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ORIGINAL PAPER

Understanding Motivation with the Progressive Ratio Task: a Hierarchical Bayesian Model

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Abstract

The progressive ratio task (e.g., Wolf et al., *Schizophrenia Bulletin*, 40(6):1328–1337, 2014) is often used to assess motivational deficits of individuals with mental health conditions, yet the number of studies investigating its underlying mechanisms is limited. In this paper, we present a hierarchical Bayesian model for the cognitive structure of the progressive ratio task. This model may be used to investigate the underlying mechanisms of human behavior in progressive ratio tasks, which can identify the factors contributing to participants' performance. A simulation study shows satisfactory parameter recovery results for this model. We apply the model to a progressive ratio data set involving people with schizophrenia, first-degree relatives of people with schizophrenia, and people without schizophrenia. Our analysis reveals that people with schizophrenia are more affected by elapsed time than people without schizophrenia, tending to lose motivation to exert effort as they spend more time and effort in the task, regardless of the effort-reward ratio. The first-degree relatives show intermediate effects of time and effort-reward optimization between people with and without schizophrenia, which indicates that first-degree relatives might share some deficits with people with schizophrenia, only not as severe.

Keywords Bayesian hierarchical modeling · Individual differences · Computational psychiatry · Schizophrenia · Progressive ratio task · Motivation

Introduction

Originally proposed by Hodos (1961) as a ratio schedule of reinforcement for animal subjects, the progressive ratio task (PRT) was introduced as a measurement test in psychiatry (e.g., Wolf et al., 2014), psycho-pharmacology (Chelonis et al., 2011) and clinical therapy development (Roane, 2008). Some studies use the PRT to evaluate the efficacy of rewards (Roane, 2008), which helps researchers in designing therapeutic interventions. Other studies use the PRT to evaluate participants' motivation, and seek to investigate the relation between motivation and certain traits of interest, such as autism (Tiger et al., 2010) or schizophrenia (Wolf et al., 2014).

The PRT asks people to make successive simple choice responses to earn rewards. A PRT schedule features a sequence of consecutive sets of trials where, in each set, the participants need to exert more effort than in the previous set to earn the same amount of reward. The task is terminated when the participants passively cease responding or actively quit the experiment. Such termination indicates that they are unwilling to exert the effort required of the current set for the reward.

The majority of PRT studies use the "breakpoint" statistic as an index of reward efficacy or motivation. The breakpoint is assumed to reflect the maximum amount of effort a participant exerted for the reward. Statistically, the breakpoint is a function of the effort/reward ratio of the last set

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completed, which has the highest effort/reward ratio of all finished sets due to the nature of PRT schedules, and the effort/reward ratio of the first set that is not completed, which is the set immediately following the last one completed. As a common practice in PRT studies, researchers compare the reward efficacy or motivation across groups by comparing their breakpoints (e.g., Wolf et al. 2014; Strauss et al. 2016).

However, the breakpoint statistic is not flawless as a measurement index. Because it is based on a single effort/reward ratio at the final set, it ignores information from previous sets in the experiment that could be meaningful, such as the earlier effort/reward ratios and RTs (Killeen et al., 2009; Bradshaw & Killeen, 2012). Relevantly, the breakpoint is sensitive to manipulations unrelated to its target trait (reward efficacy or motivation), such as response requirements (Aberman et al., 1998; Bradshaw & Killeen, 2012) and the size of increments in the sequence of effort/reward ratios (Covarrubias & Aparicio, 2008; Bradshaw & Killeen, 2012).

To overcome the shortcomings of the breakpoint statistic, it is reasonable to build a quantitative model that takes into account a participant's entire history of responses in the PRT and that also incorporates covariates that are known to influence motivation. A quantitative model can make more efficient use of data by including any covariates and participant responses available from the experiment. As the model reveals the effects of covariates, it may also reveal potential underlying mechanisms of participant behavior in PRT studies.

In this study, we built a hierarchical Bayesian model to quantify performance under the PRT test. Because the PRT test has multiple versions, we based the model on a design from Wolf et al. (2014); later we will discuss the possibility of extending the model to other tasks. Because Wolf et al. used a relatively complex PRT schedule, and most other schedules are comparatively simpler, such an extension may only require a corresponding simplification of the model.

We first describe the PRT schedule proposed by Wolf et al. (2014) and the hierarchical Bayesian model inspired by it. We then use a parameter recovery study to evaluate whether the model can successfully recover data-generating mechanisms. Then we apply the model to Wolf et al.'s (2014) data. Finally we discuss the model's potential application to other PRT schedules and possible directions of future research.

Hierarchical Bayesian Model for PRT Design

We use a geometric regression structure to construct a hierarchical Bayesian model that describes the probabilities that participants terminate their participation in a PRT as a function of available covariates, and assess its feasibility for application. Before presenting the model, we describe in detail the PRT schedule of Wolf et al. (2014) and the covariates from their study.

Wolf et al. (2014): Computer-Based PRT Schedule for Human Motivation

Wolf et al. (2014) introduced a computer-based PRT schedule for human motivation assessment. The schedule features 3 reward levels (50, 25, 10 cents), where each reward level contains 7 progressive ratio sets (see Table 1). Each set contains a fixed number of two-choice response trials where participants are asked to select the larger of two numbers presented on a computer monitor. Participants can quit at any time with a keypress and, if they decide to quit, they skip all sets in their current reward level and start the first set in the next reward level. As an example, if participants quit at Set 5, they skip Sets 6 and 7, which have the same reward level (50), and start Set 8, which has the next reward level (25). When participants complete the required number of trials in a set correctly, they get the corresponding reward, otherwise they cannot get the reward no matter how many trials they have completed. Incorrect responses are not counted toward the number of trials completed.

At the start of each set, participants see an initialization screen showing the reward level, effort level (set size), the overall reward they have earned so far, and an explicit message indicating the keypress necessary to quit (see Fig. 1, upper screen). After the set begins, participants see trial screens containing the two stimulus numbers, effort level of the set, and their current progress (see Fig. 1, lower screen).

The set sizes are shown in Table 1. From the three reward levels, we can obtain three breakpoints values — 1 from each reward level. The breakpoint in each reward level is computed as the mean between the log (base 10) of trial-per-cent (tpc) value from the last set completed and the next set immediately following it in the same reward level. For example, if a participant quits somewhere in Set 2, the breakpoint should be the mean of $\log(tpc)$ of the last completed set (Set 1) and Set 2, calculated as $-0.77 = (\log_{10}(6/50) + \log_{10}(12/50))/2$. As a special case, if the participant quits at the first set in the reward level, there is no last completed set in the same level, thus the breakpoint is taken as $\log(tpc)$ of that first set. Similarly, if a participant completes all 7 sets in a reward level, there is no incomplete set, and the breakpoint will be $\log(tpc)$ of the last set in the reward level (noted as row "completed" in Table 1) Wolf et al. (2014) used the mean of the 3 breakpoints from the 3 reward levels as an index of motivation.

Table 1 Summary of the PRT test structure from Wolf et al. (2014)

Set	Reward	Set size	Breakpoint	Set	Reward	Set size	Breakpoint	Set	Reward	Set size	Breakpoint
1	50	6	− 0.92	8	25	3	− 0.92	15	10	1	− 1.00
2	50	12	− 0.77	9	25	6	− 0.77	16	10	2	− 0.85
3	50	26	− 0.45	10	25	13	− 0.45	17	10	5	− 0.50
4	50	45	− 0.16	11	25	23	− 0.16	18	10	9	− 0.17
5	50	100	0.13	12	25	50	0.13	19	10	20	0.13
6	50	167	0.41	13	25	83	0.41	20	10	33	0.41
7	50	500	0.76	14	25	250	0.76	21	10	100	0.76
Completed			1.00	completed			1.00	completed			1.00

The sets are labeled according to the order in which they are shown in the task. The "reward" is the cents awarded to the participants should they finish the corresponding set. The "set size" is the number of two-choice trials they need to perform correctly to obtain the reward for the corresponding set. The "breakpoint" is the breakpoint value reached in the reward level should the participant quit during the corresponding set. As a breakpoint computation example, if participants complete Sets 1–7, they finish all sets in the 50-cent reward level, thus their breakpoint for the 50-cent level is "completed" 1.00; if they complete Sets 8–10, they are considered to quit at Set 11, thus his/her breakpoint for the 25-cent level is -0.16; if they don't complete any set in the 10-cent level, they are considered to quit at Set 15 with a breakpoint of -1.00; overall, the mean breakpoint in this case is $-0.05 = (1.00 - 0.16 - 1.00)/3$

Covariates and Hierarchical Bayesian Model

Based on the characteristics of the PRT schedule from Wolf et al. (2014), we constructed a hierarchical Bayesian model for the decision to quit throughout the course of the PRT schedule. Because Wolf et al.'s study also measured response times (RTs), we also separately modeled the two-choice RTs.

Hierarchical Bayesian Model

Because Wolf et al. (2014) developed the PRT to investigate participants from multiple comparable groups (e.g., participants with and without schizophrenia), we built a model suitable for C groups where group c contains I_c participants. We use c to be the group identifier ($c = 1, 2, \dots, C$) and i to be the participant identifier ($i = 1, 2, \dots, I_c$). To study a participant's decision to quit, we distinguished between whether this participant quits at the start of a set (Fig. 1, upper screen) or during the two-choice trials (Fig. 1, lower screen) — the two times at which they can press a key to quit.

If participants fail to complete a set of a certain reward level, they will not reach the remaining sets in the same level, thus every participant can reach a different number of sets. Suppose Participant i in condition c reaches N_{ic} sets. Let $X_{ic,n}$ denote a 0–1 indicator of the participant's decision of whether to quit at the start of the nth set reached ($n = 1, 2, \dots, N_{ic}$), and let $Y_{ic,n}$ denote a 0–1 indicator of whether the participant quits within the nth set. If $X_{ic,n} = 1$ then the participant chose not to initialize the set and quit at the start-of-set screen (Fig. 1, upper screen). If $X_{ic,n} = 0$ and $Y_{ic,n} = 1$ then the participant chose to start the set but quit somewhere within the set. If $X_{ic,n} = 0$ and $Y_{ic,n} = 0$ then the participant managed to complete the set and obtain the corresponding reward. Denote the probability of quitting at the set's initialization screen by $p_{ic,n}$, and the probability of quitting within the set by $q_{ic,n}$, so, assuming independence,

$$\begin{aligned} &P(X_{ic,n} = 1) = p_{ic,n}, \\ &P(X_{ic,n} = 0, Y_{ic,n} = 1) = (1 - p_{ic,n})q_{ic,n}, \quad \text{and} \\ &P(X_{ic,n} = 0, Y_{ic,n} = 0) = (1 - p_{ic,n})(1 - q_{ic,n}). \end{aligned} \tag{1}$$

We selected the model covariates making use of the information recorded in Wolf et al.'s (2014) PRT schedule. In their experiment, participants are informed of the reward level in each set and effort required in each set. The two-choice RTs were are also recorded for each trial. In our model, we considered reward value, effort level (set size), and overall elapsed time in the experiment as covariates.

Denote the reward level as $V_{ic,n}$ for the nth set from Participant i in group c. Because reward has only 3 discrete levels (50, 25, 10), we cannot infer the functional form of the reward effect from the data. Thus we use dummy variables $\hat{V}^{10}_{ic,n}, \hat{V}^{01}_{ic,n} \in \{0, 1\}$ to code reward levels:

$$\hat{V}^{10}_{ic,n} = \begin{cases} 1, & \text{if } V_{ic,n} = 25, \\ 0, & \text{otherwise,} \end{cases} \quad \text{and}$$

$$\hat{V}^{01}_{ic,n} = \begin{cases} 1, & \text{if } V_{ic,n} = 10, \\ 0, & \text{otherwise.} \end{cases}$$

So the dummy variable combinations are (0, 0), (1, 0), and (0, 1) for reward levels 50, 25, and 10, respectively. We use

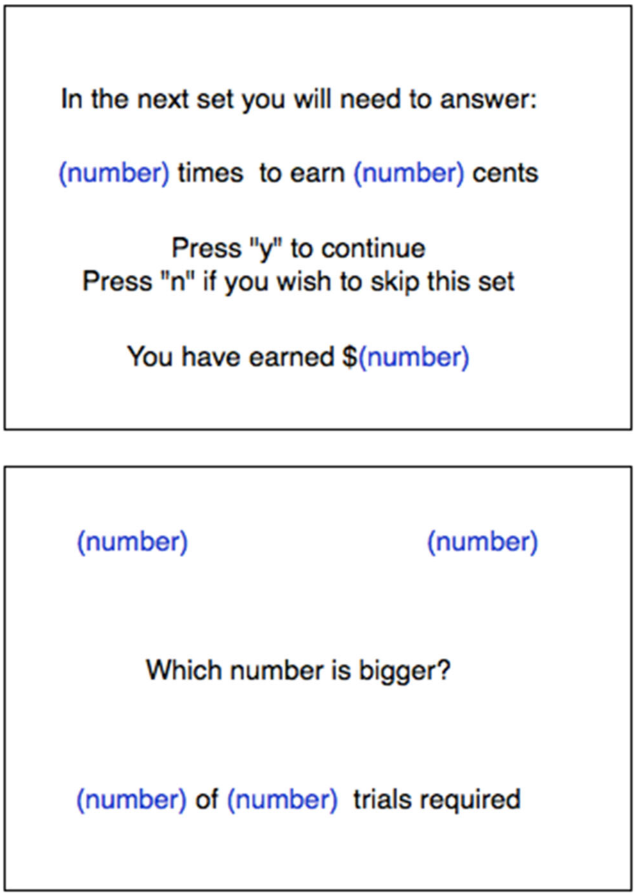

Fig. 1 Sample screens adapted from Wolf et al. (2014) experiment. The upper screen is shown at the start of each set. The lower screen shows a two-choice trial within a set

$M_{ic,n}$ to denote the current set size. Suppose the participant encounters $J_{ic,n}$ two-choice trials in set n, then we denote the participant's RT for the jth response as $t_{ic,n,j}$ $(j \leq J_{ic,n})$. Participants also spends a period of initialization time $S_{ic,n}$ at the start screen (Fig. 1, upper screen) before they make the decision to begin/skip the set. Therefore, for each set, we consider the elapsed time to be $T_{ic,n}$, where

$$T_{ic,n} = \begin{cases} S_{ic,1}, & \text{if } n = 1, \\ S_{ic,n} + \sum_{k=1}^{n-1}(S_{ic,k} + \sum_{j=1}^{J_{ic,k}} t_{ic,k,j}), & \text{otherwise.} \end{cases}$$

Because people's subjective perception of duration is likely discounted and concave with respect to objective time (Zauberman et al., 2009), we use its (natural) log discounted value $\hat{T}_{ic,n}$ as a covariate in the model. Similarly, we use the (natural) log of $M_{ic,n}$ as a covariate in the model, denoted as $\hat{M}_{ic,n}$:

$$\hat{T}_{ic,n} = \ln(T_{ic,n}), \quad \text{and} \quad \hat{M}_{ic,n} = \ln(M_{ic,n}).$$

To characterize the probabilities $p_{ic,n}$ and $q_{ic,n}$, we use a geometric regression structure where

$$\begin{aligned} p_{ic,n} &= \tfrac{1}{1+\exp(-Z^p_{ic,n})}, \text{ with} \\ Z^p_{ic,n} &= \beta_{ic} + \mu^{10}_{ic}\hat{V}^{10}_{ic,n} + \mu^{01}_{ic}\hat{V}^{01}_{ic,n} \\ &\quad + \eta^p_{ic}\hat{M}_{ic,n} + \eta^{10}_c\hat{V}^{10}_{ic,n}\hat{M}_{ic,n} + \eta^{01}_c\hat{V}^{01}_{ic,n}\hat{M}_{ic,n} \\ &\quad + \gamma^p_{ic}\hat{T}_{ic,n} + \gamma^{10}_c\hat{V}^{10}_{ic,n}\hat{T}_{ic,n} + \gamma^{01}_c\hat{V}^{01}_{ic,n}\hat{T}_{ic,n} \\ &\quad + \psi_c\hat{M}_{ic,n}\hat{T}_{ic,n}, \end{aligned} \tag{2}$$

and

$$\begin{aligned} q_{ic,n} &= \tfrac{1}{1+\exp(-Z^q_{ic,n})}, \text{ with} \\ Z^q_{ic,n} &= \beta_{ic} + \eta^q_{ic}\hat{M}_{ic,n} + \gamma^q_{ic}\hat{T}_{ic,n} + \psi_c\hat{M}_{ic,n}\hat{T}_{ic,n}. \end{aligned} \tag{3}$$

The model structure is shown in Fig. 2.

The model for $p_{ic,n}$ includes individual-level parameters: the intercept β_{ic}, the main effects of reward values μ^{10}_{ic} and μ^{01}_{ic}, the main effect of effort η^p_{ic}, and the main effect of elapsed time γ^p_{ic}. It also includes group-level parameters for pairwise interactions γ^{10}_c, γ^{01}_c, η^{10}_c, η^{01}_c and ψ_c. Because of the limited data (a maximum of 21 sets for each participant), we do not allow the interaction effects to vary at the individual level to avoid potential identifiability problems.

The model for $q_{ic,n}$ includes the individual-level intercept β_{ic}, the main effect of effort η^q_{ic}, the main effect of time γ^q_{ic}, and a group-level interaction between effort and time ψ_c. The effects of reward level are excluded because model comparison results suggest that reward level contributes little in estimating $q_{ic,n}$ (see the Appendix for the model comparison procedure and results).

Because the PRT schedule allows participants to quit whenever they want, some participants have a very small sample size N_{ic}, which results in identifiability problems and a large estimation bias if individual-level parameters are estimated in isolation. A hierarchical Bayesian model structure permits us to estimate individual-level parameters in these cases using group-level information embedded in the model's hyperparameters (Busemeyer & Diederich, 2010). To characterize the relation of individual-level parameters to the group, let $r \in \{p, q\}$ and $s \in \{10, 01\}$, we model the individual-level parameters as

$$\begin{aligned} &\beta_{ic} \sim N(\beta_c, \sigma_{\beta,c}), \quad \mu^s_{ic} \sim N(\mu^s_c, \sigma_{\mu^s,c}), \quad \text{and} \\ &\eta^r_{ic} \sim N(\eta^r_c, \sigma_{\eta^r,c}), \quad \gamma^r_{ic} \sim N(\gamma^r_c, \sigma_{\gamma^r,c}). \end{aligned}$$

Thus, each individual-level parameter is drawn from a normal distribution characterized by group-level parameters. Parameters μ_c, μ^s_c, η^r_c, γ^r_c are the group-level means, and $\sigma_{\mu,c}$, $\sigma_{\mu^s,c}$, $\sigma_{\eta^r,c}$, and $\sigma_{\gamma^r,c}$ are the group-level standard deviations. Due to the small sample size of PRT data, we reduce the number of parameters to be estimated by assigning constant values to the standard deviation parameters to avoid identifiability problems.

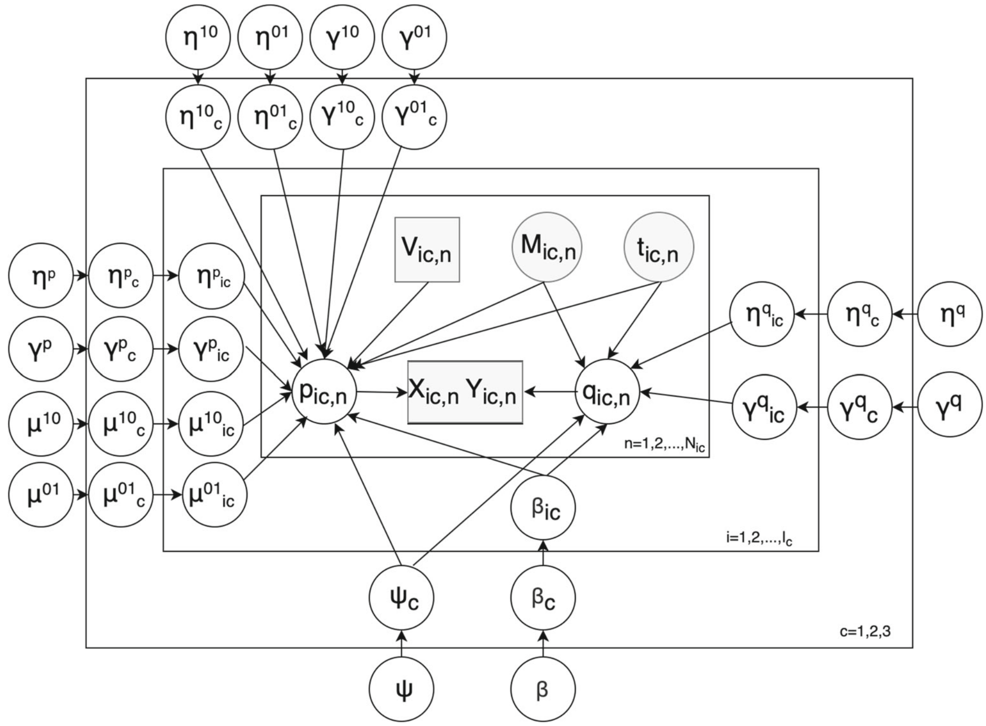

Fig. 2 The graphical structure of the hierarchical Bayesian model. Circles denote continuous quantities and rectangles denote discrete quantities. Observed variables are shaded, while parameters to be estimated are not. Arrows indicate dependence, and each plate signals repetition over trials, individuals, and experimental conditions

Similarly, the group-level parameters are modeled as

$$\begin{aligned}&\beta_c \sim N(\beta, \sigma_\beta), \quad \mu_c^s \sim N(\mu^s, \sigma_{\mu^s}),\\ &\psi_c \sim N(\psi, \sigma_\psi), \quad \text{and}\\ &\eta_c^r \sim N(\eta^r, \sigma_{\eta^r}), \quad \gamma_c^r \sim N(\gamma^r, \sigma_{\gamma^r}),\\ &\eta_c^s \sim N(\eta^s, \sigma_{\eta^s}), \quad \gamma_c^s \sim N(\gamma^s, \sigma_{\gamma^s}).\end{aligned}$$

We also assign constant values to the standard deviation parameters in this level. The hyperparameters are modeled as

$$\begin{aligned}&\beta \sim N(0, \sigma_{\beta,0}), \quad \mu^s \sim N(0, \sigma_{\mu^s,0}),\\ &\psi \sim N(0, \sigma_{\psi,0}), \quad \text{and}\\ &\eta^r \sim N(0, \sigma_{\eta^r,0}), \quad \gamma^r \sim N(0, \sigma_{\gamma^r,0}),\\ &\eta^s \sim N(0, \sigma_{\eta^s,0}), \quad \gamma^s \sim N(0, \sigma_{\gamma^s,0}),\end{aligned}$$

where the standard deviations are also fixed in model estimation.

Overall, given the entirety of model parameters θ, the likelihood of the model can be written as

$$\begin{aligned}\mathcal{L}(\theta|X,Y) &= \prod_{i,c,N_{ic}} P(X_{ic,n}, Y_{ic,n}|p_{ic,n}, q_{ic,n})\\ &= \prod_{i,c,N_{ic}} [p_{ic,n}]^{I(X_{ic,n}=1)}[(1-p_{ic,n})\, q_{ic,n}]^{I(X_{ic,n}=0, Y_{ic,n}=1)}\\ &[(1-p_{ic,n})\,(1-q_{ic,n})]^{I(X_{ic,n}=0, Y_{ic,n}=0)},\end{aligned}$$

where $I(S)$ is the indicator function that equals 1 when statement S is true and 0 otherwise.

RT Model

The hierarchical Bayesian model includes as covariates the quantities $T_{ic,n}$ that were constructed using the individual RTs $t_{ic,n,j}$. Thus, the decisions to quit are modeled conditional on (functions of) the RTs. In this section we present a model for the RTs themselves. While decisions to quit and RTs could be modeled jointly, we decided to build two separate models, mostly because the data provided to us did not include the exact values of the stimuli presented on each trial and the accuracy of each response. In light of such limitations, the results from the response time analysis are intended to be used for preliminary, exploratory purposes only.

We model the RTs using a racing diffusion model (Logan et al., 2014) based on the Wald distribution (Burbeck & Luce, 1982). We selected the racing diffusion model because it can be easily modified to the PRT and it does not require information about the above-mentioned variables that are missing from the data set. In the number comparison task used in the PRT, a participant can choose which of two numbers is bigger. The racing diffusion model proposes that evidence toward each possible response is sampled from the display and stored in two neural modules called accumulators, one dedicated to each response. In this study, the two accumulators correspond to the left and right choices. The growth of evidence on each accumulator is described by a Wiener diffusion process with drift.

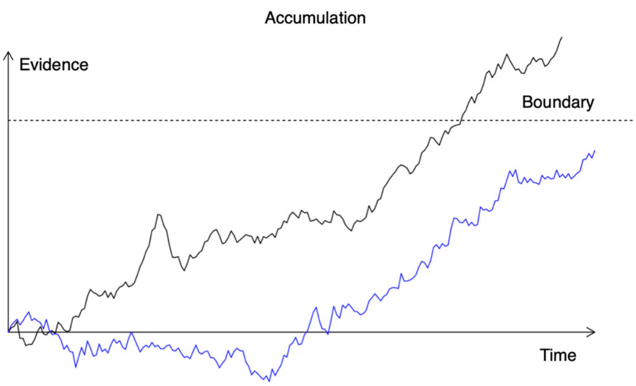

Fig. 3 The race between two accumulators corresponding to two choices (left and right stimuli in this study). The time for the first accumulator (in this case black) to reach its decision boundary is considered the RT

Each accumulator has a single absorbing boundary called the decision boundary; when the evidence level reaches the decision boundary the process ends. The boundary is the amount of evidence required for the response represented by the accumulator. The two accumulators race against each other, and the first accumulator to reach its boundary determines the response and the RT (see Fig. 3). The time for each accumulator to reach its boundary follows an inverse Gaussian (Wald) distribution. The final decision time therefore is the finishing time of the fastest accumulator, which follows the distribution of a minimum statistic of two inverse Gaussian random variables.

The finishing time distribution for each accumulator has 3 parameters: the drift rate b, the decision boundary a, and the drift coefficient σ. Because the data set provided to us does not contain information about the correct responses for each trial, we do not distinguish between the choices and use the same drift rate for both the left and right accumulators[1]. A larger drift rate b corresponds to faster information accumulation from the display. When participants have a larger decision boundary a, they need more information to reach a decision and hence are more cautious in their responses. We do not include a left/right response bias in our model, because the correct choice (a larger number) is equally likely to be on the left or the right in this task, and empirical data from the PRT (Wolf et al., 2014) does not show a left-right choice bias.

Let Participant i in Group c have a drift rate parameter given by $b_{ic} > 0$ and a decision boundary $a_{ic} > 0$. The corresponding finishing time distribution for each accumulator has density

$$f(t|a_{ic}, b_{ic}, \sigma_{ic}) = \frac{a_{ic}}{\sigma_{ic}\sqrt{2\pi t^3}} \exp\{-\frac{(a_{ic} - b_{ic}t)^2}{2\sigma_{ic}^2 t}\}, \quad t > 0,$$

where σ_{ic}, an unidentifiable parameter, is equal to 1. The density function of each RT $t_{ic,n,j}$ is then

$$2f(t|a_{ic}, b_{ic}, \sigma_{ic})(1 - F(t|a_{ic}, b_{ic}, \sigma_{ic})). \quad (4)$$

To complete the specification, we model the individual-level parameters a_{ic} and b_{ic} as gamma random variables

$$a_{ic} \sim \Gamma(1, a_c), \quad \text{and} \quad b_{ic} \sim \Gamma(1, b_c). \quad (5)$$

The group-level parameters a_c and b_c, which are the scales of the Gamma distributions, have gamma hyperpriors

$$a_c \sim \Gamma(1, a), \qquad b_c \sim \Gamma(1, b),$$

$$a \sim \Gamma(1, 1), \quad \text{and} \quad b \sim \Gamma(1, 1).$$

Given the entirety of the RT model parameters τ, the likelihood of the model is

$$\mathcal{L}(\tau|\mathbf{t}) = \prod_{i,c,N_{ic},J_{ic,n}} 2f(t_{ic,n,j}|a_{ic}, b_{ic}, \sigma_{ic}) (1 - F(t_{ic,n,j}|a_{ic}, b_{ic}, \sigma_{ic})).$$

Parameter Recovery Studies

We performed a simulation study to assess if it is possible, given an adequate amount of synthetic data generated from the breakpoint model with the parameters set at plausible values, to recover such values and reveal genuine differences across participant groups.

The PRT data set consists of the binary dependent variables $X_{ic,n}$, $Y_{ic,n}$ characterizing the decision to quit, where each participant has mostly "0"s and at maximum 3 "1"s: they can quit at most 3 times, one opportunity for each reward level. The "1"s are more likely to be in $X_{ic,n}$ than $Y_{ic,n}$ because participants have a greater tendency to quit at the start of each set rather than within.

We simulated 3 artificial groups of participants with 30 participants in each group – 90 participants overall. The simulated groups corresponds to people with schizophrenia, first-degree relatives and people without schizophrenia in the empirical data. The parameter values used to generate the data for each simulated participant were set equal to the posterior means of the corresponding parameters estimated from initial fits of the entire hierarchical Bayesian model to the data from (Wolf et al., 2014) (see Application). We followed the exact PRT schedule of Wolf et al. (2014), where each simulated participant went through the experiment as described in Section 2.1. This schedule ensured that the set size $M_{ic,n}$ and the reward $V_{ic,n}$ were

[1]When the stimuli information is available, it is more reasonable for the correct stimuli to have a larger corresponding drift rate than the incorrect one.

1: Sample $S_{ic,n}$ from the empirical pool. If n=1, the elapsed time $T_{ic,1} = S_{ic,1}$. Else, $T_{ic,n} = T_{ic,n-1} + S_{ic,n} + \sum_{j=1}^{J_{ic,n-1}} t_{ic,n,j}$.
2: Compute probabilities $p_{ic,n}$ and $q_{ic,n}$. Obtain $X_{ic,n}$ and $Y_{ic,n}$ according to Equations (2) and (3). $M_{ic,n}$ and $V_{ic,n}$ are determined by the properties of the corresponding set.
3: Determine the number of two-choice trials completed in this set, $J_{ic,n}$. If the participant quits at the start, $X_{ic,n} = 1$ and $J_{ic,n} = 0$. If the participant doesn't quit, $X_{ic,n} = 0$, $Y_{ic,n} = 0$ and $J_{ic,n} = M_{ic,n}$. If the participant quits inside the set, $X_{ic,n} = 0$ and $Y_{ic,n} = 1$, then sample $J_{ic,n}$ randomly.
4: Sample $J_{ic,n}$ two-choice RTs $t_{ic,n,j}$, from the distribution of Equation (4) of the RT model.
5: Determine which set the participant should do next. If the current set is the last set, or the participant quits and it's the last reward level, terminate. Else, if the participant quits ($X_{ic,n} = 1$ or $Y_{ic,n} = 1$), the participant should continue to do the first set in the next reward level, $n = n + 1$, go to Step 1. If the participant doesn't quit, continue to the next set in the overall schedule (Table 1), $n = n + 1$, go to Step 1.

Fig. 4 Algorithm 1 for simulation

fixed in each set according to the schedule (Table 1). For the number of sets N_{ic} and number of two-choice responses $J_{ic,n}$, we generated related covariates following Algorithm 1 (see Fig. 4). First we obtained each initialization time $S_{ic,n}$ by randomly sampling from the pool of empirical initialization times. If the simulated participant quit within the nth set after starting the set, we randomly sampled the number of two-choice responses $J_{ic,n}$ as an integer between 0 and $M_{ic,n}$. If the participant chose not to quit, we assumed $J_{ic,n}$ to be $M_{ic,n}$. Finally, for RTs $t_{ic,n,j}$ we randomly sampled from the RT distribution in Eq. (4), again using the estimated posterior means as parameter values. The procedure is shown in Fig. 5.

After obtaining the simulated data, we fit the model using Stan, via the R package Rstan (Stan-Development-Team, 2018), which uses the Hamiltonian Monte Carlo method with no-U-turn sampling (Neal, 2011) to obtain posterior samples. The standard deviation parameters were set equal to 3, except the standard deviations associated with the mean parameters $\beta_{\cdot}$ and $\mu_{\cdot}^{\cdot}$, which were set equal to 5.

We obtained two chains of length 15,000, including a burn-in period of 5000 samples. The traceplots and the Gelman-Rubin statistic $\hat{R}$ ($\hat{R} < 1.05$ for all chains) (Gelman & Rubin, 1992; Brooks & Gelman, 1998) indicated satisfactory convergence. Figure 6 shows the true data-generating parameters and summaries of their estimated posterior distributions. For most parameters, the true data-generating values are close to the posterior medians. Overall, the parameter recovery results are satisfactory, suggesting that the model is identifiable and can be applied to empirical data.

Application: Amotivation in People with Schizophrenia

We fit the hierarchical Bayesian model to Wolf et al.'s (2014) data to gain some insight about quitting in the PRT. We start by introducing the data set and discussing some possible mechanisms underlying participant behavior. We then apply the hierarchical Bayesian model to data and evaluate the model's fit. Finally, we discuss possible implications of the results.

Wolf et al. (2014): Data and Results

Wolf et al. (2014) used the PRT to study amotivation in people with schizophrenia. Amotivation is one of the common negative symptoms in psychotic disorders and may contribute to poor response to treatment (Strauss et al., 2016). Therefore, a better understanding of the mechanism of amotivation might highlight strategies through which to improve interventions for individuals with schizophrenia.

The data set from Wolf et al. (2014) includes 41 participants with schizophrenia and 37 participants without schizophrenia[2]. These participants did not differ in demographic variables except for those of socioeconomic status and education. Data from Wolf (2015) includes 40 first-degree relatives of individuals with schizophrenia. Within this set there are 264 occurrences where participants quit at the start of a set and 21 occurrences where participants quit inside an already initiated set. The common practice in applications of the PRT is to use the breakpoint as an index of motivation: a larger breakpoint corresponds to higher motivation. Wolf et al. conducted a t-test between participants with or without schizophrenia that revealed significantly lower mean breakpoint in participants with schizophrenia ($p < .05$).

Figure 7 shows the group-wise and participant-wise breakpoints from Wolf et al. (2014). The breakpoint histograms for people without schizophrenia are skewed, and those for first-degree relatives and people with schizophrenia show a potential bimodality, calling into question the validity of the assumptions underlying the parametric t-test[3]. In the participant-wise line plots, we improved visibility by jittering the breakpoints by adding $N(0, 0.02)$

[2] A few participants are excluded in this analysis because of code errors.

[3] The group difference remains significant using a non-parameteric t-test.

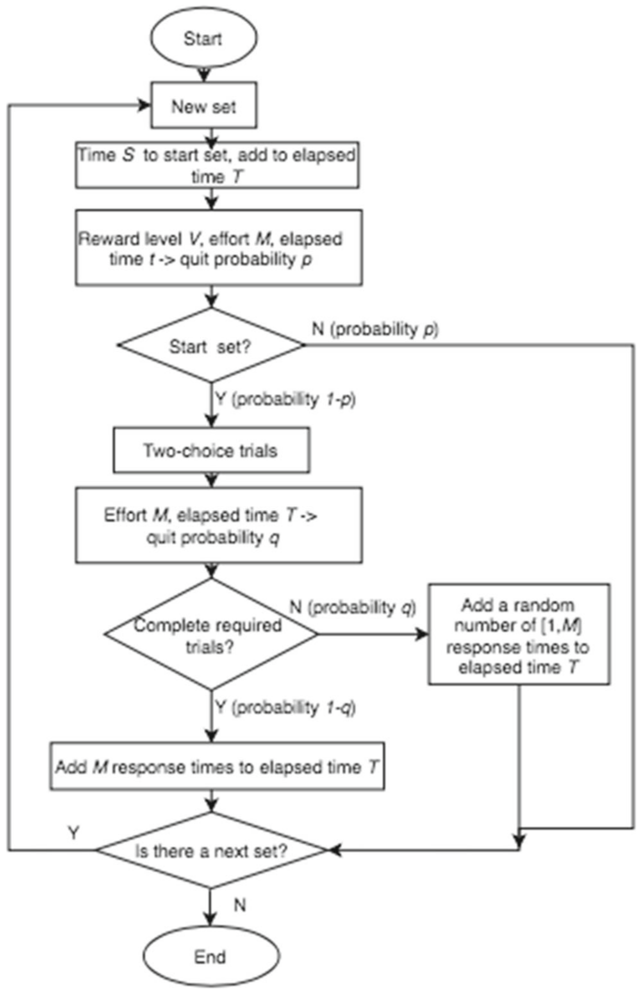

Fig. 5 A flow chart showing the procedure to sample simulated data

noise. We also divided each group by positive breakpoints (red solid lines) and negative breakpoints (blue dashed lines) for clarity. Participants without schizophrenia appear to have more consistent breakpoint values across different reward levels. Multiple first-degree relatives and individuals with schizophrenia show steep breakpoint changes across reward levels that indicate larger within-person variability. Due to the complexity of the data and the potential underlying mechanisms shown in Fig. 7, we concluded that it is appropriate to fit the hierarchical Bayesian model in an attempt to extract information from these more complicated features of the data.

Application and Model Fit

To investigate the behavioral mechanism in Wolf et al. (2014), we applied the hierarchical Bayesian model to their data. We used the same constant standard deviations as in the parameter recovery study. We obtained two chains of parameter values sampled from their posteriors using *Stan*, each consisting of 5000 burn-in samples and a total of 30000 iterations. The traceplots and Gelman-Rubin statistics $\hat{R}$ ($\hat{R}$ <1.05 for all chains) (Gelman & Rubin, 1992; Brooks & Gelman, 1998) suggested that the chains reached satisfactory convergence (Fig. 8).

For model evaluation, we first checked whether the estimated posterior predictive distributions of the frequency indicators were consistent with the data. We then checked if the estimated posterior predictive distributions of the breakpoints were consistent with the observed breakpoints.

For the hierarchical Bayesian model, denoting the entirety of the posterior parameters drawn on the kth iteration by θ^k_{ic}, we computed the probabilities of quitting $p^k_{ic,n}$ and $q^k_{ic,n}$ according to Eq. (1), then sampled the corresponding $X^k_{ic,n}$ and $Y^k_{ic,n}$ from Eqs. (2) and (3). Figure 9 shows the observed values of $X_{ic,n}$ and $Y_{ic,n}$ for a selected number of participants (i =2, 5, 8, 21, 31, 49, 92, and 98) together with their estimated posterior predictive distributions based on the sampled values $X^k_{ic,n}$, $Y^k_{ic,n}$, $k = 1, 2, \ldots, K$[4]. Here we thinned the chains by retaining every 20th iteration to reduce autocorrelation and processing time, thus $K = 3000$. The figure reveals the posterior predictive distributions to be relatively consistent with the data, and our model can accommodate the different response patterns from participants.

To evaluate whether the draws of the posterior parameters can be used to successfully recover the breakpoints, we sampled breakpoints using the procedure described by Algorithm 2 (see Fig. 8). We also thinned the chains by taking every 20th interaction to reduce autocorrelation and computation time, which resulted in a total of 3000 posterior breakpoint values. Figure 10 displays the observed breakpoints from Wolf et al. (2014) and the posterior predictive distribution of the breakpoints. The majority of true breakpoints fall in the 95% credible intervals[5]. The successful recovery of breakpoint values indicates the model's good fit to data.

Results

In this section, we present the modeling results and their implications for the quitting mechanisms for people without schizophrenia, first-degree relatives and people with schizophrenia.

We first evaluated whether any single factor among effort, reward, and elapsed time can account for the response differences between different groups of participants. Because of the presence of interaction terms in Eq. (2), we evaluated

[4]See the supplemental materials for full contrasts from all the participants.

[5]All credible intervals reported are equal-tailed intervals. The highest-posterior density intervals yield similar results and lead to the same conclusions.

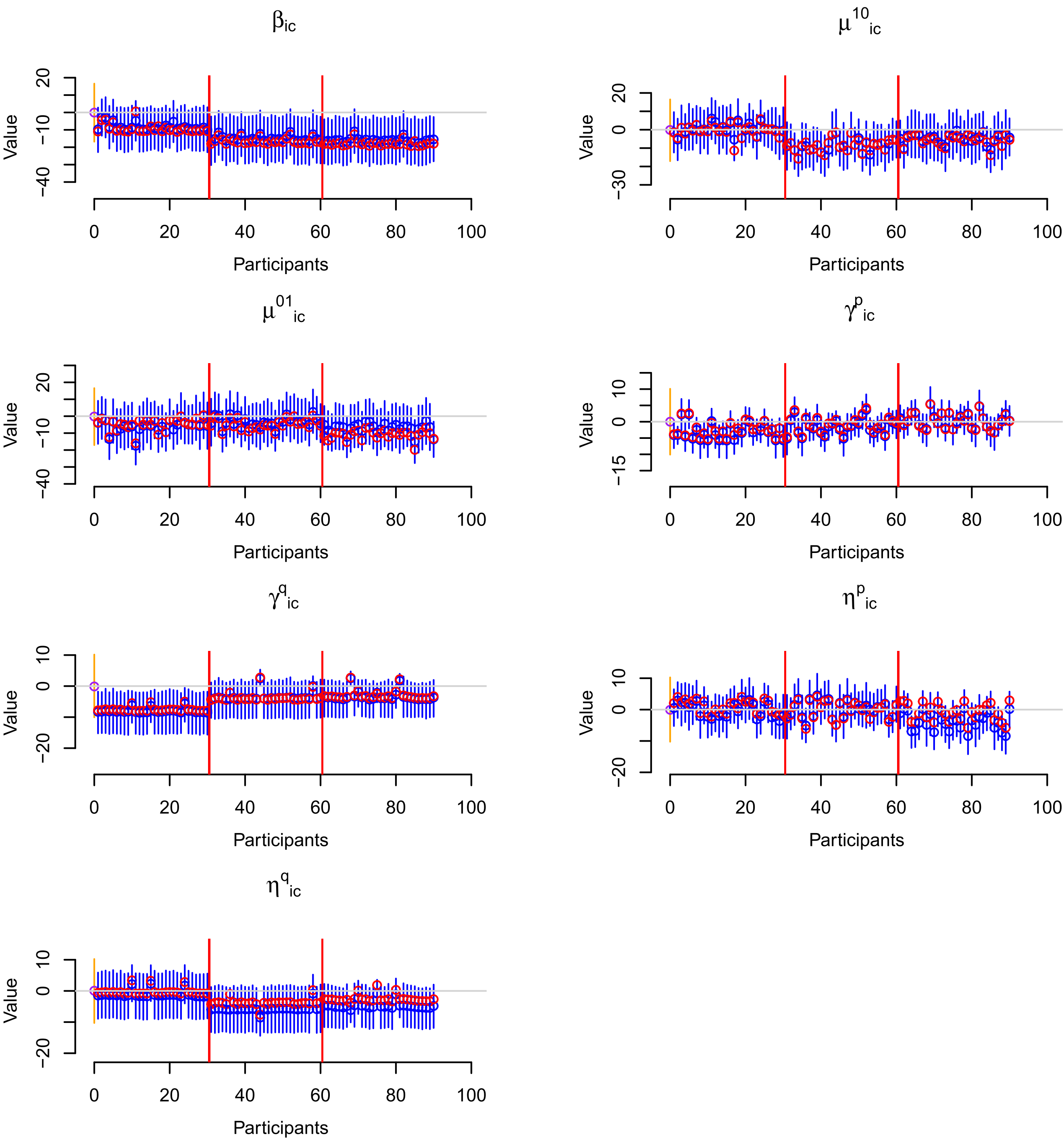

Fig. 6 Box-and-whisker plots of the estimated posterior distributions of the model parameters in the parameter recovery study together with the true data-generating values (red dots). The blue bars are the 95% credible sets and the blue dots are the posterior medians. The red vertical lines separate 3 artificial groups. The orange box-and-whisker plots at the left of the displays summarize the prior distributions for the model parameters

the effect of each factor at selected values of the other factors. We compared the group differences by comparing the estimated posterior predictive distribution of the group-level parameters. Figure 11 displays the box plots of priors and posteriors for each group-level parameter. Because participants quit less frequently within a set, we do not discuss the group-level differences regarding Eq. (3), and focus on Eq. (2).

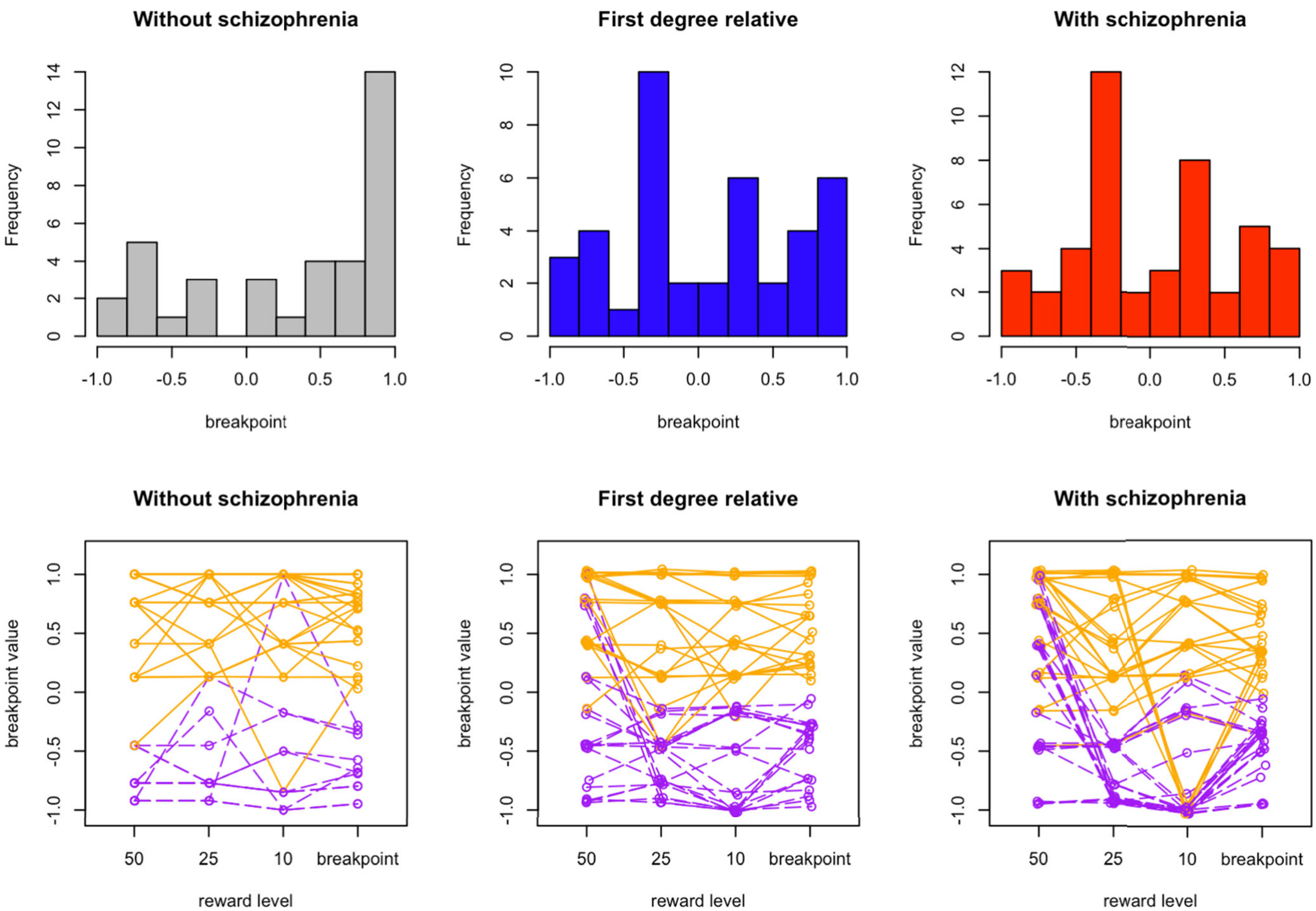

Fig. 7 Plots of the group-wise and participant-wise breakpoints from Wolf et al. (2014). The upper figures are histograms showing the distributions of breakpoints of each group. The lower figures show participant-wise breakpoints where breakpoint values of each participant are connected. Orange solid lines correspond to participant with an average breakpoint larger than 0, and purple dashed lines correspond to participants with an average breakpoint smaller than 0

For the effect of reward level, we investigated if people with schizophrenia tend to have a discounted perception of reward relative to people without schizophrenia, resulting in a larger probability to quit at the same reward level. We examined the differences between the estimated posterior probabilities of quitting between people with and without schizophrenia computed from Eq. (2) at a fixed effort level of 50 trials and a fixed elapsed time of 200 s. We selected

1: Obtain $S^k_{ic,n}$. If available in the data, use the available value. If the participant reaches a set not reached in the data, use the latest available initialization time in the same reward level. If n=1, the elapsed time $T^k_{ic,1} = S^k_{ic,1}$. Else, $T^k_{ic,n} = T^k_{ic,n-1} + S^k_{ic,n} + \sum_{j=1}^{J^k_{ic,n-1}} t^k_{ic,n,j}$.
2: Compute probabilities $p^k_{ic,n}$, $q^k_{ic,n}$. Obtain $X^k_{ic,n}$, $Y^k_{ic,n}$ according to Equation (1). $M^k_{ic,n}$ and $V^k_{ic,n}$ are determined by the property of the corresponding set.
3: Determine the number of two-choice trials completed in this set, $J^k_{ic,n}$. If the participant completes all trials, $X^k_{ic,n} = 1$, then $J^k_{ic,n} = 0$. If the participant doesn't quit, $X^k_{ic,n} = 0$ and $Y^k_{ic,n} = 0$, $J^k_{ic,n} = M^k_{ic,n}$. If the participant quits inside the set, $X^k_{ic,n} = 0$ and $Y^k_{ic,n} = 1$, then sample $J^k_{ic,n}$ randomly.
4: Sample a number of $J^k_{ic,n}$ two-choice RTs $t^k_{ic,n,j}$, from the distribution of Equation (4) from the RT model, using the posterior means of a_{ic}, b_{ic} from the RT model as parameters.
5: Determine which set the participant should do next. If the current set is the last set, or the participant quits and it's the last reward level, terminate and go to Step 5. Else, if the participant quits ($X^k_{ic,n} = 1$ or $Y^k_{ic,n} = 1$), the participant should continue to do the first set in the next reward level, $n = n + 1$, go to Step 1. If the participant completes all trials, continue to do the next set in the overall schedule (Table 1), $n = n + 1$, go to Step 1.
6: Compute the breakpoint value.

Fig. 8 Algorithm 2

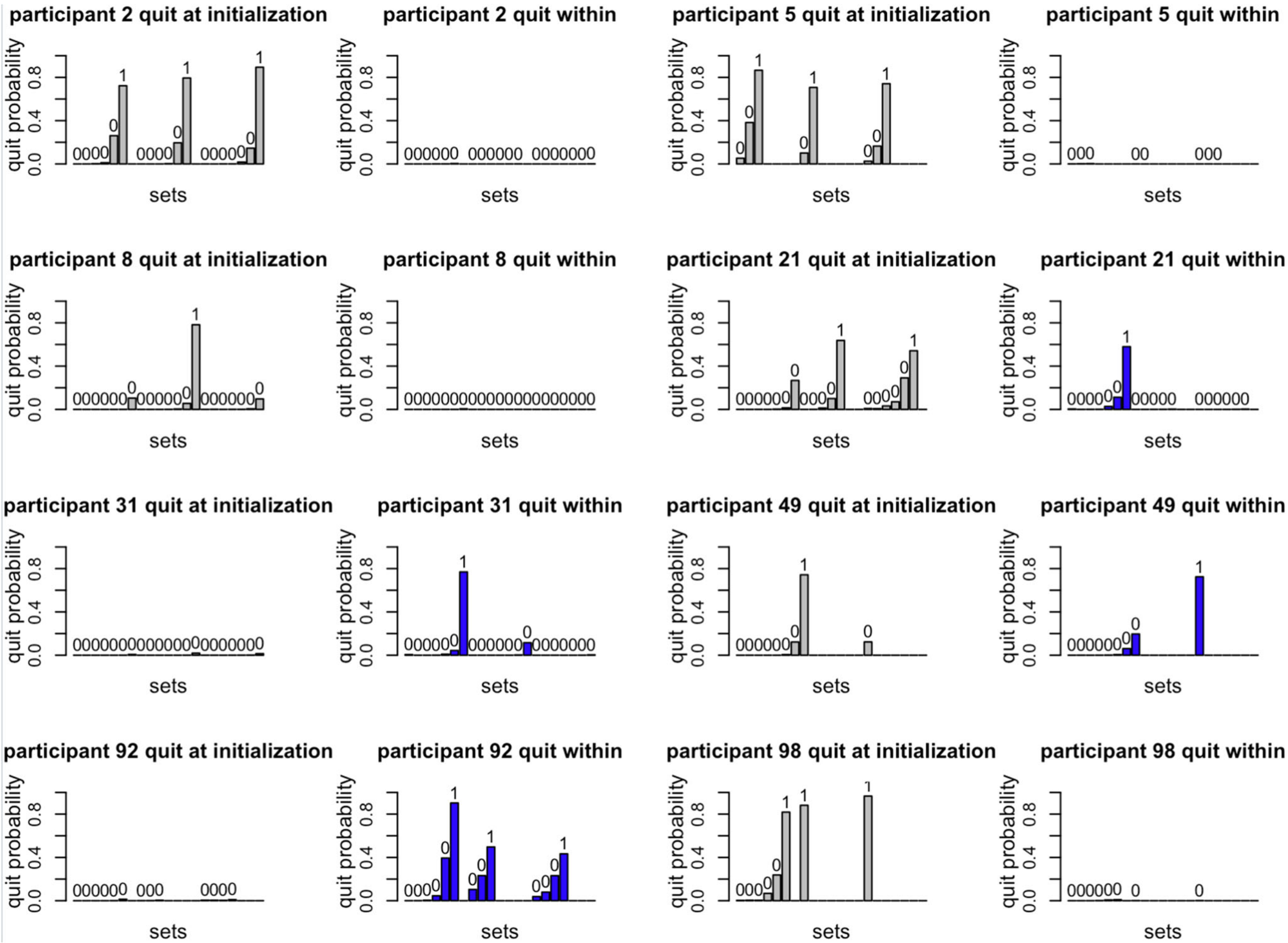

Fig. 9 Estimated posterior predictive distributions of the quitting indicators for selected participants. The bars show the proportion of "1"s among the posterior predictive draws. The number on top of each bar is the corresponding empirical value of either $X_{ic,n}$ (gray) or $Y_{ic,n}$ (blue) from Wolf et al. (2014). The empty columns correspond to sets that the participants never reached because they quit at earlier sets in the same reward level of the task

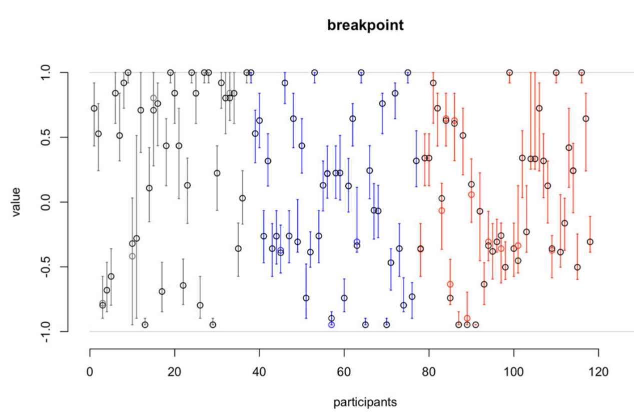

Fig. 10 Estimated posterior predictive distributions for the breakpoints together with the observed breakpoints computed from Wolf et al.'s (2014) data set for each participant. Estimated posterior predictive distributions from people without schizophrenia, first-degree relatives, and people with schizophrenia are colored gray, blue, and red, respectively. The observed breakpoints are colored black. The bars are the 95% credible intervals based on 5000 simulated breakpoints, and the points on each bar with the same color are the medians

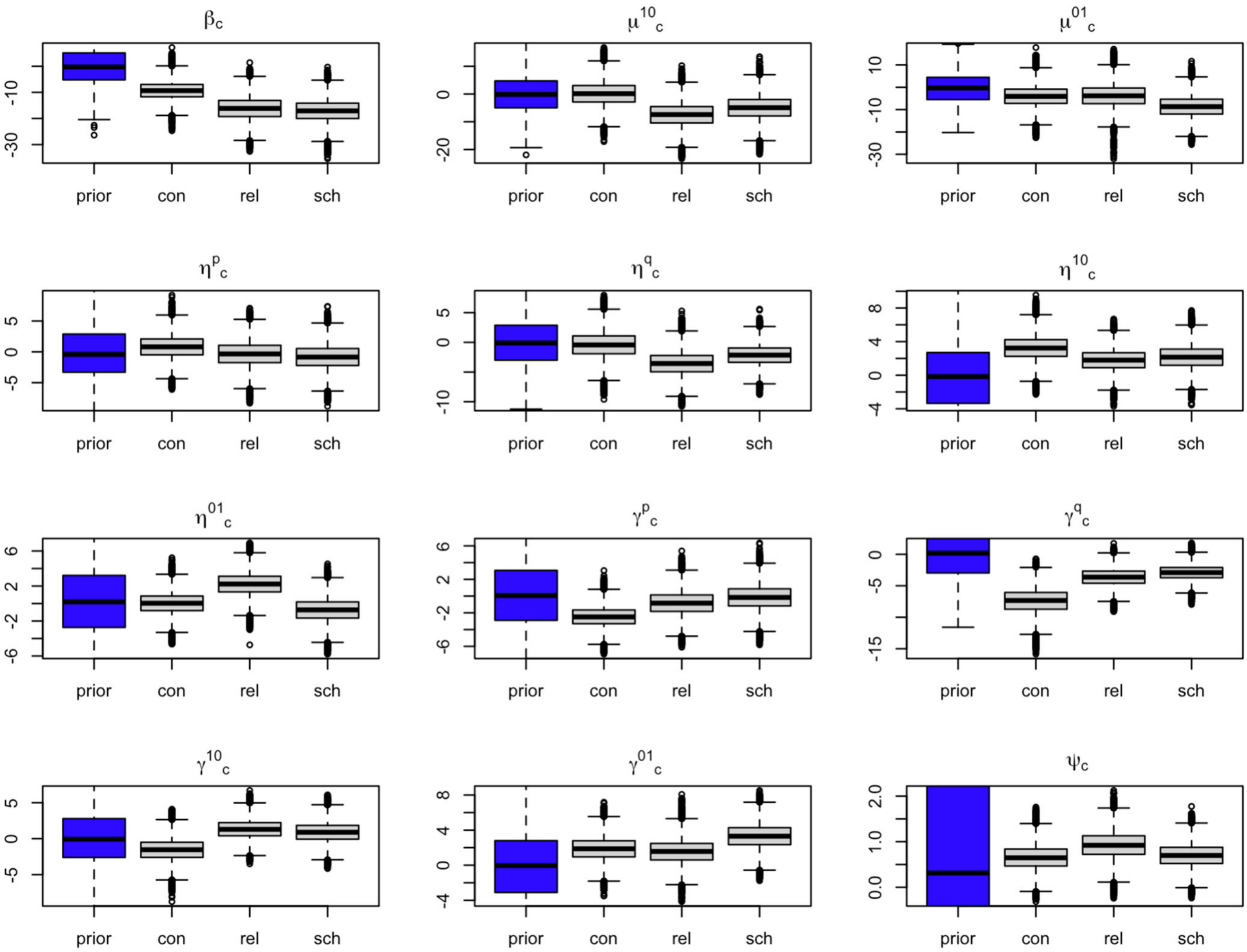

Fig. 11 Box-and-whisker plots of the estimated posterior distributions of all group-level hierarchical Bayesian model parameters. The blue box plots correspond to the model priors, and the gray box plots represent people without schizophrenia ("con"), first-degree relatives ("rel"), and people with schizophrenia ("sch")

200 s because people with and without schizophrenia are almost equally likely to quit after marginalizing over reward level at this point. The estimated posterior probabilities that the group with schizophrenia is more likely to quit than the group without schizophrenia are 0.415, 0.700, and 0.427 for the reward levels of 50, 25, and 10 cents, respectively. This provides evidence against the possibility that reward level can explain the group differences alone, as there is no strong evidence that one group has a higher probability of quitting than the other.

Similarly, for the effect of effort level, we examined the differences between groups with and without schizophrenia at a fixed reward level of 50 cents and a fixed elapsed time of 200 s. The estimated posterior probabilities that the group with schizophrenia is more likely to quit than the group without schizophrenia are 0.566, 0.415, and 0.349 for the effort level of 12, 50, and 100 trials, respectively. For the effect of elapsed time, we examined the differences at a fixed reward level of 50 cents and a fixed effort level of 50 trials. The estimated posterior probabilities that the group with schizophrenia is more likely to quit than the group without schizophrenia are 0.160, 0.415, and 0.633 for the elapsed times of 50, 200, and 800 s, respectively. These results show that the group differences cannot be explained by any single covariate among reward, effort and elapsed time, and they result from the joint effects of these covariates.

We display the interactions between reward, effort, and elapsed time in Figs. 12, 13, 14, 15, and 16. In these figures, we varied each covariate of interest across a range of values, holding the other covariates constant. For each set of covariate values, we obtained estimates of the posterior distributions of the group-wise probabilities of quitting, computed from Eq. (2). We display the posterior distributions of the group-wise probabilities of quitting with box plots, coloring people without schizophrenia, first-degree

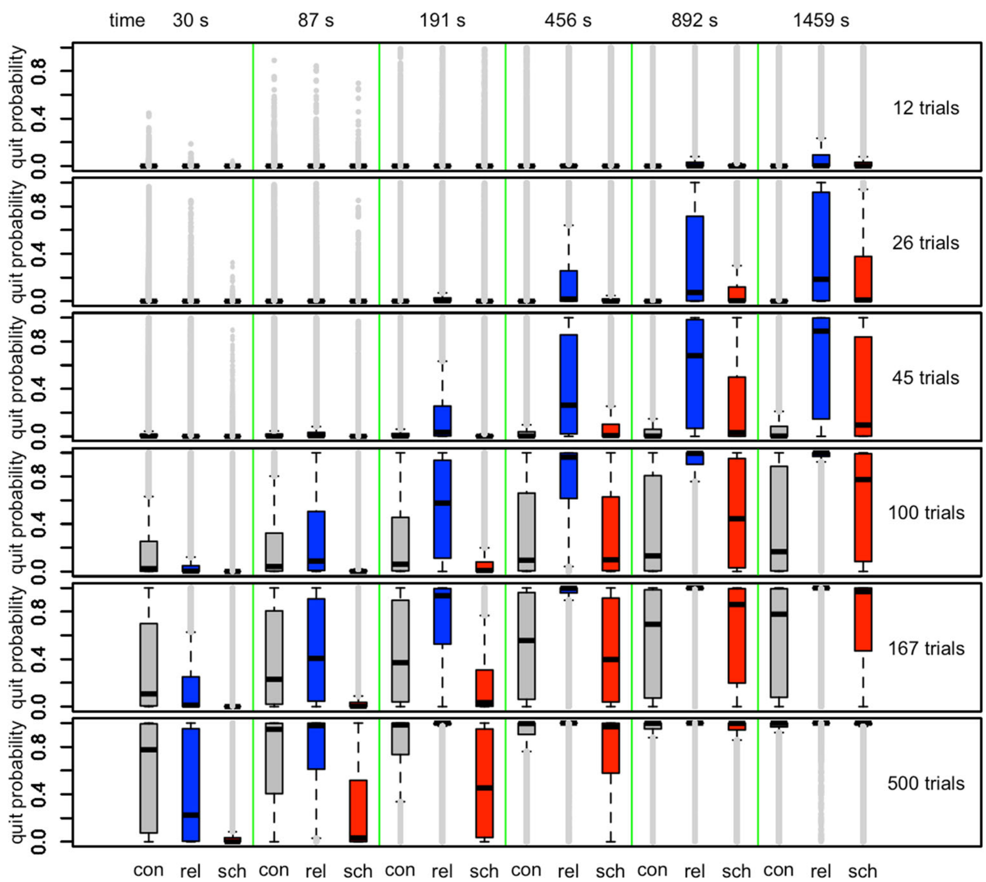

Fig. 12 Box-and-whisker plots of the estimated posterior distributions of the group-level probabilities of quitting across a number of elapsed times and effort values, holding reward level at 50 cents. Labels "con", "rel", and "sch" stand for people without schizophrenia (gray), first-degree relatives (blue) and people with schizophrenia (red). Each column block divided by green lines corresponds to elapsed time which is labeled at the top, and each row block corresponds to the set size which is labeled on the right side

relatives and people with schizophrenia in gray, blue and red, respectively. Because the PRT allows each participant to quit once in each reward level, in these figures, narrow box plots near the probability 0 indicate that most participants may choose to proceed at their corresponding covariate values; wider box plots indicate that most participants may choose to quit around their corresponding covariate values; and narrow box plots near the probability 1 indicate that most participants may have quit the task before reaching their corresponding covariate values.

In Figs. 12, 13, and 14, we held the reward levels to be 50, 25 and 10 cents respectively, and varied elapsed time and effort. These figures show that the estimated posterior distributions of the probabilities of quitting for people without schizophrenia (gray box plots) are more affected by the set size and less affected by elapsed time, and that they are more likely to quit when the effort demand is high regardless of elapsed time. In Figs. 12, 13, and 14, the wider gray box plots appear around the effort levels of 100–167 trials, 50–83 trials, and 20–33 trials respectively, which correspond to the same reward-effort ratios of 2 to 3.3 trials per cent. This pattern indicates that people without schizophrenia are more likely to consider quitting at a concentrated range of reward-effort ratio regardless of elapsed time.

In comparison, people with schizophrenia (red box plots) may base their decisions more on elapsed time. In Fig. 12, the wider red box plots appear at the effort level of 500 trials and the elapsed time of 87 s, which indicates that participants in this group are most likely to consider quitting only when the effort has reached a high level. However, at the elapsed time of 1459 s, the wider red box plots appear around the effort levels of 45–100 trials, which is a large reduction from the previous 500 trials. Similarly, in Fig. 13, the wider red box plots appear around 83–250 trials at the elapsed time of 30 s, but move up to 13 trials at the elapsed time of 1459 s. In Fig. 14, the wider red box plots appear at 100 trials at the elapsed time of 87 s, but move up to 2–5 trials at the elapsed time of 1459 s. These results indicate that people with schizophrenia may be willing to exert large efforts at the start of the experiment, but they might consider quitting at much lower effort levels later into the experiment when the elapsed time is long. This may suggest that the motivation of people with schizophrenia decreases faster as time elapses, and some individuals may quit after a certain period of time regardless of the effort level.

The group of first-degree relatives (blue box plots) is affected by both the set size and elapsed time, and appears to share the quitting mechanisms of both the groups with and without schizophrenia. Similar to the group with

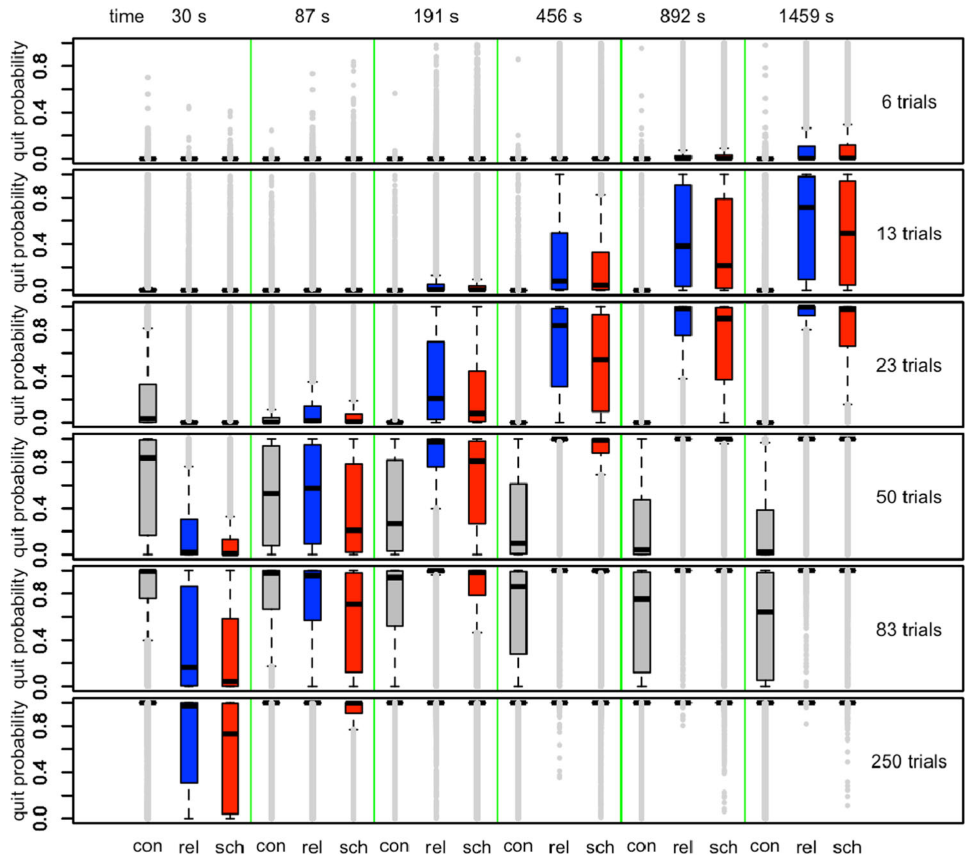

Fig. 13 Box-and-whisker plots of the estimated posterior distributions of the group-level probabilities of quitting across a number of elapsed times and effort values, holding reward level at 25 cents. Labels "con", "rel", and "sch" stand for people without schizophrenia (gray), first-degree relatives (blue) and people with schizophrenia (red). Each column block divided by green lines corresponds to elapsed time which is labeled at the top, and each row block corresponds to the set size which is labeled on the right side

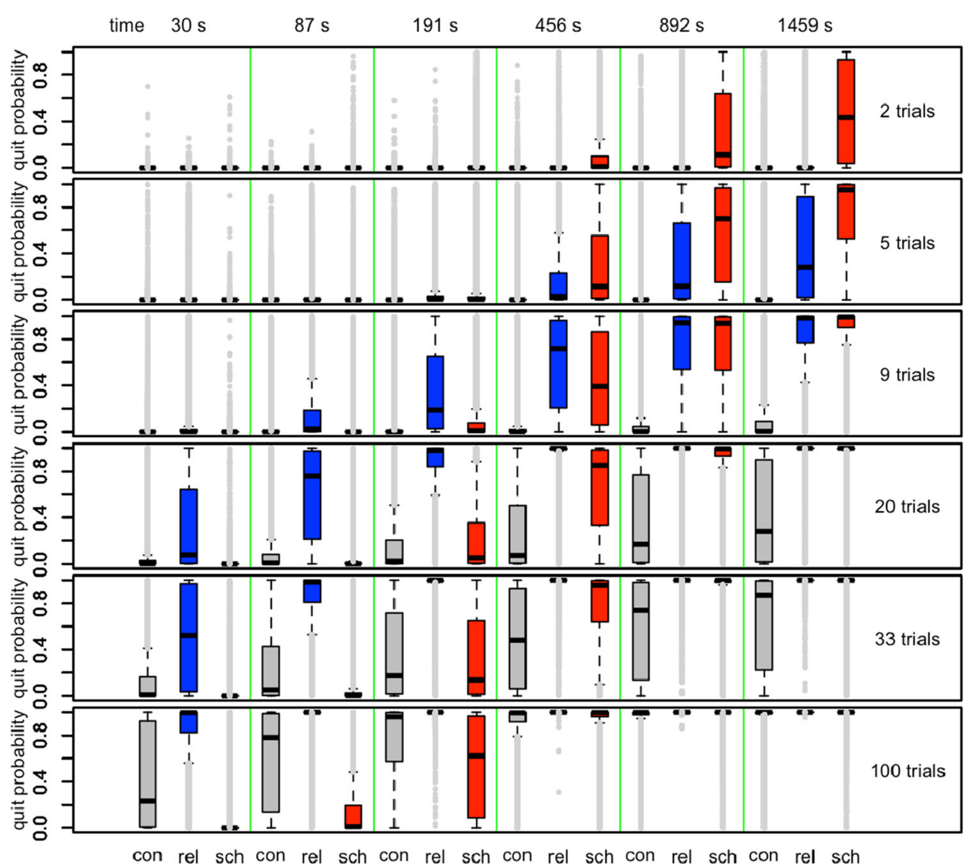

Fig. 14 Box-and-whisker plots of the estimated posterior distributions of the group-level probabilities of quitting across a number of elapsed times and effort values, holding reward level at 10 cents. Labels "con", "rel", and "sch" stand for people without schizophrenia (gray), first-degree relatives (blue) and people with schizophrenia (red). Each column block divided by green lines corresponds to elapsed time which is labeled at the top, and each row block corresponds to the set size which is labeled on the right side

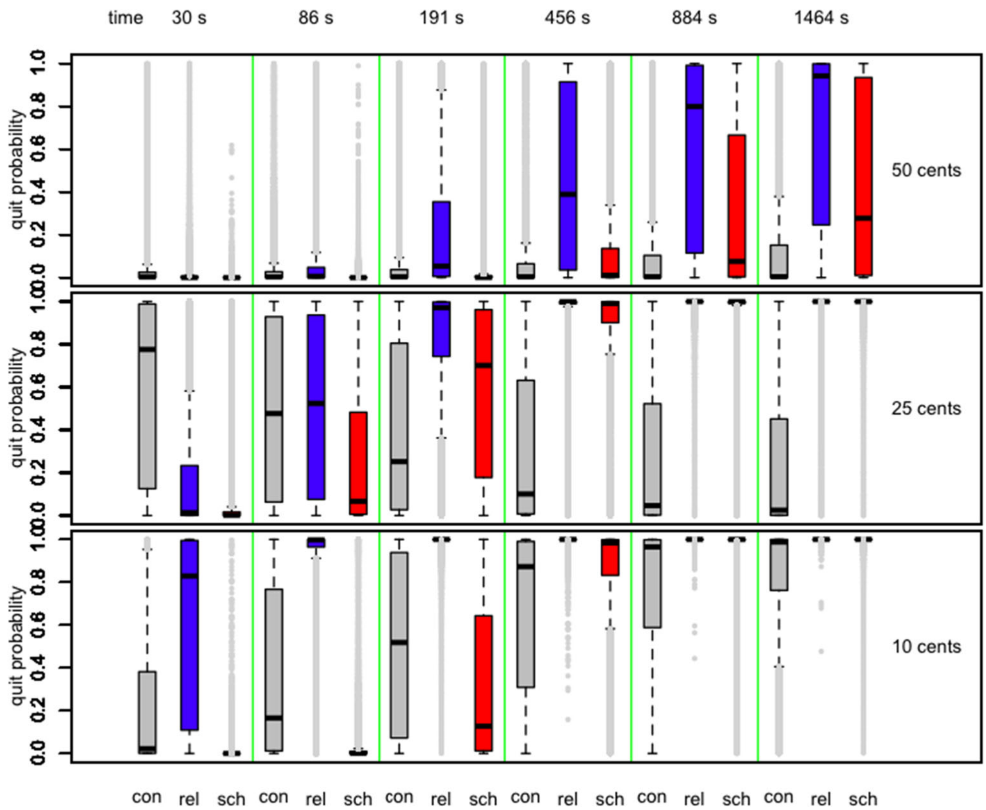

Fig. 15 Box-and-whisker plots of the estimated posterior distributions of the group-level probabilities of quitting across a number of elapsed times and reward values, holding the set size at 50. Labels "con", "rel", and "sch" stand for people without schizophrenia (gray), first-degree relatives (blue) and people with schizophrenia (red). Each column block divided by green lines corresponds to elapsed time which is labeled at the top, and each row block corresponds to the reward value which is labeled on the right side

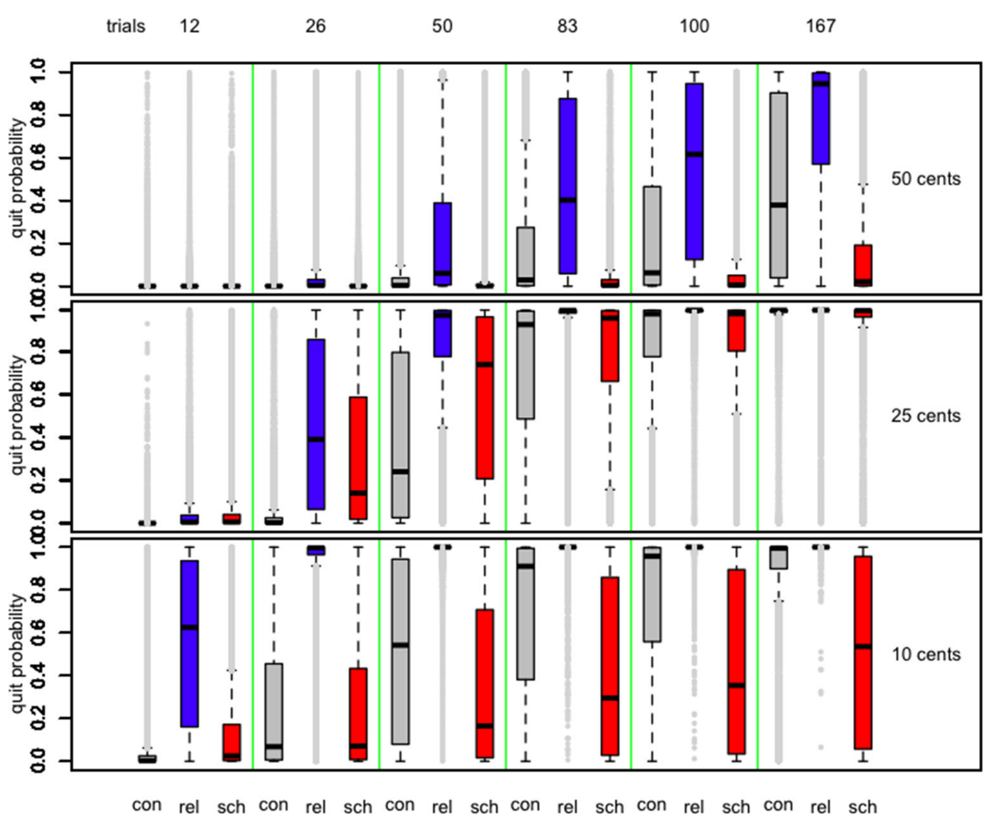

Fig. 16 Box-and-whisker plots of the estimated posterior distributions of the group-level probabilities of quitting across a number of effort values and reward values, holding the elapsed time at 200s. Labels "con", "rel", and "sch" stand for people without schizophrenia (gray), first-degree relatives (blue) and people with schizophrenia (red). Each column block divided by green lines corresponds to the set size which is labeled at the top, and each row block corresponds to the reward value which is labeled on the right side

schizophrenia, the decision to quit of first-degree relatives is largely affected by elapsed time, with the wider blue box plots appearing around the larger effort levels at the elapsed time of 30 s in Figs. 12, 13, and 14, but moving to smaller effort levels at the elapsed time of 1459 s. However, similar to the group without schizophrenia, the decision to quit of first-degree relatives is more dependent on the effort-reward ratio at the same level of elapsed time. For example, the wider blue box plots appear around the effort-reward ratio of 3.33 trials per cent at the elapsed time of 87 s in all three figures, and appear around the effort-reward ratio of 0.5 trials per cent at the elapsed time of 1459 s in all three figures. This effect of the effort-reward ratio is not found in people with schizophrenia.

Figure 15 shows the estimated posterior distributions of the probabilities of quitting when we held the set size at 50 and varied elapsed time and reward values. Figure 16 shows the estimated posterior distributions of the probabilities of quitting when we held the elapsed time at 200s and varied effort values and reward values. These figures do not show evidence for a general lack of motivation in people with schizophrenia because their probabilities of quitting are not necessarily higher than those for other groups. Neither do these plots show definite effects of reward values. The inconsistent effects of reward level may be due to the participants adjusting their response strategy as they gain increasing understanding of the task after going through more sets.

To examine individual differences, we display all box-and-whisker plots of the estimated posterior distributions of the individual-level parameters for participants with schizophrenia in Fig. 17. Results for other participants are shown in the supplemental materials. Figure 17 shows that it is possible to identify participants with divergent response patterns. For example, Participant 82 has a larger η^q_{ic} than most of the other participants. This participant may be more likely to quit in the middle of a set when the effort is large, compared to other participants in this group. To explore this hypothesis, we computed the estimated posterior probabilities of quitting in the middle of a set from

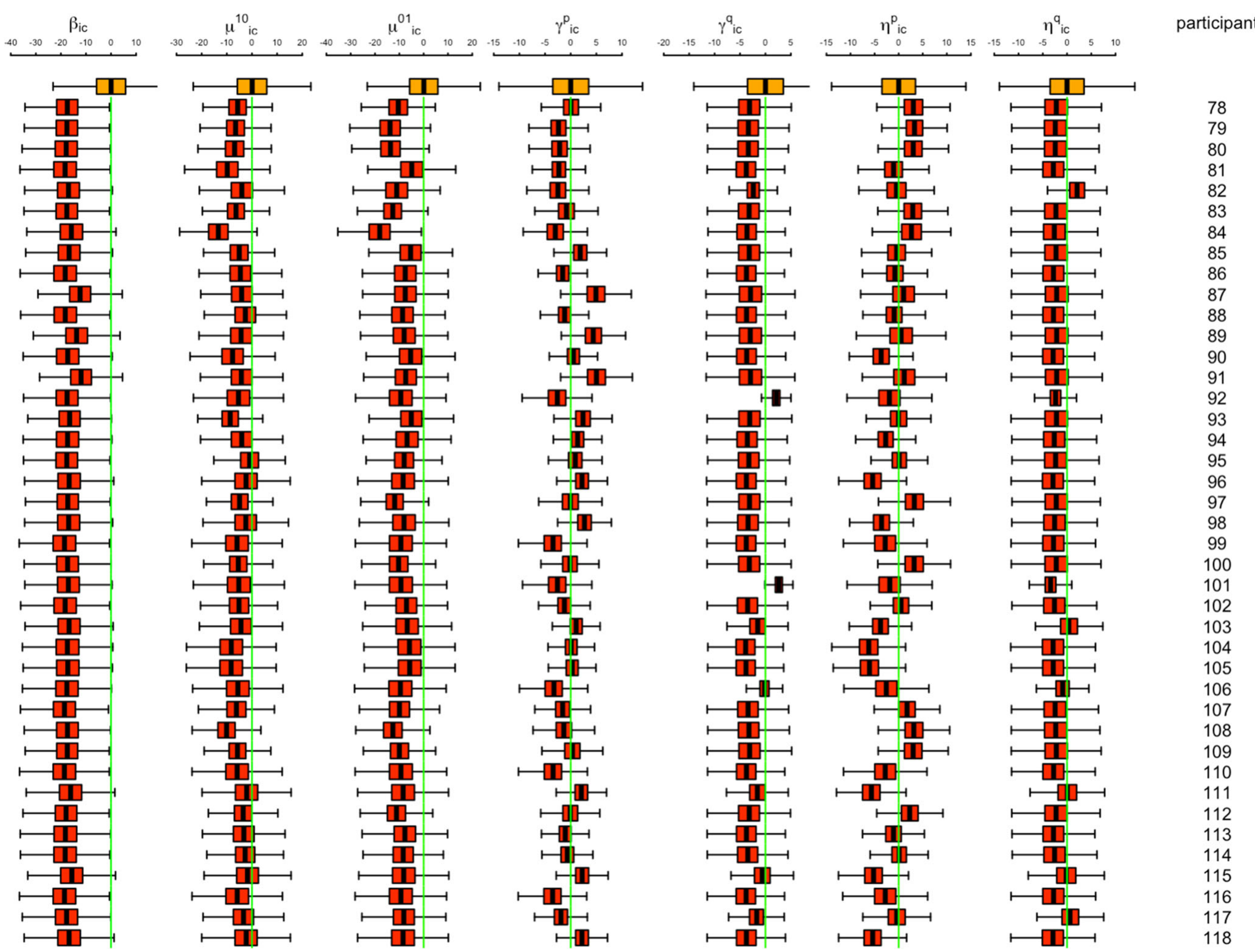

Fig. 17 The box-and-whisker plots of the participants with schizophrenia

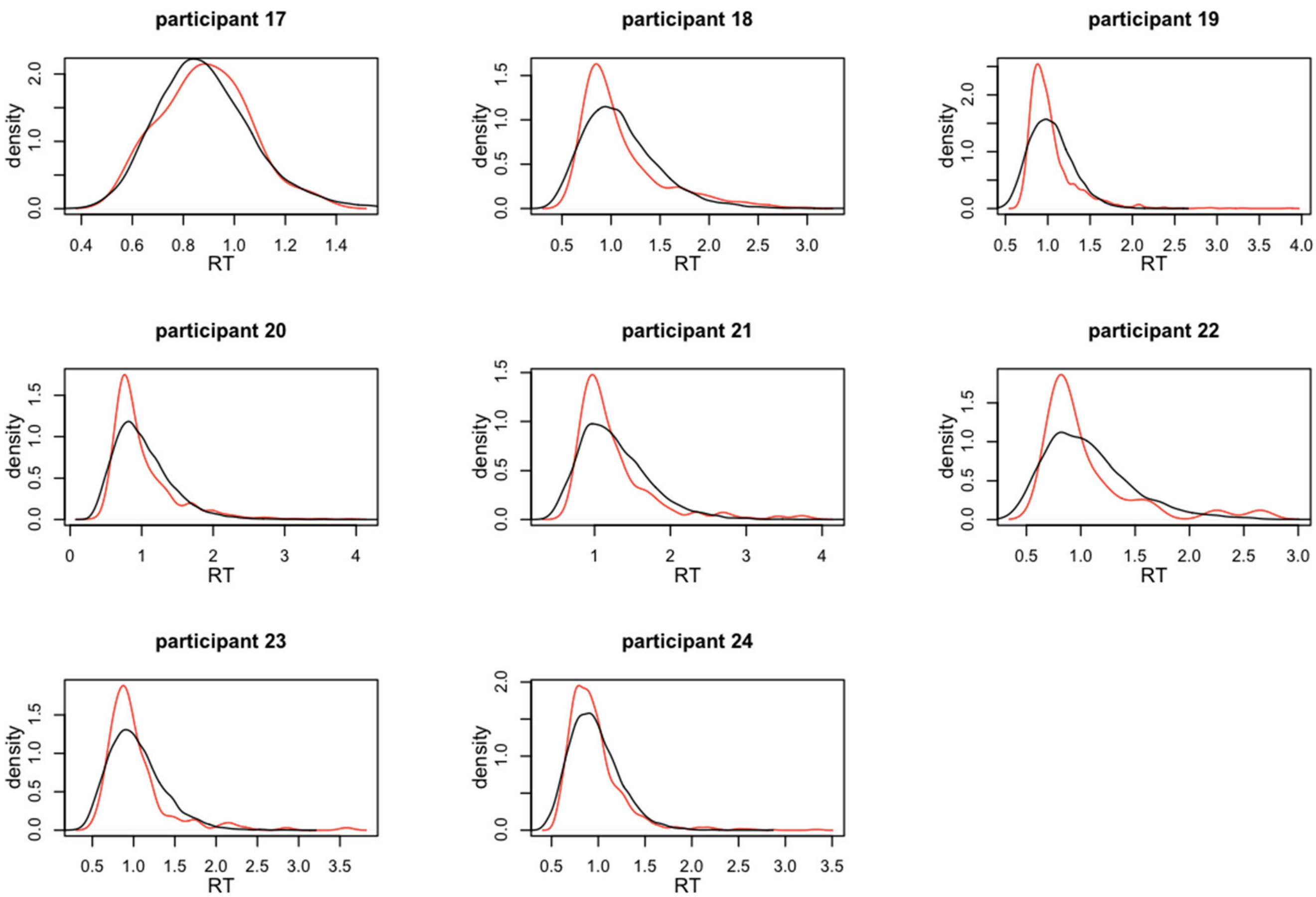

Fig. 18 Estimated posterior predictive densities (black lines) of the RTs together with estimates of the corresponding densities based on the observed RTs (red lines) from Wolf et al. (2014)

Eq. (3) for the group with schizophrenia and Participant 82 in the group in particular. For the event that Participant 82 quits in the middle of a set more than the average of the group, the estimated posterior probabilities are all more that 0.99 for effort levels of 100, 167, and 500 trials, when the reward is fixed at 50 cents and elapsed time is fixed at 500 s. Participant 82 may have a different response strategy from the other participants: enter a set first, and quit the set when the set is perceived to require too much effort for the given amount of reward. Proceeding, in this way, it may be possible to inspect the individual differences in parameter values to identify participants who are using specific strategies.

RT Modeling and Results

In data from Wolf et al. (2014, 2015), people without schizophrenia have an RT mean of 1.15 s (sd 0.51), first-degree relatives have an RT mean of 1.17 s (sd 0.48), and people with schizophrenia have an RT mean of 1.41 s (sd 0.60). To fit the RT model, we eliminated trials with RTs less than 100ms and greater than 4000ms.[6]. We obtained 2 chains for each parameter using *Stan*, each consisting of 1000 burn-in samples and 5000 total iterations. Traceplots and the $\hat{R}$ statistic ($\hat{R}$ <1.05) indicated good convergence.

For the RT model, using the posterior parameters a_{ic}^k and b_{ic}^k, we generated T_{ic}^k via the distribution in Eq. (4). Figure 18 shows the estimated density functions of the observed RTs $t_{ic,n,j}$ for Participants 17–24 together with their estimated posterior predictive densities. The model-based estimated posterior predictive densities, albeit less concentrated, have similar peaks and overall shapes as those of the kernel density estimates, indicating reasonable model fit.

From the estimated posterior distributions of the group-level parameters a_c and b_c, we noted several interesting results. For the decision boundary a_c, the estimated posterior probability that the group without schizophrenia has a

[6] A more desirable approach is to construct a mixture model including the short and long RTs as sub/supra cognitive components (Kim et al., 2017). Because the data lacked detailed two-choice trial information, we were compelled to use this simpler approach.

higher decision boundary than the group with schizophrenia is 0.488. Thus people with schizophrenia have much the same average decision boundary as people without schizophrenia. For the drift rate b_c, the estimated posterior probability that drift rate for the group without schizophrenia is higher than that of the group with schizophrenia is 0.734. Similarly, the group of first-degree relatives also has a higher group-level drift rate than that of the group with schizophrenia with an estimated posterior probability of 0.612. First-degree relatives have a lower drift rate b_c than people without schizophrenia with an estimated posterior probability of 0.640. These results may indicate that people with schizophrenia are slower to accumulate information from stimuli, resulting in a smaller drift rate and longer RTs. First-degree relatives are also slower than people without schizophrenia.

Discussion and Conclusions

In this study, we constructed a hierarchical Bayesian model for PRT data that provides reasonable parameter recovery results and good fit to a PRT data set from Wolf et al. (2014). When fit to empirical data, the model is able to reveal differences in the ways that experimental covariates influence participants' decisions during PRT experiments. Better understanding of these differences could be helpful in analyzing PRT studies and related psychological practices. Here we discuss the potential implications of our modeling results and their associations with past studies in the literature, the possibility of extending the hierarchical Bayesian model to other PRT schedules and their corresponding data sets, and possible future work to improve the Bayesian model.

Discussion

Our results may offer some insights about how motivation might be affected among individuals with schizophrenia, and they could be helpful to improve the efficacy of treatment provided to these individuals. The performance of individuals with schizophrenia appear to be more affected by elapsed time, as they are more likely to quit after spending more time and effort in the task and are less affected by reward and effort levels, while the decision of people without schizophrenia are more affected by reward and effort levels. This phenomenon could be influenced by multiple factors. One possible factor might be defective time perception in people with schizophrenia. Multiple studies have suggested that people with schizophrenia are prone to overestimation of time duration (e.g., Bonnot et al., 2011; Ortu no et al., 2011; Ciullo et al., 2016), and this overestimation might result in a faster diminishing motivation and contribute to their unwillingness to keep exerting effort after some time into the experiment. Another possible mechanism is that people with schizophrenia may suffer from impaired proactive cognitive control (Lesh et al., 2013; Ryman et al. 2018), which harms their abilities to manage their actions based on the reward-effort ratio and elapsed time. Proactive control is used to maintain goal-relevant information and keep actions goal-oriented (Ryman et al. 2018). With effective proactive control, people without schizophrenia are able to maintain a schedule that balances their effort with the average reward per time unit, whereby they exert effort when the average reward is high and quit when it is low (Otto & Daw, 2019; Guitart-Masip et al., 2011). In comparison, people with schizophrenia may rely more on reactive control due to impairments in proactive control (Mann et al., 2013). Reactive control is more transient and stimulus-driven (Braver, 2012), thus people with schizophrenia may be less able to respond based on a goal sustained across a task. They may tend to quit when they have exerted the amount of effort that they are willing to contribute, regardless of whether it maximized the reward or not. As a result, people with schizophrenia are more likely to quit within a concentrated time period corresponding to the amount of effort that they are motivated to exert.

Set size affects the performance of people with schizophrenia less than that of people in other groups. This suggests that individuals with schizophrenia might have impairments in the perception of effort and personal cost, and might be unable to optimize the gain based on the effort-reward ratio from the PRT as efficiently as other groups. This finding is consistent with findings that suggested that the lack of motivation could be related to dysfunctional integration of effort and reward in people with schizophrenia, whereby they might suffer from abnormal estimations of effort and have difficulties in maintaining an effort-reward balance (e.g., Hartmann et al., 2015; Gold et al., 2013; Fervaha et al., 2013; Barch et al., 2014).

The effect of reward value does not show consistent differences across groups. Breitborde et al. (2019) suggested that the level of extrinsic motivation in people with schizophrenia is statistically no different than that in those without schizophrenia, so the lack of consistent effects of reward value may be expected. However, due to the small number of reward values, further investigation is needed to reach more reliable conclusions.

The first-degree relatives show intermediate effects of elapsed time, effort and reward, which may align with findings by Snitz et al. (2006) that cognitive deficits are also present in first-degree relatives of people with schizophrenia, but not as severe. The larger influence of elapsed time is shared by the first-degree relatives and the individuals with schizophrenia, indicating that they may share a potential endophenotype for schizophrenia — a heritable factor that may be more clearly associated with specific, underlying genetic/biological factors than the broader diagnostic phenotype (Kendler & Neale, 2010). This endophenotype may concern defective time perception or impaired proactive control to maintain a consistent goal across the experiment. Presence of the larger influence of elapsed time in first-degree relatives also indicates that the deficits shown in people with schizophrenia are unlikely a result of antipsychotic medication, as first-degree relatives do not receive such medication.

Possible extension to other PRT studies

To apply the model to other PRT schedules used for human studies, we discuss three differences between these PRT schedules and that of Wolf et al. (2014) and the role of possible covariates on people's decision processes.

The first possible difference between other published PRT schedules and that of Wolf et al. (2014) is the number of breakpoints generated. While Wolf et al. (2014) used 3 strings of increasing sets with different reward values that generates 3 breakpoints, most other studies only have one reward value over all sets of trials, generating only one breakpoint per participant. For example, Chelonis et al. (2011) used a string of 30 sets of trials where Set n required $1 + 10n$ responses to earn a nickle; Goldstone et al. (2016) used 20 sets of trials where Set n required $10 \times 2^{n-1}$ responses to earn a sweet. Because there is only one reward value, removing the covariate $V_{ic,n}$ from the Bayesian model in Eq. (2) results in the reduced model

$$Z_{ic,n} = \mu_{ic} + \eta_{ic}\hat{M}_{ic,n} + \gamma_{ic}\hat{T}_{ic,n} + \psi_c \hat{M}_{ic,n}\hat{T}_{ic,n}.$$

We note also that if the reward levels in a PRT can be regarded as continuous, then the dummy variables $\hat{V}^{10}_{ic,n}$, $\hat{V}^{01}_{ic,n} \in \{0, 1\}$ in Eq. (2) can be replaced with a continuous variable $V_{ic,n}$. This approach can be applied to a data set with a larger sequence of reward levels, where a pattern of reward effect can be found. For example, if the reward effect is linear, the model can be specified as

$$\begin{aligned} Z_{ic,n} = &\ \mu_{ic} + \mu'_{ic}V_{ic,n} \\ &+\eta_{ic}\hat{M}_{ic,n} + \eta'_c V_{ic,n}\hat{M}_{ic,n} \\ &+\gamma_{ic}\hat{T}_{ic,n} + \gamma'_c V_{ic,n}\hat{T}_{ic,n} \\ &+\psi_c \hat{M}_{ic,n}\hat{T}_{ic,n}. \end{aligned}$$

The second possible difference in task designs is the type of responses to be made. Wolf et al. (2014) used a two-choice task that requires a relatively high level of attention, while many other studies requires much simpler responses. Chelonis et al. (2011) asked participants to press a lever until the number of lever presses reaches a required threshold. Goldstone et al. (2016) asked participants to simply click a mouse. Bismark et al. (2018) instructed participants to rotate a joystick in a specific direction. Because our hierarchical Bayesian model did not take into account the nature of the responses but only their RTs, these differences should not affect the applicability of the model.

The third difference is the way the PRT ends. In Wolf et al. (2014), the task terminates when participants have actively quit in the last reward level or have finished all trials in the last level. Other task designs terminate the experiment after a certain amount of time. For example, Chelonis et al. (2011) terminated the task when it has lasted for 10 min. This kind of task ending would not require any changes to the model we present here. It would be interesting to investigate this kind of task in our framework because passively ceasing should differ from actively quitting in a cognitive sense.

Many studies do not measure or record the RT covariates $t_{ic,n}$ thus have no exact elapsed time for each set. When the experiment run time is available, it may be reasonable to approximate the elapsed time $T_{ic,n}$ by the number of responses made before the set divided by the number of responses made in the full span of the experiment times the experiment run time.

In summary, with some slight modifications, the hierarchical Bayesian model is likely to be extensible to the majority of PRT data sets.

Future directions

There are a number of directions in which this work can be extended. The first regards the extension to other data sets. The data examined in this study did not balance socioeconomic status and education, and we could extend our analyses to cases with balanced socioeconomic status and education to eliminate these potential confounds. The two-choice trials used in the paradigm from Wolf et al. (2014) do not induce response strategies and bias, and we might consider extending the task to induce response bias to investigate the response patterns of people with schizophrenia in this scenario. Because motivation can affect performance in other cognitive tests (Fervaha et al., 2014), we could use the PRT in a cognitive battery with other tests, and assess how PRT parameters link with other cognitive functions, as well as individual characteristics such as IQ and symptoms.

The second direction is to further refine the model. Although our hierarchical Bayesian model could explain some features of participant behavior in PRT studies, its geometric regression structure is relatively arbitrary. The model offers limited theoretical implications of the participants' cognitive processes, and therefore it is rather difficult to draw conclusions about differences in information processing mechanisms across the three groups in the experiment we examined.

One obvious refinement of the model involves the RT data $t_{ic,n}$ and initialization times $S_{ic,n}$. For now we simply use these measurements to obtain the elapsed time covariate $T_{ic,n}$ in the model, and fit a model to them separately, but they are derived from meaningful underlying structures that contribute to participants' performance. If more detailed trial-by-trial information is available in future studies, the RT model can be improved with a more sophisticated structure such as the diffusion decision model (Ratcliff & McKoon, 2008) and the linear ballistic accumulation model (Brown & Heathcote, 2008). The initialization time $S_{ic,n}$ contains behavior-related information because some participants may spend more time at the start screen to rest. We may build a theoretically motivated mechanism for $S_{ic,n}$ in the model to explain behavior at the beginning of sets. For example, Bradshaw and Killeen (2012) proposed that the initialization time reflects a pause after exerting previous efforts, thus its length should be proportional to the length of the previous session. Denoting the pause at the start of session j as $S_{p,j}$, they suggested the deterministic relationship

$$S_{p,j} = S_0 + kT_{TOT,j-1}, \quad j > 1,$$

where S_0 is the initial pause and $T_{TOT,j-1}$ is the total time spent on the previous session $j-1$. Such a relationship could be incorporated into our model structure.

A third possible direction is to incorporate an effort-discounting structure from existing effort-discounting models (e.g., Klein-Flügge et al., 2015) into our hierarchical Bayesian model. Effort discounting structures are used to model the phenomenon where individuals experience a subjective discount of reward values when required effort increases (Botvinick et al., 2009), which might fit the patterns of reward-effort integration shown in PRT. However, we may need to collect responses from more reward levels to allow the use and evaluation of a discounting structure.

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Availability of data and material Not applicable

Code availability Custom code of the hierarchical Bayesian mo del and supplemental materials are available at https://github.com/Van-Zandt-Lab-at-OSU.

Declarations

Declarations

Conflict of Interest The authors declare no competing interests.

Appendix: Model comparison

We conducted model comparisons to justify the choice to leave out effects of the reward level $V_{ic,n}$ in Eq. (3) for the probability to quit within initialized sets, and to justify having individual levels in the intercept and main effect parameters.

We compared the model stated in Eqs. (2) and 3, and the following model which contains effects of the reward level,

$$\begin{aligned} p_{ic,n} &= \frac{1}{1+\exp(-Z^p_{ic,n})}, \\ Z^p_{ic,n} &= \mu_{ic} + \mu^{10}_{ic}\hat{V}^{10}_{ic,n} + \mu^{01}_{ic}\hat{V}^{01}_{ic,n} \\ &+\eta^p_{ic}\hat{M}_{ic,n} + \eta^{10}_c\hat{V}^{10}_{ic,n}\hat{M}_{ic,n} + \eta^{01}_c\hat{V}^{01}_{ic,n}\hat{M}_{ic,n} \\ &+\gamma^p_{ic}\hat{t}_{ic,n} + \gamma^{10}_c\hat{V}^{10}_{ic,n}\hat{t}_{ic,n} + \gamma^{01}_c\hat{V}^{01}_{ic,n}\hat{t}_{ic,n} \\ &+\psi_c\hat{M}_{ic,n}\hat{t}_{ic,n}, \end{aligned} \tag{6}$$

and

$$\begin{aligned} q_{ic,n} &= \frac{1}{1+\exp(-Z^q_{ic,n})}, \\ Z^q_{ic,n} &= \mu_{ic} + \mu^{10}_{ic}\hat{V}^{10}_{ic,n} + \mu^{01}_{ic}\hat{V}^{01}_{ic,n} \\ &+\eta^q_{ic}\hat{M}_{ic,n} + \eta^{10}_c\hat{V}^{10}_{ic,n}\hat{M}_{ic,n} + \eta^{01}_c\hat{V}^{01}_{ic,n}\hat{M}_{ic,n} \\ &+\gamma^q_{ic}\hat{t}_{ic,n} + \gamma^{10}_c\hat{V}^{10}_{ic,n}\hat{t}_{ic,n} + \gamma^{01}_c\hat{V}^{01}_{ic,n}\hat{t}_{ic,n} \\ &+\psi_c\hat{M}_{ic,n}\hat{t}_{ic,n}, \end{aligned} \tag{7}$$

We compared their model fit by way of the deviance information criterion (DIC), using both Spiegelhalter et al.'s (2002) and Gelman et al.'s (2013) methods to compute the effective number of parameters. A smaller DIC indicates relatively better fit.

We obtained a chain for each model containing 5000 burn-in samples and 30000 total iterations. To avoid autocorrelations, we thinned the chain by keeping every 6th iteration, resulting in 6000 samples from the posteriors to compute the model comparison statistics. The hierarchical Bayesian model from Eqs. (2) and 3 has a DIC of 615.28 according to Spiegelhalter et al.'s (2002) method, and 797.06 according to Gelman et al.'s (2013) method. The alternative model from Eqs. (6) and 7 has a DIC of 619.24 according to Spiegelhalter et al.'s (2002) method, and 838.61 according to Gelman et al.'s (2013) method. The log Bayes factor of the hierarchical Bayesian model (Eqs. (2) and (3)) over the alternative model is 19.76, according to the generalized harmonic mean estimator method (Gronau et al., 2017). Both DIC and the Bayes factor indicate a better fit of the hierarchical Bayesian model, which justifies

Table 2 Model comparison statistics

Statistic	Full	$-\mu_{ic}$	$-\mu_{ic}^{10}$	$-\mu_{ic}^{01}$	$-\eta_{ic}^{p}$	$-\gamma_i^{p}c$
DIC (Spiegelhalter)	615.28	628.56	663.37	665.85	760.6	728.74
DIC (Gelman)	797.06	834.17	815.22	850.78	1013.2	948.62
log Bayes factor	–	29.39	32.41	59.44	103.44	64.86

The "log Bayes factor" row are log Bayes factor of the full model over corresponding models in the column

leaving out the reward level effects for the probability to quit within a set.

To justify the inclusion of individual-level parameters in Eq. (2), we compared the hierarchical Bayesian model from Eqs. (2) and 3 (denoted as "full") and truncated models excluding each one of the individual-level parameters. We generated 5000 burn-in samples and 30000 total iterations for each model, and kept every 6th iteration to compute the model statistics. Table 2 shows the DICs and log Bayes factors: all model comparison statistics favor the full model.

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