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# Development of adaptive granular metamaterials for impact mitigation

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ABSTRACT

Granular crystals exhibit highly nonlinear impulse responses and are therefore excellent candidate materials for applications such as vibration absorption, impact mitigation, and shock protection. This work presents a new class of adaptive metamaterials in the form of granular crystals immersed in MR fluids which offer the unique ability to reversibly generate rheological defects at any spatial location using external magnetic fields. To demonstrate the utility and efficacy of these tunable metamaterials, a novel experimental methodology was developed to visualize the wave propagation through granular crystals immersed in opaque fluids subjected to low-speed impact loading. This experimental approach relied on a drop-tower-based setup to subject the granular crystals immersed in MR fluids to low-speed impact loading. The kinematic and strain fields in each grain at any instant during impact loading were calculated using a combination of high-speed imaging and digital image correlation. The experimental methodology was employed to illustrate the influence of "point" defects generated by using an external magnetic field on the wave propagation in granular immersed granular crystals. In this letter, a single rheological "point" defect was introduced in the center of the granular crystals using external magnetic fields of varying magnitudes, and the influence of the strength of the magnetic field on the wave dynamics was quantified. The experimental measurements demonstrated that the strength of the magnetic field has a significant influence on the wave propagation process, and it is possible to control the spatial kinetic and strain energy distribution by varying the magnetic field.


## 1. Introduction

Metamaterials are at the frontier of structural engineering owing to their unique mechanical features and extraordinary properties originating in their structure rather than their constituents [1]. The study of the mechanics of strongly nonlinear metamaterials provides a natural step from linear, weakly nonlinear to strongly nonlinear wave dynamics. Granular crystals, a popular class of non-linear periodic metamaterials, are defined as ordered aggregates of discrete solid grains, which in their dry state lack cohesion, and transmit forces via intergranular contacts. In contrast to disordered granular media (sand, regolith, iron shot granular beds, etc.), the influence of intergranular friction and rotational dynamics of grains on the wave dynamics in ordered granular crystals can be limited [2–4]. The nonlinearity in granular crystals, because of their non-linear contact interactions and zero-tensile response due to the lack of cohesion, can result in exotic dynamic characteristics, such as sonic vacuum [5–7], solitary waves [4,8–10], local resonance [11–13], energy trapping [14–16] and shock disintegration [10,14,16–20]. In the past, wide-ranging dynamic applications including vibration absorption [21,22], impact/shock mitigation [8,10,14,16,19,20], wave tailoring systems [4], acoustic logic devices [23–25] and rectifiers [26] leveraging these wave characteristics have been proposed. These unique wave characteristics can be controlled using factors, such as grain material, arrangement, and grain sizes [4,14,27–29], pre-compression [4,8,15], and the introduction of non-linear interfaces. Most numerical and experimental studies focused on the wave dynamics of granular crystals assume a standard Hertzian contact model, which ignores the damping and other dissipative phenomena. However, the presence of dissipation strongly influences the dynamic response of granular metamaterials and recent experimental works have established the need to include the influence of dissipative effects, such as viscoelasticity and viscoplasticity of the grains [23,30–33], intergranular friction [22,34–36] and viscous drag due to surrounding media (air vs. liquids) [37–39]. Although dissipative effects are generally not desirable in metamaterials, they could greatly enhance the performance of certain applications, such as wave cloaking, vibration suppression, and impact/shock mitigation. Since polymers and composites are more dissipative than metals, soft granular crystals (polymer [23,34–36,39,40],

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polymer-coated metals [32,41], composite [42]) provide tremendous potential for applications requiring high energy absorption. Viscous effects in silicone granular crystals not only can lead to wave attenuation but also significantly affect the bandgap for acoustic waves [43]. For instance, a dissipative metamaterial composed of alternating steel cylinders and toroidal Nitrile O-rings exhibited a strong stress attenuation over relatively short distances [44].

It is well understood that wave attenuation and energy dispersion can be achieved through the introduction of **(1)** intruders of different materials [14,23,37,45,46], and **(2)** mass mismatch such as tapered [47–49] or decorated chains [15,47,50] in 1D granular systems. Comparatively, the literature on the effect of intruders, dissipative effects, and force propagation during wave propagation in 2D and 3D granular media, is much less extensive [8,51–56]. Thus, features, such as geometry, size, stiffness, and arrangement of grains can be optimized to design metamaterials for a broad spectrum of operational requirements. In contrast, for a 'fixed' granular crystal design, changes in the level of pre-compression can provide only limited tunability of the nonlinear wave characteristics [57]. In the past, the limited tunability of meta-materials has been addressed through approaches such as the manual adjustment of the position of the constitutive elements of a periodic medium [58,59]. The non-linear wave characteristics of periodic lattices have been controlled via localized geometric change/ property changes, or external stimuli, such as electric fields, magnetic fields, or infrared radiation [60–63]. Recently, soft active polymers, such as dielectric [64], electrostrictive [61], and shape memory polymers [65] have been employed to achieve active tunability in auxetic mechanical metamaterials. Although there have been a few studies focusing on the active tunability of 1D granular chains using external fields [66,67], an in-depth exploration of these active tunability strategies for 2D and 3D granular crystals is lacking. In this letter, the authors present a novel strategy to induce reversible rheological point defects in granular crystals immersed in MR fluids using external magnetic fields. The fast response times and reversibility of defect generation offered by this novel strategy present tremendous potential for the metamaterials to adapt their defect patterns and effectively mitigate impact based on the amplitude, direction, and velocity of the incident wave.

## 2. Experimental methods

The drop-tower experimental setup shown in Fig. 1a and described in more detail in [34–36,39], was employed to subject the various immersed granular crystals to low-speed impact loading. This experimental apparatus includes a rigid bottom wall attached with adjustable perforated sidewalls, which helps the excess liquid to leave to avoid any potential pressure build-ups. An aluminum projectile with ∼1" (25.4 mm) diameter and 6" (152.4 mm) length is allowed to fall freely under gravity and achieves a velocity of around ∼6 m/s velocity at the point of impact with the movable top plate of the experimental setup. The kinetic energy of the projectile is transferred to the immersed granular crystal via the top wall. Since the granular system needs to be completely immersed in a liquid, the experimental setup was designed such that all four boundaries of the granular crystal are enclosed. To allow clear visualization of the wave dynamics, the front wall of the experimental setup needed to be transparent, and hence an anti-reflective coated glass (Edmund Optics) with 93%–94% transparency was employed as the front wall of the setup. This anti-reflective glass aided the high-speed imaging with a high intensity of light. This experimental procedure for low-speed impact testing of granular crystals immersed in transparent and opaque liquids has been described in [39] (see *Appendix A*).

In this letter, the drop-tower-based setup was employed to illustrate the influence of external magnetic fields on the wave dynamics of granular crystals immersed in hydrocarbon-based MR fluid (Lord MRF-122EG) with a rest viscosity of 0.042 Pa s. Like most MR fluids, the Lord MRF-122EG MR fluid was an opaque fluid which greatly complicated the visualization of the wave dynamics of the granular crystals. This issue was resolved by the inclusion of a transparent membrane of off-the-shelf medical ultrasound gel in front of the grains which allowed for monitoring the deformation of the granular crystals. Since the MR fluid was hydrocarbon-based, the ultrasound gel had a low solubility in the MR fluid. In [39], the authors observed that the inclusion of a low-friction, thin layer of ultrasound gel has no measurable influence on the wave dynamics of immersed granular crystals. Although it is essential to limit the thickness of this gel film, it is not a trivial task from a practical point of view. In some cases, about 1 mm of the grain depth was inside the gel, but this had no measurable influence on the wave dynamics. In some experiments, the presence of the gel lead to the formation of tiny bubbles in the upper portion of the granular crystals that impaired the imaging efforts in these regions. Thus, although the utilization of gel film produced desirable results in the current work, it might lead to unforeseen complications at higher velocities. These complications in imaging from bubble formation increase if multiple experiments are performed on the granular crystals without reassembly.

In this letter, to obtain the best combination of spatial and temporal resolution via high-speed imaging, images of the left half of the granular crystal have been recorded during each experiment. In [35], the authors have observed that the wave dynamics of the finite granular crystals have reasonable symmetry. The left half of the granular crystal with polyurethane grains (Durometer 80A, McMaster-Carr) with around 1/2" (12.7 mm) diameter immersed in MR fluid arranged in a cubic arrangement is shown in Fig. 1b. The mechanical and physical properties of the polyurethane grains are listed in [39]. A strong round DC electromagnet (AEC Magnetics DCA-300) with 20 W rated power and rated holding power of 440 lbs was placed behind the grains close to the center of the system, as schematically indicated in Fig. 1a, b. Although it is difficult to prescribe a constant magnetic field using an electromagnet, the low-speed impact experiments were performed at an external field with an approximate value of 0.3, 0.7, 0.9, and 1.2 Tesla at the face of the electromagnet. Multiple experiments were performed at each value of the applied magnetic field and the results were found to be repeatable. These values are estimated using the datasheets for the MR fluid and the holding power of the electromagnet. It should also be mentioned that the field these electromagnets create has a shape like a semi-ellipsoid and calculating the exact magnetic field is challenging. A Photron FASTCAM SA-Z high-speed camera was used to record the deformation of the grains at a 40,000 fps frame rate. Digital image correlation (DIC) software (VIC-2D from Correlated Solutions) was used to calculate the kinematics and strain field in each particle. More details about the DIC-based calculations for strain and kinematic fields and calculation of the strain and kinetic energy of the individual grains and the entire granular crystal can be found in [34–36,39] and *Appendix B*.

## 3. Results

The vertical displacement fields for a granular crystal immersed in MR fluids obtained using DIC are shown in Fig. 1. The (Fig. 1d) shows the vertical displacement for a granular crystal immersed in MR fluids with no external magnetic field, and (Fig. 1e) shows a vertical displacement for immersed granular crystal image subject to an external magnetic field ∼0.7 Tesla

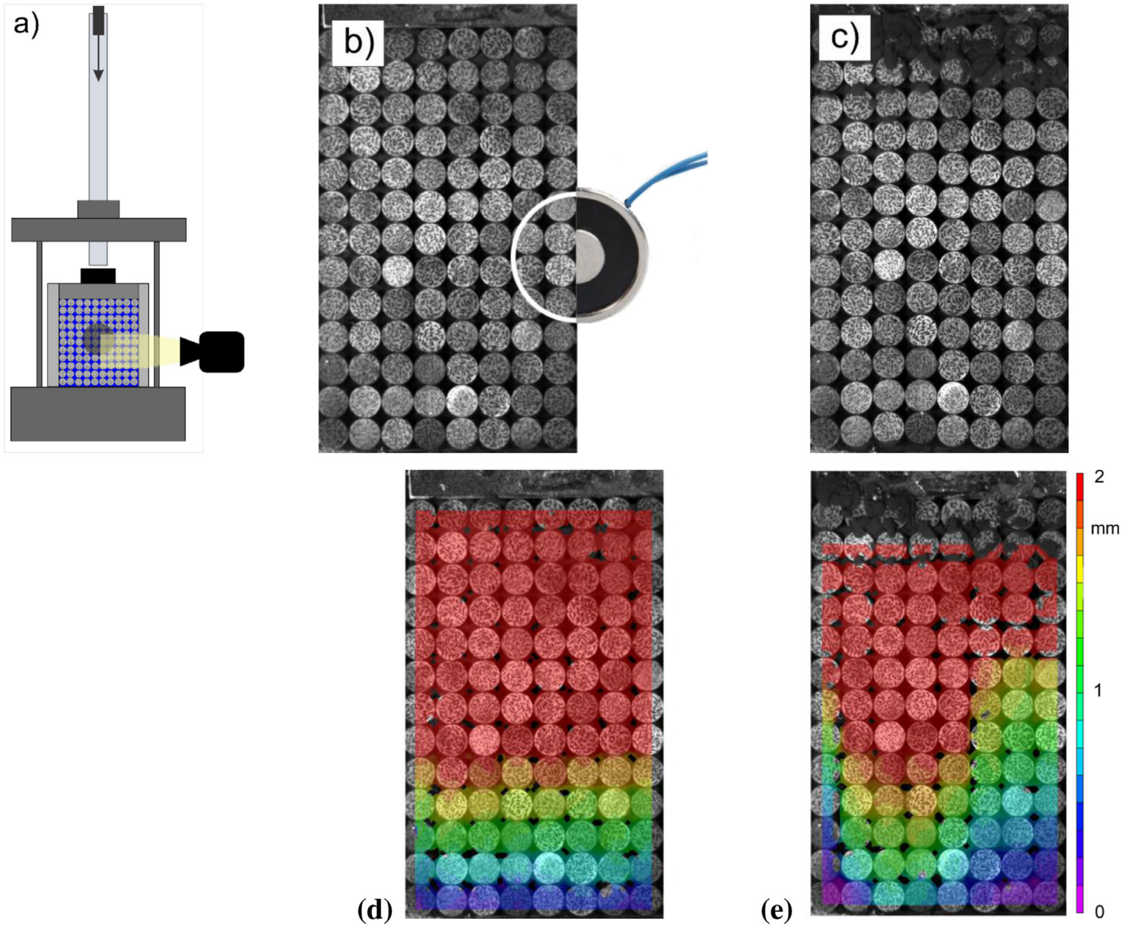


**Fig. 1.** (**a**) A schematic representation of the drop-tower experimental setup for low-speed impact testing of granular crystals immersed in MR fluids, (**b**) A typical image recorded for the left half of immersed granular crystal subjected to low-speed impact, the position of the electromagnet is indicated schematically in the left half of the granular crystal, (**c**) a typical image recorded for the granular crystal after repetitive experiments without rearranging the grains after each experiment. The vertical displacement fields in granular crystals immersed in MR fluids were calculated using DIC, (**d**) when there is no applied external magnetic field and (**e**) with an applied external magnetic field ∼0.7 Tesla (right).

field applied at the center of the granular crystal. In the case of no external magnetic field, the displacement field obtained using DIC performed for the individual grains is relatively uniform as would be expected for the finite granular crystal. However, the application of a magnetic field results in disruption of the homogeneity of the wavefront as the wave moves through the granular crystal, as the grains inside the region subjected to the external magnetic field have significantly lower displacements. MR fluids are used in energy dissipating applications such as brakes, shocks, dampers, and the MR fluid converts into a solid phase with significant and field-dependent stiffness upon application of a magnetic field. So, it is not surprising that the clamping force due to the external magnetic field seriously limits the motion of grains in the affected region.

In [36], the authors have analyzed the wave scattering and wave attenuation effects around point defects with significantly higher stiffness in granular crystals. In [34,36,53,54], it has been established that the presence of the point defects in 2D granular crystals results in significant wave scattering. However, the magnetically generated defects in the current work are quite different from the typical "point" defects/discrete breathers that have been investigated in the literature for multiple reasons. First, the size of these magnetic defects is not related to the grain size but instead is related to the size of the electromagnets. This factor creates the whole set of possibilities to control the size and magnitude of the defect by controlling the magnetic field. In contrast, typical approaches in adaptive metamaterials rely on modifying the properties of grains/lattice elements using external fields and are thus limited by the size of these elements. Second, the mechanism of these magnetic defects is also drastically different from the stiffness-based defects as it originates from the solidification of the MR fluid leading to the clamping of grains in the region affected by the magnetic field. Thus, the wave scattering and attenuation in this approach can be attributed to scattering around the stiffer defects [68] (clamping force proportional to the magnetic field) as well as dissipative processes losses in MR fluid. The evolution of the strain $\varepsilon_{yy}$ as a function of time calculated using DIC for grains at the same depth outside and inside the region affected by the external magnetic field can be seen in Fig. 2. As seen in Fig. 2, the maximum amplitude of strain in the grain inside the region affected by the magnetic field reduces by about 18% for a field of ∼0.3 Tesla, 34% for ∼0.7 Tesla, 64% for ∼0.9 Tesla and 72% for ∼1.2 Tesla. At the same depth, there is no noticeable difference in the strain magnitude outside of the magnetic field zone. However, the strain amplitude decreases quicker when there is no field applied, which may be an indication of higher strain concentration outside of the magnetic field zone. Although not the focus of this letter, the changes in rheological properties of MR fluids such as the particle coupling strength and MR fluid viscosity changes (functions of the particle concentration in MR fluids), could strongly influence the aforementioned wave scattering phenomena. Since MR fluids are non-Newtonian fluids, the nonlinear wave dynamics for these granular crystals with rheological "point defects" would depend

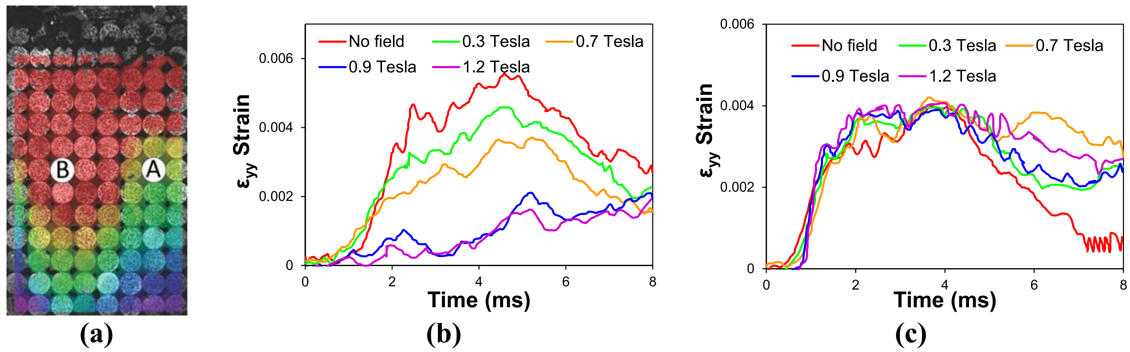


**Fig. 2.** **(a)** A comparison was performed between two grains where one grain (labeled as A) was inside the region affected by the magnetic field and the second grain (labeled as B) was outside the affected region. The evolution of strain $\varepsilon_{yy}$ as a function of time for **(b)** a representative grain A inside the region affected by the magnetic field and **(c)** representative grain B outside the affected region.

on the loading rates and impact amplitude. However, the current drop-tower based experimental setup is only capable of projectile velocities ranging from 1 m/s to 6 m/s. Based on some preliminary testing for this short range of projectile velocities, there were no measurable differences for wave dynamics of granular crystals immersed in MR fluids with or without external magnetic fields. (See *Appendix B*.)

Using an approach described in [36], the left half of the immersed granular crystals in Fig. 1b, c was divided into four equal quarters, and the strain energy in all four quarters was calculated to understand the scattering effect around local defects generated due to external magnetic field. The evolution of total strain energy for each of the four quarters in the left half of the granular crystals for varying magnitude of the magnetic field is presented in Fig. 3. As seen in Fig. 3, there is a significant difference between the strain energy distributions for different quarters with and without external magnetic fields. There is a significantly higher strain energy concentration in both upper quarters for the higher applied magnetic field values (0.9 Tesla and 1.2 Tesla), whereas the strain energy concentration in the bottom two quarters is the lowest. Contrary to this, there is only slightly higher strain energy in the upper quarters for the moderate magnetic fields (0.3 and 0.7 Tesla) and the highest strain energy concentration in the bottom left quarter is observed for these cases. As expected, the highest concentration in the bottom right quarter is observed in the absence of an applied magnetic field. In absence of an external magnetic field, most of the strain energy is concentrated directly below the point of impact in the right two quarters. The presence of external magnetic fields creates wave dispersion effects that completely alter the strain energy distribution. For the higher values of the magnetic field, there is a significant backscattering and reflection from the solidified phase, hence the high energy concentration in the upper right and left quarters, respectively. These observations of significantly backscattering for higher values of the magnetic field are consistent with strain energy distribution observations for larger point defects or two closely interacting point defects presented in [36]. The observations for moderate values of the magnetic field indicate that the size of the rheological defects is large enough to significantly alter the wave transmission. The defects generated for these moderate values result in smaller wave scattering angles and so a significant portion of the strain energy is redirected to the bottom left quarter. These observations for low scattering angles for moderate values of magnetic fields are quite consistent with those for point defects reported in [36]. Thus, it can be postulated that the size of rheological defects generated in granular crystals immersed in MR fluids can be controlled by altering the value of the magnetic field. This process of generating rheological defects has relatively low response times and is completely reversible as the granular crystal reverts to its original state upon removal of the external magnetic field (see *Appendix B*). The size of the rheological point defects generated also significantly alters the strain energy distribution in the crystal. Thus, this approach can be utilized for impact mitigation by lowering the magnitude of wave amplitude and guiding the energy to a specific part of the crystal.

It is also interesting to see how the magnetic field affects the wave decay characteristics of the granular crystal immersed in MR fluids (kinetic energy). This can be qualitatively assessed by plotting the normalized wave amplitudes (max. velocities) for grains in the rightmost column in Fig. 1a for different values of magnetic fields as a function of vertical distance of the initial point of impact (Fig. 4). As it can be seen, in the absence of a magnetic field, the velocity decay characteristics for dry granular crystals and granular crystals immersed in MR fluids have a distinct decay behavior with two regions — exponential and higher rate, explained in a detailed manner in [35,39]. Even in the absence of an external magnetic field, the presence of secondary fluid significantly enhances the wave decay as a function of depth. As seen in Fig. 4, the velocity decay characteristics are severely altered with the introduction of the magnetic field. There is a sharp and sudden drop in the wave amplitude at a certain vertical distance from the point of initial impact. As expected, this vertical distance is the approximate location of the magnet and hence the location of the rheological point defect. This sudden drop in velocity at the location of the rheological defect increased with the increasing magnitude of the applied magnetic field.

There is significant randomness in the velocity of the individual grains in various parts of granular crystals and the general trends for the total kinetic energy in different quarters of the granular crystals are consistent with strain energy data in Fig. 3. The velocity measurements for each grain are calculated by performing numerical derivatives of DIC-based displacement measurements and this measurement uncertainty is further magnified by the '$v^2$' term in kinetic energy. Thus, there was a larger scatter in the total kinetic energy data for various magnetic fields. To address this concern, the kinematic measurements were assessed qualitatively by selecting a representative grain in each quarter of the granular crystal and the velocity for each representative grain is plotted in Fig. 5. As seen in Fig. 5a, there is a clear second peak close to the initial velocity peak for the grain in the top left quarter at higher values of magnetic field (0.9–1.2 Tesla), thus indicating significant wave reflection for these higher values of the magnetic field. As opposed, the second peak is only hardly noticeable for the moderate values of magnetic fields (0.3–0.7 Tesla). The velocity measurements for the grain in

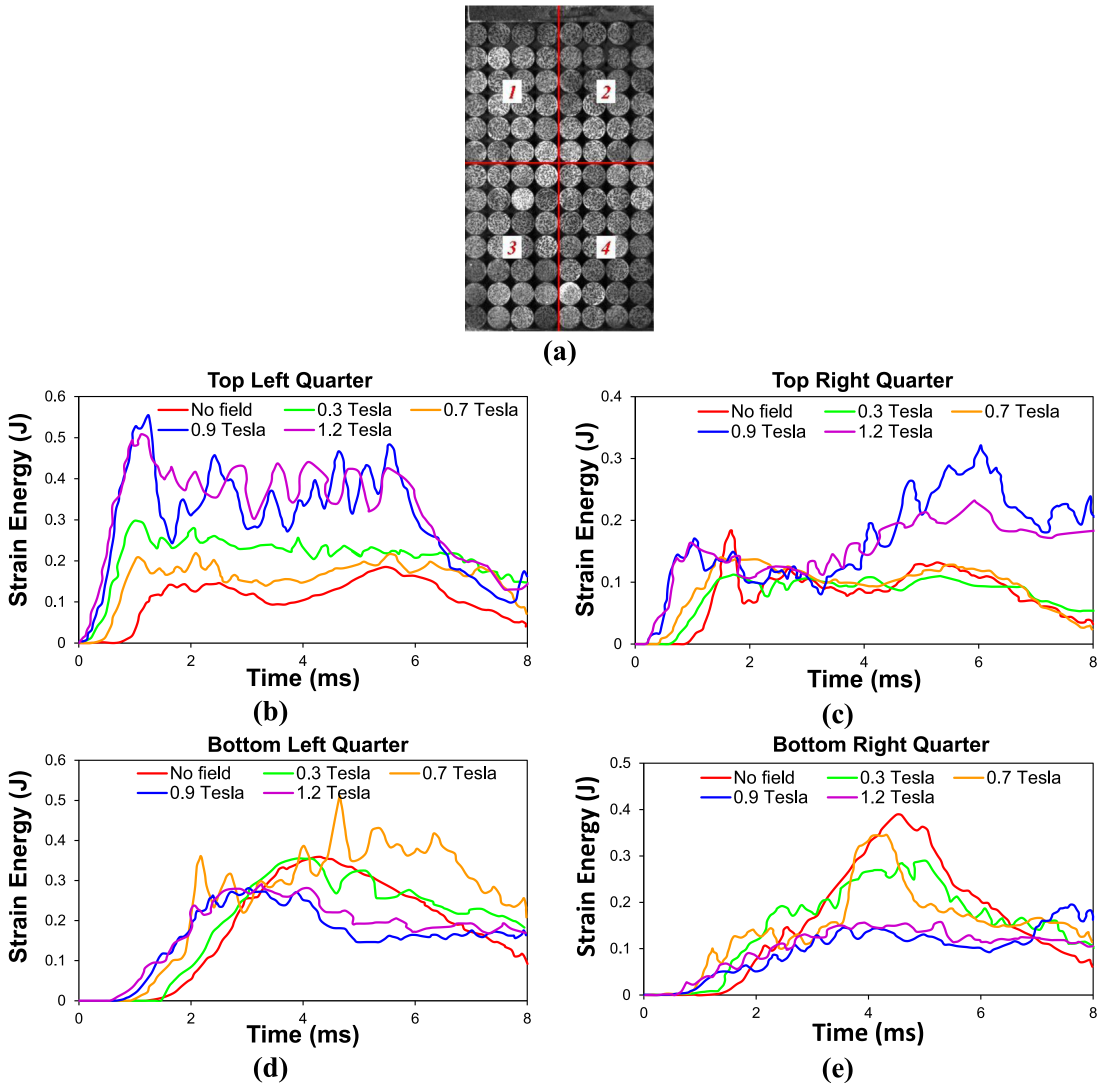


**Fig. 3.** (**a**) The left half of immersed granular crystal was divided into four quarters for analyzing the strain energy distribution. The evolution of strain energy over time in the left half of the granular crystal immersed in MR fluid for different values of external magnetic fields in **(b)** Top Left Quarter, **(c)** Top Right Quarter, **(d)** Bottom Left Quarter, and **(e)** Bottom Right Quarter.

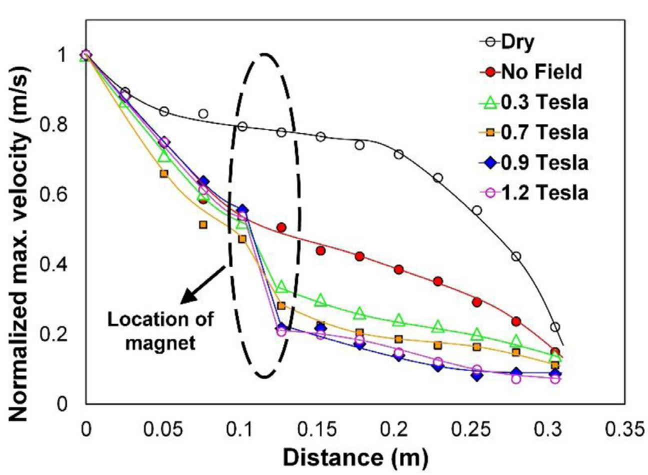


**Fig. 4.** Normalized maximum velocity of the grains in the rightmost column (central column of granular crystal) in Fig. 1b as a function of the vertical distance for different values of the external magnetic field. Note that the first grain in this central column is the point that experiences the impact loading from the low-speed projectile.

the top right quarter indicate that the first velocity peak remains at a nearly constant value for higher values of the magnetic field while the velocity profiles at moderate values of the magnetic field represent a gradual decay. These experimental observations are consistent with backscattering effects at higher values of the magnetic field. As seen in Fig. 5c, there is a significant widening of the velocity peak for the grain in the bottom left quarter at the higher magnetic field values, which sometimes can be seen as a second peak. Contrary to this, there is only a slight difference between the velocity profiles for grain in the bottom left quarter with no field and moderate magnetic field values. Thus, moderate magnetic fields result in significantly lower wave scattering as compared to higher magnetic fields. The velocity measurements for the grain in the bottom right quarter (see Fig. 5d) indicate that the application of the external magnetic field results in two distinct velocity peaks (a shorter first peak followed by a larger second peak) as opposed to the single peak for no applied field. The first velocity peak can be directly related to the portion of the wave transmitted across the rheological defect while the second peak could be related to the portion of the wave that was initially reflected from the rheological defect and likely reflected from the walls towards this section of the crystal. Thus, the kinetic energy transfer through the granular crystal is significantly affected by

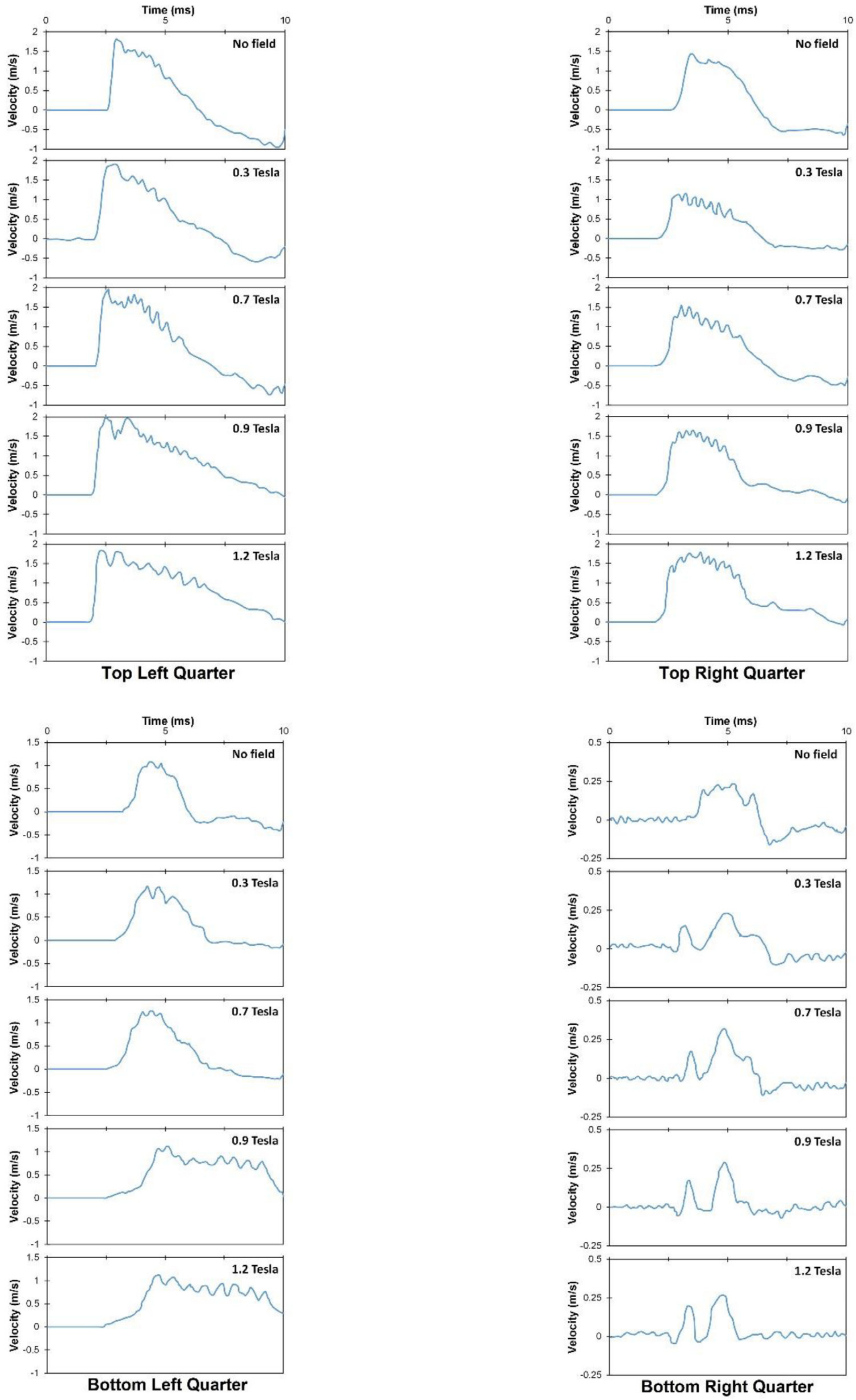


**Fig. 5.** The evolution of vertical velocity as a function of time for representative grains in each quarter for the different magnitudes of the external magnetic field.

the presence of the magnetic field. It is noteworthy that while the general trends for the strain energy and kinetic energy transfer are similar, the kinetic and strain energy transfer process are affected slightly differently by the presence of rheological defects. Some of these differences can be also attributed to the finite size of the granular crystals investigated in the current work.

## 4. Conclusions

This work demonstrates the efficacy of a new class of adaptive granular metamaterials for impact mitigation using low-speed impact loading experiments. Granular crystals immersed in MR fluids offer the unique ability to reversibly generate rheological “point” defects at any spatial location in the granular crystal with a fast response time using external magnetic fields. The location of the magnetically generated defect is fixed at the center of the granular crystals in the presented experiments. However, the location of single or multiple defects can be easily altered in future experiments by controlling the location and magnitude of the external magnetic field. Currently, acoustic metamaterials have ‘fixed’ designs for specific applications, the adaptive granular metamaterials present a drastic change in design philosophy to a ‘sense-adapt-recover’ approach where metamaterials would adapt their internal structure based on basic sensor data to achieve a specified objective (wave redirection, attenuation, etc.) and revert to its original state.

To facilitate the assessment of the performance of the adaptive granular crystals, a novel experimental methodology was developed to quantify the influence of a magnetically generated “point defect” on the low-speed impact response of granular crystals. This novel experimental approach employed a combination of high-speed imaging and DIC to visualize the changes in the kinematic fields because of external magnetic fields of different magnitudes. In this novel experiment, it was successfully demonstrated that it is possible to alter the wave propagation in granular crystals immersed in MR fluids by varying the magnetic field strength. It was observed that the magnetic field strength and the resultant rheological defects can be employed to alter the amount of transmitted energy to the bottom of the granular crystal and control the reflection and refraction of the incident wave. It was found that increasing the magnetic field significantly reduces the transmitted energy (both kinetic energy and strain energy) from the region affected by the magnetic field, and initially introduces scattering and refraction, which transforms into backscattering and reflection at higher fields. These experimental observations create new possibilities to leverage the dissipative processes involved in the scattering and attenuation around these reversible rheological defects for highly effective control of the magnitude and direction of wave propagation. The reversibility also presents the unique potential to allow a granular crystal to adapt its defect patterns based on the amplitude, direction, and velocity of the incident wave. Although a more detailed quantification of the underlying physics of the wave propagation in these adaptive granular metamaterials is warranted, the results presented in this letter clearly illustrate the great potential of adaptive granular metamaterials for dissipative applications such as shock protection, impact mitigation, and damping.

## Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Nikhil Karanjgaokar reports financial support was provided by National Science Foundation. Nikhil Karanjgaokar reports a relationship with National Science Foundation that includes: funding grants.

## Data availability

Data will be made available on request.

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## Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.eml.2022.101943.

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