} \right)^{\mu_2} e^{-\lambda_2 D_{max}} \\ 272 + N_{o3} \left(\frac{D_{max}}{D_o} \right)^{\mu_3} e^{-\lambda_3 D_{max}}. \quad (6)$$

273 where the small, middle, and large modes are represented by subscripts (1, 2, 3)
274 respectively. For each PSD, a small and large breakpoint is determined. The algorithm
275 determines the number of modes according to the number of breakpoints. If no breakpoint

276 exists, the *PSD* is unimodal and $N_{o2} = N_{o3} = 0$. If two breakpoints exist, the *PSD* is trimodal.
277 If only one breakpoint exists, the *PSD* is bimodal and $N_{o3} = 0$. A breakpoint does not mean
278 that the gamma function for a mode does not characterize crystal sizes larger or smaller,
279 but rather indicates the transition in size at which the dominance of a particular mode
280 changes. Depending on whether the breakpoint is greater 600 μm or less than 200 μm , the
281 bimodal distribution is called Bimodal1 (small breakpoint $< 200 \mu\text{m}$) or Bimodal2 (large
282 breakpoint $> 600 \mu\text{m}$). Following McFarquhar et al. (2015), the uncertainty in the number
283 of counts in each size bin was considered in determining the fit to the *PSD*. More details
284 on the development of the scheme are found in Brechner (2021).

285 Examples of different modality *PSDs* averaged for different temperature (T) ranges
286 are shown in Figure 1. The distributions are plotted for different ranges of T because T is
287 the variable that had the biggest impact on microphysical properties for the Cayenne dataset
288 (Hu et al., 2021). Visually, the fits match the observed *PSDs* well. The shapes of the same
289 modality *PSD* can vary with T because the intercept, slope and shape parameters can be a
290 function of T . As the generation of particles in different modes is dominated by different
291 processes (small mode by homogeneous nucleation, the large mode by aggregation and
292 sedimentation, and the central mode by riming), the use of three modes should allow better
293 representation of processes in numerical models without the computational expense
294 required by a bin-resolving scheme.

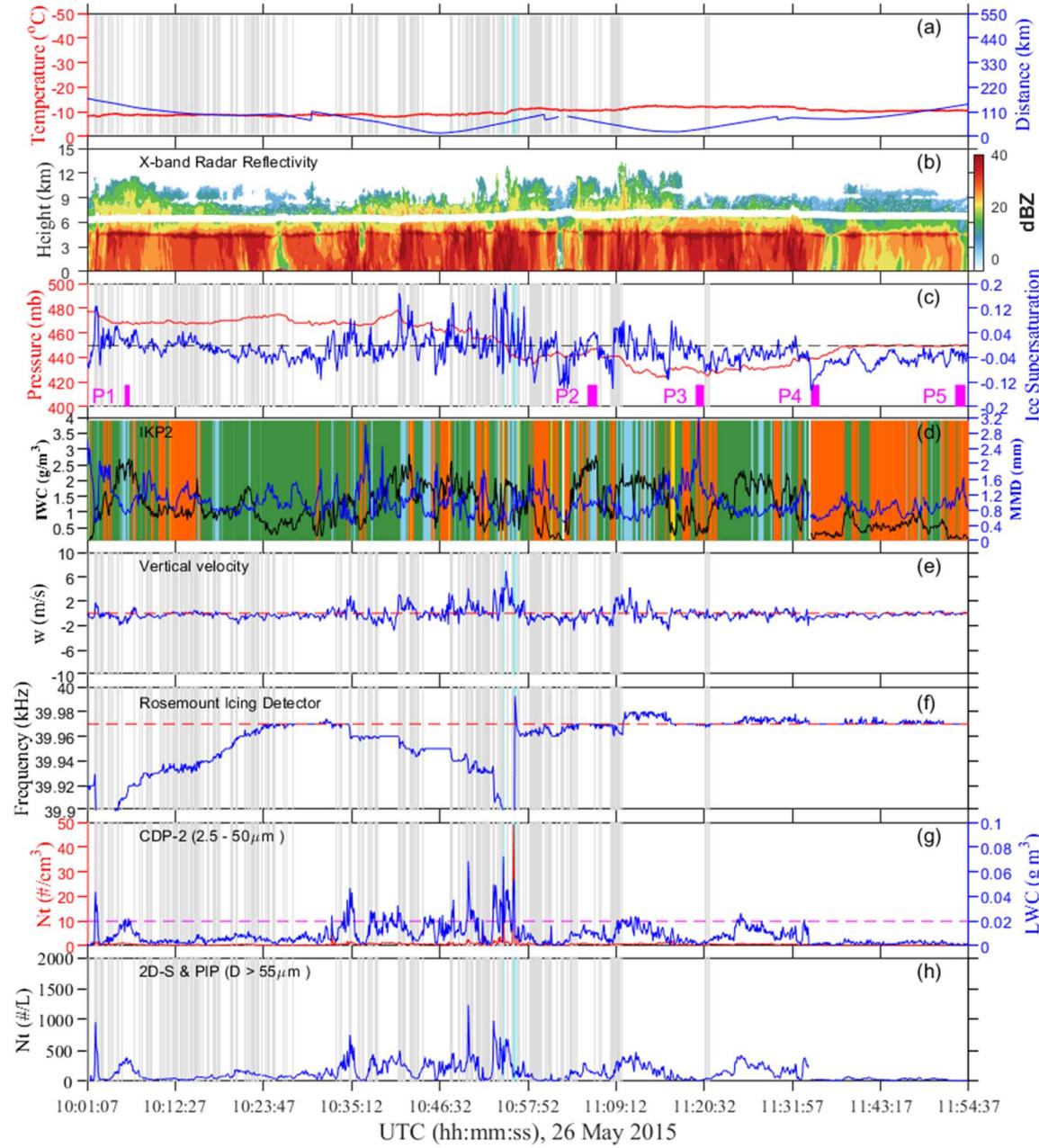
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296 2) Projections of ellipses

297 An ellipse characterizing the equally realizable solution in (N_o, μ, λ) phase space
298 for each mode was calculated using the algorithm of Moshtagh (2006) that determines a
299 Hessian matrix for the fit phase space for each *PSD*. A restriction of $-1 < \mu < 10$ and $\lambda > 0$
300 is applied so that the parameterized *PSDs* can easily be integrated when incorporated into
301 model parameterization schemes. To construct a volume of equally realizable solutions for
302 a family of *PSDs* measured in similar environmental conditions, all points contained within
303 1% of the individual *PSD* ellipsoids in the family are included. The detailed methodology
304 for determining these ellipsoids is found in McFarquhar et al. (2015).

305 3) Overlap percent

306 To determine how fit parameters vary with environmental conditions, a three-
 307 dimensional volume encompassing two ellipsoids ($S1$, $S2$) describing different
 308 environmental conditions is determined (Finlon et al. 2019; Mascio et al. 2020), where N_{s1} ,
 309 and N_{s2} are the number of points inside the $S1$ and $S2$ ellipsoids respectively, and N_{s0} is the

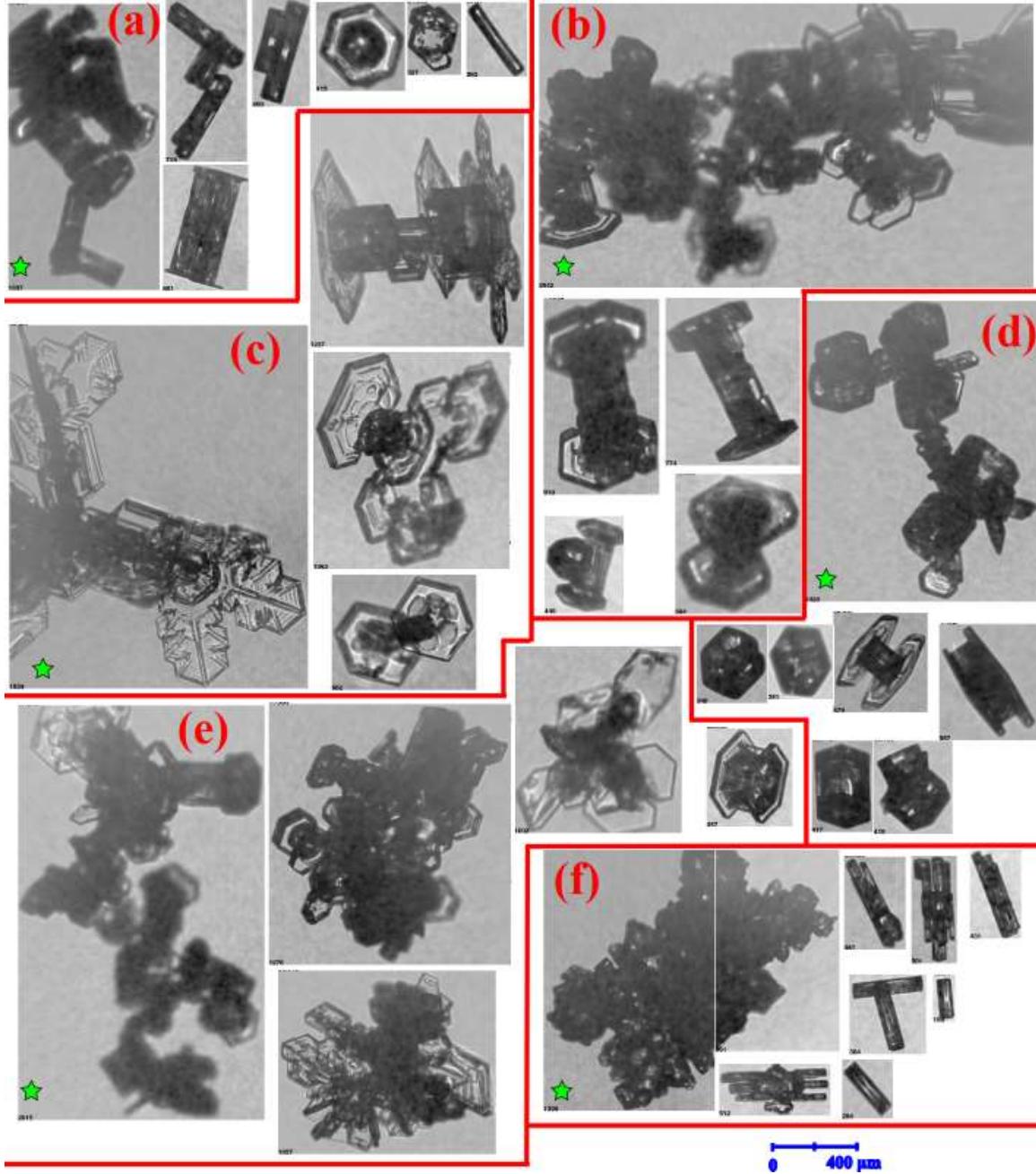


310

311 Figure 2. Time series of (a) temperature at flight altitude (red line), distance away from convective core
 312 (L , blue line, times when aircraft out of MCS not shown); (b) X-band radar reflectivity from X and W-
 313 band polarization radar mounted on Convair-580; (c) atmosphere pressure (red line) and ice
 314 supersaturation (blue line), with magenta rectangles P1 to P5 representing five periods with their width
 315 along x-axis denoting length of time period; (d) IWC from IKP2 (black line) and MMD (blue line), with

316 shaded orange boxes representing locations of Unimodal *PSDs*, sky-blue boxes Trimodal *PSDs*, yellow
 317 boxes Bimodal1, and green boxes Bimodal2 *PSDs*; (e) vertical velocity (*w*) measured by an Aventech
 318 Aircraft Integrated Meteorological Measurements System (AIMMS-20); (f) Rosemount icing detector
 319 frequency (blue line) with a threshold of 39.97 kHz indicating presence of supercooled liquid water (red
 320 line); (g) N_t (red line) and *LWC* (blue line) derived by integrating $N(D_{max})$ measured by CDP for $2.5 < D_{max} < 50 \mu\text{m}$, red line shows threshold of 10 cm^{-3} for N_t used to identify periods of liquid water; and
 321 (g) N_t from composite 2D-S ($D_{max} > 55 \mu\text{m}$) and PIP size distribution for 10:01:07–11:54:37 UTC 26
 322 May 2015 Convair-580 flight leg. Shaded gray boxes represent locations of Rosemount icing detector
 323 frequency less than 39.97 kHz, shaded cyan boxes indicated N_t measured by CDP-2 larger than 10 cm^{-3} .
 324

325



326

327 Figure 3. Examples of representative ice crystals measured by SPEC CPI at (a) 10:05:57–10:06:37, (b)
 328 11:05:32–11:06:47, (c) 11:19:27–11:20:32, (d) 11:34:17–11:35:22, and (e) 11:52:52–11:54:12 UTC on

329 26 May 2015 corresponding to periods P1–P5 in Fig. 2c. (f) Typical ice crystals measured by SPEC CPI
330 16:08:27–16:09:27 UTC on 26 May 2015. Ice crystal marked by green pentagram of each subgraph
331 means the largest particle of that whole period.

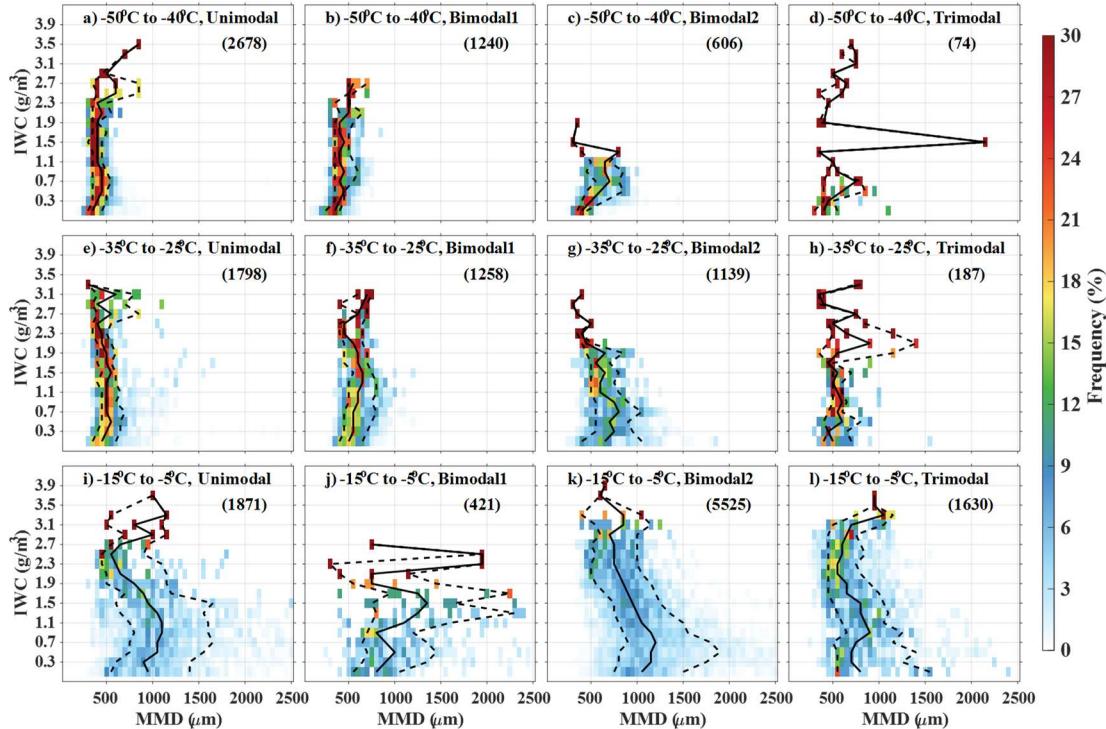
332
333 number of points inside both the $S1$ and $S2$ ellipsoids. The ratio (N_{s0}/N_{s1}) of the equally
334 plausible solutions mean number found in both ellipsoids to the number of points in the $S1$
335 ellipsoid and represents the overlap percent, with (N_{s0}/N_{s2}) representing the same for the
336 $S2$ ellipsoid.

337 **3. The distribution of four kinds of modality PSDs**

338 **a. Case study**

339 Figure 2 shows a time series of microphysical properties measured by probes on
340 the Convair-580 between 10:01:07–11:54:37 UTC on 26 May 2015. The Convair-580 flew
341 at a constant T of ~ -10 °C within an organized Oceanic MCS (Fig. 2a). Representative ice
342 crystals measured by the SPEC CPI shown in Figs. 3(a-e) correspond to P1 (10:05:57–
343 10:06:37), P2 (11:05:32–11:06:47), P3 (11:19:27–11:20:32), P4 (11:34:17–11:35:22), and
344 P5 (11:52:52–11:54:12 UTC) in the morning of 26 May 2015 in Fig. 2c. Typical ice
345 crystals measured by SPEC CPI in Fig. 3f correspond to P6 (16:08:27–16:09:27 UTC;
346 figure not shown) in the afternoon of 26 May 2015. The mean IWC and MMD are 2.34 g
347 m^{-3} and 738.8 μm respectively during P1, with some columns observed, similar to the ice
348 crystals shapes in Fig. 3f measured during P6, with mean IWC and MMD are 1.65 g m^{-3}
349 and 432.2 μm respectively. Thus, for this flight small columns dominated the mass content
350 of the $HIWC$ regions with small MMD (< 500 μm at -10 °C), with some large ice crystals
351 being aggregates of columns. The mean IWC reaches 2.53 g m^{-3} during P2 and the mean
352 MMD is 959.5 μm , with the large ice crystals being aggregates of columns and plate-like
353 crystals (Fig. 3b). Similar $IWCs$ have different MMD for P1 and P2, consistent with the
354 larger ice supersaturation in P2 (~ 0.028) compared to P1 (~ 0.008), making the P2
355 environment more favorable for the growth of ice crystals (Bailey et al., 2009). Besides,
356 the capped column, a column with a plate at either end is observed in P2, suggesting the
357 particle passed through several different growth regimes during its history. The MMD
358 reaches 1880 μm when the IWC is 0.42 g m^{-3} during P3, consistent with the large crystal
359 with broad branches shown in Fig. 3c. These crystals are typically formed at temperatures
360 around -15 °C at supersaturations greater than those at which plates form. However, ice

361 supersaturation is ~ -0.025 here, consistent with these ice crystals forming in other areas
 362 with higher relative humidity condition and falling into this region with transport by the
 363 horizontal wind. P4 with low mean IWC 0.27 g m^{-3} and MMD $597.3 \mu\text{m}$, has many small
 364 plate crystals (Fig. 3d). Some short and thick capped columns are seen here, different from
 365



366 Figure 4. Normalized frequency of MMD for each IWC range for (a) Unimodal $PSDs$, (b) Bimodal1
 367 $PSDs$, (c) Bimodal2 $PSDs$, and (d) Trimodal $PSDs$ for $-50^\circ\text{C} \leq T \leq -40^\circ\text{C}$. (e-h) As in Figures 4a-4d
 368 but for $-35^\circ\text{C} \leq T \leq -25^\circ\text{C}$. (i-l) As in Figures 4a-4d but for $-15^\circ\text{C} \leq T \leq -5^\circ\text{C}$. For each subplot,
 369 middle solid line indicates 50th percentile, whereas left and right dashed lines represent 15th and 85th
 370 MMD respectively. Numbers in brackets give numbers of sample data points.
 371

372 the long and thin capped columns in P2, consistent with the very low ice supersaturations.
 373 A region with lower IWC ($\sim 0.21 \text{ g m}^{-3}$) and large MMD ($\sim 1262 \mu\text{m}$) found during P5 has
 374 plentiful aggregates (Fig. 3e). Many small irregular ice crystals are also found in these six
 375 periods. In general, the processes of sedimentation, aggregation, mixing due to shear,
 376 vertical wind and horizontal wind transport complicate the analysis because the
 377 environment where ice particles are observed does not necessarily represent the
 378 environment in which the particles formed or grew. The mixed phase region may also
 379 potentially affect the formation of ice crystals. Mixing of air masses with different crystal
 380

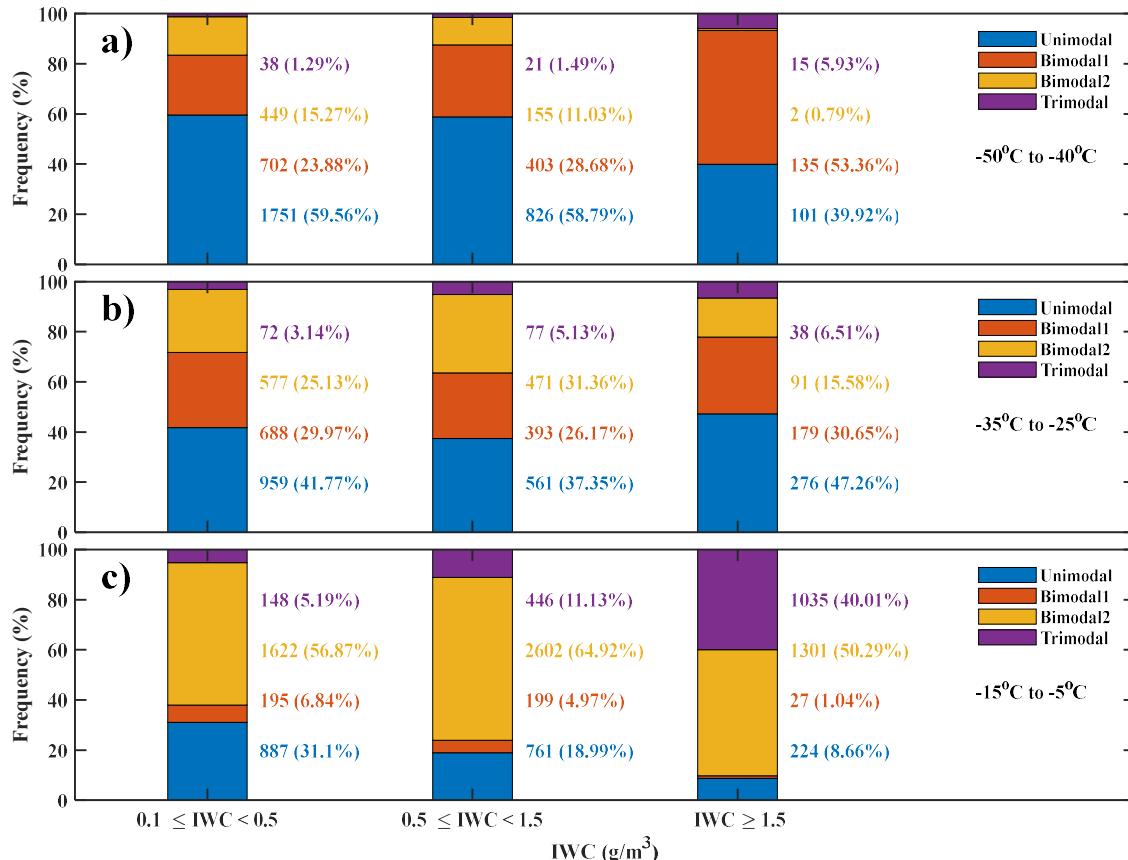
381 populations is also very important. Future analysis should try to distinguish between these
382 possibilities by using observations in combination with trajectory analysis and Large Eddy
383 Simulation (LES) modeling studies. HIWC with small *MMD* regions are full of small
384 columns and irregular ice crystals, consistent with a negative correlation between *IWC* and
385 *MMD* (Leroy et al., 2016a; Hu et al., 2021).

386 Figure 2d shows a time series of *IWC* and *MMD* with the shading indicating the
387 modality of the *PSD*. Trimodal (sky blue) and Bimodal2 *PSDs* (green) occur most
388 frequently in HIWC regions. For example, during P2 when $IWC > 1.5 \text{ g m}^{-3}$, the modality
389 of *PSDs* is trimodal. When *IWC* decreases to less than 1.5 g m^{-3} at 11:06:57 UTC, the
390 modality becomes Bimodal2 (green) and Unimodal (orange). After 11:09:07 UTC when
391 $IWC > 1.5 \text{ g m}^{-3}$, Trimodal *PSDs* again dominate. A negative correlation coefficient of -0.2
392 between *IWC* and *MMD* exists for the whole event (1,338 samples, Pearson method, *p*-
393 *value*: 10^{-13}), consistent with small columns and irregular ice crystals dominating the mass
394 of HIWC regions. Figure 2a shows HIWC regions can be as far away from the convective
395 core as 100 km for the leg at $-10 \text{ }^{\circ}\text{C}$ (e.g., $L > 100 \text{ km}$ during P1), suggesting particles
396 were generated in the convective core and then transported and fell into this region (Hu et
397 al., 2021). The lack of response from the Rosemount Icing Detector (Fig. 2f) and the $N_t <$
398 10 cm^{-3} measured by the CDP-2 (Fig. 2g) after 11:10:00 UTC confirm that most of these
399 data were collected in ice clouds. However, even though mixed phase regions were
400 excluded from the analysis, many of the ice crystals observed could have formed or grew
401 in mixed-phase regions. N_t measured by the 2D-S and PIP (Fig. 2h) are usually larger for
402 Trimodal *PSDs* (sky blue) and Bimodal2 *PSDs* (green) showing these regions are full of
403 small ice crystals and a positive correlation coefficient of 0.65 between *IWC* and N_t
404 (Pearson method, *p*-*value*: 10^{-165}) exists during the whole event.

405 **b. Distributions of *MMD* for different modality *PSDs***

406 To investigate how *MMD* depends on the *PSD* modality, the normalized frequency
407 of *MMD* for each *IWC* range for Unimodal, Bimodal1, Bimodal2, and Trimodal *PSDs* as a
408 function of *T* is shown in Figure 4. The ranges of *MMD* decrease sharply with decreasing
409 *T*, consistent with the analysis of Hu et al. (2021). The increase of *MMD* with *T* is consistent
410 with growth by vapor deposition and aggregation (Mitchell et al., 1996). Increased
411 contributions from sedimentation as larger particles fall to lower altitudes is one of

412 important factors (Jackson et al., 2015). For different modality *PSDs* at the same *T*, *MMDs*
413 for Bimodal2 and Trimodal *PSDs* are larger than those for Unimodal and Bimodal1 *PSDs*
414 for $T \leq -25^{\circ}\text{C}$, but trends for $-15^{\circ}\text{C} \leq T \leq -5^{\circ}\text{C}$ are different. The median *MMD* in
415 Unimodal *PSDs* increases with *IWC* to about 1000 μm at an *IWC* of 1.0 g m^{-3} (Fig. 4i), but
416 sharply decreases thereafter for $IWC > 1.0 \text{ g m}^{-3}$. Different trends for *MMD* for Bimodal1
417 and Bimodal2 *PSDs* are seen in Figure 4j and Figure 4k, with *MMD* increasing with *IWC*
418 for Bimodal1 *PSDs* and decreasing for Bimodal2 *PSDs*. The median *MMD* is
419 approximately constant ($\sim 800 \mu\text{m}$) when $IWC < 1.5 \text{ g m}^{-3}$ for Trimodal *PSDs* (Fig. 4l),
420 and *MMD* is the smallest when *IWC* is around 2.3 g m^{-3} . This suggests the number of large
421 ice crystals increases as *IWC* increases for Bimodal1 *PSDs*, and the number of small ice
422 crystals increases as *IWC* increases for other three types of *PSDs*.



423
424 Figure 5. Normalized occurrence frequency of four different modalities determined by Brechner (2021)
425 algorithm for three *IWC* ranges for (a) $-50^{\circ}\text{C} \leq T \leq -40^{\circ}\text{C}$, (b) $-35^{\circ}\text{C} \leq T \leq -25^{\circ}\text{C}$, and (c) $-15^{\circ}\text{C} \leq T \leq -5^{\circ}\text{C}$. Blue represents Unimodal *PSDs*, red Bimodal1 *PSDs*, yellow Bimodal2 *PSDs*, and purple Trimodal *PSDs*. Number of data points and normalized frequency percent (in brackets) of each modality labelled for each *IWC* range in same color as bar. Left column represents $0.1 \text{ g m}^{-3} \leq IWC < 0.5 \text{ g m}^{-3}$, middle $0.5 \text{ g m}^{-3} \leq IWC < 1.5 \text{ g m}^{-3}$, and right $IWC \geq 1.5 \text{ g m}^{-3}$.

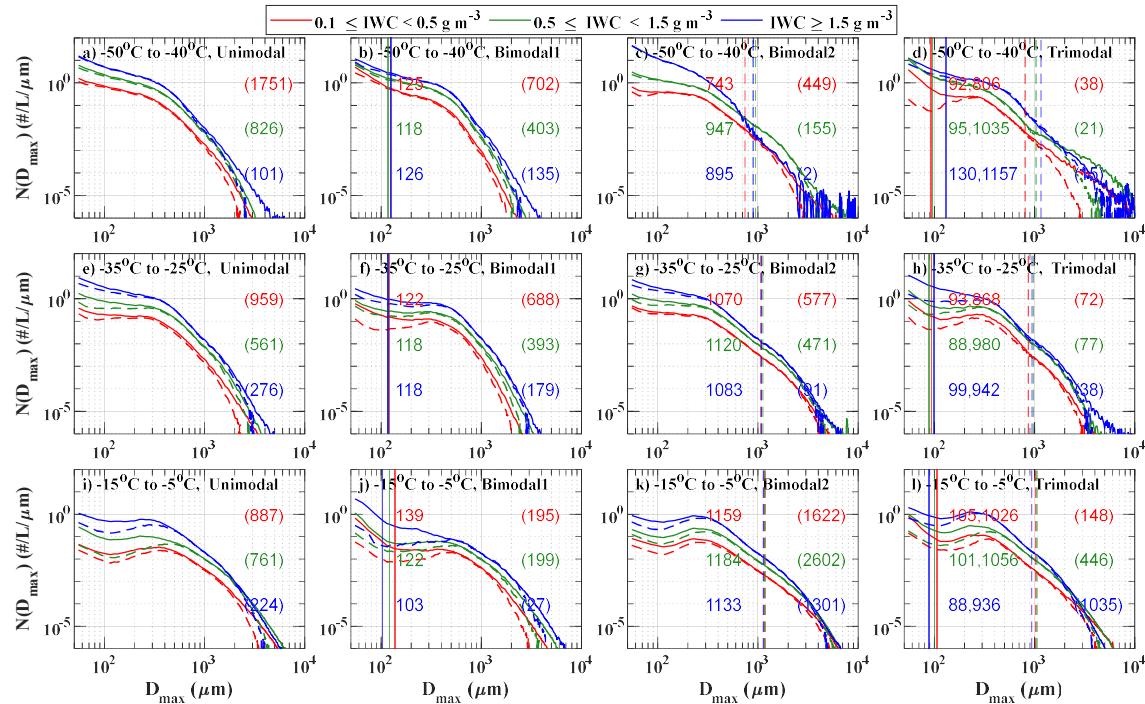
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431 **4. $N(D)$ as a function of environmental conditions**

432 In this section, the characteristics of the functional fits of the observed *PSDs* are
 433 examined as a function of environmental conditions (e.g., cloud types, underlying surface
 434 characteristics, MCS ages and distance away from the convective core). As T primarily
 435 influences the distributions of *IWC* and *MMD* (Hu et al., 2021), *PSDs* were first sorted into
 436 three T ranges ($-15^{\circ}\text{C} \leq T \leq -5^{\circ}\text{C}$; $-35^{\circ}\text{C} \leq T \leq -25^{\circ}\text{C}$; $-50^{\circ}\text{C} \leq T \leq -40^{\circ}\text{C}$) before
 437 examining the influence of other environmental conditions. Most of the data were collected
 438 at three different constant altitude legs at T of -10 , -30 , and -45°C .

439 **a. Different *IWC* regions**

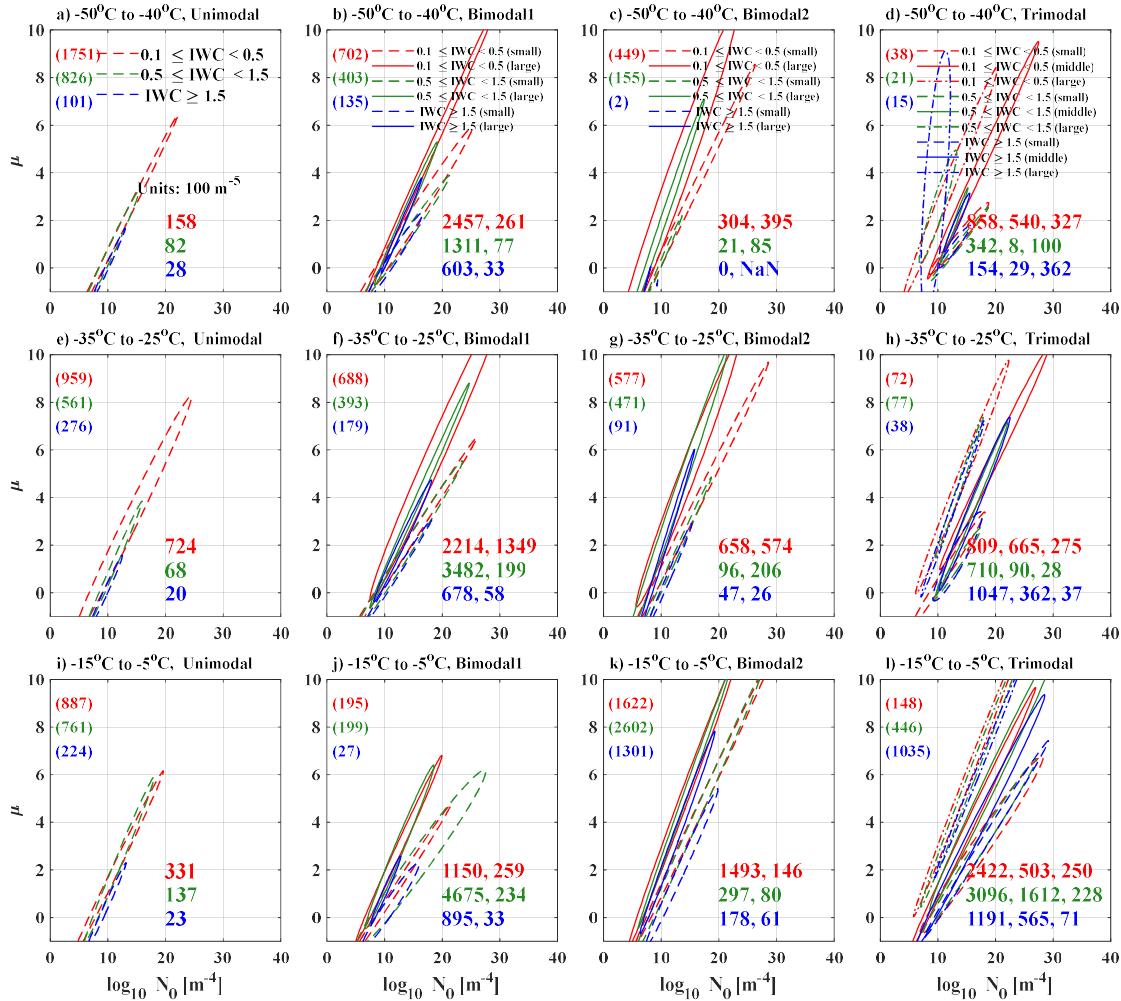
440 To compare *PSDs* in high *IWC* regions with regions without high *IWC*, *PSDs* were
 441 sorted into three *IWCs* ($0.1 \text{ g m}^{-3} \leq \text{IWC} < 0.5 \text{ g m}^{-3}$, $0.5 \text{ g m}^{-3} \leq \text{IWC} < 1.5 \text{ g m}^{-3}$, $\text{IWC} \geq$



442
 443 Figure 6. The Mean (solid line) and median (dashed line) distribution of (a) Unimodal, (b) Bimodal1,
 444 (c) Bimodal2, and (d) Trimodal *PSDs* for $-50^{\circ}\text{C} \leq T \leq -40^{\circ}\text{C}$. (e–h) As in Figures 6a–6d but for $-35^{\circ}\text{C} \leq T \leq -25^{\circ}\text{C}$. (i–l) As in Figures 6a–6d but for $-15^{\circ}\text{C} \leq T \leq -5^{\circ}\text{C}$. Red lines indicate $0.1 \text{ g m}^{-3} \leq$
 445 $\text{IWC} < 0.5 \text{ g m}^{-3}$, green $0.5 \text{ g m}^{-3} \leq \text{IWC} < 1.5 \text{ g m}^{-3}$, and blue $\text{IWC} \geq 1.5 \text{ g m}^{-3}$. For multimodal *PSDs*,
 446 solid vertical lines represent small breakpoints and dashed lines indicate large breakpoints. Values of
 447 breakpoints shown in same color as lines. Numbers in brackets give number of sample data points.
 448

449
 450 1.5 g m^{-3}). Figure 5 shows the normalized frequency of occurrence of the four different
 451 modality *PSDs* for the three *IWC* ranges as a function of T . It is apparent that the modality
 452 of *PSDs* is strongly dependent on T and is related to *IWC*, the frequency of multimodal

453 *PSDs* increases with T , and the frequency of Unimodal *PSDs* decreases with T . For
454 example, the frequency of Unimodal *PSDs* can be larger than 50% for $IWC < 1.5 \text{ g m}^{-3}$
455 when $-50^\circ\text{C} \leq T \leq -40^\circ\text{C}$ (Fig. 5a), but decreases to less than 32% when $-15^\circ\text{C} \leq T \leq -5^\circ\text{C}$ (Fig. 5c). This is consistent with small ice crystals experiencing aggregation when
456 falling from higher to lower altitude, and heterogeneous nucleation in the presence of
457 particles descending from the higher altitude (Zhao et al., 2010), leading to the varying
458 shapes of *PSDs* and frequency of modality of *PSDs* with T . The changes of modality with
459 IWC are strongly dependent on T . For $-15^\circ\text{C} \leq T \leq -5^\circ\text{C}$ (Fig. 5c), the frequency of
460 Unimodal *PSDs* decreases from $\sim 31\%$ to $\sim 8.7\%$ with increasing IWC , while the frequency
461 of Trimodal *PSDs* increases from $\sim 5.2\%$ to $\sim 40\%$. The frequency of Bimodal2 *PSDs*
462

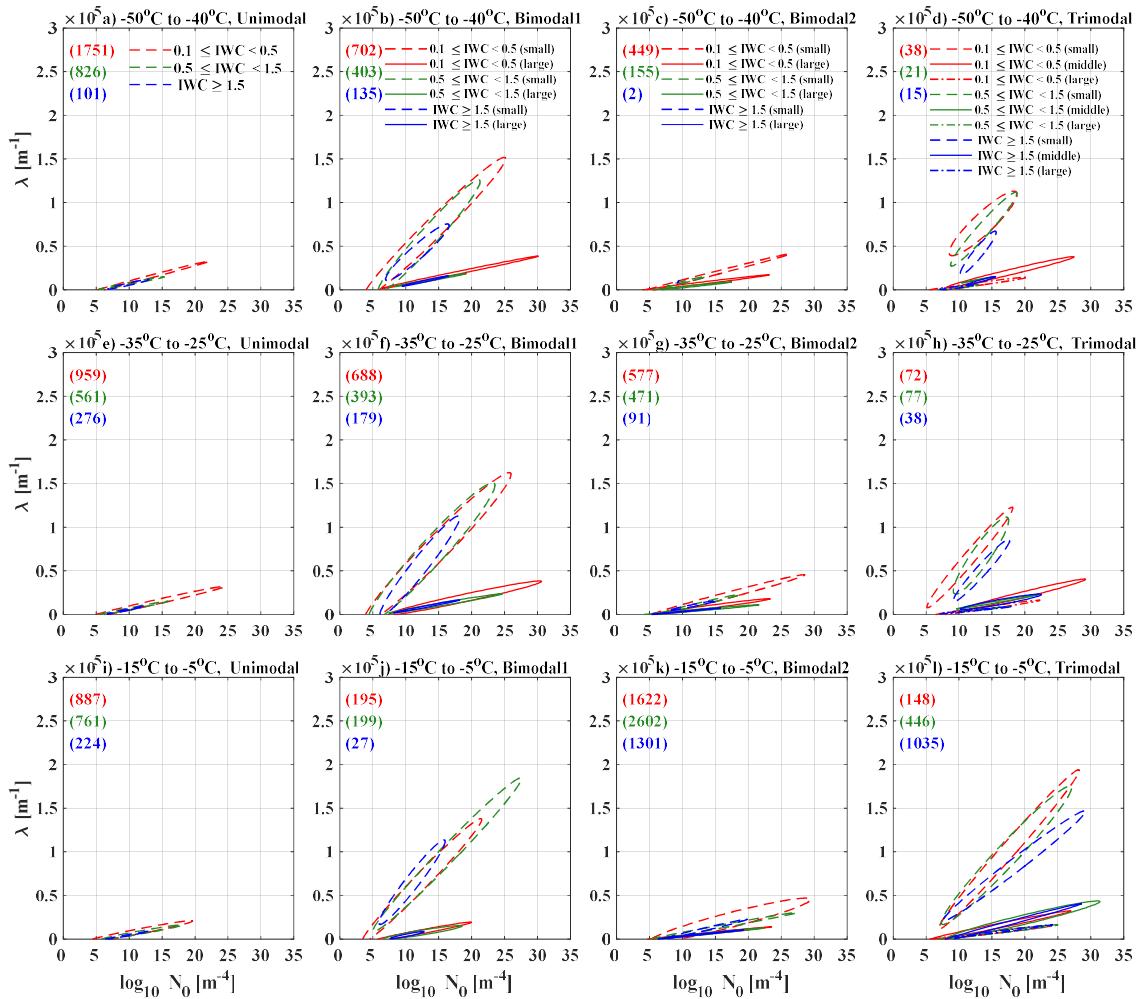


463
464 Figure 7. Projection of three-dimensional ellipses characterizing distributions of equally realizable
465 solutions for fit parameters in (N_o, μ) phase space for (a) Unimodal, (b) Bimodal1, (c) Bimodal2, and
466 (d) Trimodal *PSDs* at $-50^\circ\text{C} \leq T \leq -40^\circ\text{C}$. (e–h). As Figures 7a–7d but for $-35^\circ\text{C} \leq T \leq -25^\circ\text{C}$. (i–l)

467 As Figures 7a–7d but for $-15^{\circ}\text{C} \leq T \leq -5^{\circ}\text{C}$. Red lines indicate $0.1 \text{ g m}^{-3} \leq \text{IWC} < 0.5 \text{ g m}^{-3}$, green $0.5 \text{ g m}^{-3} \leq \text{IWC} < 1.5 \text{ g m}^{-3}$, and blue $\text{IWC} \geq 1.5 \text{ g m}^{-3}$. Different line types represent projections of 468 parameters for different modes in *PSDs*. Numbers in brackets give number of sample data points. The 469 volumes of ellipsoid are denoted by colorful numbers and shown in bottom right corner of each subplot 470 (units: 100 m^{-5}). Unimodal *PSDs* have one column, Bimodal *PSDs* have two columns and three columns 471 for Trimodal *PSDs*. The number of columns from left to right represents the mode from small to large. 472 For each column, top row represents $0.1 \text{ g m}^{-3} \leq \text{IWC} < 0.5 \text{ g m}^{-3}$, middle row $0.5 \text{ g m}^{-3} \leq \text{IWC} < 1.5 \text{ g}$ 473 m^{-3} , and bottom row $\text{IWC} \geq 1.5 \text{ g m}^{-3}$.

475

476 increases with increasing *IWC* for $-15^{\circ}\text{C} \leq T \leq -5^{\circ}\text{C}$, but the frequency of Bimodal1 477 *PSDs* decreases with increasing *IWC*. However, for $-35^{\circ}\text{C} \leq T \leq -25^{\circ}\text{C}$ and $-50^{\circ}\text{C} \leq T$ 478 $\leq -40^{\circ}\text{C}$ (Figs. 5a and 5b), the frequency of *PSDs* modalities does not show obvious 479 changes with increasing *IWC*. In summary, the frequency of Unimodal *PSDs* decreases and 480 the frequency of multimodal *PSDs* increases with increasing T , and different trends for the 481 frequency of the modality of *PSDs* with varying *IWC* for different T exist.

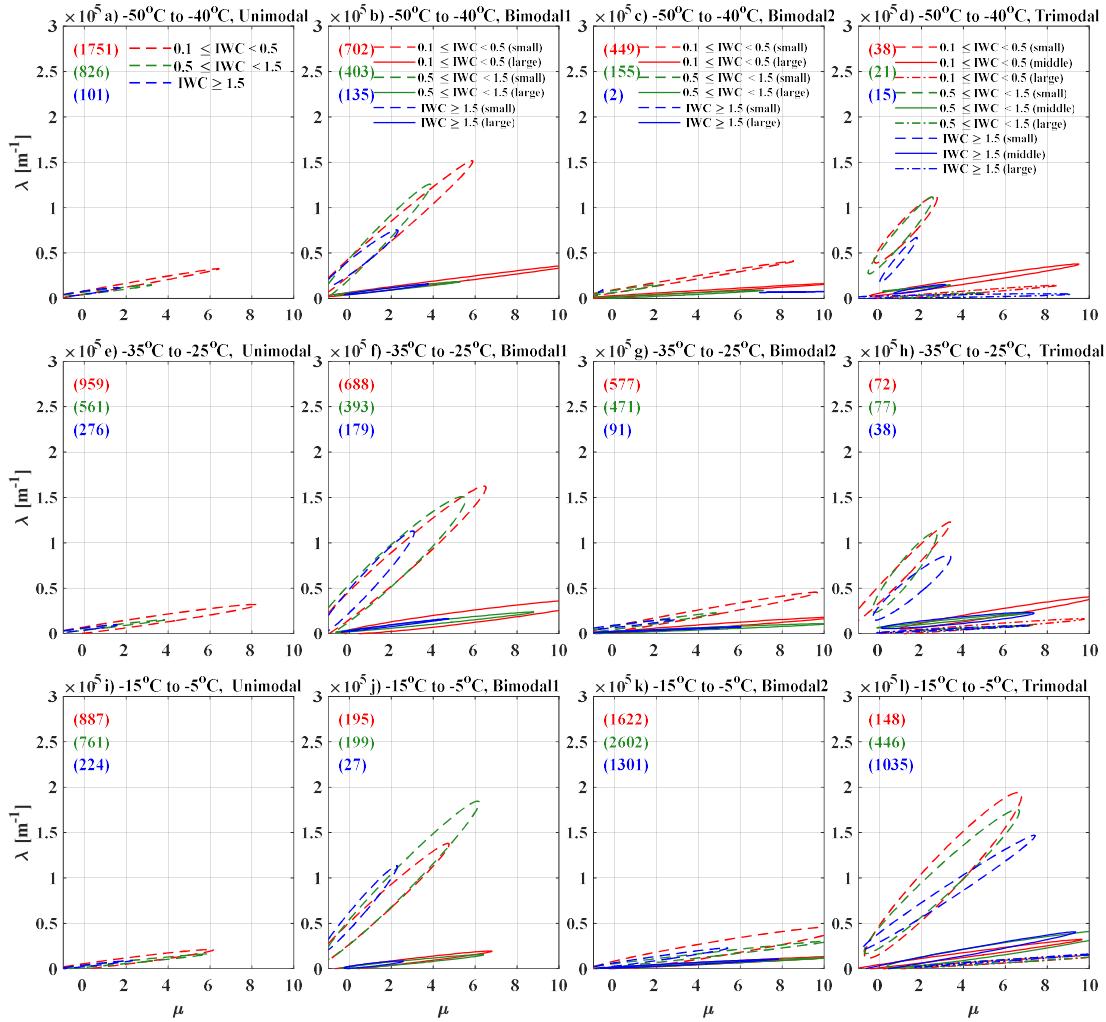


482

483 Figure 8. Same as Figure 7 but for (N_o, λ) phase space.

484 Figure 6 shows the mean (solid line) and median (dashed line) distributions of *PSDs*
485 for different *IWC* regions as a function of *T*, with breakpoints between modes marked. The
486 shapes of *PSDs* can change a lot even for the same modality with the changes heavily
487 dependent on *T*. The *N_t* of small ice crystals (< 200 μm) decreases with increasing *T*,
488 consistent with *MMD* increases with increasing *T* and a negative correlation between *T* and
489 *N_t* (Jackson et al., 2015; Hu et al., 2021). This is consistent with ice crystals experiencing
490 more aggregation and sedimentation at lower altitudes. Small ice crystals can completely
491 evaporate in subsaturated environments to keep the concentrations of ice crystals with *D_{max}*
492 < 100 μm relatively low (Korolev et al., 2011, 2013). There are usually more small ice
493 crystals (< 200 μm) and fewer large ice crystals (*D_{max}* > 2000 μm) as *IWC* increases for
494 Bimodal2 and Trimodal modality at the same *T*, which is consistent with the *MMD* being
495 negatively correlated with *IWC*. However, for Bimodal1 *PSDs*, the opposite trend can be
496 seen (Figs. 6b, 6f and 6j), as both the number of small and large ice crystals increase with
497 increasing *IWC*, which is consistent with the *MMD* being positively correlated with *IWC*
498 for $-15^\circ\text{C} \leq T \leq -5^\circ\text{C}$ (Fig. 4j) and the *MMD* being constant for $-35^\circ\text{C} \leq T \leq -25^\circ\text{C}$ and
499 $-50^\circ\text{C} \leq T \leq -40^\circ\text{C}$ (Figs. 4b and 4f). The breakpoint between the two modes for
500 Bimodal1 *PSDs* averages $120 \pm 20 \mu\text{m}$ and exhibits little dependence on *T*. Even though
501 this breakpoint occurs over a size range where the sample volume of the optical array
502 probes is rapidly increasing with particle size (Baumgardner et al., 2017; McFarquhar et
503 al., 2017), the existence of the bimodal distribution does not appear to be instrument related
504 because the bimodal distributions do not occur all the time. The mean breakpoints between
505 the two modes for bimodal2 *PSDs* range between 743 to 1184 μm , with the breakpoints
506 typically increasing with *T*, and minimal dependence on *IWC*. For the Trimodal *PSDs*, the
507 mean breakpoints between the small mode and middle mode range from 88 to 130 μm , and
508 the mean breakpoints between the middle mode and large mode range from 806 to 1157
509 μm . The mean breakpoints increase with increasing *T* for the regions with $0.1 \text{ g m}^{-3} \leq IWC$
510 $< 0.5 \text{ g m}^{-3}$ and decrease with increasing *T* for the HIWC regions with $IWC \geq 1.5 \text{ g m}^{-3}$.
511 The small breakpoints and large breakpoints in Trimodal *PSDs* correspond to the
512 breakpoints in the Bimodal1 *PSDs* and the breakpoints in Bimodal2 *PSDs* respectively.
513

514 To construct volumes of equally realizable solutions characterizing a family of
 515 *PSDs*, McFarquhar et al. (2015) defined a single ellipsoid around all (N_o, λ, μ) contained
 516 within at least 1% of the volumes for the *PSDs* contained within a family. The volumes
 517 characterizing the different modes in *PSDs* were similarly defined for each mode separately.
 518 To visualize the relationship of the gamma fit parameters with *IWC* as a function of T , two-
 519 dimensional projections of the volumes in (N_o, μ) phase space are shown in Figure 7.
 520 Different color lines represent different *IWC* groups, and different types of lines represent
 521 the distribution of parameters for the different modes. Regardless of the modality, μ is
 522 directly correlated with and increases with N_o . The slopes of the long axis of the ellipse in
 523 (N_o, μ) phase space are the largest for the small mode in the Trimodal *PSDs*, while slopes
 524 are smallest for the large mode in the Trimodal *PSDs*. The slopes of the long axis of the



525
 526 Figure 9. Same as Figure 7 but for (λ, μ) phase space.
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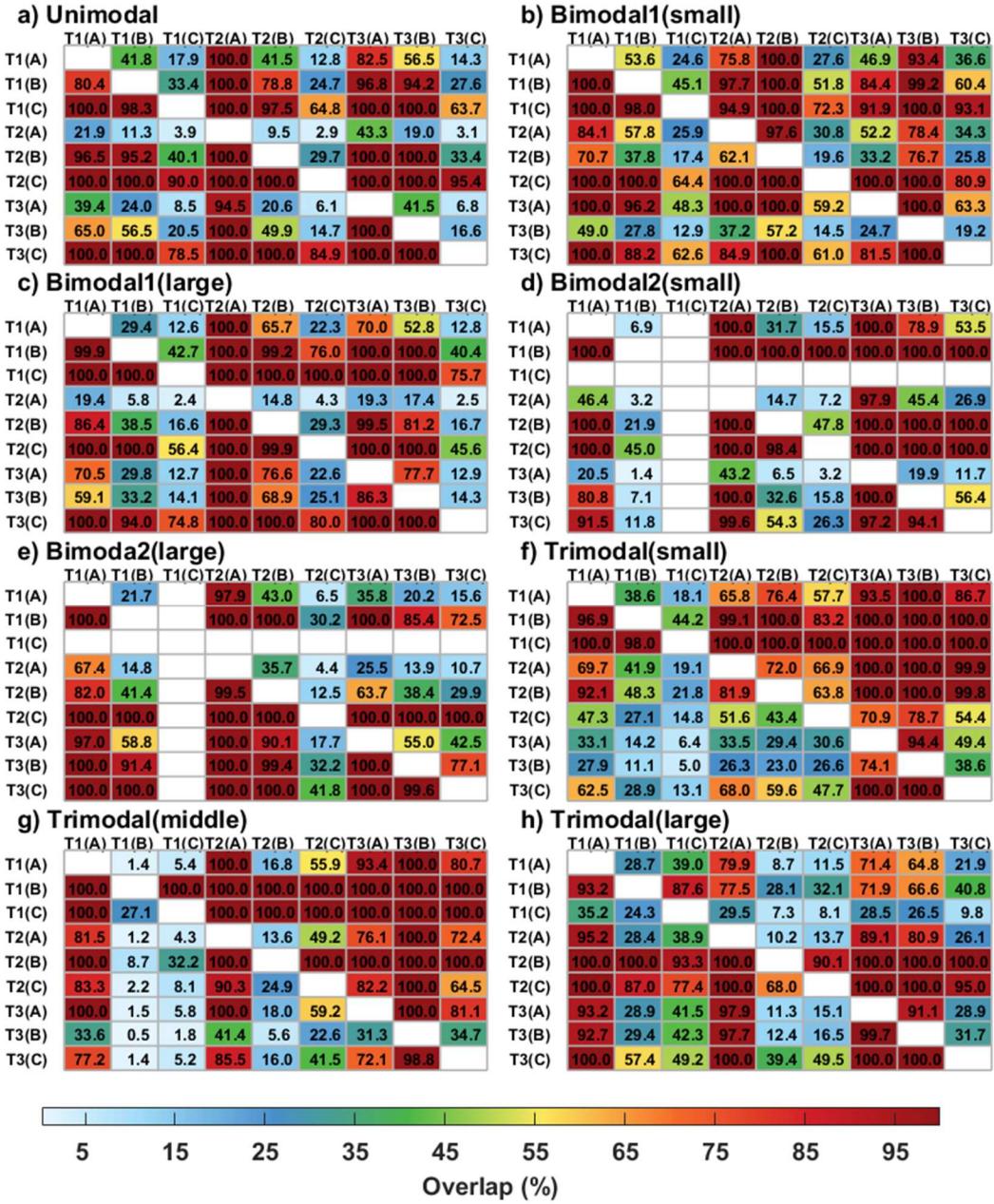
528 ellipse in (N_o, μ) phase space for Bimodal2 are usually larger than those for Bimodal1. In
529 general, the slopes of the long axis of the ellipse in (N_o, μ) phase space in multimodal *PSDs*
530 are larger for the small mode than for the large mode. For the same N_o, μ usually decreases
531 with increasing *IWC*. For example, N_o decreases from 10^{22} m^{-4} to 10^{14} m^{-4} and the
532 maximum of μ decreases from 6.5 to 1.8 for Unimodal *PSDs* for increases in *IWC* from 0.1
533 $\text{g m}^{-3} \leq \text{IWC} < 0.5 \text{ g m}^{-3}$ to $\text{IWC} \geq 0.5 \text{ g m}^{-3}$ when $-50 \text{ }^{\circ}\text{C} \leq T \leq -40 \text{ }^{\circ}\text{C}$. Consistent with
534 previous findings of Mascio et al. (2020) showing that μ and λ tend to decrease with
535 increasing *IWC*. This means the volumes of ellipses for small *IWC* regions are usually
536 larger than those for large *IWC*. For example, at $-50 \text{ }^{\circ}\text{C} \leq T \leq -40 \text{ }^{\circ}\text{C}$, the volume of
537 ellipsoid for $0.1 \text{ g m}^{-3} \leq \text{IWC} < 0.5 \text{ g m}^{-3}$ is $\sim 1.6 \times 10^4 \text{ m}^{-3} \mu\text{m}^{-2}$, while it decreased to 2.8
538 $\times 10^3 \text{ m}^{-3} \mu\text{m}^{-2}$ when $\text{IWC} > 1.5 \text{ g m}^{-3}$. The smaller μ as *IWC* increases is consistent with
539 more small ice crystals with increasing *IWC*, and with *MMD* decreasing with increasing
540 *IWC*. The range of N_o is from 10^5 m^{-4} to above 10^{20} m^{-4} values that cover more than 15
541 orders of magnitude, consistent with previous studies (e.g., McFarquhar et al., 2015;
542 Jackson et al., 2015; Mascio et al., 2020). In addition, these figures also show that there are
543 less prominent impacts on how *T* affects the parameters of *PSDs*.

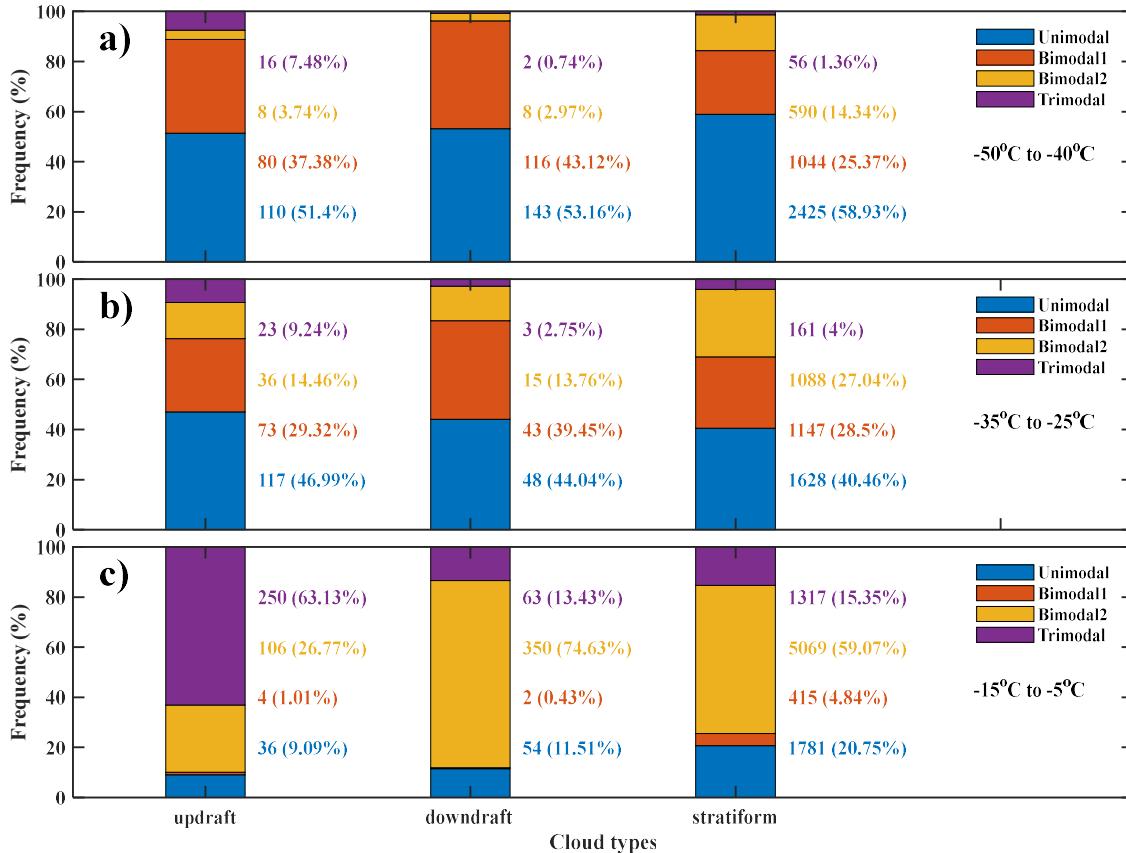
544 Figure 8 shows the ellipse distributions in (N_o, λ) phase space for different modes
545 as a function of *T* and *IWC*. Regardless of modality, λ increases with N_o . Similar to (N_o, μ)
546 phase space, the slopes of the long axis of the ellipses in (N_o, λ) phase space are largest for
547 the small mode in Trimodal *PSDs*, and smallest for the large mode in Trimodal *PSDs*. The
548 slopes of the long axis of the ellipse and values of λ in (N_o, λ) phase space for Bimodal1
549 *PSDs* are usually larger than those in Bimodal2 *PSDs*, due to the existence of more large
550 crystals in the Bimodal2 *PSDs*. This is expected because the breakpoint occurs at a larger
551 D_{max} . For the same N_o , there are less obvious changes for λ with increasing *T* for Unimodal,
552 Bimodal1 and Bimodal2 *PSDs*, while the range of λ increases for the small mode of
553 Trimodal *PSDs*. The range of λ tend to be smaller with increasing *IWC* regardless of the
554 modality of *PSDs* and the ellipses of small *IWCs* are usually larger than those of larger
555 *IWCs*.

556 Figure 9 shows the ellipse distributions in (μ, λ) phase space. Regardless of
557 modalities, λ increases with increasing μ . Similar to the (N_o, λ) and (N_o, μ) phase spaces,

558 the slopes of the long axis of the ellipse in (μ, λ) phase space are largest for the small mode
559 in the Trimodal *PSDs*, and smallest for the large mode. The slopes of the long axis of the
560 ellipse in (μ, λ) phase space for Bimodal1 *PSDs* are usually larger than those in Bimodal2
561 *PSDs*. That means for the same μ, λ is usually smaller in Bimodal2 *PSDs* than in Bimodal1
562 *PSDs*, consistent with the presence of more larger ice crystals in Bimodal2 *PSDs* due to
563 contributions from processes such as aggregation. Similar to the other projections, there is
564 no strong dependence of the slopes of μ and λ on *IWC*, but the volumes of the ellipses of
565 small *IWCs* are usually larger than those of large *IWCs*.

566 In order to develop a size distribution parameterization for use in models and to
567 learn more about processes occurring in these clouds, it is necessary to quantify the
568 dependence of fit parameters on environmental conditions. Figure 10 shows the fractional

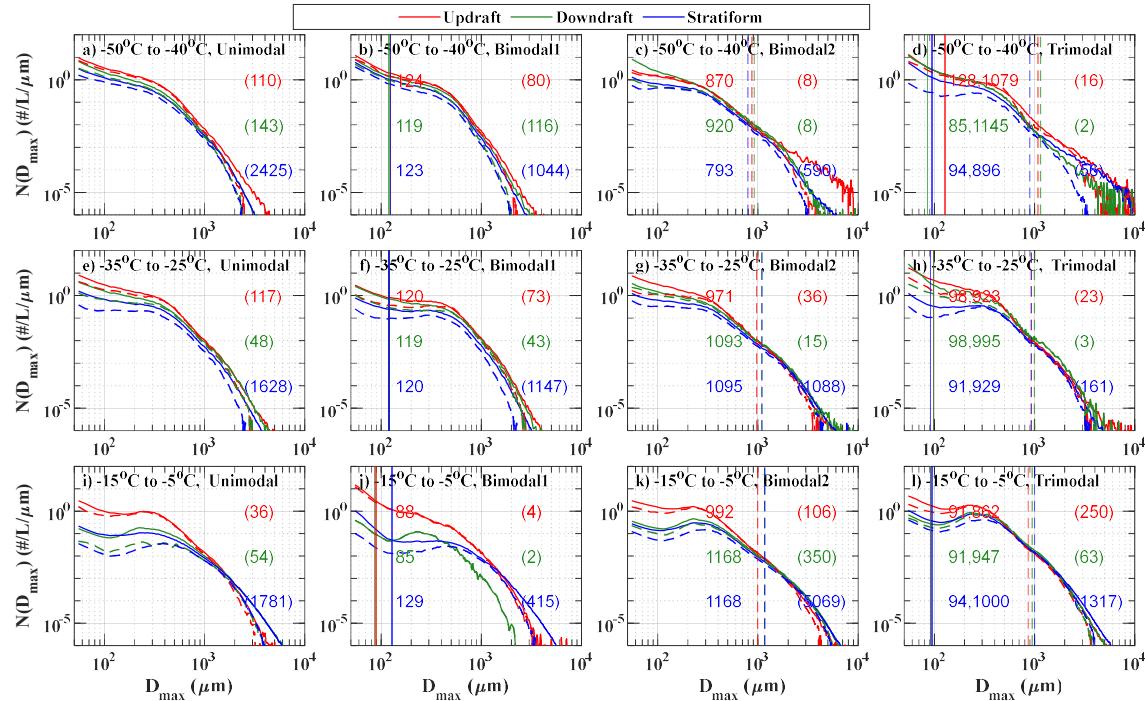




582
583 Figure 11. As Figure 5 but for updrafts, downdrafts and stratiform regions.
584
585 *IWC* families at the same T , the volumes in regions with $IWC \geq 1.5 \text{ g m}^{-3}$ are usually
586 smaller than the regions with $IWC < 1.5 \text{ g m}^{-3}$ regardless of modality. For example, for the
587 temperature range $-15^{\circ}\text{C} \leq T \leq -5^{\circ}\text{C}$ the blue box in the first row and third column of
588 figure 10a means 17.9% of the equally plausible solutions with $0.1 \text{ g m}^{-3} \leq IWC < 0.5 \text{ g}$
589 m^{-3} are found in the family with $IWC \geq 1.5 \text{ g m}^{-3}$, whereas the dark red box in the third
590 row and first column indicates all solutions with $IWC \geq 1.5 \text{ g m}^{-3}$ are contained in the
591 family with $0.1 \text{ g m}^{-3} \leq IWC < 0.5 \text{ g m}^{-3}$. The matrices are not symmetric, because the
592 overlap between any two ellipsoids is unique, and each ellipsoid has its own special central
593 position and range. A prominent feature in Fig. 10a is that for the Unimodal *PSDs* at $-15^{\circ}\text{C} \leq T \leq -5^{\circ}\text{C}$, less than 20% of the equally plausible solutions found in regions with $IWC < 1.5 \text{ g m}^{-3}$ are contained in the families with $IWC \geq 1.5 \text{ g m}^{-3}$ while $\sim 100\%$ of the points
595 in the family with $IWC \geq 1.5 \text{ g m}^{-3}$ are found in the regions with $0.5 \text{ g m}^{-3} \leq IWC < 1.5 \text{ g}$
596 m^{-3} . Similar trends for volume overlap can be seen for Bimodal1 (small) and Bimodal1

598 (large). In general, the ellipsoids of $0.1 \text{ g m}^{-3} \leq IWC < 0.5 \text{ g m}^{-3}$ families have the largest
 599 volumes for all modalities, while volumes for $IWC \geq 1.5 \text{ g m}^{-3}$ families have the smallest
 600 volumes. Except for the large mode in Trimodal *PSDs*, the percent overlap is usually more
 601 than 60% and even up to 100% for regions with $IWC \geq 1.5 \text{ g m}^{-3}$ when $-50^\circ\text{C} \leq T \leq -40$
 602 $^\circ\text{C}$. This make sense as the volume of ellipsoid for $-15^\circ\text{C} \leq T \leq -5^\circ\text{C}$ and $IWC \geq 1.5 \text{ g}$
 603 m^{-3} is usually the smallest, consistent with the ranges of parameters shown in Figures 7, 8,
 604 and 9, indicating that the parameters (N_o , μ , and λ) are constrained for HIWC regions
 605 compared to other regions.

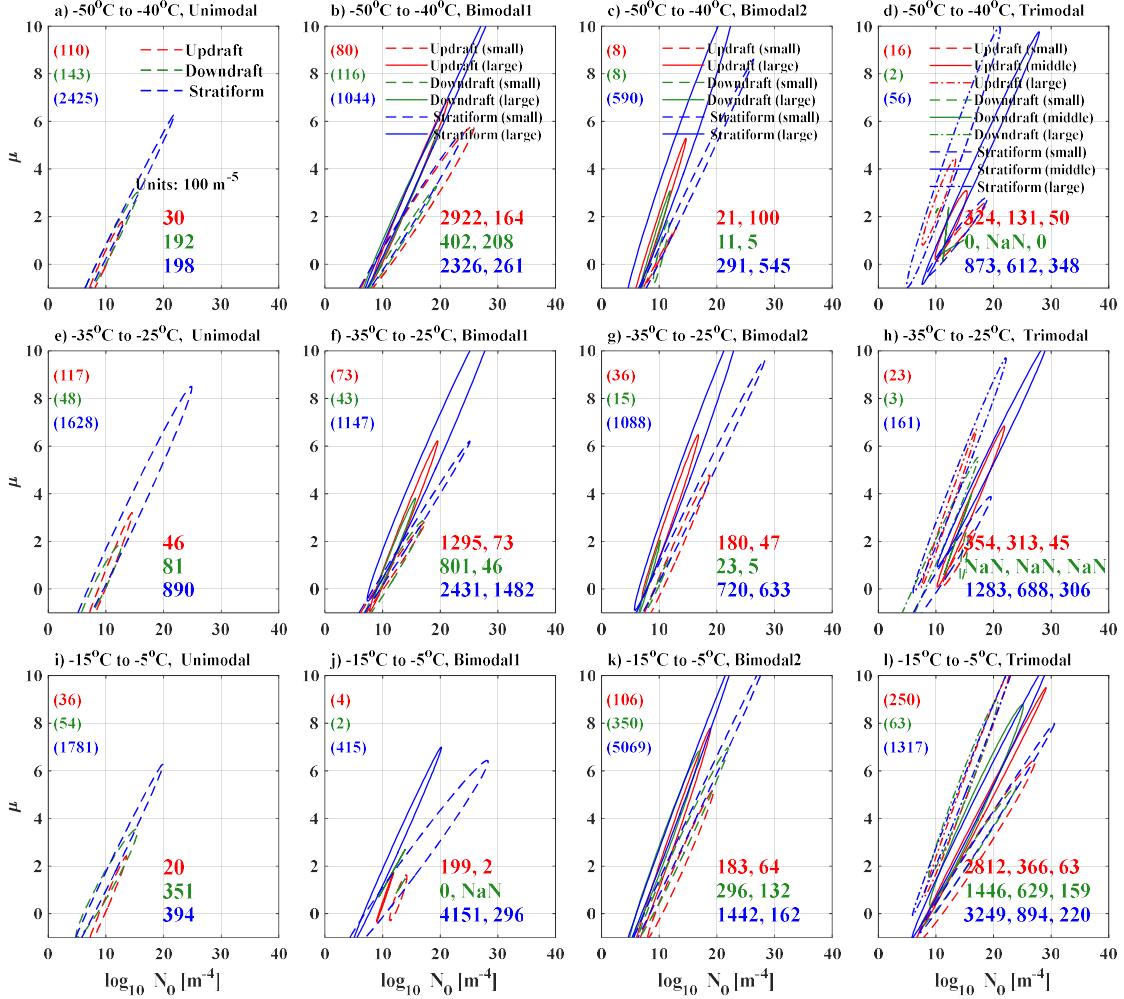
606 In summary, the volumes of the ellipsoids decrease with increasing *IWC* because
 607 there are more small ice crystals in the higher *IWC* regions. λ and μ both increase with N_o ,
 608 and λ increases with increasing μ , consistent with the trends noted by McFarquhar et al.
 609 (2007), Heymsfield et al. (2002), and Heymsfield et al. (2013). And, the N_o , λ and μ exhibit
 610 mutual dependence and are not truly independent parameters, consistent with previous
 611 studies (e.g., McFarquhar et al, 2015; Mascio et al., 2020).



612
 613 Figure 12. As Figure 6 but segregated according to whether observations obtained in updrafts,
 614 downdrafts or stratiform regions.

615
 616 **b. Vertical velocity**

617 To investigate the impact of vertical motion on *PSDs*, data were divided according
 618 to whether they were obtained in updrafts, downdrafts, or stratiform regions. A convective
 619



620
 621 Figure 13. As Figure 7 but segregated according to whether observations obtained in updrafts,
 622 downdrafts or stratiform regions. Different line types represent projections of parameters for different
 623 modes in *PSDs*. Numbers in brackets give number of sample data points. The volumes of ellipsoid are
 624 denoted by colorful numbers and shown in bottom right corner of each subplot. Unimodal *PSDs* have
 625 one row, Bimodal *PSDs* have two rows and three rows for Trimodal *PSDs*. The number of columns
 626 from left to right represents the mode from small to large. For each column, top row represents updrafts,
 627 middle row downdrafts, and bottom row stratiform regions.

628
 629 updraft (downdraft) was defined as any 5-second period when $w > 1 \text{ m s}^{-1}$ ($< -1 \text{ m s}^{-1}$) was
 630 sustained for at least four consecutive seconds (Jorgensen et al., 1985; McFarquhar and
 631 Black, 2004; Murphy et al., 2017). A stratiform region (i.e., $-1 \text{ m s}^{-1} \leq w \leq 1 \text{ m s}^{-1}$) was a
 632 period that had neither an updraft nor a downdraft present. Figure 11 shows the normalized

frequency of the different modalities for the different vertical motion categories. An apparent feature is that the frequency of Unimodal *PSDs* at $-50^{\circ}\text{C} \leq T \leq -40^{\circ}\text{C}$ is larger than those at $T \leq -35^{\circ}\text{C}$ regardless of vertical motion. Further, the frequency of Bimodal2

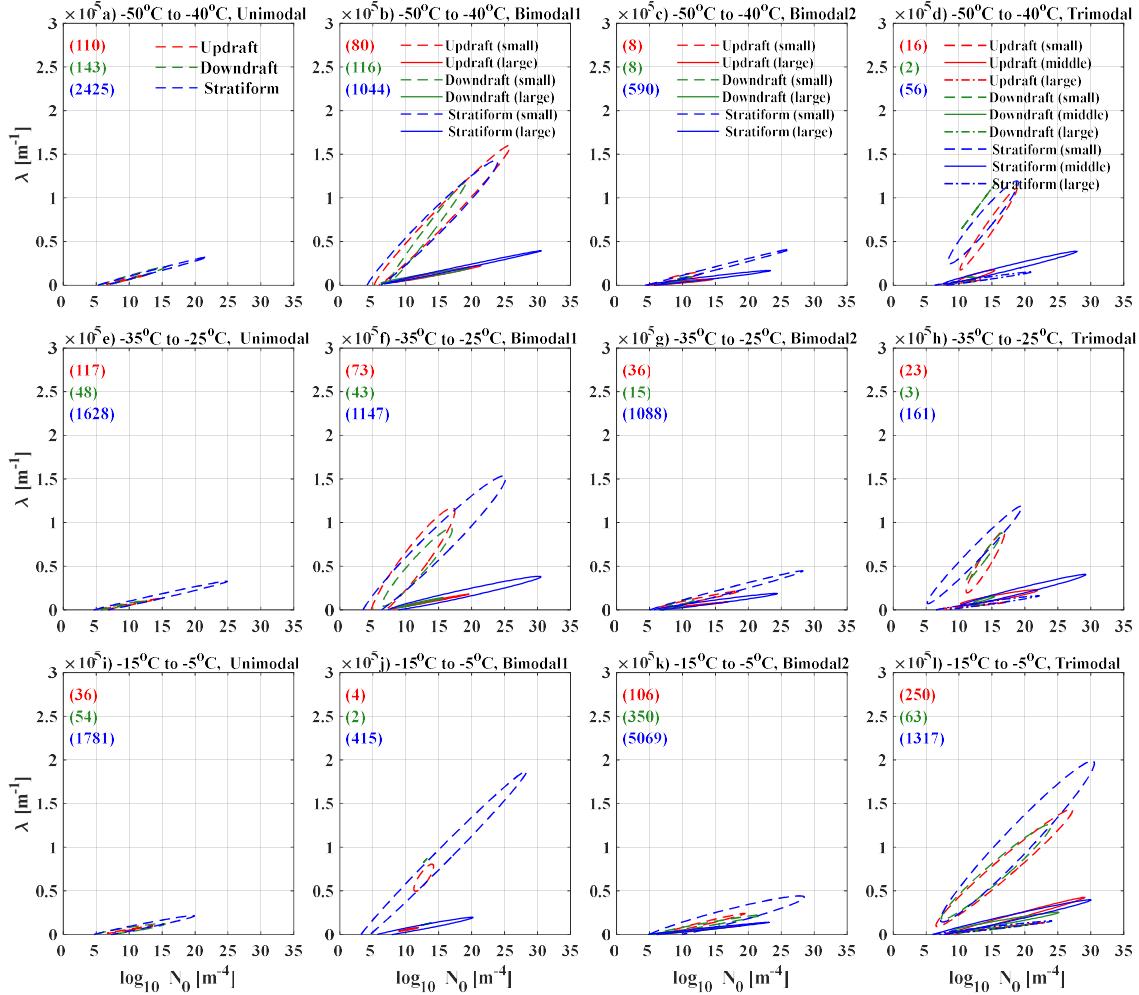


Figure 14. As Figure 8 but segregated according to whether observations obtained in updrafts, downdrafts or stratiform regions.

PSDs and Trimodal *PSDs* both increase as T increases, while the frequency of Bimodal1 *PSDs* decreases with increasing T regardless of vertical motion. Despite trends with T dominating any trend with vertical motion, there is still a clear trend in that the frequency of Trimodal *PSDs* in updrafts is more frequent than for downdrafts or stratiform regions, especially for $-15^{\circ}\text{C} \leq T \leq -5^{\circ}\text{C}$ (Fig. 11c). This is consistent with previously noted trends in *IWC* (Fig. 5) because the *IWCs* in updrafts are larger than those in the other areas. The

frequency of Bimodal *PSDs* is the largest ($\sim 75\%$) in downdrafts $-15^{\circ}\text{C} \leq T \leq -5^{\circ}\text{C}$ (Fig. 11c).

Figure 12 shows the mean (solid line) and median (dashed line) *PSDs* for different vertical velocities as a function of T for all modality *PSDs*. Consistent with trends in Fig.

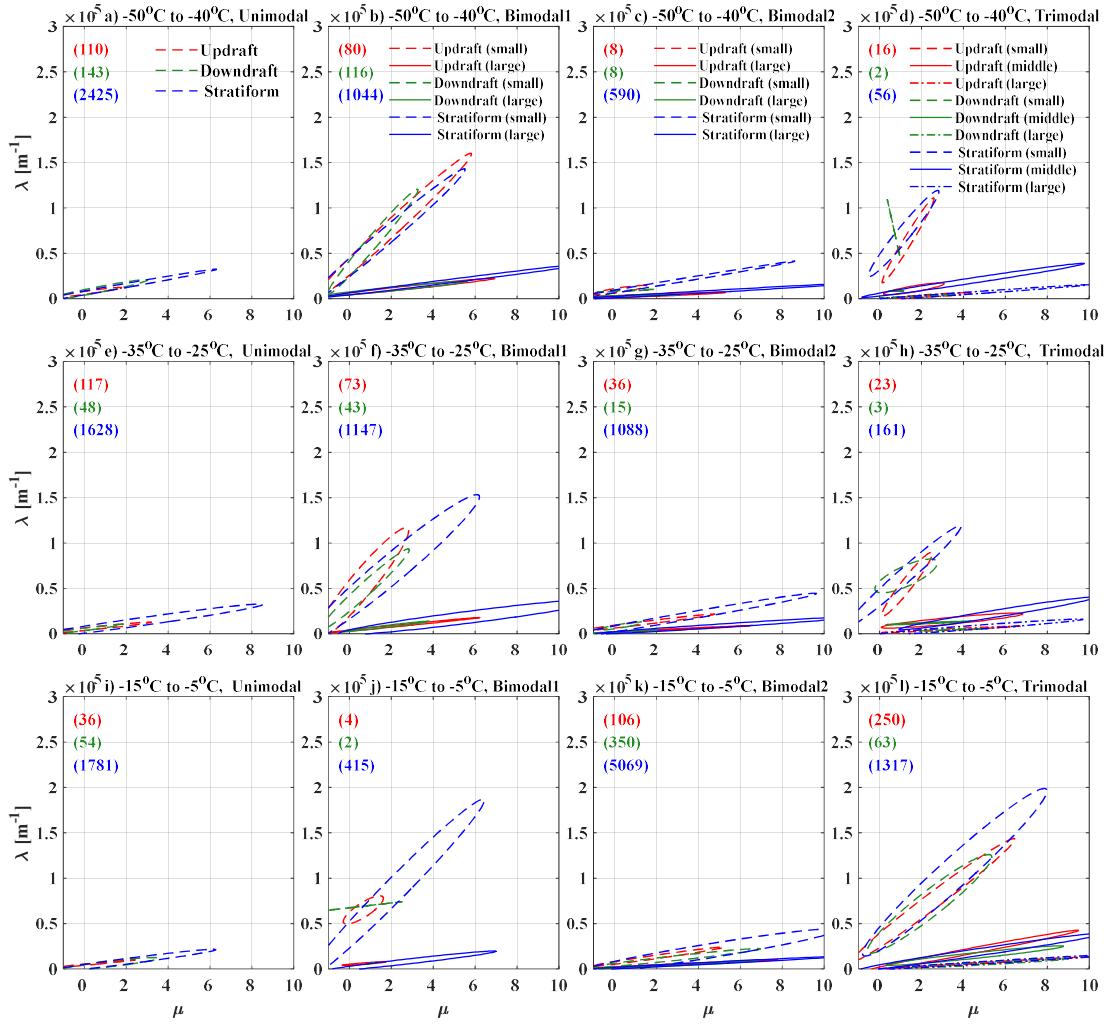
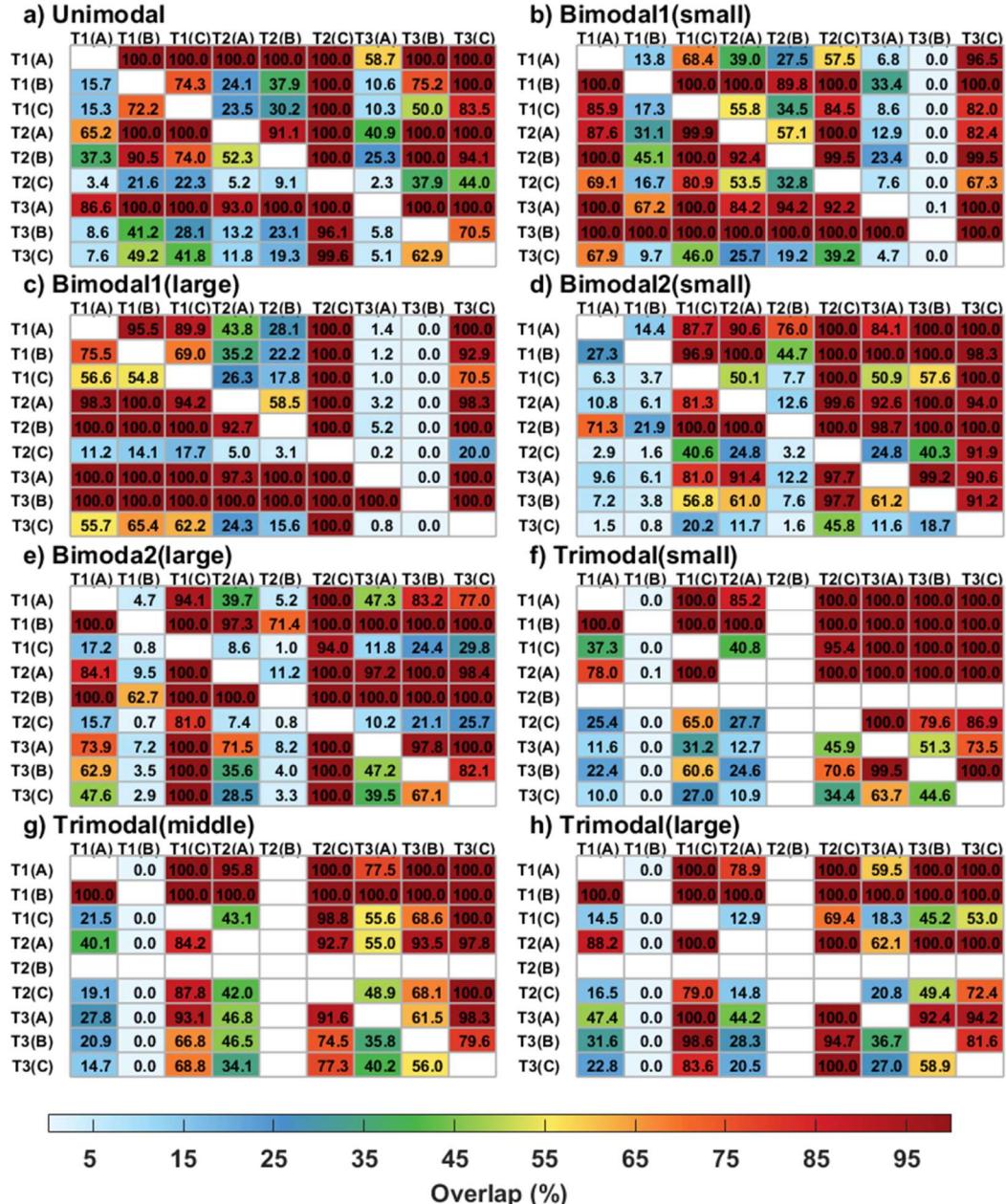


Figure 15. As Figure 9 but segregated according to whether observations obtained in updrafts, downdrafts or stratiform regions.

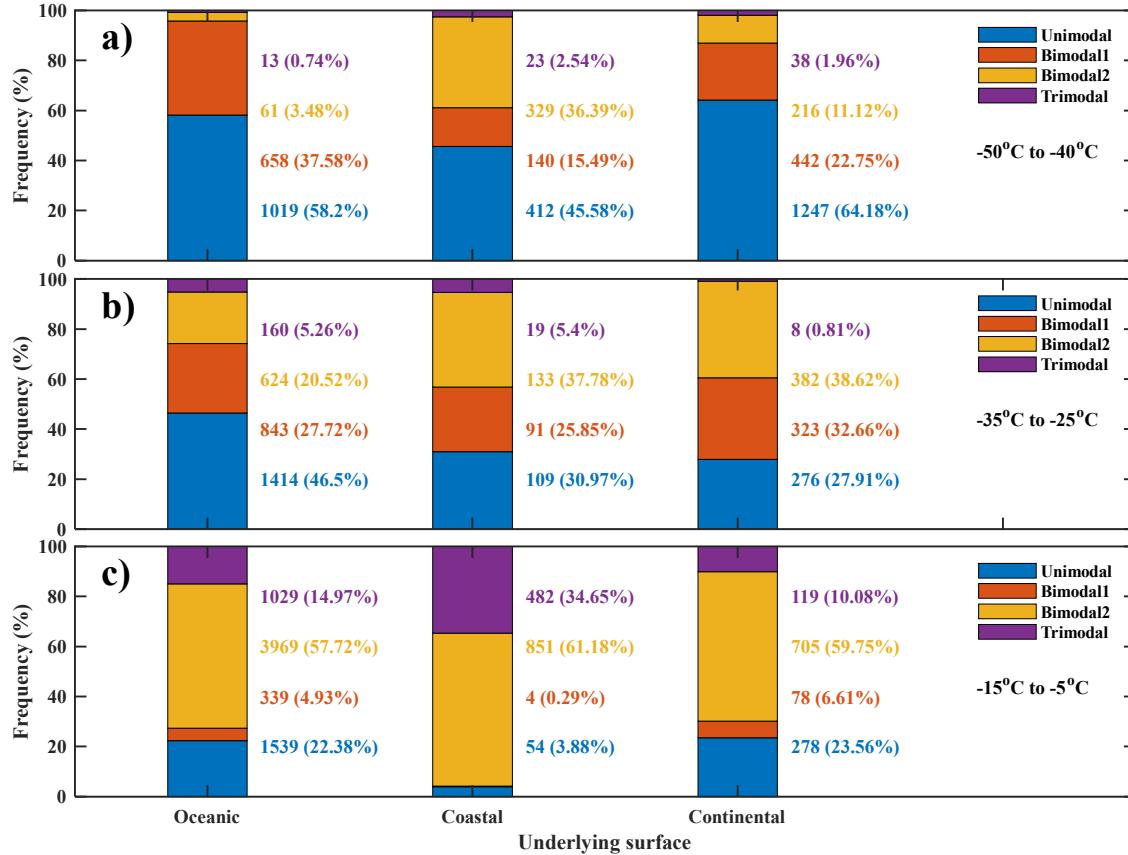
11, the *PSDs* are more dependent on T than on vertical motion. But there is some dependence on vertical motion. There are typically more small ice crystals ($< 500 \mu\text{m}$) in updrafts than in other regions for the same modality and T , similar to the finding of Mascio et al. (2020) showing that $N(D)$ is smallest for stratiform regions, consistent with updrafts regions being the source region of small ice crystals. The mean breakpoints between the two modes for Bimodal1 *PSDs* are on the average of $100 \pm 30 \mu\text{m}$ with no obvious

660 dependence on T . The mean breakpoints between the two modes for Bimodal2 *PSDs* are
661 in the range of 992 to 1168 μm when $-15^\circ\text{C} \leq T \leq -5^\circ\text{C}$, but decrease with decreasing T ,
662 becoming less than 950 μm for $-50^\circ\text{C} \leq T \leq -40^\circ\text{C}$. There is no obvious dependence of
663 the breakpoints on vertical velocity. For the Trimodal *PSDs*, the mean breakpoints between



664
665 Figure 16. As Figure 10 but for different vertical velocities. A, B and C represents three cloud types:
666 updrafts, downdrafts and stratiform regions respectively. $T1$, $T2$ and $T3$ represent $-50^\circ\text{C} \leq T \leq -40^\circ\text{C}$,
667 $-35^\circ\text{C} \leq T \leq -25^\circ\text{C}$ and $-15^\circ\text{C} \leq T \leq -5^\circ\text{C}$ respectively.
668

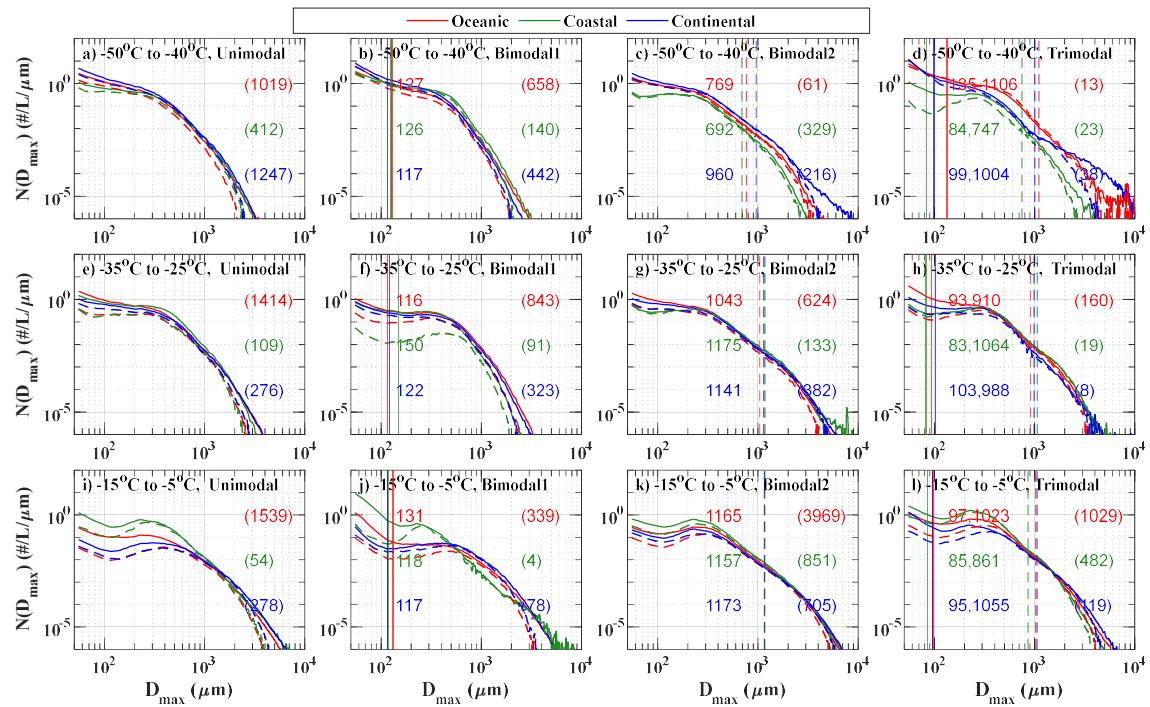
669 the small mode and middle mode are in the range of 85 to 128 μm , and the mean
 670 breakpoints between the middle mode and large mode are in the range of 850 to 1200 μm ,
 671 with no strong dependence on either T or vertical motion. The small breakpoints and large
 672 breakpoints in Trimodal *PSDs* are similar to the breakpoints in Bimodal1 *PSDs* and in
 673 Bimodal2 *PSDs* respectively, with the same trends for the varying vertical motion.



674
 675 Figure 17. As Figure 5 but segregated according to whether observations obtained in MCSs over
 676 different underlying surface characteristics (land, ocean, or coastal line).

677
 678 Figure 13 shows the ellipse projections of the fit parameters in (N_o, μ) phase space
 679 for different modalities and vertical velocities as a function of T . The slopes of N_o with λ
 680 in the small mode of the Trimodal *PSDs* are largest, and in the large mode of the Trimodal
 681 *PSDs* are smallest. There are less obvious changes of the slopes as T changes. However,
 682 the ellipses of stratiform regions are larger than those for updrafts and downdrafts. N_o and
 683 μ can reach larger values in stratiform regions compared to values within updrafts and
 684 downdrafts because the *IWC* is smaller in stratiform regions than in updrafts and
 685 downdrafts (Hu et al., 2021).

686 Figure 14 shows the ellipse projections of the fit parameters in (N_o, λ) phase space
 687 for different modalities and vertical velocities as a function of T . For all modalities, λ
 688 increases with increasing N_o . Although there is no strong difference in the slopes of N_o and
 689 λ on modality for the same T , the ellipses describing stratiform regions tend to be larger
 690 than those associated with other vertical motions. The N_o and λ have smaller values in
 691 updrafts, consistent with the more likely occurrence of HIWC regions in updrafts and the
 692 smaller ellipsoid volumes for the HIWC regions. However, Mascio et al. (2020) found N_o
 693 and λ are the largest for updrafts. This difference probably occurs because data were
 694 sampled under different meteorological conditions.

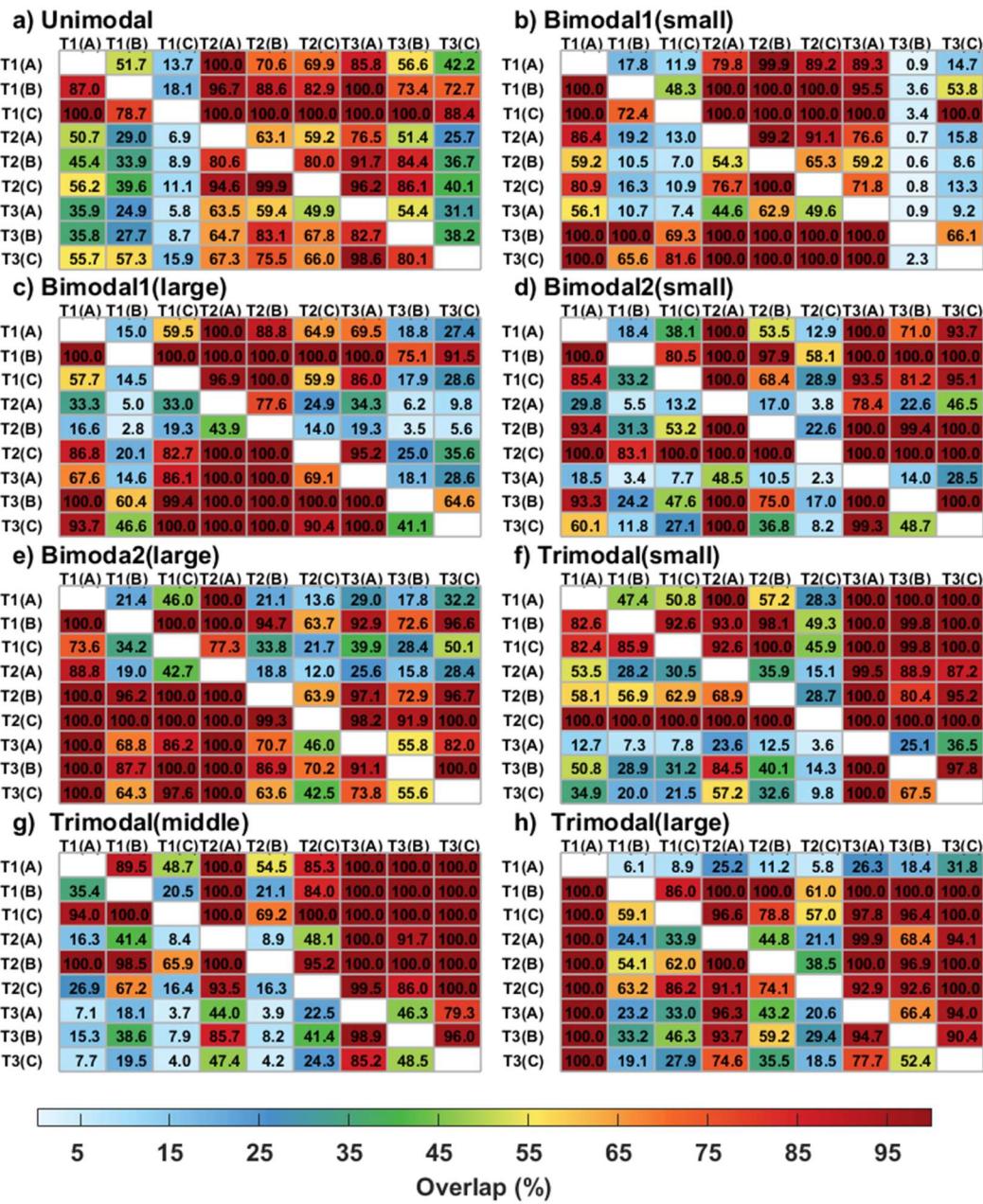


695
 696 Figure 18. As Figure 6 but segregated according to whether observations obtained in MCSs over
 697 different underlying surface characteristics (land, ocean, or coastal line).
 698

699 Figure 15 shows the projections of fit parameters in (μ, λ) phase space. Similar to
 700 the (N_o, λ) and (N_o, μ) phase spaces, the slopes of the long axis of the ellipse in (μ, λ) phase
 701 space are positive regardless of modality or T . The ellipses of stratiform regions are usually
 702 larger than those for updraft and downdraft regions for most situations. For Trimodal PSDs,
 703 there are obvious difference of slopes between the small mode and middle mode, and less
 704 obvious difference between middle mode and large mode.

705 To quantify how the fit parameters vary with vertical motion, Figure 16 shows the
 706 overlap fraction between the ellipsoid volumes for different T and vertical motions. For the
 707 same T , volumes in stratiform regions are usually larger for updrafts and downdrafts
 708 regardless of modality. For example, the percent overlap is usually less than 70% for
 709 stratiform regions when $-15^{\circ}\text{C} \leq T \leq -5^{\circ}\text{C}$. Further, the volumes in downdrafts are usually
 710 larger than for updrafts regions.

711 c. Surface characteristics

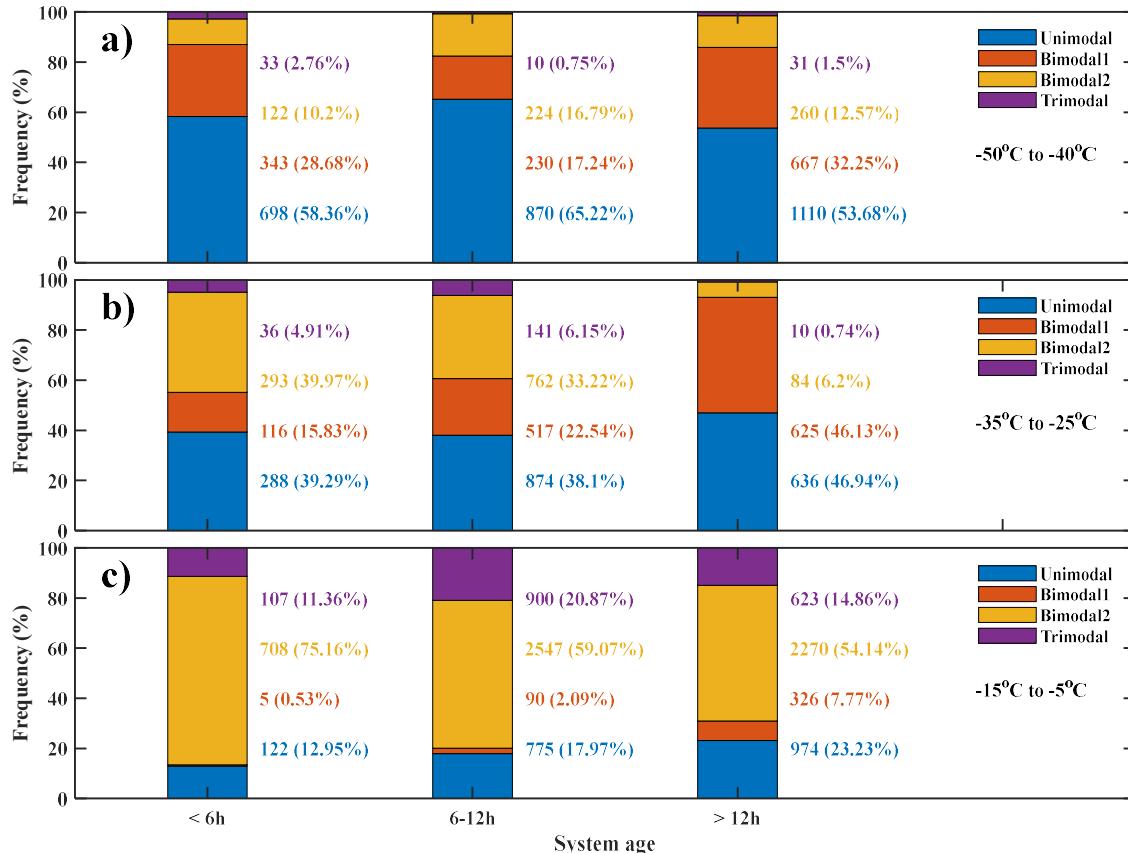


713 Figure 19. As Figure 10 but for different underlying surface characteristics. A, B and C represents
714 underlying surface characteristics: oceanic, coastal and continental MCSs respectively. $T1$, $T2$ and $T3$
715 represent $-50^{\circ}\text{C} \leq T \leq -40^{\circ}\text{C}$, $-35^{\circ}\text{C} \leq T \leq -25^{\circ}\text{C}$ and $-15^{\circ}\text{C} \leq T \leq -5^{\circ}\text{C}$ respectively.
716

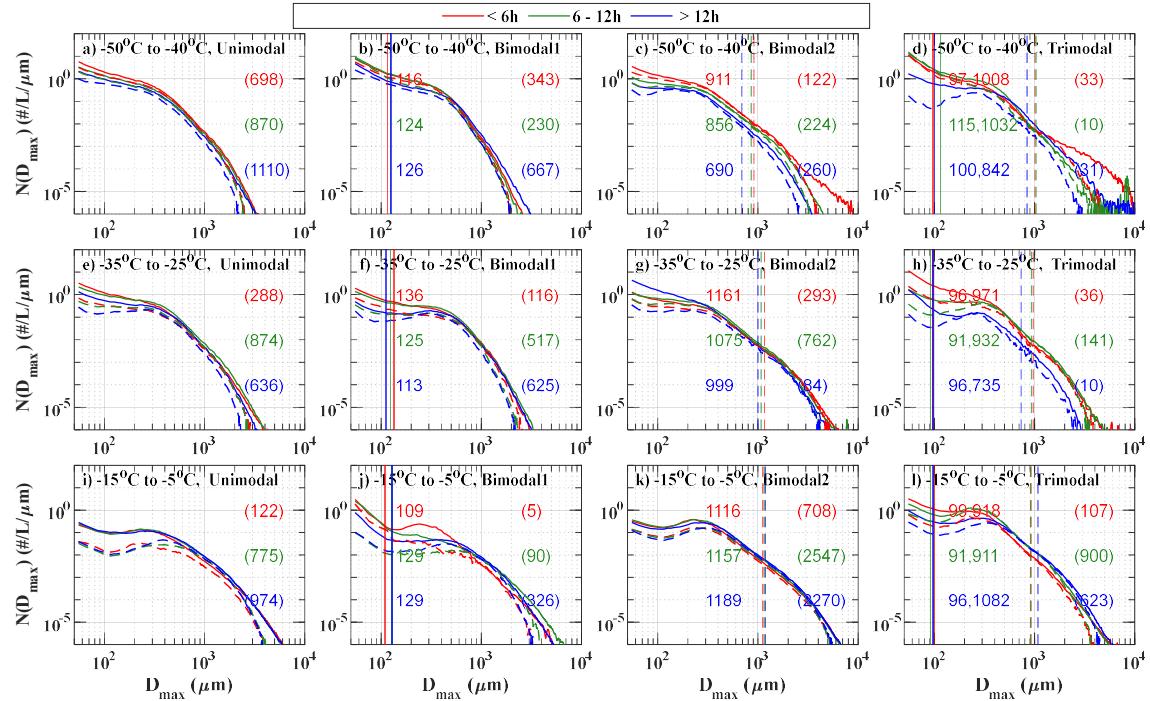
717 The MCSs were separated into three groups according to whether the convection
718 was observed over the ocean, coastline, or land to compare *PSDs* as a function of surface
719 characteristics. Figure 17 shows the normalized frequency of the different modalities of
720 these regions as a function of T . The dependence on T seems to be stronger than any
721 dependence on surface characteristics. For example, the frequency of Unimodal *PSDs*
722 increases and the frequency of multimodal *PSDs* decreases with decreasing T regardless of
723 surface type. The frequency of multimodal *PSDs* in coastal MCSs is usually larger than for
724 oceanic or continental convection at the same T , especially for $-15^{\circ}\text{C} \leq T \leq -5^{\circ}\text{C}$ when
725 multimodal *PSDs* occur $\sim 96.1\%$ of the time (Fig. 17c). Among the multimodal *PSDs*, the
726 frequency of Bimodal2 *PSDs* in coastal MCSs is usually largest reaching $\sim 61.5\%$ of the
727 time when $-15^{\circ}\text{C} \leq T \leq -5^{\circ}\text{C}$ (Fig. 17c). The frequency of Trimodal *PSDs* for continental
728 MCSs are smaller than those for oceanic MCSs when $T \leq -35^{\circ}\text{C}$, but an opposite trend is
729 noted when $-50^{\circ}\text{C} \leq T \leq -40^{\circ}\text{C}$. This is consistent with the findings that higher *IWCs* are
730 found in oceanic MCSs at lower altitude and higher *IWCs* in continental MCSs at higher
731 altitude (Hu et al., 2021).

732 Figure 18 shows the mean (solid line) and median (dashed line) *PSDs* for different
733 surface characteristics as a function of T . The number of small ice crystals with $D_{max} < 100$
734 μm in continental MCSs is smaller than for the other surface characteristics when -15°C
735 $\leq T \leq -5^{\circ}\text{C}$, but larger when $-50^{\circ}\text{C} \leq T \leq -40^{\circ}\text{C}$, because MCSs over the mainland are
736 usually stronger than over the ocean or coast (e.g., Lucas et al., 1994; Zipser et al., 2006;
737 Matsui et al., 2016), and thus produce more small ice crystals. The mean breakpoints
738 between the two modes for Bimodal1 *PSDs* are larger than 100 μm , reaching a maximum
739 of 150 μm for Bimodal1 *PSDs* in Coastal MCSs at $-35^{\circ}\text{C} \leq T \leq -25^{\circ}\text{C}$. The average
740 breakpoint is 692 μm for Bimodal2 *PSDs* in Coastal MCSs at $-50^{\circ}\text{C} \leq T \leq -40^{\circ}\text{C}$, smaller
741 than values noted for other T and surface characteristics. The mean small breakpoints (83
742 to 85 μm) and large breakpoints (747 to 1064 μm) for coastal MCS are usually smaller than
743 those in oceanic MCS (93 to 135 μm and 910 to 1106 μm) and continental MCS (95 to 103
744 μm and 988 to 1055 μm).

745 To quantify how the ellipsoids describing the fit parameters vary with surface
 746 characteristics, Figure 19 shows the overlap between the three-dimensional volumes of
 747 equally realizable fit parameters for different surface characteristics. Ellipsoids describing
 748 continental MCSs have the smallest volumes for Unimodal *PSDs* at different T , consistent
 749 with the parameter ranges being smaller (Figure not shown). The ellipsoid volume overlaps
 750 in coastal MCSs at $-50^{\circ}\text{C} \leq T \leq -40^{\circ}\text{C}$ have the smallest values, and the ellipsoid volume
 751 overlaps in oceanic MCSs regions at $-15^{\circ}\text{C} \leq T \leq -5^{\circ}\text{C}$ have the smallest values for



752
 753 Figure 20. As Figure 5 but segregated according to age of MCS where observations obtained.

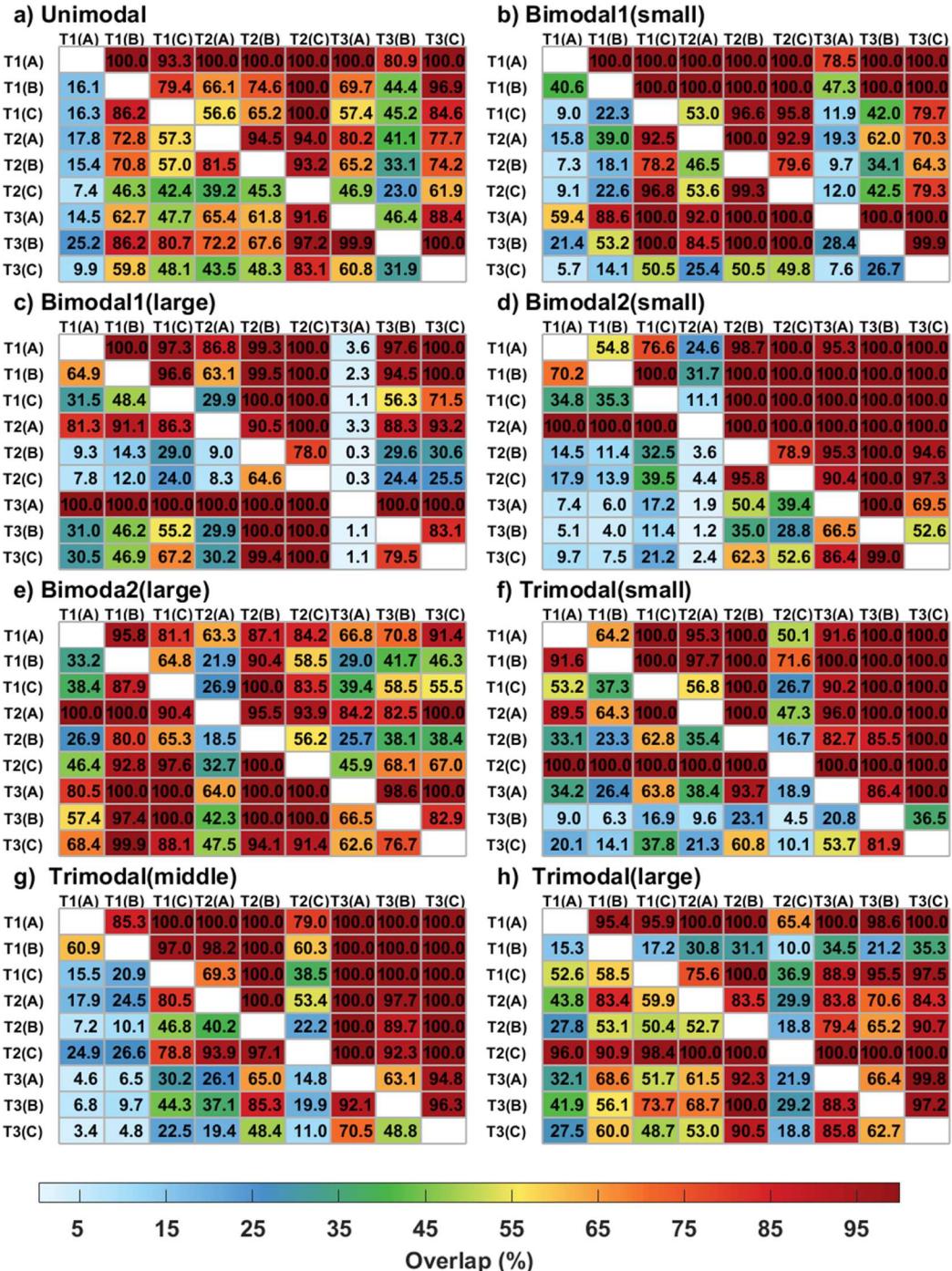


754

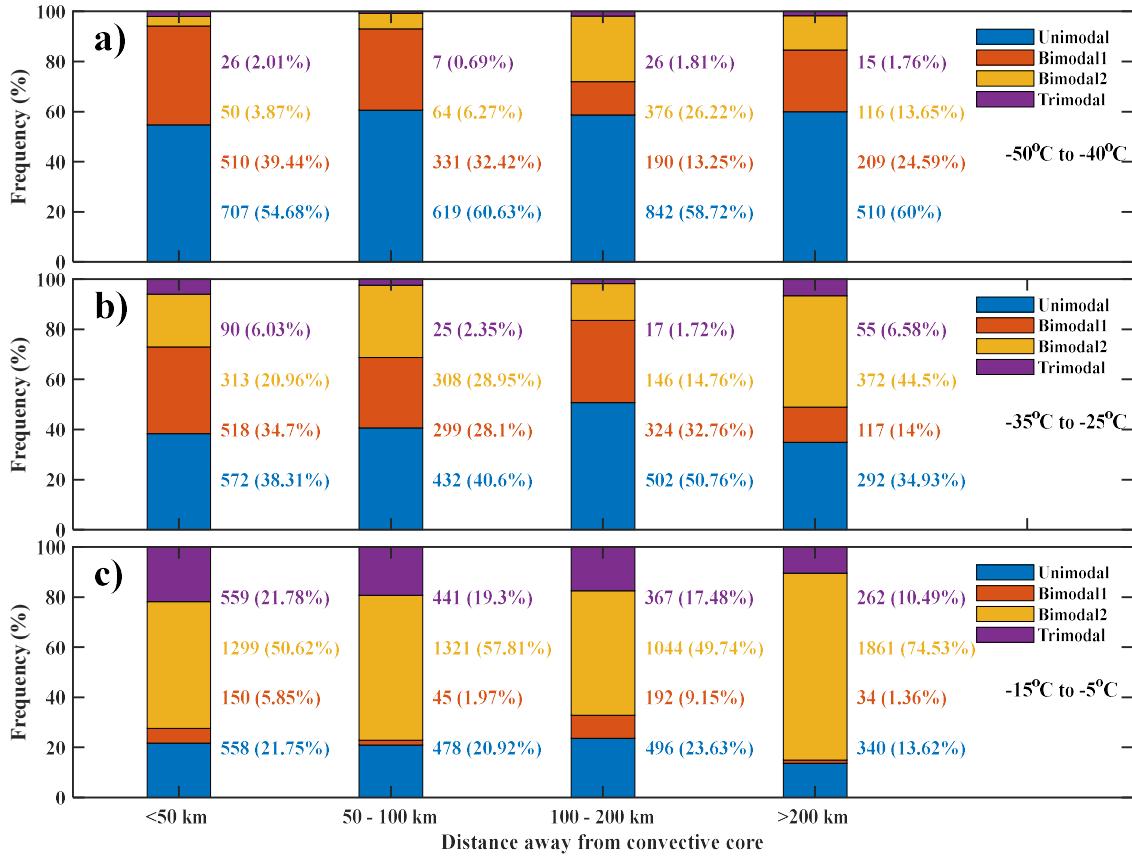
755 Figure 21. As Figure 6 but segregated according to age of system where observations obtained.

756

757 most conditions. And the ellipsoids for coastal MCSs are usually smaller than for oceanic
758 MCSs for different modality *PSDs*.



759
760 Figure 22. As Figure 10 but segregated according to MCS age. A, B and C represents MCS ages within
761 < 6 h, 6–12 h, and > 12 h respectively. T1, T2 and T3 represent $-50^{\circ}\text{C} \leq T \leq -40^{\circ}\text{C}$, $-35^{\circ}\text{C} \leq T \leq -25^{\circ}\text{C}$ and $-15^{\circ}\text{C} \leq T \leq -5^{\circ}\text{C}$ respectively.
762
763

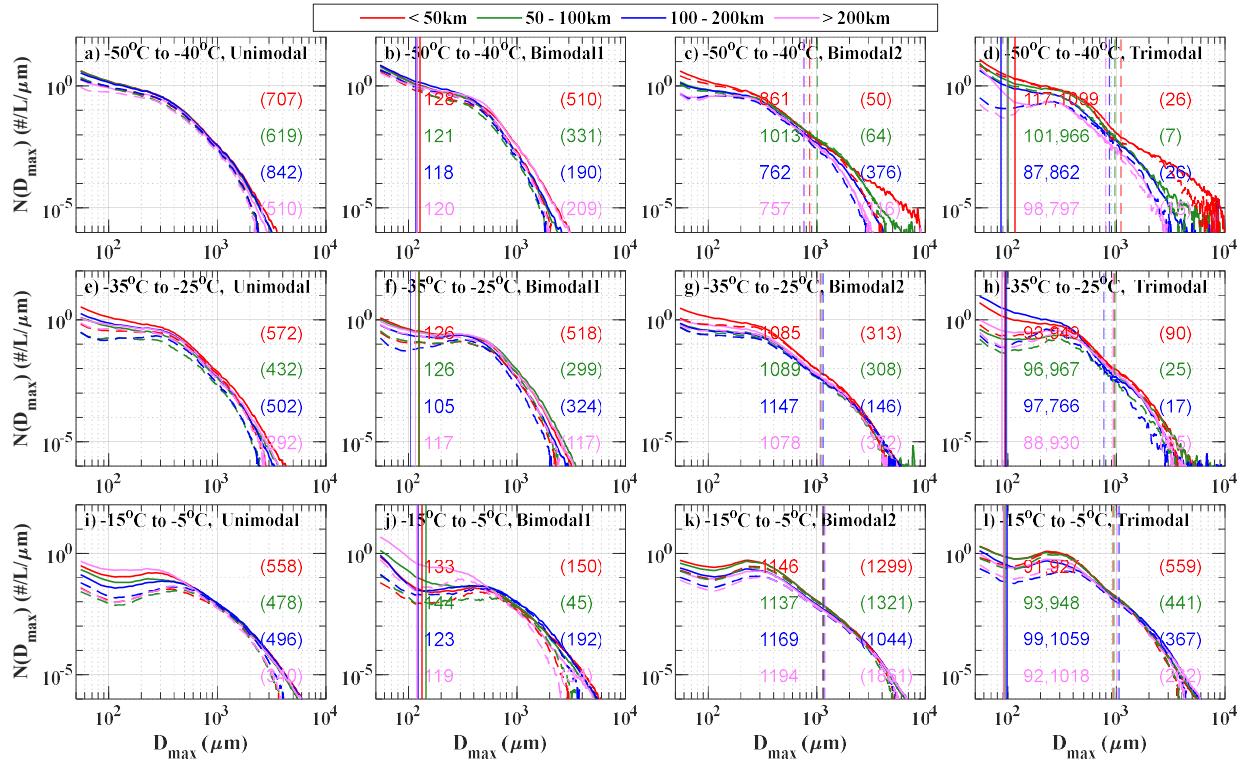


764
765 Figure 23. As Figure 5 but segregated according to distance away from convective core where
766 observations obtained.

767
768 **d. System age**

769 To compare *PSDs* for different system ages, the MCS age was sorted into three
770 groups (< 6 h, 6–12 h, > 12 h). Figure 20 shows the normalized frequency of the different
771 modalities for MCSs with different ages as a function of T . There is a stronger dependence
772 of the modality with T than on the system age. Nevertheless, the frequency of Unimodal
773 *PSDs* increases with system age for $T \geq -35$ °C, while the frequency of Trimodal *PSDs* is
774 the largest for system age between 6–12 h (Figs. 20b and 20c). This is consistent with the
775 6–12 h stage representing the developing or mature stage after ice has started being injected
776 into the anvil (Leary and Houze, 1980), with many ice crystals being created and growing
777 by aggregation (Hu et al., 2021). For the multimodal *PSDs*, the frequency of Bimodal1
778 *PSDs* in MCSs age > 12 h is largest for each T , reaching to $\sim 46.1\%$ when -35 °C $\leq T \leq$
779 -25 °C (Fig. 20b). On the other hand, the frequency of Trimodal *PSDs* in MCS ages > 12
780 h is usually the smallest for $T \leq -25$ °C, due to the *IWC* in the > 12 h age is the smallest,

781 consistent with sublimation and a weakening and dissipation of MCSs not being conducive
 782 to the generation and growth of ice particles (Hu et al., 2021).

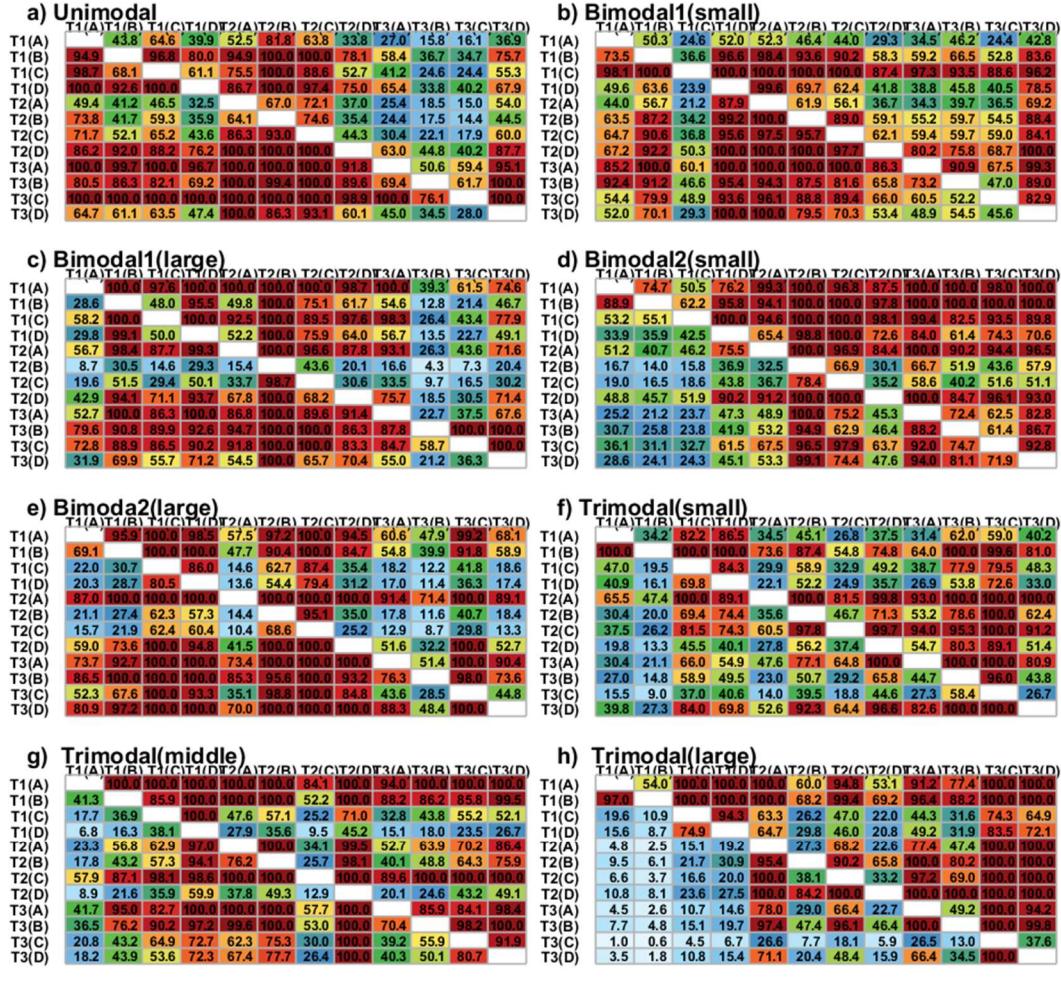


783
 784 Figure 24. As Figure 6 but segregated according to distance away from convective core where
 785 observations obtained.

786
 787 Figure 21 shows the mean (solid line) and median (dashed line) *PSDs* as a function
 788 of MCS age and T . For most conditions, the $N(D_{max})$ for younger system ages are usually
 789 larger than those in older MCSs for $D_{max} < 200$ because more ice crystals are produced in
 790 the developing and mature stages. The differences between Trimodal *PSDs* for different
 791 MCS ages are more obvious than for other modality *PSDs* at all T . Breakpoints between
 792 the two modes in Bimodal2 *PSDs* increase with increasing T for all MCS ages. The
 793 breakpoints in Bimodal2 and the breakpoints between the second and third mode in
 794 Trimodal *PSDs* are the smallest for MCS ages > 12 h when $T \leq -25$ °C.

795 To quantify the difference between the overlapping ellipsoids of equally plausible
 796 solutions for different T and MCS ages, Figure 22 shows the overlap between the three-
 797 dimensional volumes of equally realizable fit parameters for different MCS ages and T for
 798 the four modalities. The most apparent feature is that for different modality *PSDs*, the
 799 ellipsoid volumes for MCSs with ages < 6 h at -50 °C $\leq T \leq -40$ °C are largest. For example,

800 for the small mode of Bimodal2 *PSDs*, the overlaps are mostly less than 801 80% when MCS 802 ages are in < 6 h at $-50^{\circ}\text{C} \leq T \leq -40^{\circ}\text{C}$, while more than 80% are found in other ellipsoids 803 (Fig. 22d).



803 Figure 25. As Figure 10 but segregated according to distance away from convective core where 804 observations obtained. A, B, C and D represent four regions within $L \leq 50$ km, $50 \text{ km} < L \leq 100$ km, 805 $100 \text{ km} < L \leq 200$ km, $L > 200$ km respectively. $T1$, $T2$ and $T3$ represent $-50^{\circ}\text{C} \leq T \leq -40^{\circ}\text{C}$, $-35^{\circ}\text{C} \leq T \leq -25^{\circ}\text{C}$ 806 and $-15^{\circ}\text{C} \leq T \leq -5^{\circ}\text{C}$ respectively.

807 **e. Distance away from the convective core**

808 In order to compare the *PSDs* with distance away from the convective core (L), 809 *PSDs* were divided into four groups ($L \leq 50$ km, $50 \text{ km} < L \leq 100$ km, $100 \text{ km} < L \leq 200$ 810 km, $L > 200$ km). Figure 23 shows the normalized frequency of the four different modalities 811

813 as a function of L and T . The dependence on T is stronger than the dependence on L .
814 However, there are still some trends with L . The frequency of Trimodal $PSDs$ decreases
815 slowly with increasing L when $-15^{\circ}\text{C} \leq T \leq -5^{\circ}\text{C}$, and the frequency of Bimodal2 $PSDs$
816 is larger for regions away from the convective core when $T \leq -25^{\circ}\text{C}$, while the opposite
817 trends are realized for Bimodal1 $PSDs$ (Fig. 23a and 23b) as the frequency of Bimodal1
818 $PSDs$ is smallest for these regions at the same T . In summary, changes in the frequency of
819 PSD modality with L is less obvious, probably due to horizontal wind transport. T is still
820 the important factor, the frequency of Unimodal $PSDs$ increases, and multimodal $PSDs$
821 decrease with decreasing T for different L .

822 Figure 24 shows the mean (solid line) and median (dashed line) $PSDs$ as a function
823 of T and L . There are less obvious differences for Unimodal, Bimodal1 and Bimodal2 $PSDs$
824 between different L at the same T . Breakpoints between the two modes in Bimodal1 $PSDs$
825 show less obvious changes with increasing T , the breakpoints between the two modes in
826 Bimodal2 $PSDs$ within different L increase with increasing T . For example, the mean
827 breakpoint is 757 μm for Bimodal2 $PSDs$ with $L > 200 \text{ km}$ when $-50^{\circ}\text{C} \leq T \leq -40^{\circ}\text{C}$,
828 while it increases to 1194 μm when $-15^{\circ}\text{C} \leq T \leq -5^{\circ}\text{C}$. Less obvious difference between
829 the breakpoints within different L for the same modality $PSDs$ and same T .

830 To quantify the difference between the ellipsoids of equally plausible solutions for
831 different L , Figure 25 shows the overlap between the three-dimensional volumes of equally
832 realizable fit parameters for different L and T for the four modalities. Less obvious regular
833 trends between different L at the same T , consistent with the finding of Mascio et al. (2020),
834 due to the distributions of $IWCs$ with L are different at different T (Korolev et al., 2018;
835 Hu et al., 2021).

836 5. Conclusions

837 To quantitatively describe the difference of particle size distributions ($PSDs$) as a
838 function of environmental conditions, data obtained during the second HAIC-HIWC flight
839 campaign conducted from 9–29 May 2015 out of Cayenne, French Guiana were analyzed.
840 Data were separated according to temperature (T , $-15^{\circ}\text{C} \leq T \leq -5^{\circ}\text{C}$; $-35^{\circ}\text{C} \leq T \leq -25^{\circ}\text{C}$;
841 $-50^{\circ}\text{C} \leq T \leq -40^{\circ}\text{C}$), vertical velocity (updrafts, downdrafts, and stratiform cloud
842 regions), surface conditions (oceanic, coastal and continental), MCS age ($< 6 \text{ h}$, $6\text{--}12\text{h}$, $>$
843 12 h), and distance away from the convective core ($L \leq 50 \text{ km}$, $50 \text{ km} < L \leq 100 \text{ km}$, 100

844 km $< L \leq 200$ km, $L > 200$ km). The difference of *PSDs* between HIWC regions was also
845 contrasted against those obtained in regions without HIWCs. The principal findings of this
846 study are as follows:

- 847 1. HIWC regions ($IWC \geq 1.5 \text{ g m}^{-3}$) with small *MMD* ($< 500 \mu\text{m}$) are full of
848 small columns and irregular ice crystals at $-15^\circ\text{C} \leq T \leq -5^\circ\text{C}$, consistent
849 with negative correlation between *IWC* and *MMD* within the HIWC regions.
850 The distributions of ice crystals shapes are different for different *IWC*
851 regions.
- 852 2. Data from the Cayenne campaign were used to determine how many modes
853 were required to represent the *PSD* using a new methodology that
854 automatically determines the number of modes and gamma fit parameters
855 for each mode in a multimodal *PSD*. Four kinds of modalities of *PSDs* were
856 found by this method.
- 857 3. The *T* has the largest effect on *PSD* shape, and hence on the number of
858 modes and the fit parameters describing each mode of the *PSDs*. The
859 frequency of multimodal *PSDs* increases and the frequency of unimodal
860 *PSD* decreases with increasing *T*, consistent with heterogeneous nucleation
861 in the presence of particles sedimenting from above (Zhao et al., 2010) and
862 the aggregation process.
- 863 4. The number and location of the modes in a *PSD* (modalities) are also related
864 to *IWC*. The frequency of multimodal *PSDs* increases with *IWC* and the
865 frequency of unimodal *PSDs* decreases with *IWC*. There was no strong
866 trend in how frequency distributions of *IWC* and *MMD* varied with the
867 modality of the *PSD*.
- 868 5. The frequency of Unimodal and multimodal *PSDs* depends on vertical
869 velocity (*w*), consistent with the impact of *w* on *IWC*. The largest frequency
870 of Trimodal *PSDs* occurs in updrafts. There is some dependence on the
871 distributions of modality on other environmental conditions (e.g.,
872 underlying surface characteristics, MCS age, the distance away from the
873 convective core), which is not as strong as the dependence on *T* and can be
874 related to the dependence of *IWC* on environmental conditions.

875 6. The breakpoints between the two modes in the Bimodal1 *PSDs* occur at a
876 D_{max} of $100 \pm 20 \mu\text{m}$ with no strong dependence on T . Breakpoints between
877 the two modes in the Bimodal2 *PSDs* are located at a D_{max} of around 1000
878 $\pm 300 \mu\text{m}$ and increase with T . The breakpoints that exist in Trimodal *PSDs*
879 are similar to the breakpoints in the two types of bimodal *PSDs*.

880 7. The three fit parameters (N_o - λ - μ) characterizing the gamma distribution of
881 each mode are interdependent. For the same λ , N_o and μ are larger in the
882 large modes with larger D_{max} compared to the small modes, consistent with
883 particles with smaller D_{max} consisting of recently nucleated particles that
884 have not significantly grown by vapor deposition, and particles with larger
885 D_{max} having undergone sorting by sedimentation and deposition (Jackson et
886 al., 2015).

887 8. The μ decreases with *IWC* for the same N_o , and the ranges of λ and N_o both
888 narrow with increasing *IWC*. This consistent with more smaller ice crystals
889 being found in the high *IWC* regions.

890 9. The HIWC regions have the smallest ellipsoids, indicating that the
891 parameters (N_o , μ , and λ) are constrained for HIWC regions compared to
892 other regions and also meaning the least uncertainty in parameters for such
893 conditions.

894 The findings presented here apply only to data collected in the vicinity of Cayenne, French
895 Guiana during HAIC-HIWC. The techniques developed here can be applied to data
896 collected elsewhere to determine the robustness of the findings. Future studies should seek
897 to obtain data on more diverse geographic locations, and implement parameterizations of
898 *PSDs* developed here in ensemble or stochastic simulations that can be used to test
899 processes responsible to occurrence of large numbers of small ice crystals. Model
900 simulations using parameterizations of *PSDs* with three modes should be conducted to see
901 if they improve upon simulations conducted using one-mode parameterizations as such
902 simulations should better represent processes without the additional computational expense
903 of a bin-resolved scheme. Future work also needs to better determine reasons for the
904 variability of surfaces characterizing the *PSDs*, including examining whether the surfaces
905 and the modality vary with concentrations of cloud condensation nuclei and ice nucleating

906 particles. In addition, care should be taken to isolate the differences in the *PSD* surfaces
907 that are caused by variations in meteorological and aerosol conditions, as opposed to
908 differences that arise due to differences in probes used to collect the data and algorithms
909 used to process the data.

910

911

912 **Data Availability Statement.**

913 Observation data are available at https://data.eol.ucar.edu/master_lists/generated/haic-
914 [hiwc_2015](#). The GOES-13 data are available over the domain of (0-10°N, 45-60°W) where
915 the aircraft flew during the Cayenne campaign (<https://doi.org/10.5065/D6NC5ZX6>).

916

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