Proton Radius: A Puzzle or a Solution!?

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Abstract. The proton radius puzzle is known as the discrepancy of the proton radius, obtained from muonic hydrogen spectroscopy (obtained as being roughly equal to 0.84 fm), and the proton radius obtained from (ordinary) hydrogen spectroscopy where a number of measurements involving highly excited states have traditionally favored a value of about 0.88 fm. Recently, a number of measurements of hydrogen transitions by the Munich (Garching) groups (notably, several hyperfine-resolved sublevels of the $2S$–$4P$) and by the group at the University of Toronto ($2S$–$2P_{1/2}$) have led to transition frequency data consistent with the smaller proton radius of about 0.84 fm. A recent measurement of the $2S$–$8D$ transition by a group at Colorado State University leads to a proton radius of about 0.86 fm, in between the two aforementioned results. The current situation points to a possible, purely experimental, resolution of the proton radius puzzle. However, a closer look at the situation reveals that the situation may be somewhat less clear, raising the question of whether or not the proton radius puzzle has been conclusively solved, and opening up interesting experimental possiblities at TRIUMF/ARIEL.

1. Introduction

We present a brief account of the current status of measurements of the proton radius, based on recent hydrogen spectroscopy [1–8] and scattering experiments [9, 10]. Also, we point out that a current discrepancy of the deuteron radius, based on deuterium spectroscopy and scattering experiments, merits further investigation [11]. Attention is drawn to the fact that in 1969, coindicidentally on the same day on which mankind set foot on the moon (21st of July, 1969), two Letters were published [12, 13], which report on an observation of a larger slope of the Sachs form factor of the proton, when measured using electron versus muon scattering, consistent with a (roughly) 2% difference in the proton radii derived from either scattering method. The current situation opens interesting experimental possibilities for the MUSE experiment [14–16] at the Paul–Scherrer Institute (PSI) and for a potential muon option at the Advanced Rare Isotope Laboratory (ARIEL) at TRIUMF (Vancouver).

2. Brookhaven Experiment

It might be useful to recall that scattering experiments on proton targets, using either electron or proton projectiles, have a considerable history. One example dates from about five decades ago [12, 13]. It was observed that muon-proton (μ-p) versus electron-proton (e-p) scattering data for the Sachs form factor taken in the range $0.15\,(\mathrm{GeV}/c)^2 < q^2 < 0.85\,(\mathrm{GeV}/c)^2$ differ from each other by a relative factor $N = 0.960 \pm 0.006$. It is stated, in the right text column on p. 154

of Ref. [12], that the "4% suppression of the form-factor ratio represents an 8% difference in the cross section since $\mathrm{d}\sigma/\mathrm{d}q^2 \propto G^2$".

One should note that the authors of Ref. [13] base their analysis on a number of assumptions. Let us consider the Rosenbluth formula [see Eq. (1) of Ref. [12]],

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}q^2}\right|_{\mu,e} \left.\frac{\mathrm{d}\sigma}{\mathrm{d}q^2}\right|_{\mathrm{NS}} \frac{1}{\cot^2(\theta/2)} \left[2\tau G_M(q^2) + \frac{G_E^2(q^2) + \tau\, G_M(q^2)}{1+\tau} \cot^2(\theta/2)\right] , \tag{1}$$

where $\tau = -q^2/(4m_p^2) = \vec{q}^2/(4m_p^2)$ (for purely space-like momentum transfer), θ is the scattering angle, and $\mathrm{d}\sigma/\mathrm{d}q^2\big|_{\mathrm{NS}}$ is the differential cross section for spinless particles. The proton radius is inferred in Ref. [13] as follows [see also Eq. (11) of Ref. [17]],

$$r_p = \sqrt{\langle r^2 \rangle_p}\,, \qquad \langle r^2 \rangle_p = r_p^2 = 6\hbar^2 \left.\frac{\partial G_E(q^2)}{\partial q^2}\right|_{q^2=0} \tag{2}$$

The experiment [12, 13] has been carried out in a region of small scattering angle θ, where the second (slope) term in square brackets in Eq. (1) dominates the first (intercept), and the electric and magnetic form factors cannot be determined separately. The authors of of Refs. [13] base their analysis on the assumption that $G_E = G_M/\mu \equiv G$, where $\mu \approx 1$ is the ratio of the magnetic and electric form factors of the proton. This assumption is rather well justified and is in fact confirmed by more recent experiments [18].

In the abstract of Ref. [13], it is stated that the "apparent disagreement [of the μ-p versus e-p data] can possibly be accounted for by a combination of systematic normalization errors". This statement highlights a possible problem in the experiment [12, 13]: Namely, it is experimentally difficult to accurately normalize the flux of incoming particles, which could in principle account for the tentative muon-proton nonuniversality observed in Refs. [12,13]. Yet, as evident from the rather detailed analysis present in the two Letters [12, 13], it is evident that considerable efforts were made by the authors of Refs. [12,13] to avoid such normalization errors. In fact, in Ref. [12], the scattering data were checked for the possible presence of inelastic and multiple-scattering contributions, and also, the validity of the functional form (1) was verified against experimental data, for both muon as well as anti-muon projectiles. Let us also record an observation: The differential cross section is roughly proportional to the square of the Sachs electric G_E form factor (under the assumptions made above), and the slope of the Sachs form factor is again proportional to the square of the proton radius. So, the observed (roughly) 8% difference of the μ-p scattering cross sections [12, 13] translates into a 4% difference of the form factor, which translates into a 2% difference of the proton radii derived from μ-p versus e-p scattering. Tentatively, one could argue that, if the trend seen in the 1969 experiment were to be exclusively due to a normalization error, the relative systematic error would have to be on the order of 8% for the cross section, which is quite large.

Coincidentally, this is exactly the difference (both sign and magnitude agree) between the proton radii derived from the most recent $2S$–$8D$ measurement on atomic hydrogen [8] and the result from muonic hydrogen spectroscopy [4]. This situation gives a very pronounced motivation for the MuSE experiment [14–16], and also for adding a possible muon option to TRIUMF/ARIEL.

3. Brief Review of Hydrogen Experiments

In contrast to scattering experiments, the proton radius is derived from spectroscopic experiments via the finite-size energy shift, which, in leading order and applying the nonrelativistic approximation, reads as

$$\Delta E = \frac{2}{3} \frac{(Z\alpha)^4\, \mu c^2}{\pi\, n^3}\, \delta_{\ell 0} \left(\frac{\mu\, c\, r_p}{\hbar}\right)^2 , \tag{3}$$

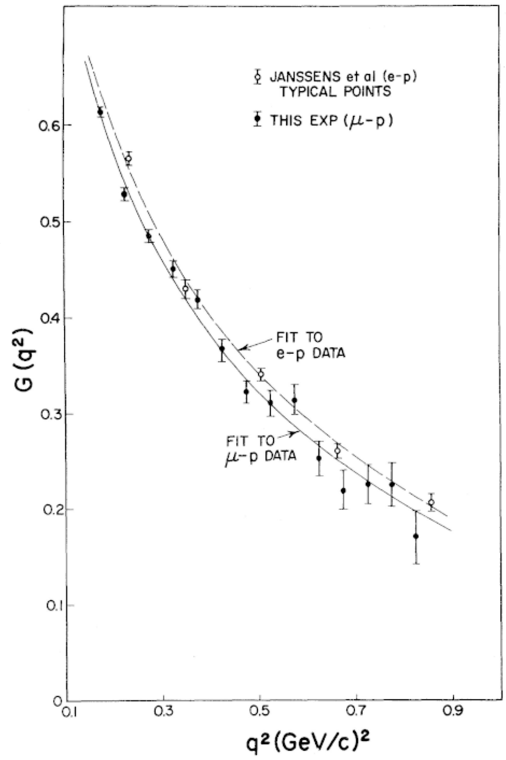

Figure 1. [Reproduced with permission.] Cross section data from Refs. [12, 13].

where Z is the nuclear charge number of the hydrogenlike ion, α is the fine-structure constant, μ is the reduced mass, c is the speed of light, and ℓ is the orbital angular momentum. The Kronecker δ implies that the energy correction is nonvanishing only for S states with $\ell = 0$. The relation (3) is only the leading approximation to the finite-size effect. Higher-order terms involve convoluted moments of the nuclear-charge distribution, as discussed in Chap. 5 of Ref. [19] and in great detail in Ref. [20].

Let us discuss Eq. (3). We have $Z = 1$ for both hydrogen and muonic hydrogen. The reduced mass μ is roughly 200 times larger for muonic bound systems as compared to ordinary hydrogen. The finite-size effect is proportional to μ^3, and thus, muonic hydrogen energy levels constitute a very sensitive probe of the proton radius.

In chronological order, let us discuss some recent results for the proton radius from spectroscopic [1–8], as well as scattering, experiments [9, 10]. We first recall that a least-squares fit of all experimental data available up to 2005 for both hydrogen as well as deuterium led to the results $r_p = 0.8750(68)\,\mathrm{fm} \approx 0.88\,\mathrm{fm}$ and $r_d = 2.1394(28)\,\mathrm{fm}$, based on a total of twenty-three measured transitions, including transitions to highly excited states [3].

In 2010, the muonic hydrogen $2S$–$2P_{1/2}$ Lamb shift measurement at PSI (Ref. [4]) led to a proton radius of $r_p = 0.84184(67)\,\mathrm{fm} \approx 0.84\,\mathrm{fm}$. The result was in disagreement with the results of an electron scattering experiment carried out in Mainz (in 2010, see Ref. [9]), which led to the result of $r_p = 0.879(8)\,\mathrm{fm} \approx 0.88\,\mathrm{fm}$, consistent with the value obtained from atomic hydrogen spectroscopys [3].

Since 2010, the discrepancy between the proton radius obtained from hydrogen spectroscopy ($r_p \approx 0.88\,\mathrm{fm}$) and from muonic hydrogen spectroscopy ($r_p \approx 0.84\,\mathrm{fm}$) has been termed the proton radius puzzle. In 2017, a measurement of the $2S$–$4P$ transition in atomic hydrogen (Ref. [6], taking into account various fine-structure and hyperfine sublevels) led to a result of

$r_p = 0.8335(95)$ fm, consistent with the smaller proton radius, otherwise obtained from muonic hydrogen spectroscopy. In the analysis of the experiment [6], emphasis is laid on so-called cross-damping terms, which had been analyzed (for differential cross sections) in Ref. [21].

A recent hydrogen measurement (in 2018, by the French group) of the $1S$–$3S$ transition [22] is consistent with the larger proton radius, with a result of $r_p = 0.877(13)\,\text{fm} \approx 0.88\,\text{fm}$. In Ref. [5], it had been pointed out that the experimental approach taken by the Paris group should be largely independent of cross-damping terms.

The PRad Scattering Experiment, in 2019, obtained a value of $r_p = 0.831(14)$ fm, as reported in Ref. [10], based on scattering data. In 2019, a spectroscopic measurement in Toronto, of the $2S$–$2P_{1/2}$ Lamb shift in atomic hdrogen, resulted in a proton radius of $r_p = 0.833(10)$ fm [7], consistent with the smaller value of the proton radius. One might point that the $2P_{1/2}$–$2P_{3/2}$ fine-structure is nearly independent of the proton radius and can be calculated to very high precision [23]; its measurement would constitute an important consistency check for the smallness of the "Canadian protons" measured in Ref. [10]. (Of course, the Einstein equivalence principle (EEP) implies that the outcome of an experiment should be independent of where and when in the Universe it is performed. Assuming that protons are indistinguishable, identical elementary particles, one might be tempted to include the tongue-in-the-cheek remark that any conceivable size difference in "French versus Canadian and German Protons" would violate the EEP.)

In Ref. [5], it had been pointed out that the so-called cross-damping terms, which had played an important role in the analysis of the $2S$–$4P$ experiment [6], should be suppressed in $2S$–nD transitions in atomic hydrogen [see the discussion surrounding Eqs. (28)—(31) of Ref. [5]]. It had also been pointed out in Ref. [5] that cross-damping terms are "not likely to shift the experimental results reported in Refs. [1] and [2] on a level commensurate with the proton radius puzzle energy shift." We recall that Refs. [1] and [2] report on measurements of $2S$–nD transitions with $n = 8$ and $n = 12$. In consequence, it is of significance that a very recent (2022) measurement of the $2S_{1/2}$–$8D_{5/2}$ transition in atomic hydrogen (see Ref. [8]) has led to a proton radius of $r_p = 0.8584(51)\,\text{fm} \approx 0.86\,\text{fm}$, which lies in between the previously accepted value of $r_p \approx 0.88$ fm from atomic hydrogen spectroscopy [3] and the value of $r_p \approx 0.84$ fm derived from muonic hydrogen spectroscopy [4].

The proton radius puzzle is exacerbated by the fact that the raw data for the Sachs form factor derived from the 2010 Mainz experiment [9] and the 2019 PRad data [10] are discrepant [24] (see also [25]). Clearly, further investigations are indicated.

4. Conclusions

In view of interesting possibilities with muon physics, one might ask if one should add a muonic beam to TRIUMF/ARIEL. One idea for low-cost muon sources can be mentioned, being based on an 8 GeV proton beam on a current-carrying target followed by a lithium lens and a quadrupole decay channel, where the decaying ions generate muons [26].

It is highly unlikely that muonic hydrogen theory, and Lamb shift theory, could provide explanations for the proton radius puzzle, since they are well under control. From the experimental side, the situation regarding the proton radius may be less clear than commonly thought. Electron versus muon scattering experiments could shed light. A confirmation of the 1969 experiment [12, 13], which could point to a small electron-muon nonuniversality, would be very desirable. A comparison of electron and muon scattering on the same apparatus would shed light on the issue [14]. Finally, let us remember that an even more striking 7σ disagreement persists for the deuteron radius, when comparing the results from electronic and muonic bound systems [11].

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