BARRIER ISLAND MODELING INSIGHTS FROM APPLIED GLOBAL SENSITIVITY ANALYSES

STEVEN W.H. HOAGLAND[1], JENNIFER L. IRISH[2], ROBERT WEISS[3]

1. *Department of Civil and Environmental Engineering, Virginia Tech, 750 Drillfield Drive, 200 Patton Hall, Blacksburg, VA 24061. shoagland@vt.edu.*
2. *Department of Civil and Environmental Engineering & Center for Coastal Studies, Virginia Tech, 750 Drillfield Drive, 221E Patton Hall, Blacksburg, VA 24061. jirish@vt.edu.*
3. *Department of Geosciences & Center for Coastal Studies, Virginia Tech, 926 West Campus Drive, 1068A Derring Hall, Blacksburg, VA 24061. weiszr@vt.edu.*

Abstract: Barrier island models that include marsh and lagoon processes are highly parameterized. To constrain model uncertainty, those desiring to use these models should seek a robust understanding of the parameter sensitivities. In this study, global sensitivity analysis was performed on a long-term barrier island model to yield insights into the modeled barrier-backbarrier system. Given that a variety of global sensitivity analysis methods exist, each one appearing to differ in its implementation, computational burden, and output, three methods (i.e., the Two-Level Full Factorial Method, Morris Method, and Sobol Method) were applied to the model for the purposes of comparison. Key influential parameters (e.g., sea level rise rate, equilibrium/critical barrier width, and reference wind speed) were consistently identified by all three sensitivity analysis methods. Despite the relatively low number of simulations required by the Morris Method, the Two-Level Method computationally outperformed the others, warranting further exploration of the Morris Method's parallelization structure.

Introduction

Barrier island models that include backbarrier (i.e., marsh and lagoon) processes are highly parameterized. To constrain model uncertainty, those desiring to use these models should seek a robust understanding of the parameter sensitivities. To gain additional insights into the modeled morphodynamics of a coupled barrier island-backbarrier system, the parameter space of a long-term barrier model published by Lorenzo-Trueba and Mariotti (2017), hereafter the 'LTM17' model, was explored by means of conducting a global sensitivity analysis (GSA). However, there are a variety of GSA methods, each of which appears to differ in its implementation, computational burden, and output. Therefore, in addition to gaining additional insights into the LTM17 model, three GSA methods were applied to the model for comparison purposes.

The three GSA methods are the Two-Level Full Factorial Method (overviewed in Saltelli et al., 2008), the Morris Method (Morris, 1991), and the Sobol Method (Sobol, 1993). Using these methods, we evaluated the influence of 20 input parameters on the LTM17 model results over a 100-year period. Each method is briefly discussed and select results from the sensitivity analyses are presented.

Coastal Barrier Island System Model

The LTM17 model, published by Lorenzo-Trueba and Mariotti (2017), is a 1D idealized barrier transect model that was developed by coupling two previous models: a 2014 barrier island model from Lorenzo-Trueba and Ashton (2014), hereafter the 'LTA14' model, and a 2014 marsh-lagoon model from Mariotti and Carr (2014), hereafter the 'MAC14' model. The model simulates temporal changes to 10 state variables, including the transect boundaries, barrier height, and marsh and lagoon depths. These variables are modified through sediment flux calculations, which are based on the system's deviation from prescribed equilibrium states. A graphical representation of the model transect and state variables is displayed in Figure 1.

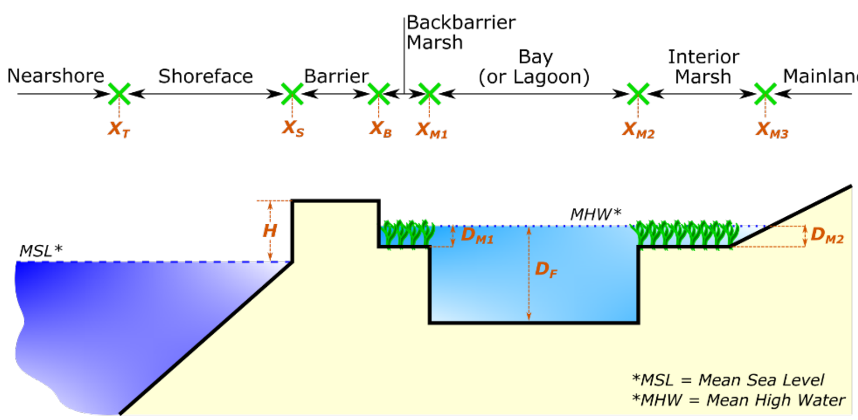

Fig. 1. LTM17 Model Transect and State Variables. *Figure reproduced and modified Lorenzo-Trueba and Mariotti (2017).*

The 20 LTM17 model input parameters shown in Table 1 were selected for GSA. Each parameter's sensitivity range was set at plus or minus 50 percent of the base value in Table 1, with the exceptions of tidal range (r), which has a lower limit of 0.7 meters, and sea level rise rate ($\dot{z}$), which has a defined range of 3-20 *mm/yr* based on global sea level rise projections through 2100 by Sweet et al. (2022).

Global Sensitivity Analysis Methods

Three GSA methods were integrated into the LTM17 model code: the Two-Level Full Factorial Method (hereafter the 'Two-Level' Method), the Morris Method,

Table 1. LTM17 Model Input Parameters

Parameter (Symbol)	**Base Value**	**Units**
Mainland Slope (β)	$7e^{-4}$	$[m/m]$
Toe Depth (D_T)	10	$[m]$
Equil./Crit. Width (W_{cr})	175	$[m]$
Equil./Crit. Height (H_{cr})	3	$[m]$
Equil. Shoreface Slope (α_{cr})	$6.5e^{-3}$	$[m/m]$
Critical Marsh Width ($B_{M1,cr}$)	70	$[m]$
Marsh Profile Shape Const. (d)	10	$[m]$
Max. Annual Overwash Flux ($Q_{OW,*}$)	60	$[m^3/m/yr]$
Sea Level Rise Rate ($\dot{z}$)	5	$[mm/yr]$
Shoreface Flux Const. (K)	$2e^3$	$[m^3/m/yr]$
Tidal Range (r)	0.87	$[m]$
Wind Speed (U_{ref})	8	$[m/s]$
Sediment Settling Velocity (ω_s)	0.4	$[mm/s]$
Sediment Erodibility Constant (λ)	$1e^{-4}$	[...]
Critical Bed Shear Stress (τ_{cr})	0.1	$[Pa]$
Marsh Accretion Const. (K_a)	2	[...]
Marsh Erosion Const. (K_e)	0.16	[...]
Ocean Sediment Conc. (C_o)	30	$[mg/l]$
Peak Biomass Production (B_P)	2.5	$[kg/m^2]$
Fraction Accumulated Carbon (χ_{ref})	0.15	[...]

and the Sobol Method. These methods are graphically represented in Figure 2 for a hypothetical model with 3 input parameters, where the cube represents the 3-dimensional parameter space, the axes represent the hypothetical input parameters (X_1, X_2, X_3), and each dot represents a single model simulation with distinct input parameter values (Note: only parameter constraints are represented in Figure 2c).

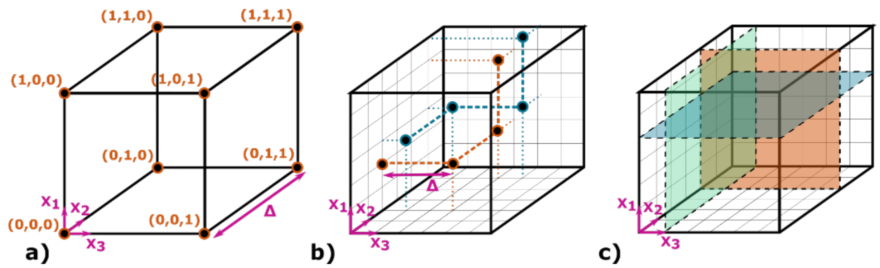

Fig. 2. Graphical representation of a) the Two-Level Full Factorial Method, b) the Morris Method, and c) the Sobol Method, for a hypothetical 3-parameter model.

In the Two-Level Method (Figure 2a), simulations were run for every extreme value combination of the input parameters (i.e., at the 'corners' of the parameter space), whereas simulations in the Morris Method (Figure 2b) were run from randomized sample trajectories (represented by the dashed lines) throughout the interior of the parameter space, making discrete parameter value 'jumps' according to a defined delta (Δ) value. Both methods follow the 'One-At-a-Time' (or OAT) approach, where sensitivity of a specific input parameter is represented by the average change in model results, or average of all 'elementary effects,' across all simulations. The averaged results are referred to herein as the 'average effect.' Thus, the average effect of a parameter 'X_i' (AE_{X_i}) is calculated by Equation 1:

$$AE_{X_i} = \frac{1}{N} \sum_{N} Y|_{X_i=B} - Y|_{X_i=A} \tag{1}$$

where N is either half the number of possible 2-parameter combinations (when using the Two-Level Method) or the number of user-defined trajectories (when using the Morris Method), Y is the model output, and A and B represent two distinct values for parameter X_i.

Oppositely, the Sobol Method (Figure 2c) follows a variance-based approach, which computes sensitivity indices based on the ratio of model output variance, constrained by fixed parameter values (the colored planes in Figure 2c), to the total unconstrained model variance. The two indices computed in this study were the first-order Sobol index (S_i) and the *k*th-order Sobol index (S_{T_i}), where *k* represents the number of input parameters. Following the notation of Saltelli et al. (2008), these index values are calculated using Equations 2 and 3:

$$S_i = \frac{V(E(Y|X_i))}{V(Y)} \tag{2}$$

$$S_{T_i} = 1 - \frac{V(E(Y|X_{\sim i}))}{V(Y)} \tag{3}$$

where S_i and S_{T_i} are the first and *k*th-order index values, respectively, for the *i*th input parameter (X_i), V is the variance, E is the mean or expected value, and Y is the model output. The first-order index indicates the percentage of model variance accounted for by each parameter, while the *k*th-order index accounts for the percentage of model variance from each parameter and its interactions with other parameters. For additional details on the methods, readers are referred to Morris (1991), Sobol (1993), and Saltelli et al. (2008), who provides an accessible summary of all three methods.

Results and Discussion

Each GSA method that was used to evaluate the sensitivity of the LTM17 model's 20 input parameters varied in its number of simulations and computational burden. The Two-Level Method completed its 2^k (i.e., 1,048,576) simulations and sensitivity calculations in 30 minutes. The Sobol Method, which runs a user-defined number of simulations per input parameter, took 2.3 hours to run its 5.5 million simulations. The Morris Method, which was set up with 3 elementary effects per parameter range (i.e., $\Delta = 3/5$), required the greatest computational burden with a run time of 3 hours; however, this was not expected since the total number of simulations was only 210,000. This suggests that there may be a more efficient way to parallelize the Morris Method computations.

Select results from the sensitivity analyses are presented in Figure 3, where Figures 3a, 3c, and 3e show the results for change in barrier width, and Figures 3b, 3d, and 3f show the results for change in backbarrier marsh width. Additionally, Figures 3a and 3b show the Two-Level Method results, Figures 3c and 3d show the Morris Method results, and Figures 3e and 3f show the Sobol Method results. The sensitivity indicators associated with each method are plotted on the y-axes for each input parameter on the x-axes. The background colors in each subplot of Figure 3 identify a broad categorization for the input parameters: dark grey for parameters associated with system geometry, white for forcing conditions, and light grey for parameters related to sediment transport.

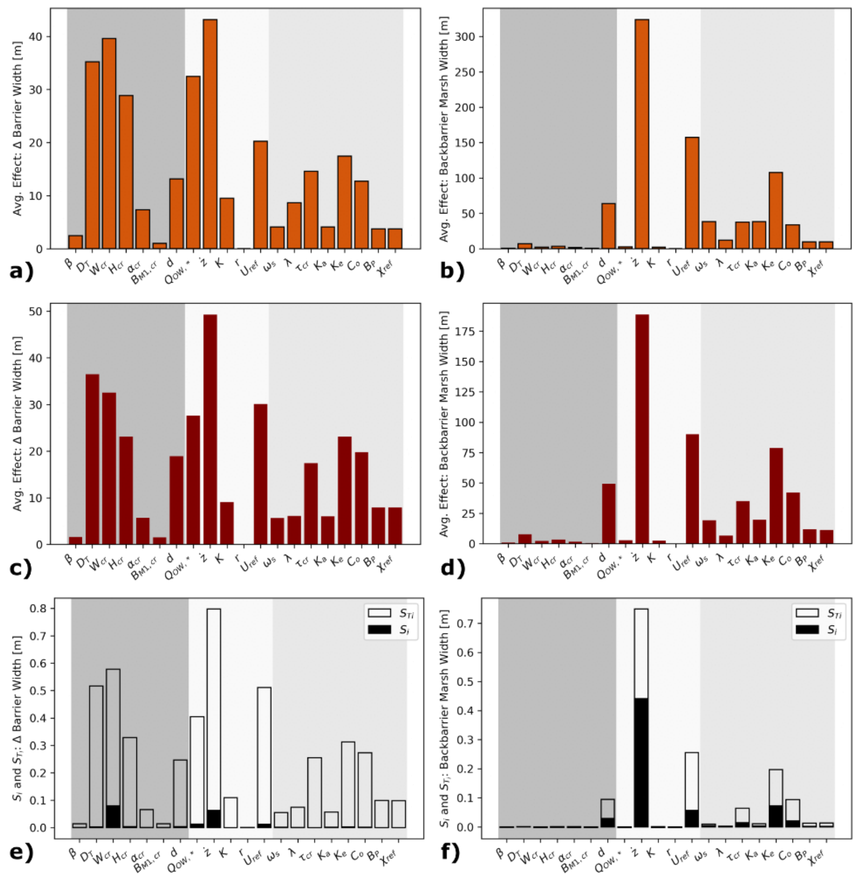

Fig. 3. Sensitivity analysis results for Barrier Width (3a, 3c, 3e) and Backbarrier Marsh Width (3b, 3d, 3f) for the Two-Level (3a, 3b), Morris (3c, 3d), and Sobol (3e, 3f) Methods

Figure 3 displays remarkable consistency between the results of the three GSA methods. Comparing the barrier width results in Figures 3a, 3c, and 3e, each method consistently identifies the most influential parameters (i.e., those with the highest sensitivity indicators) and non-influential parameters. Sea level rise rate ($\dot{z}$) and equilibrium/critical width (W_{cr}) were consistently two of the most influential parameters, while critical backbarrier marsh width ($B_{M1,cr}$) and tidal range (r) were consistently the least influential parameters. These results suggest that parameters such as $\dot{z}$ and W_{cr} are the most important parameters for modelers to understand and to constrain for future projections. The most and least influential parameters were also identified for the backbarrier marsh width results from Figures 3b, 3d, and 3f. It should be noted that $\dot{z}$ and the reference wind speed (U_{ref}) were consistently influential for both the barrier and backbarrier results.

Given the consistency in results between methods, the Two-Level Method proved to be the least computationally burdensome method to identify the most sensitive parameters in this study. However, future work in optimizing the Morris Method parallelization or the Morris and Sobol Method setups may improve performance.

Comparing the barrier width and backbarrier marsh width results (e.g., Figure 3a and 3b) suggests parameter sensitivity can vary significantly from one set of results to the next. For example, the geometrical parameters (i.e., those with a dark grey background), most of which were associated with the original LTA14 barrier island model, have substantial influence on the barrier width results, but little to no influence on the backbarrier marsh width results. The sediment transport parameters (i.e., those with a light grey background), which were associated with the original MAC14 model, influence both the barrier width and backbarrier marsh width results. These observations also suggest that backbarrier component of the model has more influence on the barrier results than vice versa.

Conclusion

The sensitivity analysis results from this study provide a few modeling insights. First, the most influential parameters were sea level rise rate ($\dot{z}$), equilibrium/critical barrier width (W_{cr}), and reference wind speed (U_{ref}), while the least influential parameters were critical backbarrier marsh width ($B_{M1,cr}$) and tidal range (r). Second, the relative sensitivity of input parameters was found to be highly dependent on the results of interest. Third, the Two-Level Method identified the parameter sensitivities with the least computational burden for this study. However, given the differences in the number of simulations, run time, and output of each method, there remains future work in comparing method performance and improving the parallelization of the Morris Method code. These results may be used to help identify parameter constraints and characterize model uncertainty toward more confident predictions and management decisions for coastal barrier systems.

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