RELATIONSHIPS BETWEEN TEACHER QUESTIONING AND STUDENT GENERALIZING

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This study shares two frameworks for analyzing teacher actions that support students in generalizing and examines how those frameworks align with teacher questioning. One classroom teaching episode focused on the mathematical activity of generalizing is shared to illustrate effective generalizing promoting practices. We found several patterns of productive and unproductive generalizing promoting actions and questioning. Repeating generalizing promoting actions in succession were needed to produce student generalizations. Priming actions that set up for later generalizing promoting were helpful when students struggled to identify and state generalizations. Connection questions promoted generalizing, but justification and concept questions did not. Further research will explore the additional strategies to support teachers in fostering student-created generalizations.

Keywords: Instructional Activities and Practices, Classroom Discourse, Algebra and Algebraic Thinking

The mathematical practice of generalizing, identifying a relationship to describe multiple examples or instances of a phenomenon, is fundamental to learning in mathematics (Carraher & Schliemann, 2002; Kaput, 1999) and engages students in algebraic thinking (Blanton et al. 2011; Kieran et al. 2016), which requires students identify, investigate, and represent relationships. Understanding how to support students in developing, articulating, and refining generalizations is critical to mathematics teaching and learning. Teacher questioning plays a pivotal role in fostering students' generalizations, especially when students' reasoning does not lead to formal general statements (Radford, 2010). However, research on teacher questioning and generalizing remains distinct. This study is aimed at better understanding the relationship between teacher questioning and students generalizing. We describe our questioning framework and its relationship to two frameworks for analyzing actions to promote generalizing. We address how a high school mathematics teacher's questioning aligns with her actions to foster students generalizing and describe the patterns in student-teacher interactions that promoted student-created generalizations.

Literature

Generalizing skills contribute to algebraic understanding (Carpenter & Franke, 2001) and are identified as a key mathematical practice across all math content domains in the Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Here, we adopt Kaput's (1999) definition of generalization as "lifting" and communicating reasoning to a level where the focus is no longer on a particular instance but rather on patterns and relationships of those instances. A formal generalization is the product of the mental activity of generalizing (Font & Contreras, 2008). In generalizing students must recognize quantities that vary and remain constant and represent these relationships using symbols or words. When students move to generalizing symbolically without having adequate time to understand and reason about quantities and their relationships in a variety of contexts beforehand, they can become dependent on procedures (Kieran, 2007). Teachers value, and thus place an instructional focus on, formal algebra such as symbols, notations, and procedures (Nathan & Koedinger, 2000). A focus on procedural approaches to algebra can obscure attention to engaging students in mathematical practices such as generalizing that build a conceptual understanding of mathematics. Given student difficulties in generalizing (e.g., Blanton & Kaput, 2002; Lannin, 2005; Lee & Wheeler, 1987; Stacey, 1989; Stacey & MacGregor, 1997), students' failure to justify generalizations (Breiteig & Grevholm 2006), and secondary math teachers' challenges in responding productively to student generalizations (Demonty et al., 2018), determining what instructional actions promote generalizing activities is warranted.

Ellis (2007) proposed an actor-oriented generalization taxonomy that consists of generalizing actions, which include students' activities while generalizing and their statements of generalization. To better understand the classroom interactions and discourse that promote generalizing actions, we use a modified version (Strachota, 2020) of Ellis' (2011) generalizing-promoting actions (GPAs) and Strachota's (2020) priming actions (PAs). Priming actions set the stage for more explicit attention to generalizing; they prepare students to build on an idea or refer to an idea later. Generalizing promoting actions, on the other hand, prompt immediate activities that have the potential to produce generalizations. Priming can include making the critical ideas of an individual public to the work of the class, making evident tools needed for generalizing, asking students to consider ideas or examine specific key examples, or setting up to extend an idea later. For example, a teacher who introduces x to represent a varying quantity in a pattern or who displays similar expressions for comparison is using a priming action, *reviewing a critical tool* and *constructing searchable and related situations*, respectively. Generalizing promoting requires students to identify a relationship, state a generalization, extend beyond cases available to them, or justify a general statement. For example, prompting students to describe a pattern algebraically is a generalizing promoting action that *encourages reflection*. Table 1 illustrate the codes we used for priming and generalizing promoting actions (Ellis, 2011; Strachota, 2020).

Table 1: Priming Actions and Generalizing-Promoting Actions

Priming Actions (PAs)[1]	
Naming a phenomenon, clarifying critical terms and tools	"Offering a common way to reference a phenomenon or emphasizing the meaning of a critical term or tool."
Constructing or encouraging constructing searchable and relatable situations	"Creating or identifying situations or objects that can be used for searching or relating. Situations that can be used for searching or relating involve particular instances or objects that students can identify as similar."
Constructing extendable situations	"Identifying situations or objects that can be used for extending. Extending involves applying a phenomenon to a larger range of cases than from which it originated."
Generalizing-Promoting Actions (GPAs)[2]	
Encouraging relating and searching	"Prompting the formation of an association between two or more entities; prompting the search for a pattern or stable relationship."

Encouraging Extending	"Prompting the expansion beyond the case at hand."
Encouraging Reflection	"Prompting the creation of a verbal or algebraic description of a pattern, rule, or phenomenon."
Encouraging Justification	"Encouraging a student to reflect more deeply on a generalization or an idea by requesting an explanation or a justification. Includes asking students to clarify a generalization, describe its origins, or explain why it makes sense."

[1]These categories and descriptions are from Strachota (2020, p. 7).
[2]These categories and descriptions are from Ellis (2011, p. 316). We adapted encouraging relating and searching, following Strachota (2020), by combining these into one category.

Teachers often struggle with asking 'good' questions that are cognitively demanding, involve higher-order thinking, and follow up on student input and explanations (Boaler & Brodie, 2004; Franke et al., 2009). In addition to asking good questions, teachers must be able to effectively interpret and make sense of students' questions during class to use student thinking to move the mathematics of a lesson forward (Hufferd-Ackles et al., 2004). The follow-up questions teachers ask after hearing student thinking are of critical importance. Franke et al. (2009) found that students benefited most when teachers asked a probing sequence of specific questions. This process of probing helped teachers better understand student thinking, helped the students who were responding to teacher questions to solidify their ideas and thinking, and helped other students connect ideas to their own thinking and address misconceptions. When a teacher asked only one question, they were often not able to obtain enough information to understand the student's thinking. Boaler and Brodie (2004) concluded that it is important for teachers to ask higher-order types of questions, so students have more opportunities to engage meaningfully with mathematics in ways that go beyond performing procedures. The finalized questioning framework used in this study is based on Boaler and Brodie (2004), Hallman-Thrasher & Spangler (2020), and Chen (2021) shown in Table 2 and described in the methods section.

Table 2: Question Types to Support Generalizing

Definition of Question Type	Example
Rhetorical	
Does not generate responses (teacher answers them or does not provide time for students to answer)	"Everyone else get that? Yeah? Ok."
Gathering Information	
Requires only a single short answer	' "How many flowers are in step 5?"
Concept	
Attends students to underlying mathematical relationships and meanings	(No example from data)
Strategy	
Elicits descriptions of students' strategies, solutions, or procedures	"Do you want to come up here and show how you got that?"
Clarification	
Clarifies student input that has been shared or is known	"So, these would be the step numbers?"
Connection	
Seeks a connection across ideas, representations, or strategies	"Which part [of the picture] would be *x*?"

Justification	
Defends the appropriateness of a particular strategy, augmenting with connections that validate reasoning.	"Why does this [expression] not work [for other cases]?"
Extending Thinking	
Extends to examples beyond what is available or to examples where similar ideas could be used	"What about the 10th step?"

Adapted from Boaler and Brodie (2004) and Chen (2021).

Methods

The participant for this study was Ms. Patton, a teacher candidate enrolled in a one-year master's program with licensure for individuals with STEM degrees. She had earned an undergraduate degree in mathematics the previous year. At the time of data collection, she was in the Fall semester of a year-long teaching placement in an Algebra II classroom and enrolled in her only mathematics teaching methods course. In this lesson, Ms. Patton was supported by Mr. Dayton, her experienced mentor teacher. For this study, Ms. Patton planned, taught, and reflected on her video data from two episodes of teaching a pattern task with grades 9-10 students as part of an assignment for her mathematics teaching methods course. By pattern task we mean, a visual representation of objects that grows over instances of time (Figure 1). The data analyzed for this paper is the video recording of Ms. Patton's teaching where the most student-created generalizations were shared. In methods class, Ms. Patton first engaged in completing pattern tasks as a learner and analyzing videos of other teachers using pattern tasks. We carefully structured her planning for this task to attend to teacher questioning to elicit, understand, and make connections to student thinking.

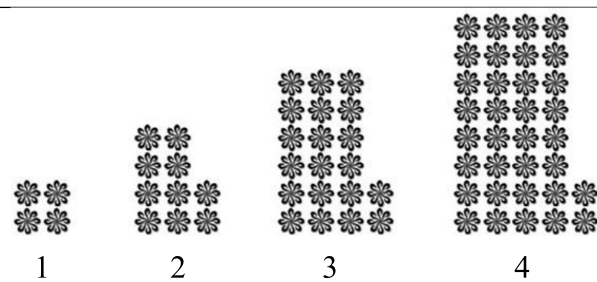

Figure 1. Ms. Patton's Pattern Task (Nguyen, 2020).

For our initial analysis, we coded the video data of teaching in 15-second segments using two established frameworks for generalizing (Table 1, Ellis, 2011; Strachota, 2020). Next, to capture teacher moves we modified Boaler and Brodie's (2004) question types. The question types (Boaler & Brodie, 2004) did not align well with the specific nature of supporting students in the creation of generalizations. Not all of the question types were applicable for this study and some were not sufficiently specific. None of the lessons we reviewed involved linking to concepts outside of mathematics so "linking and applying" and "establishing context" questions were not used. The "orienting and focusing" questions were not adequately specific, so we defined other categories that helped us classify the strategies a teacher would use to orient or focus (e.g., clarifying or justifying questions). "Probing" questions did not describe all the different ways a teacher might follow up on student thinking. We drew on Hallman-Thrasher & Spangler's (2020) broad categories of questions to develop a more comprehensive list which we compared against Chen (2021) to search for overlaps, gaps, and types needing more or less specificity. For example, Chen's (2021) "elicit thinking questions" were broken into strategy and concept questions in a new framework (Table 2). To establish a more descriptive framework specific to the evaluation of questioning that supports generalization, we also used thematic coding (Saldana, 2013) to identify questioning types not addressed by our existing frameworks. We defined a new question type "rhetorical" for questions that did not require a response or served as a means to garner attention to a thought, such as, "Does that make sense to everybody?" We also carefully considered what counted as a question: statements that functioned a question (e.g., "Find all the expressions you can for step 5") counted as questions, as did the questions that were not answered or were not intended to be answered (e.g., "You said that was x, right?"). To more closely examine the nuanced turns in conversation, we re-coded the data line-by-line for generalizing promoting actions, priming actions, student generalizations, and questioning with our revised framework for questioning. Interrater reliability was established by having all four researchers review and code all data. When disagreements arose, we discussed them using the frameworks to reach consensus (Syed & Nelson, 2015).

To better understand how the conversation developed over the course of the lesson, we divided the 45-minute lesson transcript into 13 blocks with each block representing a different instructional goal (e.g., launching the task, generating expressions for a particular step, applying a numeric expression for one step to a different step). We identified which blocks of conversation were productive for producing student generalizations. We wrote a description for each block and examined it in order to isolate characteristics of instruction that supported generalizing. We examined the coded data to identify patterns that signaled productive and unproductive questioning strategies and teacher moves to develop students' generalizations.

Results

We focus our results on Ms. Patton's most productive lesson; the one that included the most student-created generalizations. From our blocked classroom interactions, those producing at least one student generalization were identified as productive. Blocks that did not produce a student generalization were considered either missed opportunities for having had the potential to produce a student-created generalization or unrelated when the purpose of the interaction was not directly related to generalizing (e.g., clarifying directions, checking in with a small group). Students shared 13 generalizations spread over 7 productive blocks. Three blocks were unrelated to generalizing and three blocks were missed opportunities for generalizing. Table 3 shows which questioning types were used with priming actions (PAs) and generalizing promoting actions (GPAs). Ms. Patton relied primarily on connection, clarification, and extending thinking questions to develop generalizing actions. Two PAs did not involve questioning and eight GPAs did not involve questioning. Priming was associated with gathering information, clarification, strategy, and connection questions, whereas generalizing promoting incorporated all question types, relying most heavily on connection and extending thinking questions.

Particular types of questioning were more or less helpful for engaging students in conversations related to generalizing. Ms. Patton's use of connection, clarification, and rhetorical questions exceeded all other questioning types. She repeatedly used connection questions when encouraging students to relate between figural and algebraic representations of the pattern. When she asked, "Which part [of the picture] would be x?" her focus on a specific part of an expression prompted students to discover that the width of the large rectangle was the same as the step number. All of the connection questions she posed, were priming or generalizing promoting.

Ms. Patton used clarifying questions to have students further explain terminology or ideas and in doing so students revised an idea or stated it more precisely. For example, in prompting students to unpack an expression related to viewing the pattern as two equal-sized squares, Ms. Patton prompted, “Now what were you saying about perfect squares?” Clarifying questions often served to provide an opportunity for students to repeat an important point to which Ms. Patton wanted the class to attend. Clarifying questions, though important to understanding student thinking, did not consistently relate to generalizing; only 8 of her 17 clarifying questions functioned as priming or generalizing promoting actions.

Table 3: Alignment of Question Types, Priming, and Generalizing-Promoting Actions

Question Types	Priming Actions	Generalizing Promoting	Total Questions
Rhetorical	---	4	10
Gathering Information	1	1	5
Strategy	3	2	7
Clarification	4	4	17
Concept	---	---	---
Justification	---	3	3
Connection	4	13	17
Extending Thinking	---	5	6

Ms. Patton used rhetorical questions to state generalizations; the four teacher-stated generalizations were shared in the form of a rhetorical question. She did not, however, provide an opportunity for students to respond to these rhetorical questions. For example, she revoiced a generalization saying, “Does everyone see how he got that? For every picture you have this two right here [the constant two flowers on the rightmost column in each image] and then across [the width of the rectangle] is just the step number plus one.” This question had the potential to ensure that all students attended to and understood a key generalization shared earlier. However, by not providing students an opportunity to agree or disagree and justify their decisions she limited students’ opportunities to actively engage with another’s ideas.

Other questioning types, while not as prevalent as connection, clarification, and rhetorical, were more consistently associated with generalizing promoting. Extending thinking and justifying questions nearly always functioned as generalizing promoting actions. Extending thinking questions and justifying questions did not immediately lead to student generalizations. They always required follow-up supporting questions. Though justifying questions were not used often, they were effective at promoting generalizations. For example, Ms. Patton asked, “Why wouldn’t it work for say step number 3?” and followed with strategy, clarification, justification, and connection questions before a student correctly generalized that an expression for the pattern in step 4 would not work for any other step of the pattern. Justification questions, while present in this lesson, were less than we might have expected and may have established a tendency not to justify claims which could have limited opportunities for students to discover, refine, and state generalizations. We also noted that Ms. Patton did not use concept questions which may have contributed to the missed opportunities for students’ generalizing. By not consistently making underlying concepts and justifications evident, Ms. Patton may have focused more on what strategies were developed and less than the underlying structure that would have supported students in making their own generalizations.

Ms. Patton often used clarification, justification, and connection questions to make students’ generalizations accessible to the whole class. In clarifying to *encourage reflection*, she revoiced a student idea and pressed for detail that prompted a student to state a generalization more precisely; in connecting she *encouraged relating and searching*. Justifying questions such as “Why wouldn’t that work for step 3?’ served to *encourage justification* and in response a student produced a new general statement about how the formula could not extend to all instances of the pattern. To *encourage extending*, she repeatedly asked students to consider cases beyond those shared and to apply ideas developed from one step of the pattern to earlier or later steps.

Within each productive block we looked at the sequence and density of priming and generalizing promoting actions to identify patterns of actions that were productive towards producing student-created generalizations. One productive pattern for producing student generalizations was using repeated instances of generalization promoting over a short duration. The first such productive block included three GPAs (*encouraging extending*, *encouraging extending*, *encouraging justification*) over a one-minute time span. The questioning types that were used to accomplish these actions varied. She first gathered information and extended thinking to *encourage extending*, and then followed with a strategy question to *encourage justification*. By *encouraging extending* twice with two different question types, she first elicited the information she needed to build on in order to extend and justify. A student was then able to provide a general description of the pattern’s structure. Ms. Patton also used repeated GPAs to shift students’ attention to articulate a complete version of their generalizations.

A second related pattern, not consistently productive, but which Ms. Patton consistently employed, was to immediately follow every student generalization with another generalizing promoting action. For example, she followed the productive block described above with another GPA to build on that student’s thinking by asking the class to translate the student’s description into a generalized formula. These GPAs following a student generalization were productive in developing a new generalization when they took the form of a clarifying, connection, or justification question and functioned as *encouraging reflection*, *encouraging relating and searching*, or *encouraging justifying*. They were not productive when the teacher and students focused on different perspectives. Ms. Patton used a connection question, “How could we get it to be minus 6 in terms of *x*?” followed by a rhetorical question, “[*x*] Plus 2. Will that work?.. Distribute….Not quite, right?” as generalizing promoting actions to *encourage reflection* to generalize missing flowers in terms of step number. Yet, because students did not approach the problem from a ‘what’s missing’ perspective, they did not readily generalize this strategy.

Within blocks that we labeled a missed opportunity, we identified a third pattern. This pattern involved a failure to use priming and generalizing promoting actions together to build towards a conclusion. When an initial PA or GPA failed to produce a student generalization, Ms. Patton abandoned the line of questioning. For example, Ms. Patton asked students to apply their numeric expressions created for step 4 to the first step of the pattern. Because the general structure is not evident in the first step of the pattern, students had trouble applying their formulas and because she failed to use generalizing promoting and priming together students could not meaningfully respond to this prompt.

A fourth pattern resulted from a missed opportunity where repeated GPAs produced no student generalizations. To scaffold students’ thinking, Ms. Patton employed repeated priming before returning to GPAs to produce a student generalization. For example, over a two-minute span she used nine GPAs to try to help students develop a generalization for the number of missing flowers in the rightmost column. Students were able to recognize that she was looking for the missing flowers but were unable to make any general claims about the structure of the missing flowers relative to the full picture. Ms. Patton then used PAs to *construct and search for* *relatable situations.* Aware of the “tricky” part of representing missing flowers algebraically, she primed students by asking for clarification about the missing flowers. She continued priming by noting that there were 6, then 4, then 2 missing flowers (working backwards through the steps), and primed again to prompt students about the goal “We need to have something minus 6 and we need to end up with 2*n* squared plus 2, right? Cause right now we have…” [teacher draws attention to current representation prompting students to complete her sentence]. With this groundwork laid she used GPAs of *encouraging reflection and justification* which prompted students to state incorrect generalizations of the 6 missing flowers in step 4 and then to identify that those generalizations did not work for other steps of the pattern. A student generalized why the expression did not work and then Ms. Patton again primed to build on this idea. Finally, her mentor, Mr. Dayton used a connection question as a GPA to help students notice the connection between the missing flowers and the height of the rectangle. This lengthy exchange spanned three blocks and demonstrates how Ms. Patton’s and Mr. Dayton’s repeated questioning led to student generalizations for the blank spot of missing flowers.

Conclusion

We set out to determine what teacher moves were most productive in promoting students’ generalizing through a close examination of line-by-line student-teacher interactions situated within blocks of classroom dialogue with common purposes. It is not surprising that extending thinking questions and justifying questions aligned with generalizing promoting actions of *encouraging extending* and *encouraging justification,* respectively. However, it is surprising that these actions did not consistently produce student generalizations. Other questioning types (connection, clarification, and strategy) aligned with generalizing promoting and priming actions and, when used in particular patterns of promoting and priming, did lead to student-created generalizations. Our results confirmed the benefits of using connection questions to relate visual, numerical, and algebraic representations and to help students identify and articulate generalizations. Clarifying questions often provided an opportunity for students to repeat an important point critical to precisely state a generalization. Results also point to the difficulty of using justification and concept questions to promote generalizing even when teachers have expressly prepared to use these types of questions. Though justifying was used less than we expected, when employed it resulted in a generalizing promoting action each time.

There is not a one-to-one relationship between questioning and generalizing promoting or priming. A variety of questioning types served as priming and generalizing promoting actions. We expected to see a linear sequence beginning with teacher priming and generalizing promoting actions followed by student generalizations. But, in fact, generalizing promoting actions did not always lead to student generalizations; patterns leading to student generalizations were more complex. Repeated generalizing promoting actions and priming in conjunction with generalizing promoting were needed to produce a student generalization. When generalizing promoting actions did not produce student generalizations, priming followed by additional generalizing promoting was helpful. In this lesson, as with any lesson, questioning occurs in response to student contributions making the planning of all questioning challenging. Accepting this fact means that for questioning to support students in creating generalizations, teachers must possess the ability to respond in the moment with questioning and generalizing promoting moves that are likely to be productive. We suggest that this method of examining common pathways that lead students to generalize can be applied in other lessons to further develop suggestions for teachers as they work to support students as they develop their own generalizations.

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