The Bicycle Network Improvement Problem

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Abstract: Using a bicycle for commuting is still uncommon in US cities, although it brings many benefits to both the cyclists and to society as a whole. Cycling has the potential to reduce traffic congestion and emissions, increase mobility, and improve public health. To convince people to commute by bike, the infrastructure plays an important role because safety is one of the primary concerns of potential cyclists. This paper presents a method to find the best way to improve the safety of a bicycle network for a given budget and maximize the number of riders that could now choose bicycles for their commuting needs. This optimization problem is formalized as the bicycle network improvement problem (BNIP): it selects which roads to improve for a set of traveler origin–destination pairs, taking both safety and travel distance into account. The BNIP is modeled as a mixed-integer linear program (MIP) that minimizes a piecewise linear penalty function of route deviations of travelers. The MIP is solved using Benders decomposition to scale to large instances. The paper also presents an in-depth case study for the Midtown area in Atlanta, GA, using actual transportation data. The results show that Benders decomposition algorithm allows for solving realistic problem instances and that the network improvements may significantly increase the share of bicycles as the commuting mode. Multiple practical aspects are considered as well, including sequential road improvements, uneven improvement costs, and how to include additional data. **DOI: 10.1061/JTEPBS.0000742.** *© 2022 American Society of Civil Engineers.*

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Introduction

Using a bicycle for transportation is still uncommon in US cities, but it brings many benefits to both the cyclists and to society as a whole (Handy et al. 2014). Cycling has the potential to reduce traffic congestion and emissions, increase mobility, and improve public health (Northrop 2011). Additionally, bikes can serve as an economical alternative to a car, especially for short trips (Ryu et al. 2018). Promoting cycling as an alternative to using a car has been studied extensively, with systematic reviews provided by Ogilvie et al. (2004) and Yang et al. (2010).

The benefits of cycling as a mode of transportation have also been recognized by policy makers, and more and more cities have started promoting bicycle usage. An example in Atlanta is the *Walk, Bike, Thrive!* plan, which provides a recipe for a more walkable and bikable city (Atlanta Regional Commission 2020). Plans like these can play a key role in promoting bicycle usage, as was found by Lanzendorf and Busch-Geertsema (2014), who studied four German cities. Effective cycling policy may also benefit modes that are similar to bicycles, such as e-scooters and e-bikes. E-scooters and e-bikes both substitute for travel by car (Kroesen 2017; Gössling 2020), and evidence from the Netherlands suggests that car owners are more willing to use e-bikes than conventional bikes (Kroesen 2017).

The low number of cyclists is not due to a lack of interest. Dill and McNeil (2016) questioned 3,000 people in the 50 largest US metropolitan areas about their attitudes towards cycling, and they found that 56% of the population can be classified as *interested but concerned*. One of the key barriers for this group is traffic safety: although most feel comfortable riding on a protected bike lane that is part of a major street, only 16% would be somewhat comfortable without the bike lane. The willingness to cycle is also demonstrated by the surge in US bike ridership during the COVID-19 pandemic (Bryant 2020). Many people have started cycling for recreation, but also as a socially distant alternative to public transit. Policy makers hope that this trend continues and that these new cyclists start commuting by bike when they return to work after the pandemic.

To convince people to commute by bike, the infrastructure plays an important role. The study by Dill and Carr (2003) suggested that, if a city provides the proper infrastructure for cycling, commuters are likely to make use of it. Hull and O'Holleran (2014) studied selected European cities to identify whether good design can encourage cycling. They found that the design may indeed have a significant impact on mode choice and that *safety*, *comfort*, and *continuity* were the most influential factors. Although safety is especially important to cyclists, safety improvements in the last decades have often focused on motorized vehicles, as highlighted by CIVITAS Initiative (2020) for the European Union. This situation can be improved by investing in bicycle infrastructure, which additionally improves safety for noncycling road users (Walljasper 2016).

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Another important factor, which was not explicitly considered in the previous study, is the *proximity*: the distance between the origin and destination of the trip (Saelens et al. 2003). Heinen et al. (2010) studied the determinants for bicycle commuting and found that the built environment affects traveler choice, among which distance is probably the most important factor. A study for British cities and towns by Cervero et al. (2019) suggested that safe connections that are as close to the shortest path as possible are most likely to encourage bicycle commuting. Ospina et al. (2020) arrived at a similar conclusion for Medellin city in Colombia: cyclists are willing to take a detour to ride on dedicated lanes, but only up to an extent. Wang et al. (2021) found that similar factors affect the choice to use shared bicycles.

This paper presents optimization models to find the best way to improve an existing bicycle network for a given budget. Policy makers may use this method to guide their investments in cycling infrastructure and to obtain the advantages that come with it. The optimization problem is formalized as the bicycle network improvement problem (BNIP) that selects how best to spend a given budget for road improvement in order to minimize the total penalty for a set of traveler origin–destination pairs (ODs). The penalties are calculated from the distance deviations from travelers' shortest paths. The optimization problem uses piecewise linear penalty functions, which are flexible enough to model different human preferences and numerous other factors.

This paper modeled the BNIP as a mixed-integer linear program (MIP), which was solved through Benders decomposition. The optimization method was then used to conduct an in-depth case study for the Midtown area in Atlanta, GA, based on real transportation data. As the 10th most congested city in the US, and with a bicycle infrastructure scored in the red category (Reed 2019), Atlanta makes for an interesting test case. Compared to the city of Delft in the Netherlands, cyclists in Atlanta are over two times more likely (78% versus 32%) to report poor road infrastructure as a cause of stress (Gadsby et al. 2021). Furthermore, a survey by the Atlanta Department of City Planning mentions that 70% of people in the city currently feel uncomfortable riding a bike (Bottoms 2018).

The paper contains four main contributions:

1. From a methodology standpoint, the paper formalizes the bicycle network improvement problem and shows that Benders decomposition is able to find optimal improvement plans for realistic instances, whereas the problem is computationally intractable for state-of-the-art black-box solvers.
2. From a case study standpoint, the paper shows that even small investments in infrastructure may allow many additional commuters to travel safely by bike.
3. At the intersection of methodology and case study, the paper demonstrates the value of optimization, which produces improvement plans that are significantly better than those obtained by heuristics. Moreover, and this is important for cities, the paper compares the benefits of optimal long-term plans with those obtained by upgrading the infrastructure incrementally. The paper shows that, in the case study, successive improvements using the BNIP lead to a network that is very close to optimal in the long term, which simplifies decision-making.
4. From a computational perspective, this paper compares a wide range of different penalty functions and reports consistent results across all of them.

The rest of the paper is organized as follows. The paper first reviews prior work. Then, the BNIP is formally introduced, followed by describing a Benders decomposition algorithm to solve it. Then, the paper discusses the current conditions in Midtown Atlanta, which also motivates this research. Next, the Midtown Atlanta case study is presented, and the following section explores the use of different penalty functions. This paper also discusses how the methods in this paper can be adapted for future work, and then ends with the conclusion and possible directions for future research.

Review of Prior Work

There are several studies that consider bicycle infrastructure improvement planning. Duthie and Unnikrishnan (2014) presented a network design formulation to connect all OD pairs with a lower bound on the bicycle level of service and an upper bound on the maximum travel length expressed as a function of the corresponding shortest path. Their objective and the BNIP objective are similar in that they limit the worst service for travelers with respect to the travel distances. There is, however, a fundamental difference between their work and the BNIP: the former mandates that all ODs admit feasible bicycle travel regardless of the improvement budget, where the BNIP has a limited budget to serve as many OD pairs as possible. The benefit of having a finite budget is that it abides by realistic scenarios, such as urban planners developing new bicycle infrastructure. Indeed, budgets for infrastructure improvements are almost always limited, and their effective use is a key aspect for decision-makers.

Mauttone et al. (2017) introduced another MIP model to minimize the overall travel cost of riders, where the cost primarily consisted of travel distances. This formulation included a budget constraint but still required all OD pairs to be served. This was made possible by allowing for bicycle trips that were not 100% safe and penalizing the usage of unsafe roads. Because safety is the primary concern for many potential cyclists, as argued in the introduction, the BNIP does not sacrifice the requirement for completely safe bicycle routes; rather it imposes limits on maximum travel distances to model realistic trips and provides an outside option for those riders who do not have a realistic safe route. It is also important to mention that Mauttone et al. (2017) only provided suboptimal solutions in reasonable time for their real-life case studies, and they used a heuristic to report results for large cases with more accuracy. The Benders decomposition algorithm proposed in this paper, however, solves the large Atlanta instances to optimality.

Liu et al. (2020) presented a MIP model to plan bicycle networks using objectives for coverage and continuity of travels. They assumed that the MIP model receives bicycle paths as input. Their adjacency–continuity utility function, which incorporated both safety and trip length, selected one of the precalculated paths to route each traveler while maximizing the utility of the network. Their work is similar to the BNIP because it improves both safety and proximity of the trips. The BNIP, however, has full flexibility in routing cyclists; this simplifies modeling for decision-makers and may produce solutions of higher quality because the optimization can choose the best routes for riders and is not constrained by preselected paths.

In addition to the previous works, a number of studies incorporate more diverse characteristics in the problem modeling. For example, Lin and Yu (2013), Lin and Liao (2016), and Zhu and Zhu (2020) used multiobjective optimization to include various objectives such as road connections, accessibility, and service level. These formulations can model more customized bicycle experiences but become more computationally intensive. The BNIP is a single-objective optimization problem with an objective that is flexible enough to model realistic applications. Most importantly, the proposed Benders decomposition algorithm is capable of performing studies in much larger instances and areas than those multiobjective programs. It is an interesting avenue for future research to study if the techniques in this paper can be generalized to multiple objectives.

A number of other studies have relied on heuristic methods instead of mathematical optimization techniques. Bao et al. (2017) used large-scale bicycle trajectory data to define a flexible objective that combined the population covered by the network and the distances of their trajectories. It was solved with greedy-based heuristics that included steps to initiate road segments, expand the network, and terminate the improvement when the budget limit is met. Also, Hsu and Lin (2011) used the shortest paths of ODs like those of Duthie and Unnikrishnan (2014) to evaluate the quality of the network. They used a variety of algorithms, some of which were greedy, to compare the shortest paths to bicycle routes. Orozco et al. (2020) introduced two greedy algorithms to connect bicycle network components. They also compared the shortest paths to bicycle routes, but as opposed to the BNIP, they did not optimize any objective.

Bicycle Network Improvement Problem

This section introduces the bicycle network improvement problem to find the best improvement of an existing bicycle network within a given budget. Let the current road network be represented by a directed graph $G = (V, W)$ with nodes V and arcs W. The arcs are referred to as *ways*. Ways are partitioned into two distinct sets $W = W^{\text{safe}} \cup W'$, with W^{safe} the set of ways that are safe for cycling, and W' the set of unsafe ways. Every way $(i, j) \in W$ has a length, given by the parameter $d_{ij} \geq 0$. The total length of ways that can be improved is limited by the budget B. The set of sample trips that travel through the network is given by T. Each OD $k \in T$ consists of an origin $o_k \in V$ and a destination $d_k \in V$. Additionally, let $s_k \geq 0$ be the length of the (possibly unsafe) shortest path between o_k and d_k, and let $p_k \geq 1$ be the number of travelers completing this travel.

Modeling Bicycle Travel

Safety is critical to increase cyclist participation. Accordingly, two characteristics are taken into account when modeling bicycle trips. First, potential cyclists would like completely safe routes from origin to destination: if the safety requirement is met for a certain OD, then the route is labeled as *safe*. If the network cannot provide a safe route, the BNIP assumes that the potential rider will select another mode of transportation, which is referred to as the *outside option*. Second, the travel should not take much longer than the alternative transportation mode, such as driving by car. Hence, if there is no safe path of length smaller than L_k (a parameter for rider k), the BNIP assumes that rider k will not travel by bicycle.

To model the appeal of short bicycle trips for rider k, the BNIP uses a penalty function f_k that is nondecreasing and satisfies $f_k(0) = 0$. For an OD pair k and a path of length l_k, the penalty is given by $f_k(u_k)$ where $u_k = l_k - s_k$ denotes the deviation from the (potentially unsafe) shortest path. If rider k cannot be provided a short trip, the outside option is used, and the penalty is the objective function is defined as $L_k - s_k$ (an optimization model that purely maximizes the number of cyclists is also presented in the paper). As such, this trip is assigned the same penalty as a path of length L_k, which is the tipping point at which the rider starts preferring the outside option.

Mathematical Formulation

The mathematical model for the BNIP is depicted as follows:

$$\min \sum_{k \in T} p_k f_k(u_k) \tag{1a}$$

$$\text{s.t.} \sum_{(i,j) \in W'} d_{ij} y_{ij} \leq B \tag{1b}$$

$$\sum_{(i,j) \in W} x_{ij}^k - \sum_{(j,i) \in W} x_{ji}^k = \begin{cases} 1 - z_k & \text{if } i = o_k \\ z_k - 1 & \text{if } i = d_k \\ 0 & \text{otherwise} \end{cases} \quad \forall\, k \in T, i \in V \tag{1c}$$

$$x_{ij}^k \leq y_{ij} \quad \forall\, k \in T, (i,j) \in W' \tag{1d}$$

$$u_k \geq \sum_{(i,j) \in W} d_{ij} x_{ij}^k + L_k z_k - s_k \quad \forall\, k \in T \tag{1e}$$

$$y_{ij} \in \mathbb{B} \quad \forall\, (i,j) \in W' \tag{1f}$$

$$x_{ij}^k \in \mathbb{B} \quad \forall\, k \in T, (i,j) \in W \tag{1g}$$

$$z_k \in \mathbb{B} \quad \forall\, k \in T \tag{1h}$$

For every way $(i, j) \in W'$, variable $y_{ij} \in \mathbb{B}$ indicates whether (i, j) is upgraded to safe conditions (value one) or remains unchanged (value zero). The shortest safe path for every OD is determined by variables x and z: variable $x_{ij}^k \in \mathbb{B}$ represents whether trip $k \in T$ uses way (i, j). Variable $z_k \in \mathbb{B}$ indicates whether trip $k \in T$ uses the outside option. As explained in the previous section, the variable u_k represents the argument of the penalty function for every trip $k \in T$.

The objective in Eq. (1*a*) minimizes the total penalty of the riders $\sum_{k \in T} p_k f_k(u_k)$ on the network. The penalties for cyclists are computed in terms of the deviation from the shortest-path distance. Riders who are not cycling or have safe paths that are too long incur the penalty associated with a path of length $l_k = L_k$, as will become clear shortly. The constraint in Eq. (1*b*) limits the budget for improving the network. The constraints in Eq. (1*c*) impose the path conservation conditions: each OD has either a unit flow (if $z_k = 0$), in which case the x-variable describes the path, or uses the outside option ($z_k = 1$). The constraints in Eq. (1*d*) make sure that unsafe ways can only be used if upgraded. The constraints in Eq. (1*e*) compute the deviation. If the shortest path for trip k exceeds length L_k, it is optimal to set z_k to 1; that is, trip k uses the outside option. Infeasible trips are also assigned the penalty associated with a path of length L_k. Constraints in Eqs. (1*f*)–(1*h*) capture the integrality conditions.

Solution Methods for the BNIP

Solving the BNIP directly with a MIP solver, such as CPLEX or Gurobi, is computationally intractable for the scale of the case study considered in this paper. Observe however that, for a given design (i.e., when the y-variables are fixed), the formulation reduces to a set of independent minimum-cost flow problems, one for each OD. By total unimodularity, this implies that the integrality conditions in Eqs. (1*g*) and (1*h*) can be relaxed. This makes the problem ideally suited for Benders decomposition (Benders 1962).

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Benders Decomposition

The Benders decomposition for the BNIP has a master problem to determine how to upgrade the network and subproblems to return the safe path or the outside option for each rider for a given upgraded network. The master problem generates a network design, and the subproblems find the paths in the proposed network for each trip. The optimal solutions of the subproblems are then used to derive the Benders cuts that are added to the master problem. These two steps are iterated until no more violated Benders cuts are generated by the subproblems, at which point the network is optimal.

The Benders Master Problem The master problem is presented as follows:

$$\min \sum_{k \in T} p_k f_k(u_k) \tag{2a}$$

$$\begin{aligned} \text{s.t. } & (1b), (1f), \\ & u_k \geq \Phi_k(y) \quad \forall\ k \in T \end{aligned} \tag{2b}$$

Its objective given in Eq. (2*a*) is the same objective as in Eq. (1). The constraints in Eqs. (1*b*) and (1*f*) ensure that the network improvement plan is within budget and valid. When solving the master problem, the constraints in Eq. (2*b*), where $\Phi_k(y)$ is the minimum objective value for trip k given a design y, are replaced by the Benders cuts generated from the solutions to the subproblems.

Benders Subproblem The subproblem for a trip k generates Benders cuts for each network produced by the master problem. The z-variables (the outside option) ensure *complete recourse*: the subproblem is feasible for any network because ODs can always use the outside option. This implies that the optimality cuts in [Eq. (2*b*)] are sufficient and no feasibility cuts are needed.

For a given network, the subproblem decomposes into many independent subproblems, and the subproblem of the BNIP for each $k \in T$ is defined as follows:

$$\begin{aligned} \Phi_k(y) = & \min \sum_{(i,j) \in W} d_{ij} x_{ij}^k + L_k z_k - s_k, \\ & \text{s.t. } (1c), (1d), (1g), (1h) \end{aligned} \tag{3}$$

Because the subproblem is a standard minimum-cost flow problem, it is totally unimodular and can be solved by linear programming. This implies that the Benders subproblem can be solved by optimizing many small and independent linear programs, which is the prime reason the Benders decomposition provides a significant computational benefit.

With dual variables λ and μ associated with the constraints in Eqs. (1*c*) and (1*d*), respectively, the dual subproblem is defined as follows:

$$\Phi_k(y) = \max \lambda_{o_k}^k - \lambda_{d_k}^k - \sum_{(i,j) \in W'} \mu_{ij}^k y_{ij} - s_k \tag{4a}$$

$$\text{s.t. } \lambda_i^k - \lambda_j^k \leq d_{ij} \quad \forall\ (i,j) \in W^{\text{safe}} \tag{4b}$$

$$\lambda_i^k - \lambda_j^k - \mu_{ij}^k \leq d_{ij} \quad \forall\ (i,j) \in W' \tag{4c}$$

$$\lambda_{o_k}^k - \lambda_{d_k}^k \leq L_k \tag{4d}$$

$$\lambda_i^k \in \mathbb{R} \quad \forall\ i \in V \tag{4e}$$

$$\mu_{ij}^k \geq 0 \quad \forall\ (i,j) \in W' \tag{4f}$$

The Benders cuts are obtained as

$$u_k \geq \bar{\lambda}_{o_k}^k - \bar{\lambda}_{d_k}^k - \sum_{(i,j) \in W'} \bar{\mu}_{ij}^k y_{ij} - s_k \tag{5}$$

where $\bar{\lambda}$ and $\bar{\mu}$ are optimal dual values for the corresponding trips.

Pareto-Optimal Cuts It is well known that network flow problems often suffer from dual degeneracy. Magnanti and Wong (1981) addressed this issue by generating *Pareto-optimal cuts* that are not dominated by any other Benders cut. This requires solving a Pareto subproblem that uses the result from the standard subproblem. Pareto-optimal cuts need a *core point*, that is, a point in the relative interior of the feasible region of the master variables. For the BNIP, the following point y' is selected as the core point

$$y'_{ij} = \frac{1}{2} \min\left\{\frac{B}{|W'| d_{ij}}, 1\right\} \quad \forall\ (i,j) \in W' \tag{6}$$

The point y' is in the relative interior because $y'_{ij} \in (0,1)$ and $\sum_{(i,j) \in W'} d_{ij} y'_{ij} \leq \sum_{(i,j) \in W'} d_{ij} [B/(2|W'| d_{ij})] = (B/2) < B$, which strictly satisfies the budget constraint in Eq. (1*b*).

Using the core point y', the Pareto subproblem is defined as follows:

$$\max \lambda_{o_k}^k - \lambda_{d_k}^k - \sum_{(i,j) \in W'} \mu_{ij}^k y'_{ij} - s_k \tag{7a}$$

$$\text{s.t. } \lambda_{o_k}^k - \lambda_{d_k}^k - \sum_{(i,j) \in W'} \mu_{ij}^k y_{ij} - s_k = \Phi_k(y), \tag{7b}$$

$$(4b) - (4f)$$

To use Pareto-optimal cuts, each Benders iteration is changed as follows. For every trip, the value of $\Phi_k(y)$ is calculated by solving the subproblem. Next, the Pareto subproblem is solved to produce new optimal dual values $\bar{\lambda}$ and $\bar{\mu}$, and those variables are used to generate cuts as in Eq. (5).

Two-Phase Benders McDaniel and Devine (1977) observe that it is not necessary to solve the master problem to optimality at every iteration to obtain valid Benders cuts. They propose to apply Benders decomposition in two phases. In phase one, Benders decomposition is applied to the relaxed master problem. For the BNIP, this amounts to relaxing the integrality conditions in Eq. (1*f*) in the master problem and solving the subproblems for fractional values of y. In phase two, the integrality conditions are reinstated, and the Benders decomposition algorithm continues with the original master problem. The Benders cuts that are added in phase one are maintained, which ensures a better starting lower bound, which often improves the overall performance of the algorithm. Moving from phase one to phase two is possible at any point, and this paper switches over when the relaxed problem is solved, or when a time limit is reached.

Greedy Heuristic

This section also introduces a greedy heuristic that only relies on the ability to solve shortest paths to demonstrate the value of optimization. The heuristic greedily computes the next way to upgrade in the network. For every OD, it computes a shortest path that minimizes the total distance on unsafe roads, that is, the cost for traveling safe or unsafe way $(i,j) \in W$ is 0 or d_{ij}, respectively. Only the shortest paths with a distance of at most L_k are considered for all $k \in T$. The relative importance of an unsafe way is determined by counting the total number of riders whose shortest paths

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include this way. The greedy heuristic selects the most "important" way to upgrade and repeats the process until the budget is exhausted.

Cycling in Midtown Atlanta

This paper was motivated by improving bicycle travel conditions in the Midtown area in Atlanta, GA. Midtown is a neighborhood of Atlanta that consists of a commercial core and a residential neighborhood. The Midtown core is characterized by high-rise buildings and functions as a major employment center, including offices of large companies such as NCR, Google, Equifax, and Honeywell. The commercial core has around 28k residents, and 70k people per day travel to the area for work (Midtown Alliance 2019). The residential neighborhood is to the east of the business district and mainly consists of single-family residences. Every workday, a large number of commuters drive to Midtown and cause a significant amount of traffic congestion. To better this situation, the case study aimed at improving the bicycle infrastructure to provide these commuters safe and short cycling trips as an alternative to commuting by car.

Travel Data

To gain insight into current commuting behaviors, this paper used travel data provided by the Atlanta Regional Commission (ARC). The ARC used an activity-based model, calibrated with survey data collected in 2007–2011, to simulate trips at the individual level (Atlanta Regional Commission 2017). The data revealed a low share of cyclists among commuters to Midtown (about 0.7%) but also a significant potential for improvement: many of the trips to Midtown are short, and over 70% of commutes are completed by people driving alone, who could potentially switch to cycling if the infrastructure were improved.

The case study focused on one particularly interesting group: the group of white-collar workers coming in from Virginia-Highland, an affluent neighborhood immediately to the east of Midtown. Fig. 1 shows the daily number of trips originating from Virginia-Highland for both cyclists and solo drivers, categorized by trip purpose. Many trips are taken by white-collar commuters, but, despite the close proximity of Virginia-Highland to Midtown, the number of cyclists is less than 5% that of the number of solo drivers.

To study how improving the bicycle network affects travelers, a sample of travels was generated to represent the white-collar commuters from Virginia-Highland. The eight traffic analysis zones (TAZs) that cover Virginia-Highland were selected as the departure zones, and 72 TAZs that cover Midtown and Virginia-Highland

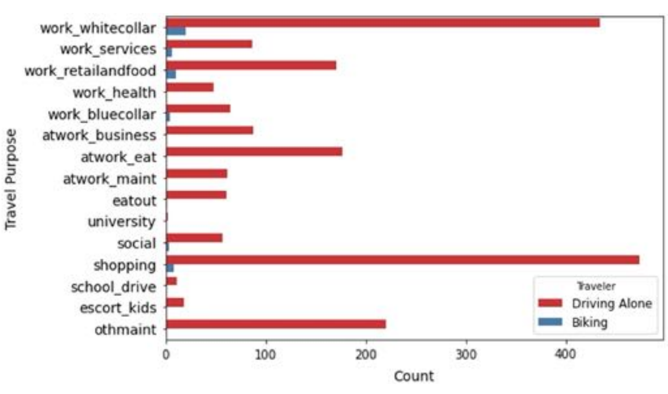

Fig. 1. Travelers from Virginia-Highland to Midtown.

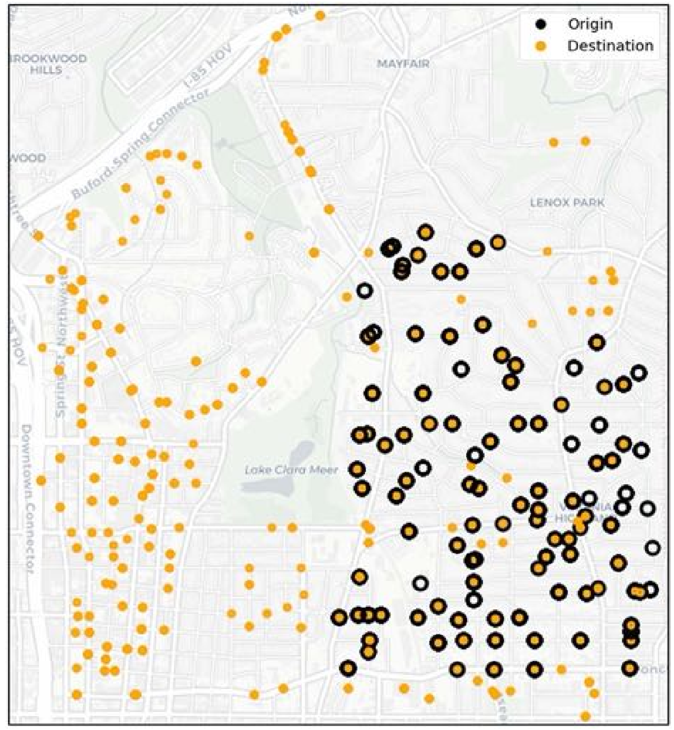

Fig. 2. Origins and destinations of 1,039 sample OD pairs. (Data from Atlanta Regional Commission 2017, © OpenStreetMap contributors.)

were chosen as the destination zones. Destinations in Virginia-Highland were included because these trips may benefit from the same infrastructure improvements. The ARC provided travel data between the TAZs, and to obtain a more realistic sample, the origins and destinations were randomly assigned to the centers of the smaller census blocks, weighted by the population counts. Five samples were generated for every OD pair, and samples that did not connect to the existent road network were filtered out. The result is a set of 1,039 representative trips, covering 110 origins and 256 destinations, presented by Fig. 2. The origins on the map are in Virginia-Highland in the east, and most destinations are in the business area in the west. The center of the map shows Piedmont Park, with the Midtown residential neighborhood to its south.

Current Bicycle Network

The current road network in the case study area was retrieved from OpenStreetMap (OpenStreetMap 2020). The network consisted of 5,815 nodes and 11,329 directed ways. The ways made up 667 roads and had a total length of 339 km (212 mi). The roads were classified into three types: roads with a dedicated bike lane, residential roads, and unsafe roads, where dedicated and residential roads were assumed to be safe for cyclists.

Fig. 3 shows the current bicycle network in the case study region, where there are three types of roads: dedicated, unsafe, and residential. Unsafe roads do not have the proper infrastructure for cyclists and were the target for conversion to dedicated roads. There were 450 bicycle-unsafe roads, with a total length of 180 km (113 mi). The conditions on the unsafe roads were assumed to be similar throughout the area, and the cost to realize dedicated bike lanes was assumed to be the same per unit length everywhere. Among the sampled ODs, only 170 trips (16%) had bicycle-safe routes that are completely safe. Moreover, many of them required a significant detour to complete those safe paths. For example, only

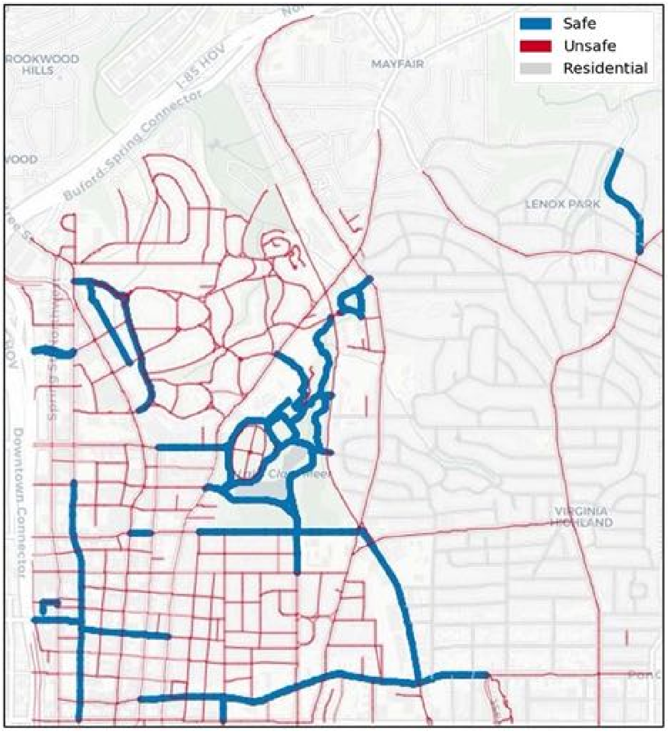

Fig. 3. Current bicycle network. (©OpenStreetMap contributors.)

89 ODs (9%) had access to a bicycle path with a detour of less than 10% of the shortest path.

Experimental Results

This section presents experimental results on the case study. It first shows that the Benders decomposition algorithm allows for solving the BNIP to optimality for realistic instances that cover a whole neighborhood. It also analyzes the effectiveness of the optimal improvement plans to provide safe and short cycling trips to commuters and demonstrates the benefits of optimization by comparing the optimal improvement plans to those obtained with the greedy heuristic.

Experimental Settings

The experimental results use the case study region, the existing network, and the 1,039 sample trips from the previous section. For each sample, the number of passengers $p_k = 1$ because the case study targeted solo drivers who commute to their workplaces. The budget B ranged from 6.4 km (4 mi) up to 44.8 km (28 mi) of improvements, in 6.4 km (4 mi) increments. For the case study, it is useful to consider network improvements on the road level, rather than on the way level, and to improve both directions at the same time. These solutions are more practical to implement and also contribute to safe return trips. In total, 450 roads were identified. To improve all ways of a road at the same time, additional constraints were added to the BNIP: if $(i,j) \in W'$ and $(p,q) \in W'$ were part of the same road, then $y_{ij} = y_{pq}$. These constraints were added to the Benders master problem without changing the main steps of the algorithm. For the greedy heuristic, the relative importance of a road was calculated by averaging the usage counts for the individual ways.

The core experimental results used a linear penalty function with $f_k(u_k) = u_k$; that is, the objective maximized the number of cyclists and minimized the overall average distance over the shortest paths, and the resulting BNIP was denoted as BNIP-L. Section "Alternative Penalty Functions" provides experiments with alternative penalty functions, and the "Discussion" section discusses how additional data can be included in the model to complement travel distance. The threshold L_k was computed in terms of a deviation factor $R \geq 1$, that is, $L_k = s_k R$. The full Benders algorithm was implemented in Python, and Gurobi 9.0.2 was used to solve the master problem and subproblems. All computations were performed with an Intel Core i7-8565U CPU and 16 GB of RAM.

Efficiency of the Benders Algorithm

This section compares three Benders decomposition algorithms: the traditional Benders decomposition algorithm (TB), the algorithm that uses Magnanti and Wong Pareto-optimal optimal cuts (MW), and the algorithm that uses both Pareto-optimal cuts and the McDaniel and Devine two-phase strategy (MW-McD). In the MW-McD experiments, phase one was limited to 20 min.

Fig. 4 reports the experimental results for various budgets and $R = 1.2$. The left charts show the optimality gap, that is, the difference between the best feasible solution and the lower bound over time, and the right charts show how the upper and lower bounds approach each other until the optimal solution is found. The three rows correspond to the TB, MW, and MW-McD strategy, respectively. In the MW-McD case, the vertical dashed line indicates the switch from phase one to phase two.

The first observation is that TB was significantly outperformed by MW and MW-McD. When fractional cuts were added prior to integral cuts (MW-McD), the initial optimality gap was much smaller and the number of iterations was significantly reduced compared to MW. Table 1 compares MW and MW-McD on the number of Benders iterations to reach optimality. MW solved each BNIP instance optimally in under 150 min, whereas MW-McD only took around an hour per instance. Fig. 5 verifies that the good performance of MW-McD was consistent for different budgets B and different distance thresholds R. Overall, the solution time of at most five hours was short for creating an improvement plan for months or years into the future. This contrasts with the MIP model, which could not solve the instances in reasonable time. The difficulty came from the size of the problem: the network consisted of $|W| = 11{,}329$ arcs, and the BNIP introduced a flow variable for each arc and for each of the 1,039 ODs, resulting in a MIP with over 10 million variables. The Benders decomposition exploited the problem structure and solved a significantly smaller problem for each OD at every step.

Impact of Bicycle Network Improvement Plans

It is interesting to study the type of improvements produced by optimal plans. Figs. 21–24 in the Appendix present the full series of improvement plans for the different settings of B and R; this section discusses the most important observations. The experiments were conducted with $R = 1.1$, 1.2, 1.3, and 1.5.

Plan Characteristics: The bicycle network improvement plans showed several notable trends. First, the problem searched for subregions that could be served with minimum improvements by maximizing the usage of preexisting infrastructure. The left map of Fig. 6 shows the optimal plan when only 6.4 km (4 mi) of road can be improved. Two crucial improvements (circled) were selected: a short segment that links the park and Virginia-Highland to the east and roads northwest of the park that connect to the business area. These connections are essential to allow commuters from Virginia-Highland to commute to their workplaces.

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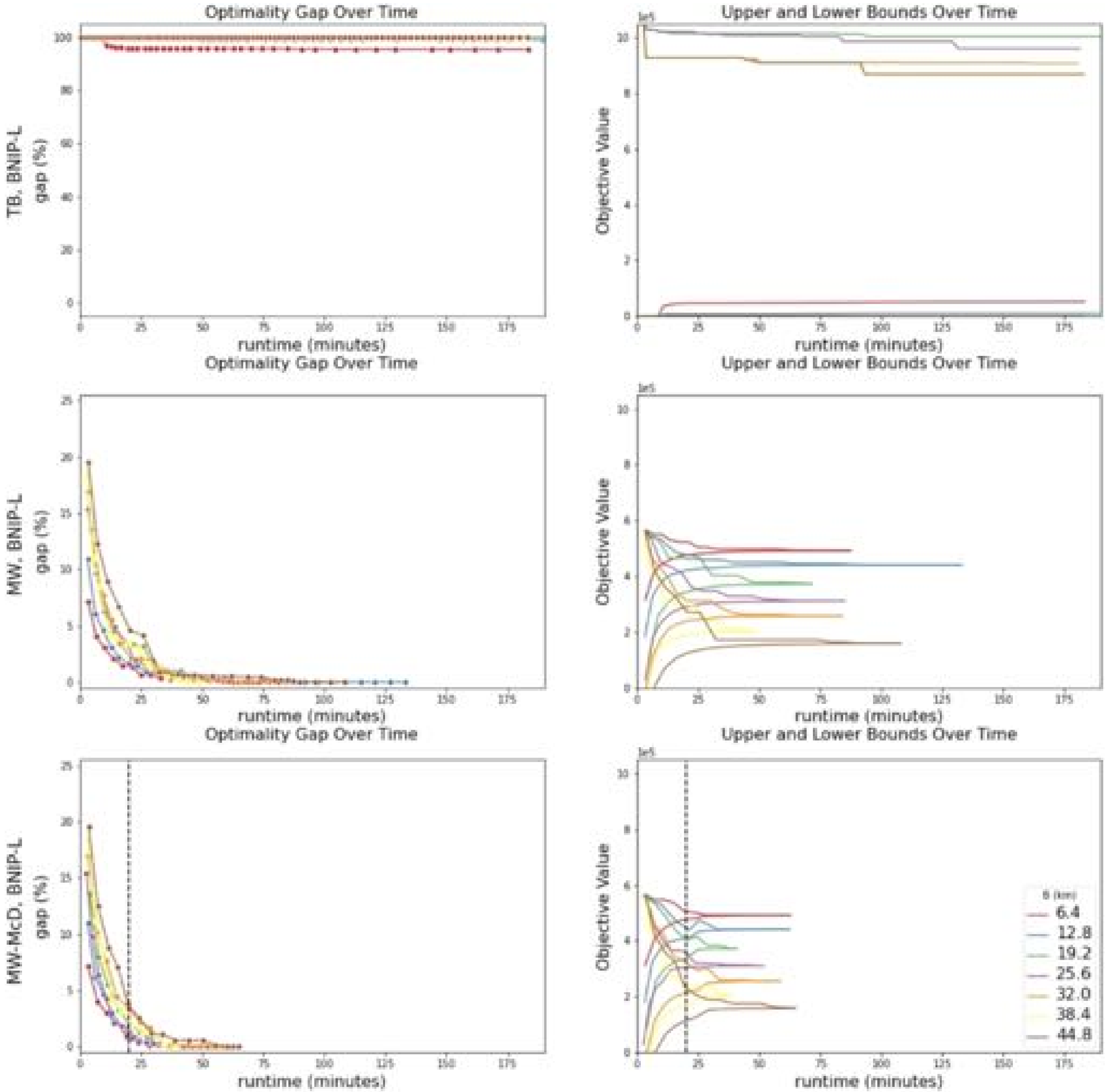

Fig. 4. The performance of Benders decomposition algorithms ($R = 1.2$).

Table 1. Number of iterations for MW and MW-McD (fractional and integral cuts)

	Iteration count	
B (km)	MW	MW-McD
6.4	24	$6 + 10 = 16$
12.8	28	$6 + 10 = 16$
19.2	16	$5 + 5 = 10$
25.6	20	$4 + 13 = 17$
32.0	17	$6 + 7 = 13$
38.4	13	$6 + 4 = 10$
44.8	19	$6 + 8 = 14$

A second observation is that the optimal plans did not waste budget to improve multiple roads that serve a similar purpose. For instance, consider the short discontinuity of safe roads between the park and the roundabout located north of the park. The optimization algorithm did not remove this discontinuity because the short segment connecting Virginia-Highland to the park serves the same purpose. Plans generated by the greedy heuristic (Fig. 25 in the Appendix) did not recognize this. This is the value of optimization that provides globally optimal plans. This will have the consequence that some trips are better served on the heuristic network, but overall the optimization will produce significantly better plans.

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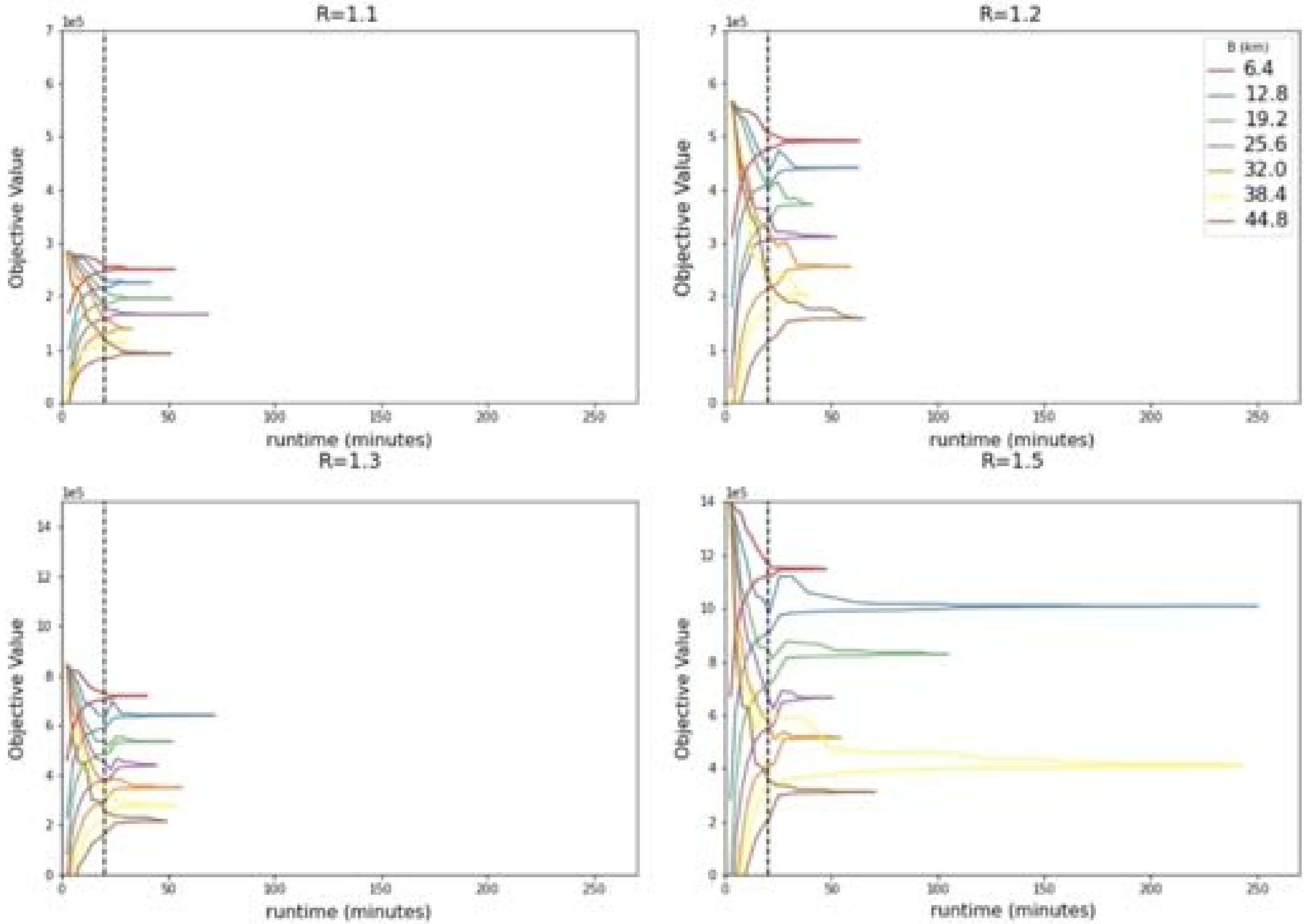

Fig. 5. The performance of MW-McD algorithm with different distance thresholds.

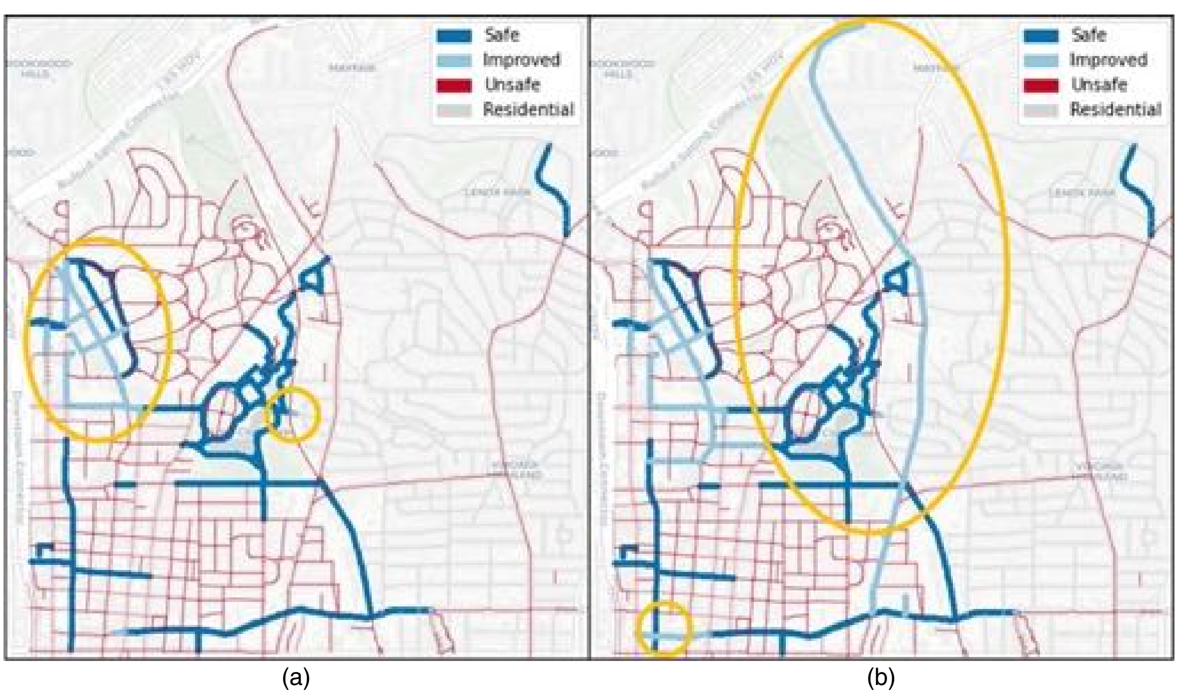

Fig. 6. Optimal improvement plans ($R = 1.1$): (a) $B = 6.4$ km (4 mi); and (b) $B = 19.2$ km (12 mi). (©OpenStreetMap contributors.)

A third observation concerns increased budgets: here the BNIP prioritized the improvement of the backbone of the network rather than providing sporadic developments. The 19.2-km (12-mi) map in Fig. 6 shows the investments on Monroe Drive (a hook-like vertical road) to provide safe north–south travel and on missing segments of Ponce De Leon Avenue (a horizontal road located southwest) that may connect all southern demands.

The order in which the backbone is constructed depends on the deviation factor. For instance, Fig. 10, which includes 38.4 km (24 mi) of improvement plans with four deviation factors, shows that North Highland Avenue (a vertical road located east) was not improved for $R = 1.5$, unlike three other plans of shorter distance thresholds. That is because the longer deviation allowance allowed service to origins located on the east with residential roads despite causing some detours and improved more roads in the western business area to complete more last miles of the ODs.

Although the order of its construction may be different, the backbone of the network converged as B increased. To follow up on the previous case, the 44.8-km (28-mi) plan for $R = 1.5$ (Fig. 24 in the Appendix) improved North Highland Avenue and exhibited practically identical road improvements regardless of different travel length allowance.

Effectiveness: Fig. 7 shows the effectiveness of the optimal improvement plans for different parameters. For each R, the percentage of potential cyclists grew as the budget and average trip distance over the shortest paths decreased. Potential cyclists are those riders with a safe bicycle trip whose length does not exceed the maximum distance (i.e., L_k for trip k). The number of people who benefit from the improvements was similar for all values of R. Improving only 6.4 km of bicycle lanes already doubled the amount of potential cyclists at the minimum. Moreover, the number of potential cyclists increased almost linearly with the budget, suggesting that, in the case study, further investments in bicycle infrastructure deliver similar value and keep increasing the number of potential cyclists with a safe and short route.

Fig. 8 presents an example of how the cycling path can change as the bicycle network is extended. The corresponding OD had a shortest path length of 4,250 m (2.64 mi, s_k), and the maximum allowed length for a bicycle safe path was 6,375 m (3.96 mi, $L_k = 1.5s_k$). On the current network, there was no safe bicycle path that was sufficiently short. However, the OD achieved a safe bicycle path of 6,009 m (3.73 mi, $1.41s_k$) when $B = 12.8$ km of roads were improved. Increasing the budget further to 19.2 km provided a shorter bicycle path of 4,976 m (3.09 mi, $1.17s_k$).

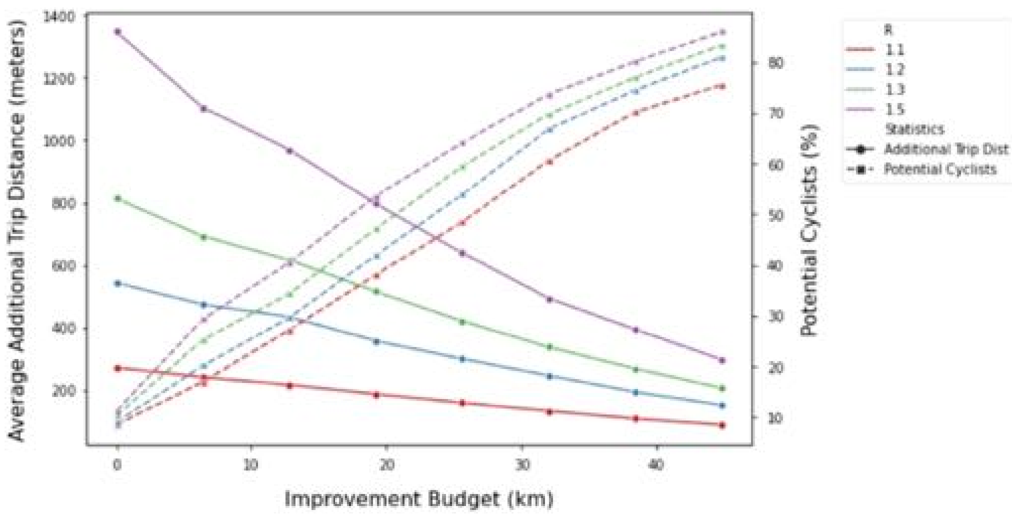

Fig. 7. Effectiveness of the BNIP improvements.

Fig. 8. Example routes on optimal improvement plans, R = 1.5. (©OpenStreetMap contributors.)

Benefits of Optimization

This section compares the optimal improvement plans (OPT) to those obtained with the greedy heuristic (HEU). For instance, Fig. 9, which includes 38.4 km (24 mi) HEU plans of four deviation factors, demonstrates that heuristic improvements focus less on constructing the backbone, compared to Fig. 10, and often unnecessarily provide multiple connections that serve a similar purpose. The optimal and heuristic plans were compared on the percentage of potential cyclists and the average additional trip distance. An example set of heuristic solutions for $R = 1.1$ is presented by Fig. 25 in the Appendix.

Individual Travel Comparison: By definition, a heuristic plan cannot be better than the optimal plan, but the heuristic may improve some trips more than the optimal plan. For example, Table 2 counts the number of travels with a smaller trip distance when $R = 1.1$: it shows that, for all budgets, the optimal plans produced shorter routes more often than the heuristic.

Fig. 11 shows a single OD that is evaluated both on the heuristic and optimal plan. The corresponding OD had a shortest path length of 3,784 m (2.35 mi, s_k), and the maximum allowed length for a bicycle safe path was $L_k = 1.3s_k$. The optimal plan provided a short route of 3,973 m (2.47 mi, $1.05s_k$) that is below the threshold distance. The heuristic also produced a safe path; however, the path length was 5,286 m (3.28 mi, $1.39s_k$), which exceeds the threshold, so the heuristic plan still required the outside option to serve the OD.

Potential Cyclists: Fig. 12 compares the heuristic plans and the optimal plans in terms of the number of potential cyclists. The results demonstrate that the optimal plans produced significant benefits in the number of potential cyclists. This is consistent over all budget values, and the difference may be more than 20%. This is a compelling demonstration of the value of sophisticated optimization for infrastructure improvement.

Average Additional Trip Distance: Fig. 13 compares the average additional trip distance of the optimal and heuristic plans for different values of B and R. Again, the optimal plans produced significant benefits compared to the heuristics for all budget values. They paralleled the improvements in potential cyclists and demonstrated the significant value of optimization for the BNIP.

Fig. 14 shows the distribution of additional trip distances of safe routes for the optimal and heuristic plans under two example settings. Among the safe routes, the heuristic plans tended to produce fewer trips with smaller additional trip distances. In contrast, the optimization plans, which optimized both the number of potential cyclists and average additional trip distances, had more trips with smaller additional trip distances as well as more feasible bicycle travels.

Sequential Incremental Improvements

In practice, policy makers cannot necessarily predict future availability of budget or other resources for the infrastructure development.

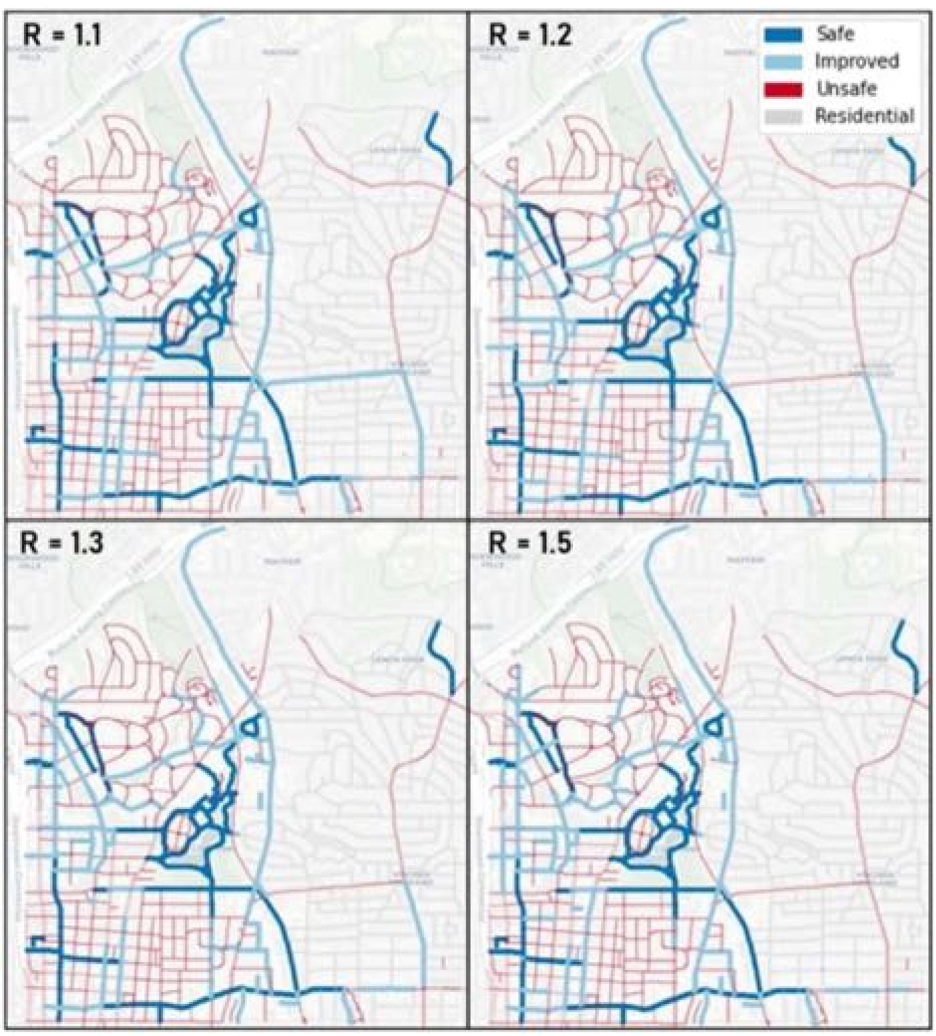

Fig. 9. Heuristic improvement plans, $B = 38.4$ km (24 mi). (©OpenStreetMap contributors.)

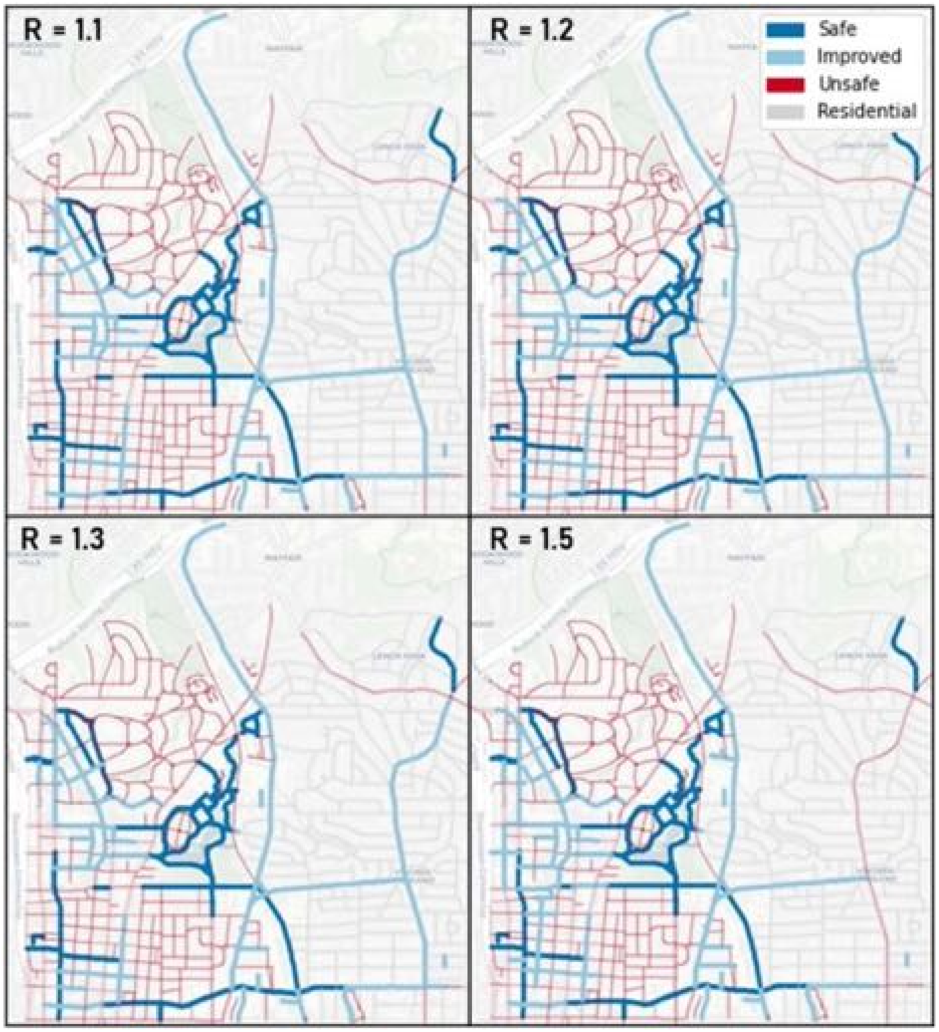

Fig. 10. Optimal improvement plans, $B = 38.4$ km (24 mi). (©OpenStreetMap contributors.)

Table 2. Travels with smaller penalties on different improvement plans

	Travel count		
B (km)	OPT map	HEU map	Equal
6.4	89	43	907
12.8	123	25	891
19.2	142	7	890
25.6	255	54	730
32.0	321	38	680
38.4	338	65	636
44.8	275	69	695

Therefore, the improvement plans may be prepared over time and not in advance. Furthermore, there may be an incentive to implement solutions that are optimal in the short term but may not necessarily be good in the long run. To investigate the effect of small myopic improvements, it is interesting to study the cumulative effect of a succession of individual improvement plans that optimally extend the bicycle infrastructure by 6.4 km at a time. Fig. 15 presents one example of these plans, which illustrates how the sequential approach progressively produced the same overall structure as the optimal plan using $R = 1.1$.

Fig. 16 shows that for all R, the additional average trip distances in the sequential approach were essentially similar to the overall optimal plans. This is of great practical importance because it indicates that incrementally improving the network over time is practically identical to a strategic planning approach.

Table 3 provides more details on the differences between the plans. The largest difference of additional trip distances between an optimal and a myopic plan was only 4,950 m in total, or 5 m per travel on average. The optimal and myopic plans did not lead to the same networks, but the difference in penalty was very small, especially as the network was improved further over time.

Uneven Improvement Costs

Another practical consideration for the BNIP is to consider improvement costs that depend on road properties, such as the number of lanes, pavement type, traffic volume, and so on. Uneven improvement costs can be supported easily by replacing the distance parameters d_{ij} in the budget constraint [Eq. (1b)] with some other cost parameters b_{ij} that combine multiple factors. Calculating the real cost for each way would take a significant amount of data and is out of the scope of this paper. However, a sensitivity analysis based on the road type is provided in this section.

For this analysis, the ways $(i, j) \in W'$ were partitioned into three distinct sets based on the *cycling-condition labels* retrieved from OpenStreetMap (2020). First, any road that had a cycling-related label, such as "cycleway" or "bicycle", but was known not to have a dedicated cycle lane was classified as a *bike-friendly* road,

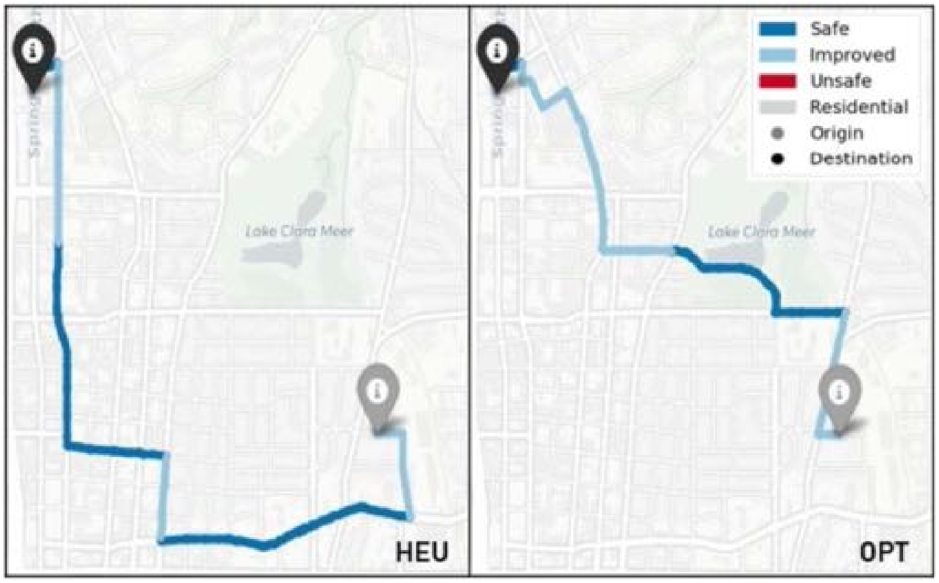

Fig. 11. Example routes on OPT and HEU maps, $R = 1.3$, $B = 38.4$ km (24 mi). (©OpenStreetMap contributors.)

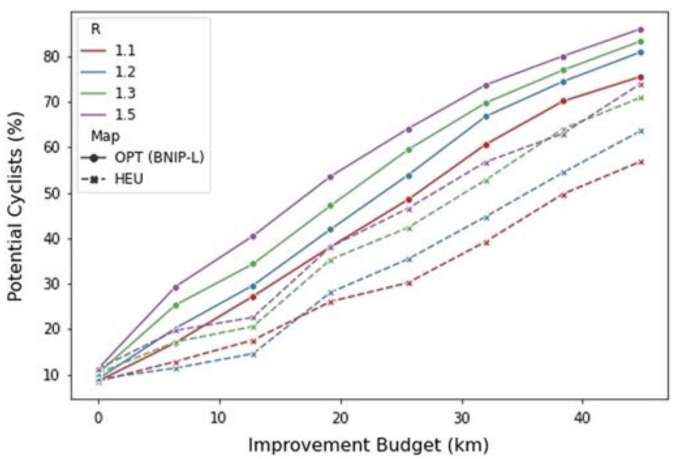

Fig. 12. Percentage of cyclists among population.

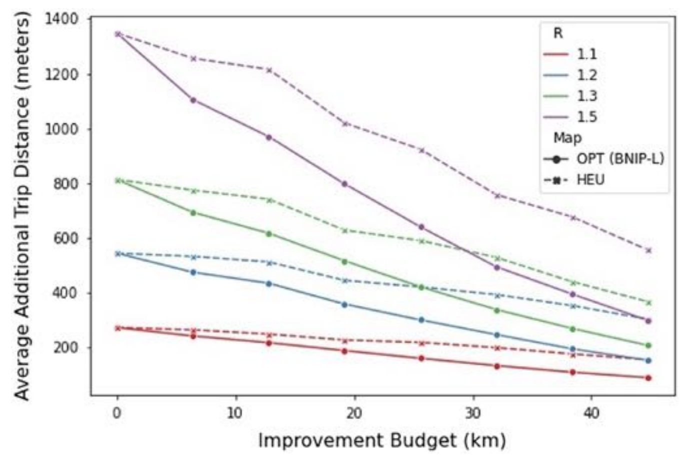

Fig. 13. Average additional trip distance.

and $b_{ij} = (1/2)d_{ij}$ was used as the weight. The remaining roads were ranked by the number of lanes and how important the road was in the local system based on the following labels: motorway, trunk, primary, secondary, tertiary, residential, and unclassified. The roads with the first four labels were classified as *significant* roads with higher improvement costs $b_{ij} = 2d_{ij}$, and the others were set to incur distance values as costs ($b_{ij} = d_{ij}$).

New improvement plans were prepared using these new costs for the same budget, under two deviation factors. Figs. 26 and 27 in the Appendix present these results. For the lower budget cases, it was observed that the algorithm took advantage of cheaper improvement costs by adding more lanes near the business area rather than improving major backbone roads. When the budget was increased, the results eventually converged to include all backbone roads, and then the maps were essentially identical to the original solutions. Figs. 17 and 18 further compare the additional average trip distances and percentage of potential cyclists for improvement plans designed by distance or road condition. The trends were very similar, which suggests that the results are robust to uneven improvement costs.

Alternative Penalty Functions

The results in the case study were generated with a linear penalty function, but other choices may lead to different bicycle network improvement plans. The BNIP supports different penalty functions, which makes it a flexible tool to use in practice. This section considers two alternative models aimed at minimizing the penalty of lost travelers and maximizing the number of cyclists, respectively. The former resulted in a piecewise linear penalty function, and the latter required a modification of the program formulation. Additional possibilities are discussed in the next section.

Minimizing the Penalty of Lost Cyclists

This section presents a model that minimized the penalty incurred by the travelers who chose *not* to use the improved bicycle network. Compared to the linear penalty functions, the new model specifically focused on potential cyclists who were lost to the system rather than on the average additional trip distance.

The new model assumed that potential cyclists had a probability to drop out that increased with the deviation u_k from the shortest

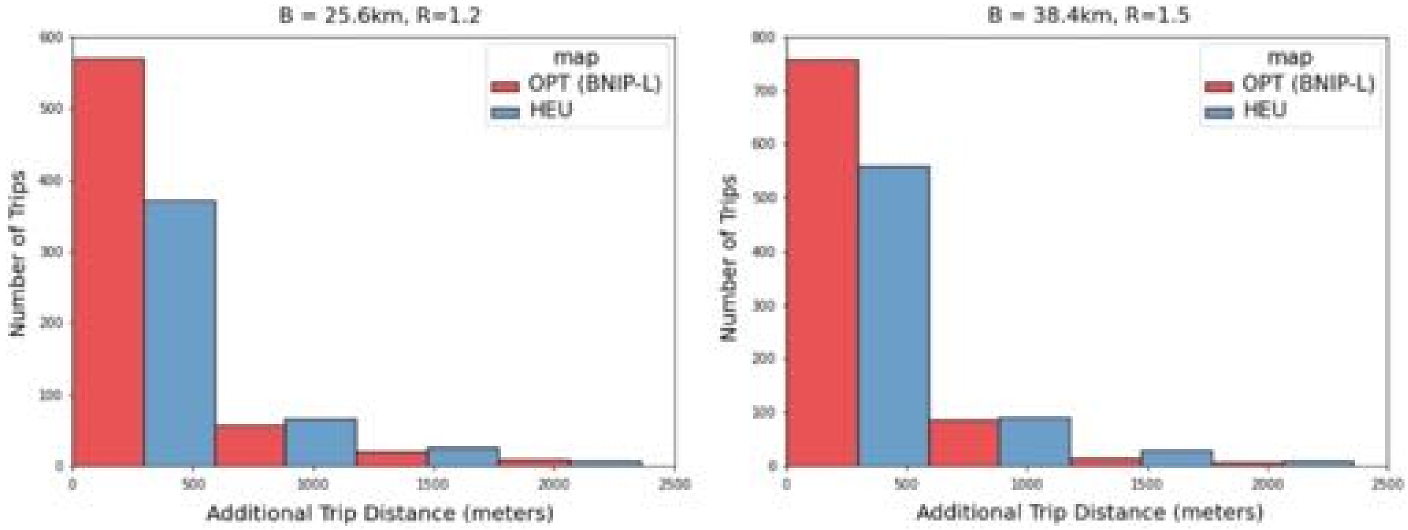

Fig. 14. Distribution of additional trip distances in bicycle travels.

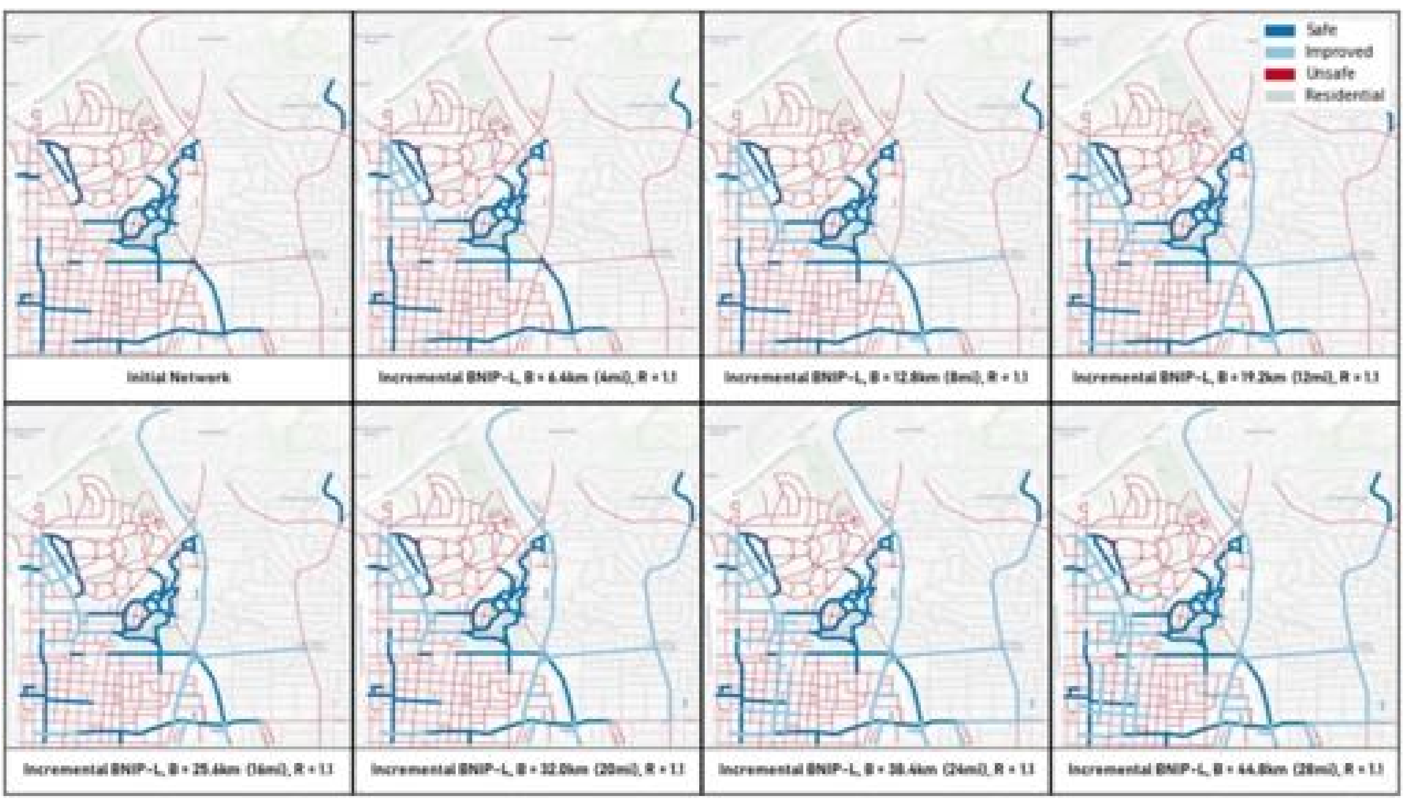

Fig. 15. Sequential incremental improvement plans using BNIP-L, $R = 1.1$. (©OpenStreetMap contributors.)

safe trip. For short deviations $u_k \leq 0.2s_k$, the probability was assumed to be zero, and no penalty was incurred. After that, the dropout probability increased linearly until the maximum trip length of $L_k = 1.5s_k$ was reached, at which point all travelers chose the outside option. The expected penalty for lost travelers at deviation $u_k \in [0.2s_k, 0.5s_k]$ followed from multiplying the dropout penalty of $u_k = L_k - s_k$ by the probability of dropping out. This results in the following penalty function:

$$f_k^P(u_k) = \begin{cases} 0 & \text{if } u_k \leq 0.2s_k, \\ \frac{5}{3}u_k - \frac{1}{3}s_k & \text{if } 0.2s_k \leq u_k \leq 0.5s_k \end{cases} \quad (8)$$

The value of f_k^P is piecewise linear and convex. The endpoint $f_k^P(0.5s_k) = 0.5s_k$ corresponds to a 100% dropout rate and penalty $L_k - s_k = 0.5s_k$.

To use f_k^P in the Benders decomposition algorithm, the master problem defines continuous υ-variables, uses $f_k^P(\upsilon_k) = \upsilon_k$ in the objective, and includes the following two additional constraints:

$$\upsilon_k \geq 0 \quad \forall\ k \in T \quad (9)$$

$$\upsilon_k \geq \frac{5}{3}u_k - \frac{1}{3}s_k \quad \forall\ k \in T \quad (10)$$

The optimization with piecewise linear penalty function f_k^P is labeled BNIP-P, and Fig. 28 in the Appendix presents the full series of the BNIP-P improvement plans.

Improvement Results: The BNIP-P improvement plans were compared with the BNIP-L plans for $R = 1.5$, which had equal travel distance thresholds.

Table 4 summarizes the improvements using each model. The two penalty functions produced very similar improvement plans; the percentage of network difference—a sum of percentages of unique improvement lengths on each improvement plan—shows that the two plans converged as more budgets were allowed.

The deviation of an individual traveler was shorter for the BNIP-L, which was anticipated because the BNIP-P assigned an equal penalty for short routes and focused more on increasing the probability of bicycle participation. However, the difference in cyclist percentages between the two optimized networks was very small. This is an important observation because it shows that the linear objective did not forfeit the advantage of the piecewise objective.

Maximizing the Number of Cyclists

Rather than assuming a probabilistic model for cyclists dropping out, it is also possible to directly maximize the number of potential cyclists, that is, to maximize the number of ODs with a short and safe trip or, equivalently, minimize the use of the outside option z_k. The base formulation was modified to accommodate that objective, labeled BNIP-Z, and the change is as follows:

$$\min \sum_{k \in T} p_k z_k \quad (11a)$$

$$\text{s.t.} \sum_{(i,j) \in W'} d_{ij} y_{ij} \leq B \quad (11b)$$

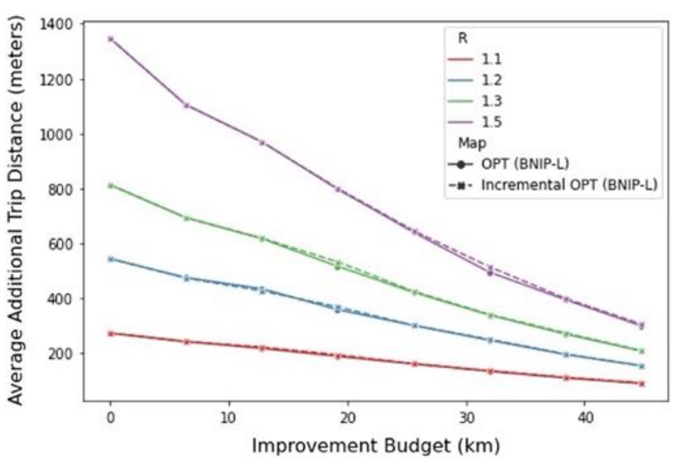

Fig. 16. Average additional trip distance for the sequential improvement approach.

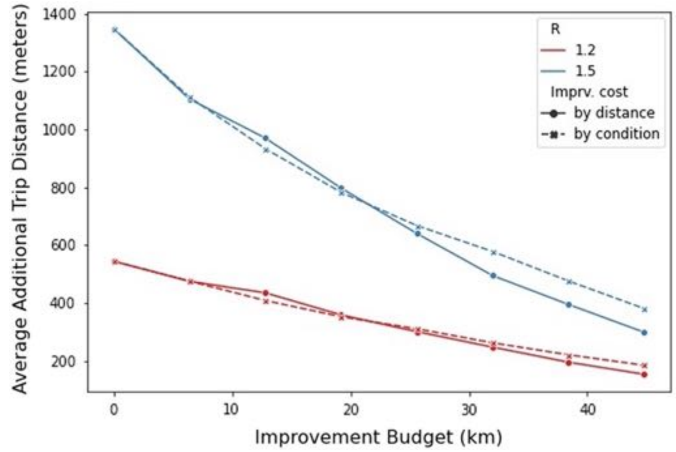

Fig. 17. Average additional trip distance with uneven improvement costs.

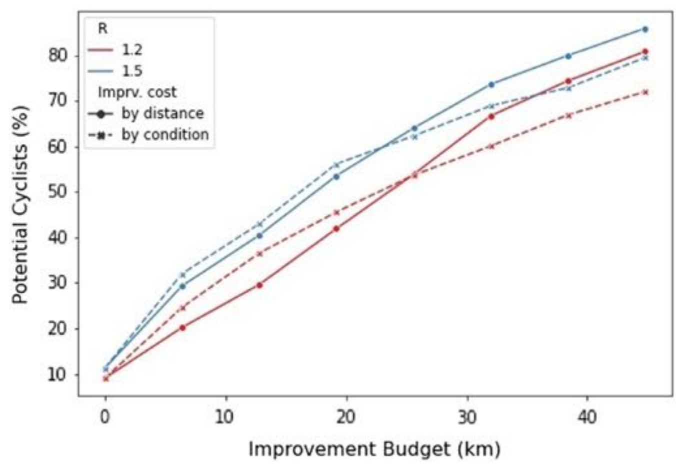

Fig. 18. Percentage of cyclists among population with uneven improvement costs.

Table 4. Network improvement comparison of BNIP-P and BNIP-L ($R = 1.5$)

B (km)	Network difference (%)	Average additional distance (m) BNIP-P	Average additional distance (m) BNIP-L	Cyclist percentage BNIP-P (%)	Cyclist percentage BNIP-L (%)
6.4	0	1,105	1,105	29.36	29.36
12.8	4.20	974	970	41.00	40.52
19.2	8.56	972	797	53.90	53.61
25.6	8.66	870	640	65.06	64.10
32.0	3.86	515	494	75.65	73.72
38.4	3.00	408	394	81.61	80.08
44.8	2.94	672	298	87.68	86.04

Table 3. Strategic planning versus incremental improvements: average additional trip distance (m), $R = 1.1$

	B						
Improvement plans	6.4 km	12.8 km	19.2 km	25.6 km	32.0 km	38.4 km	44.8 km
Strategic planning	40.49	65.07	93.90	122.12	148.88	173.10	192.99
Sequential approach	40.49	61.22	89.13	121.54	147.64	171.40	191.56
Difference	0%	5.92%	5.07%	0.48%	0.83%	0.98%	0.74%

$$\sum_{(i,j)\in W} x_{ij}^k - \sum_{(j,i)\in W} x_{ji}^k = \begin{cases} 1-f_k & \text{if } i = o_k \\ f_k - 1 & \text{if } i = d_k \\ 0 & \text{otherwise} \end{cases} \quad \forall\ k \in T, i \in V \tag{11c}$$

$$x_{ij}^k \le y_{ij} \quad \forall\ k \in T, (i,j) \in W' \tag{11d}$$

$$\sum_{(i,j)\in W} d_{ij} x_{ij}^k + (L_k + M_k) f_k \le L_k + M_k z_k \quad \forall\ k \in T \tag{11e}$$

$$y_{ij} \in \mathbb{B} \quad \forall\ (i,j) \in W' \tag{11f}$$

$$x_{ij}^k \in \mathbb{B} \quad \forall\ k \in T, (i,j) \in W \tag{11g}$$

$$f_k \in \mathbb{B} \quad \forall\ k \in T \tag{11h}$$

$$z_k \in \mathbb{B} \quad \forall\ k \in T \tag{11i}$$

The objective in Eq. (11a) minimizes the use of the outside option on the network. A binary variable f_k is introduced for every trip $k \in T$ to ensure complete recourse, and the f-variables replace the z-variables in Eq. (11c). That is, if no safe bicycle path exists, f_k can be set to one instead. The constraint in Eq. (11e) uses the constant $M_k > 0$ to ensure that the outside-option variables z_k are set correctly: preferably, the outside option is not used ($z_k = 0$), which implies that $f_k = 0$ and that the x-variables represent a safe and short bicycle path. If the outside option is used, setting $f_k = 1$ removes this requirement. As a result, BNIP-Z indeed minimizes the use of the outside option. Finally, the constraint in Eq. (11h) defines the newly added variables.

It is notable that the Benders decomposition solution method can be used for the BNIP-Z. The modified master problem for the BNIP-Z is as follows:

$$\begin{aligned} &\min\,(11a), \\ &\text{s.t. } (11b), (11f), (11i), \\ &\quad \Psi_k(y) \le L_k + M_k z_k \quad \forall\ k \in T \end{aligned} \tag{12}$$

The modified subproblem for the BNIP-Z is as follows:

$$\begin{aligned} \Psi_k(y) = \min \sum_{(i,j)\in W} d_{ij} x_{ij}^k + (L_k + M_k) f_k, \\ \text{s.t. } (11c), (11d), (11g), (11h) \end{aligned} \tag{13}$$

The new function calculating the shortest path, $\Psi_k(y)$, is introduced by the modified subproblem. The master problem now includes the z-variables for the minimization of the objective in Eq. (11a). The subproblem can be solved by linear programming because it is a standard minimum-cost flow problem. Pareto-optimal cuts and two-phase Benders can be used in the same way as before, and both y-variables and z-variables may be relaxed in phase one.

Fig. 29 in the Appendix presents the full series of the BNIP-Z improvement plans using $L_k = 1.5 s_k$.

Improvement Results: Due to their objectives, the BNIP-L produced shorter deviations in average in its routes, and the BNIP-Z collected more potential cyclists. Nevertheless, Fig. 19 illustrates that the difference of performances between the two programs is inconsequential. This shows that the use of the linear penalty, which examines both travel safety and proximity, did not sacrifice cyclists count to provide shorter deviation for riders.

Efficiency of Alternative Models

To evaluate the methodological efficiency of the three BNIP formulations, the optimality gap and the upper and lower bounds of the Benders solutions were compared over time. Using algorithm MW-McD, the BNIP-Z had the fastest computation times in many cases, as shown in Fig. 20. The BNIP-L also exhibited fast convergence to a small optimality gap for most cases. This is noteworthy because BNIP-L considers both traveler safety and deviation.

All three formulations reached optimality within a reasonable time, and they are shown to produce equally attractive improvement plans. Recall the BNIP-Z disregards travel distances, and the BNIP-P has a piecewise linear objective that is harder to interpret. The linear objective of the BNIP-L reports the quality of the network in distance values, allowing for a more direct and meaningful evaluation of the network.

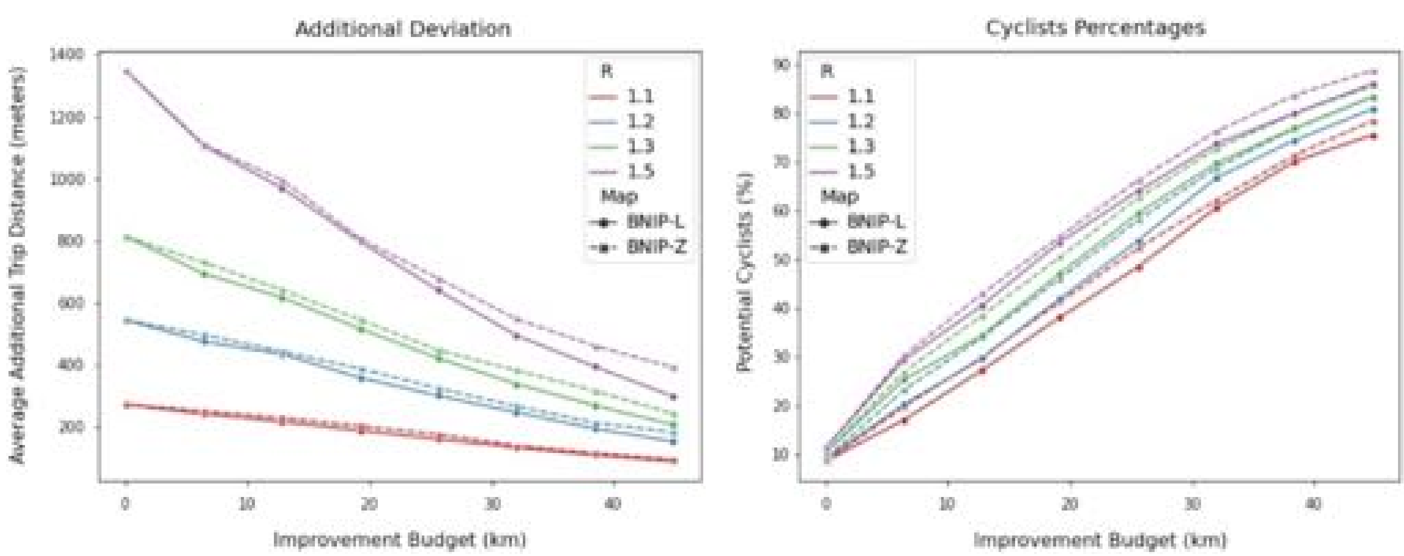

Fig. 19. Performances of the BNIP-L and BNIP-Z.

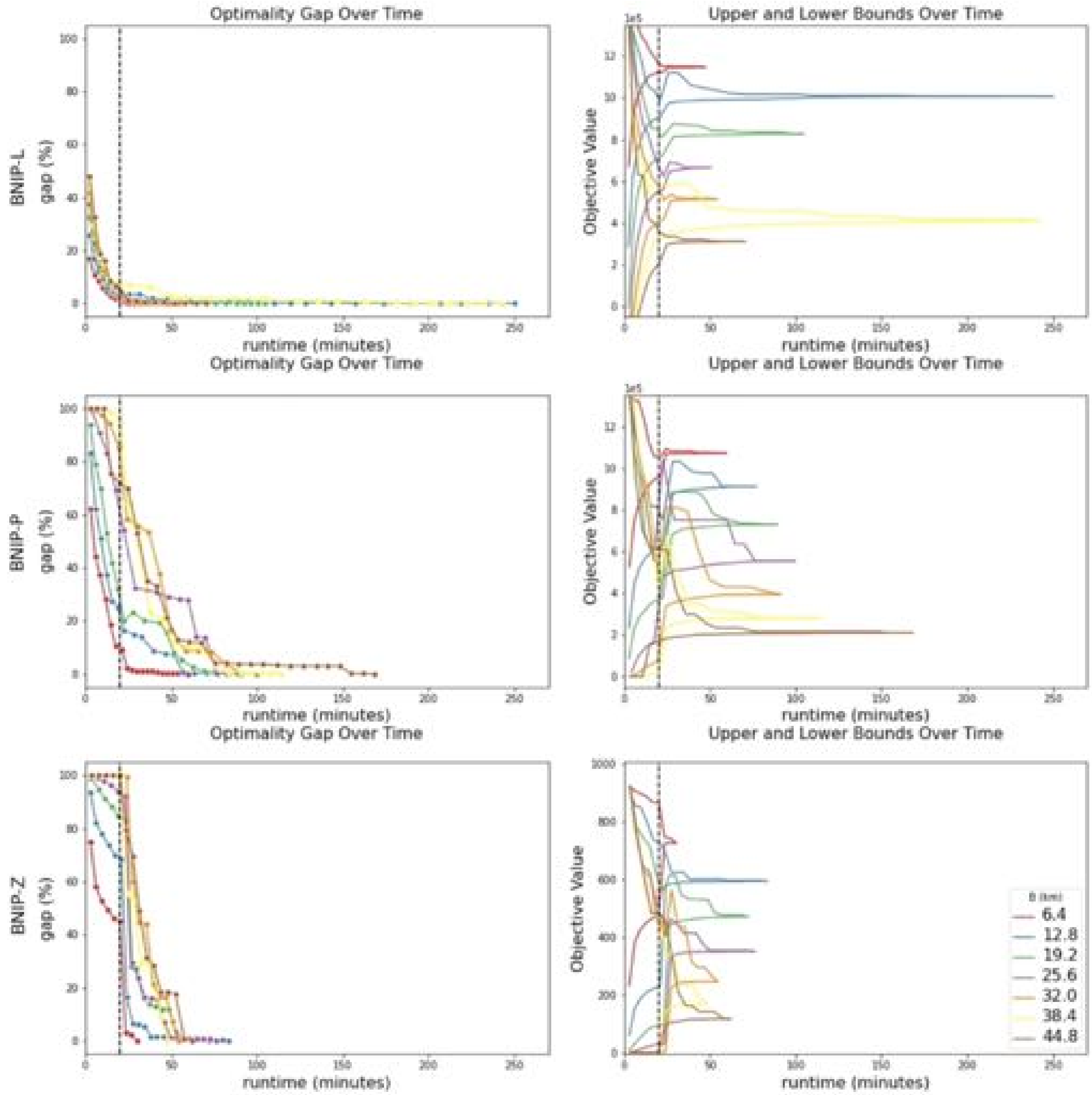

Fig. 20. Benders decomposition efficiency of the BNIP with different objectives, $L_k = 1.5s_k$.

Discussion

The models in this paper focus on safety and travel distance, which are among the most important determinants for bicycle commuting (Heinen et al. 2010; Cervero et al. 2019; Ospina et al. 2020). The case study is based on realistic data, and the conclusions are robust under uneven improvement costs and alternative penalty functions. This section discusses how the models can be extended to accommodate additional data through alternative penalty functions, cost structures, and choice models.

Penalty Functions: The BNIP defined a term $p_k f_k(u_k)$ in Eq. (1*a*) for every trip $k \in T$. The parameter p_k models the number of travelers completing this travel, but it can be used more generally to model any nonnegative trip weights. This allows policy makers to assign more weight to certain areas or certain subsets of the population. From a model perspective, the only real requirement on the penalty function is that f_k is nondecreasing [the requirement $f_k(0) = 0$ is not restrictive]. The penalty function is trip specific, which means that external data (e.g., census data) can be used to help shape this function.

From a computational perspective, the methods in this paper are expected to be effective for two general classes of penalty functions. The first class is that of convex penalty functions, which includes both BNIP-L (linear) and BNIP-P (piecewise linear).

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Linear and quadratic penalty functions can be handled directly by modern solvers (as for BNIP-L), and piecewise-linear functions can be modeled with additional variables and constraints [similar to Eqs. (9) and (10) for BNIP-P]. General convex functions can be handled with a classical cutting plane method (Kelley 1960). The second class is that of MIP representable functions, that is, functions that can be modeled with mixed continuous and integer variables and additional constraints. BNIP-Z falls in this class because it uses binary variables z to indicate whether a traveler cycles. Modeling details are discussed by Croxton et al. (2003). Alternative penalty functions only affect how the master problem is solved, and the overall method stays the same. As such, similar computational performance is expected when the number of additional variables and constraints is small.

Cost Structures: The BNIP includes two types of costs: the path length l_k of trip $k \in T$, and the cost d_{ij} for improving way $(i,j) \in W'$. Both are currently based on travel distance, but they can be used to reflect any combination of attributes.

Path length can be replaced by a more general path cost. The path cost c_k is defined as the sum of a trip-specific constant α_k and trip-specific cost parameters $c^k_{ij} \geq 0$ for every way $(i,j) \in W$ on the path. The cost per way may combine any number of properties, including traffic stress, road gradient, activity density, mean rainfall, and even trip-specific sociodemographic attributes. Many of these attributes are discussed by Cervero et al. (2019). The maximum acceptable length L_k is replaced by the maximum acceptable cost C_k, accordingly. Formulation is updated by replacing Eq. (1*e*) by

$$u_k \geq \sum_{(i,j)\in W} c^k_{ij} x^k_{ij} + C_k z_k + \alpha_k \quad \forall\ k \in T \tag{14}$$

The original model reappears when the cost is distance ($c^k_{ij} = d_{ij}, C_k = L_k$), and the constant $\alpha_k = -s_k$ is used to calculate the deviation from the shortest path. Switching from path length to path cost makes the model more expressive and allows for including additional data without affecting the solution method.

A similar argument can be made for the cost of improving ways. The distance d_{ij} in the budget constraint in Eq. (1*b*) may be replaced by a general budget cost, as is done in the section "Uneven Improvement Costs" to account for different road types. Furthermore, additional constraints may be added without affecting the solution method. For example, it can be enforced that the budget be spread out evenly over different areas.

Choice Models: This paper uses a simple choice model based on distance: if path length $l_k \leq L_k$, then trip $k \in T$ is completed by cycling, and if $l_k > L_k$, the outside option is used. Based on the previous discussion, a more expressive choice model is also supported: if cost $c_k \leq C_k$, then trip $k \in T$ is completed by cycling, and the outside option is used if $c_k > C_k$.

More generally, the methods in this paper support *any* classifier that predicts cycling when $c_k \leq C_k$ for some cut-point C_k and no cycling otherwise. This includes logit and machine learning models (Zhao et al. 2019). For example, the logistic regression model (Kleinbaum and Klein 2010) is given by

$$\pi(c_k) = \frac{1}{1 + e^{c_k}} \tag{15}$$

where $\pi(c_k)$ = probability of using the outside option. Fitting the model amounts to combining the different attributes into a cost c_k that best explains traveler behavior. The model predicts cycling when the probability of using the outside option is low, that is, $\pi(c_k) \leq \Pi$ for some value Π. This probability cut-point can be translated into a cut-point for the traveler cost

$$\pi(c_k) \leq \Pi \Leftrightarrow c_k \leq \log\left(\frac{\Pi}{1 - \Pi}\right) = C_k \tag{16}$$

This results in a setting that is directly supported by the methods in this paper.

Conclusion

Cycling brings many benefits to both the cyclists and society as a whole, and the emergence of e-bikes may make this transportation mode attractive for a larger population segment. However, safety is a critical issue faced by commuters when deciding their transportation mode. This paper considered the problem of improving the bicycle infrastructure to allow more people to travel by bicycle. This optimization problem was formalized as the bicycle network improvement problem. As opposed to the literature, the BNIP supports a budget for improvement, provides completely safe routes, allows full flexibility in routing cyclists, and has an exact solution approach. Solving the BNIP directly is computationally intractable for large instances, so the paper presented a Benders decomposition to remedy this issue by exploiting the problem structure and considering each rider independently in the subproblems.

The paper demonstrated the effectiveness of the method on an in-depth case study for Midtown Atlanta, based on real transportation data of white-collar commuters from Virginia-Highland. The computational results show that the proposed Benders decomposition algorithm with Pareto-optimal cuts and two-phase Benders was very effective in solving the realistic case study instances. Further analysis revealed that the optimal bicycle network improvement plans for Midtown Atlanta were very powerful in providing access to safe and short bicycle routes. The increase in the number of travelers with access to a safe and short trip was almost linear in the available budget, indicating that more investments in bicycle infrastructure may keep attracting additional commuters to switch to cycling. The Benders decomposition method was compared to a greedy heuristic and shown to lead to significantly better plans, which shows the value of optimization to produce mathematically optimal solutions.

The paper also considered practical aspects of the bicycle network extension. It showed that repeated myopic extensions of the network led to an almost optimal result in the long run. This is of great practical importance because it indicates that myopically improving the network over time is practically identical to in-advance planning. It was shown that changing the road improvement costs to take the type of road into account did not affect the main conclusions of this paper. In addition, the paper demonstrated that the results are robust with respect to different objective functions.

Future work may incorporate additional data and more complicated choice models into the BNIP. The "Discussion" section provides guidance on how this may be done. Another interesting direction is to investigate how to extend bicycle network improvement to multimodal transit systems, where the goal is not necessarily to offer a bicycle path for the complete trip but to let bicycles play a role in addressing the ubiquitous first and last mile problem.

Appendix. Bicycle Network Improvement Plans

This section presents all bicycle network improvement plans created for the case study. In Figs. 21–29, existing bicycle infrastructure is indicated by blue roads, and light-blue roads indicate the proposed expansion. Red roads remain unsafe, and gray roads are residential roads that are safe to use without improvement.

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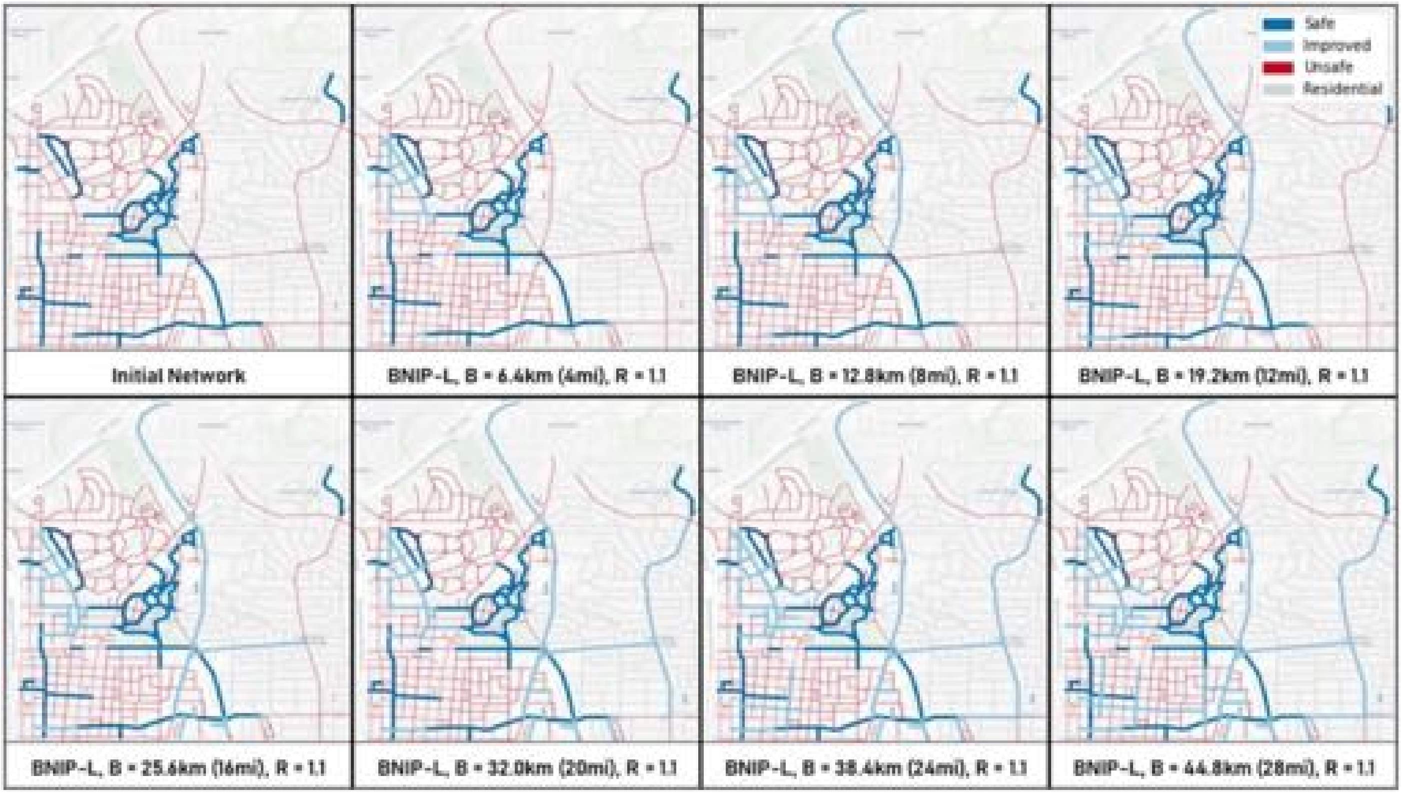

Fig. 21. Optimal bicycle network improvement plans using BNIP-L, $R = 1.1$. (©OpenStreetMap contributors.)

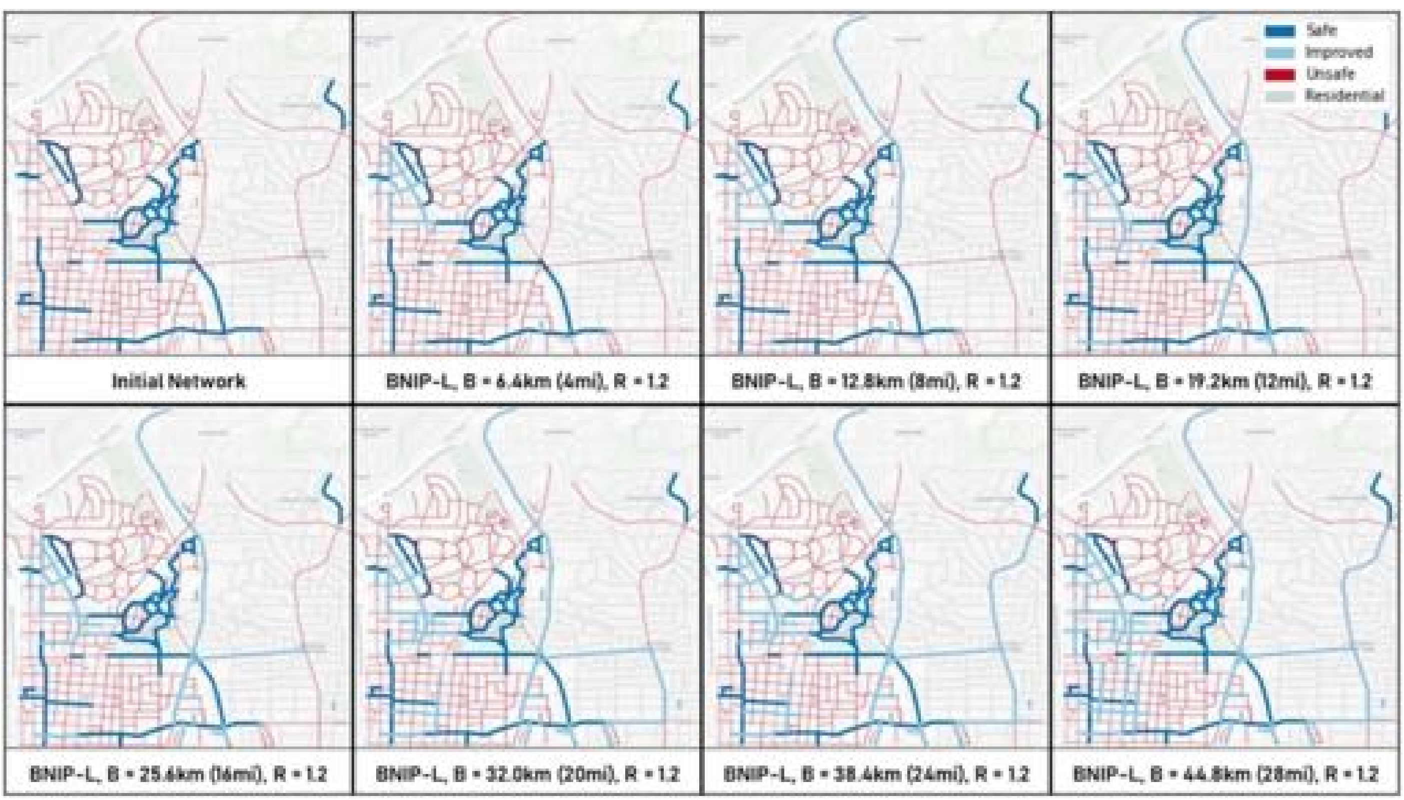

Fig. 22. Optimal bicycle network improvement plans using BNIP-L, $R = 1.2$. (©OpenStreetMap contributors.)

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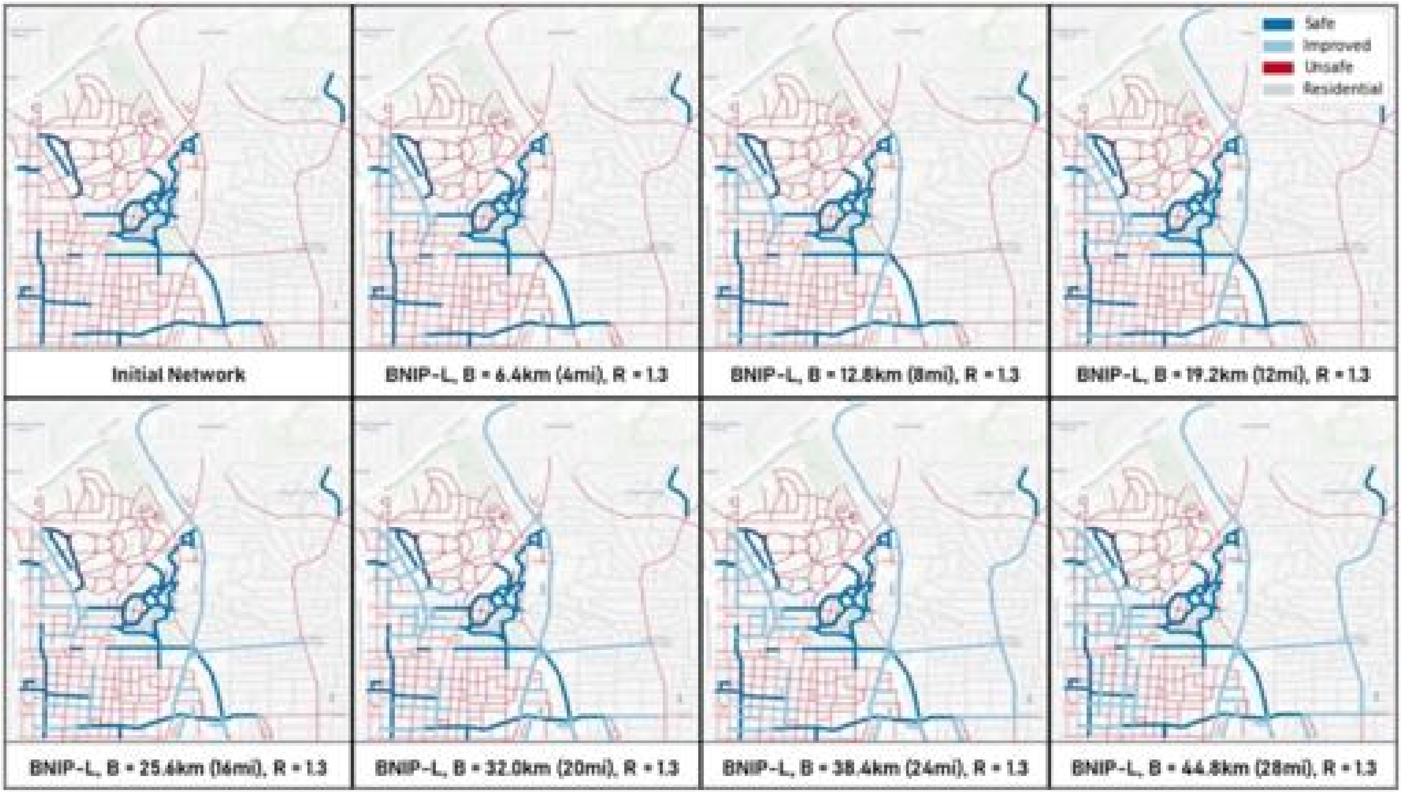

Fig. 23. Optimal bicycle network improvement plans using BNIP-L, $R = 1.3$. (©OpenStreetMap contributors.)

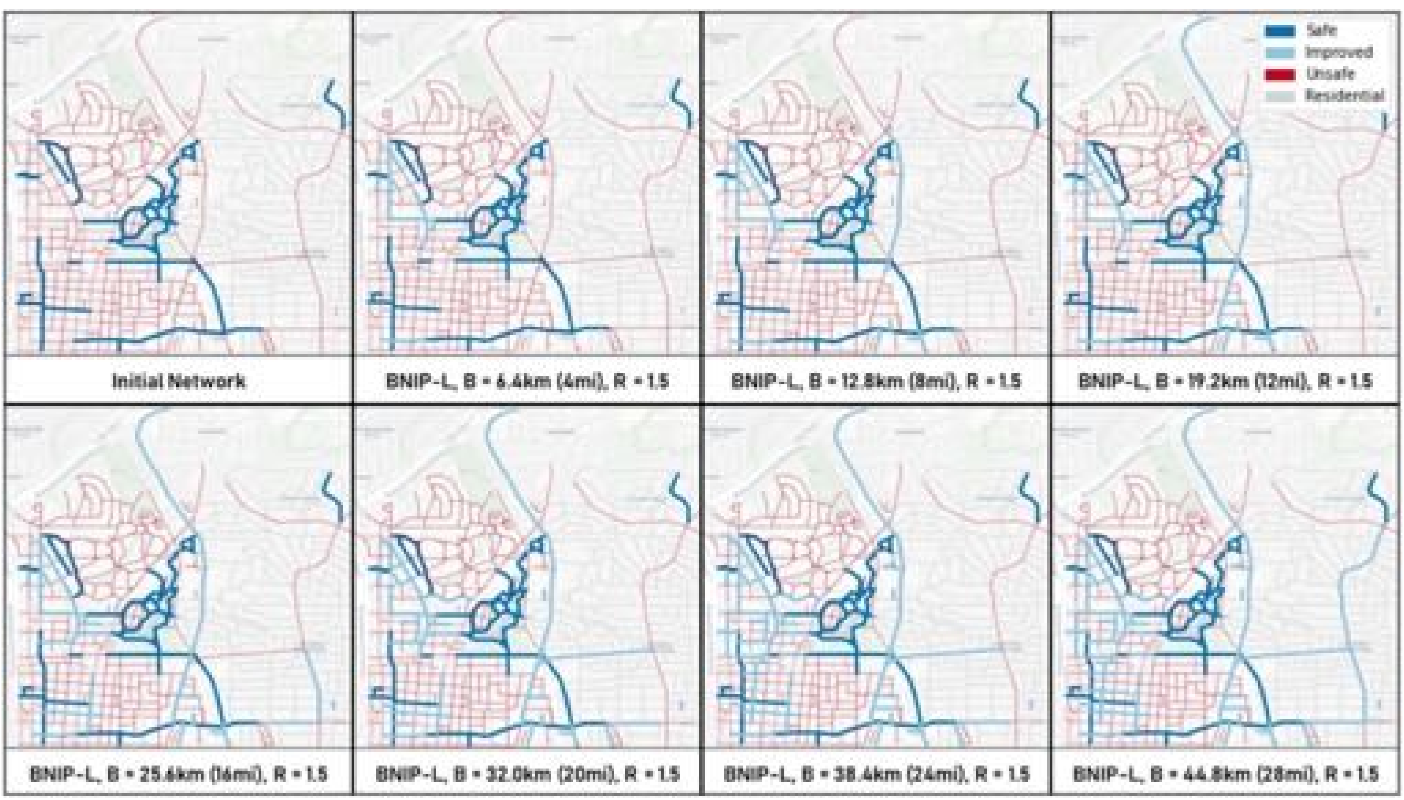

Fig. 24. Optimal bicycle network improvement plans using BNIP-L, $R = 1.5$. (©OpenStreetMap contributors.)

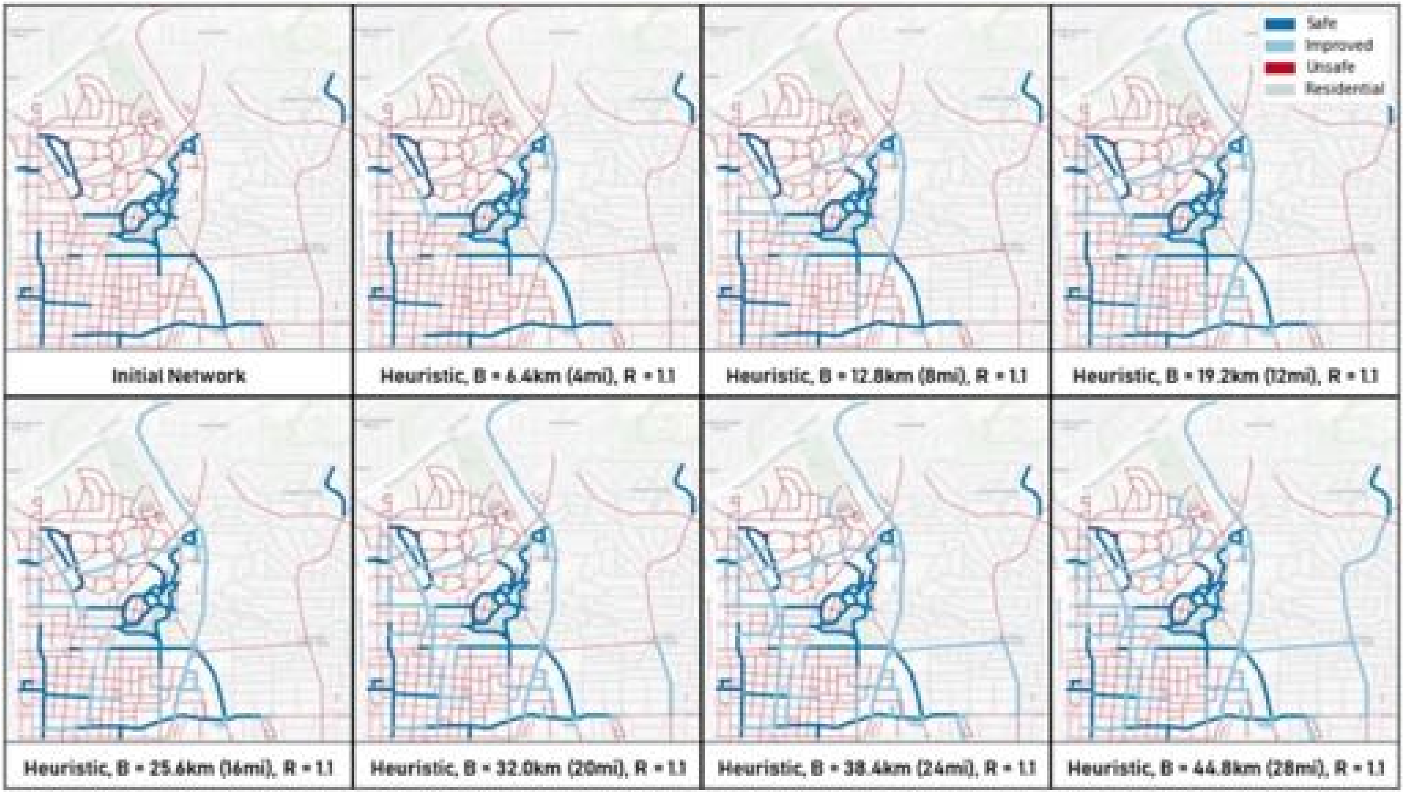

Fig. 25. Heuristic bicycle network improvement plans $L_k = 1.1 s_k$. (©OpenStreetMap contributors.)

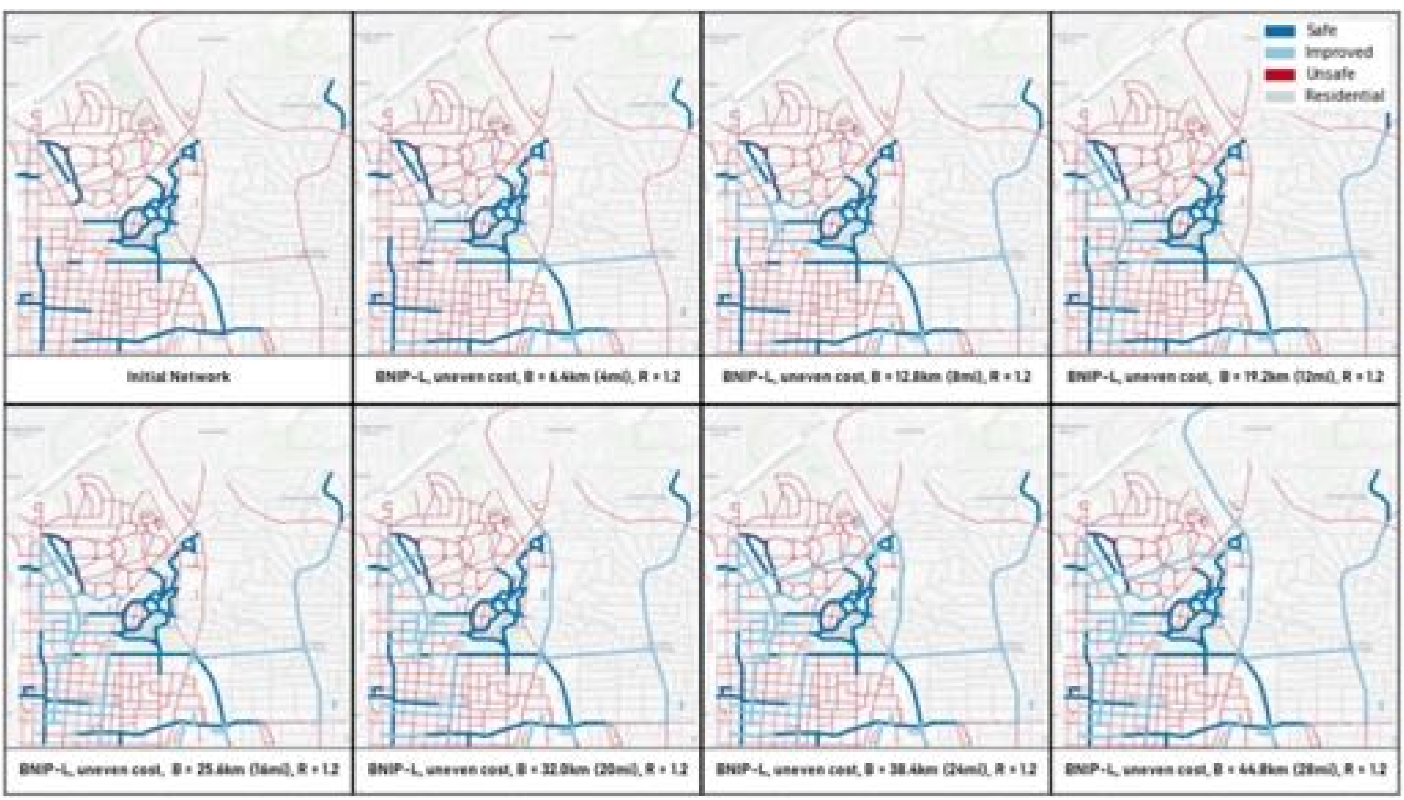

Fig. 26. Optimal bicycle network improvement plans using BNIP-L and uneven improvement costs, $R = 1.2$. (©OpenStreetMap contributors.)

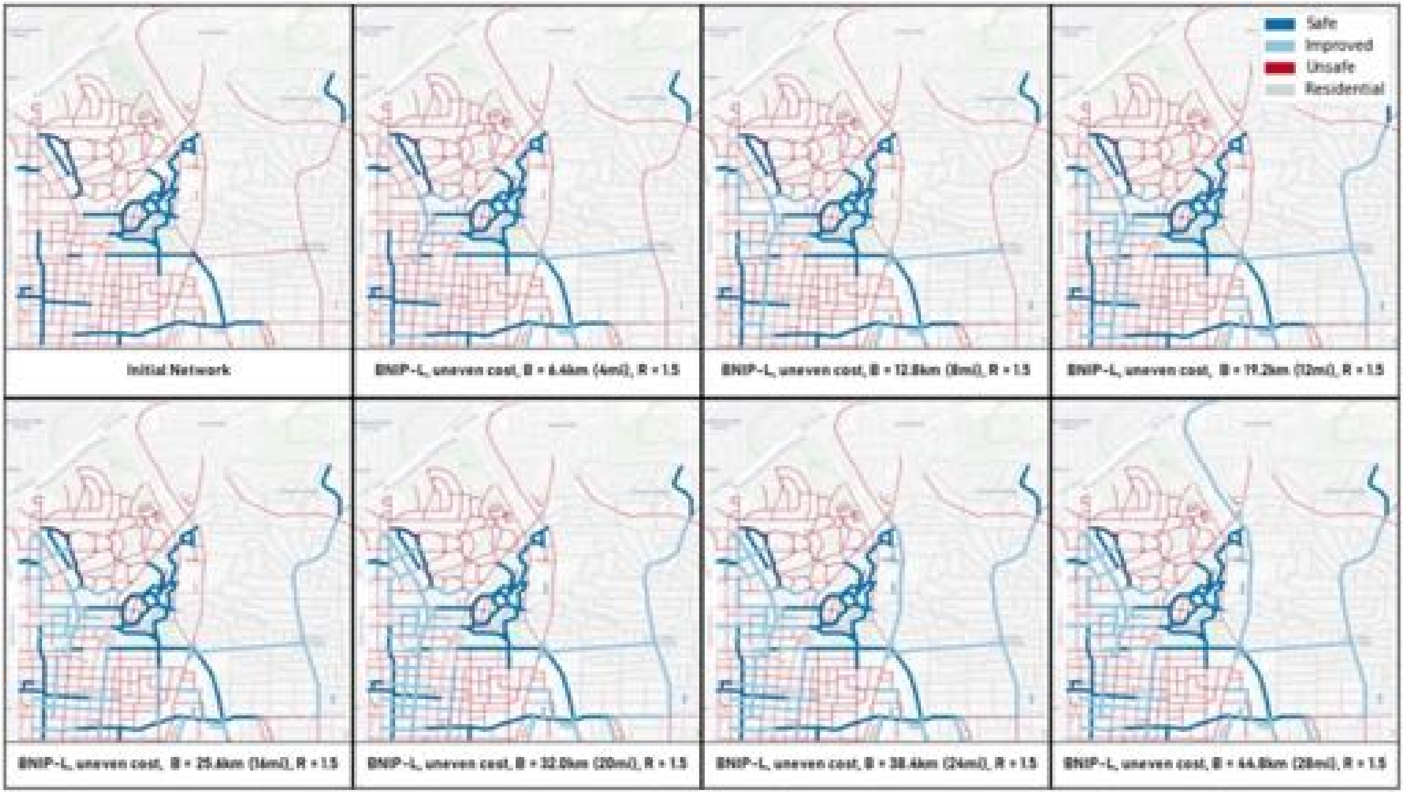

Fig. 27. Optimal bicycle network improvement plans using BNIP-L and uneven improvement costs, $R = 1.5$. (©OpenStreetMap contributors.)

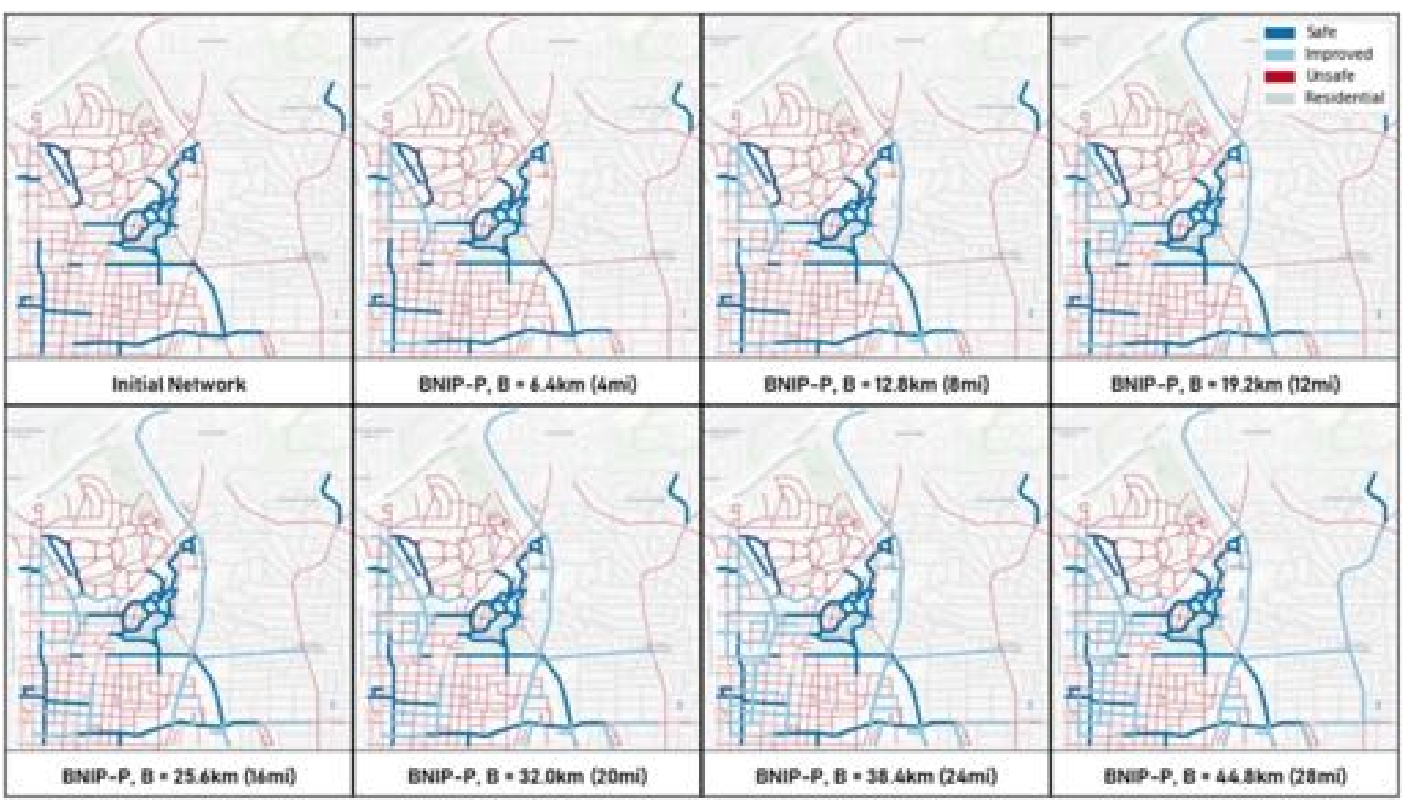

Fig. 28. Optimal bicycle network improvement plans using BNIP-P. (©OpenStreetMap contributors.)

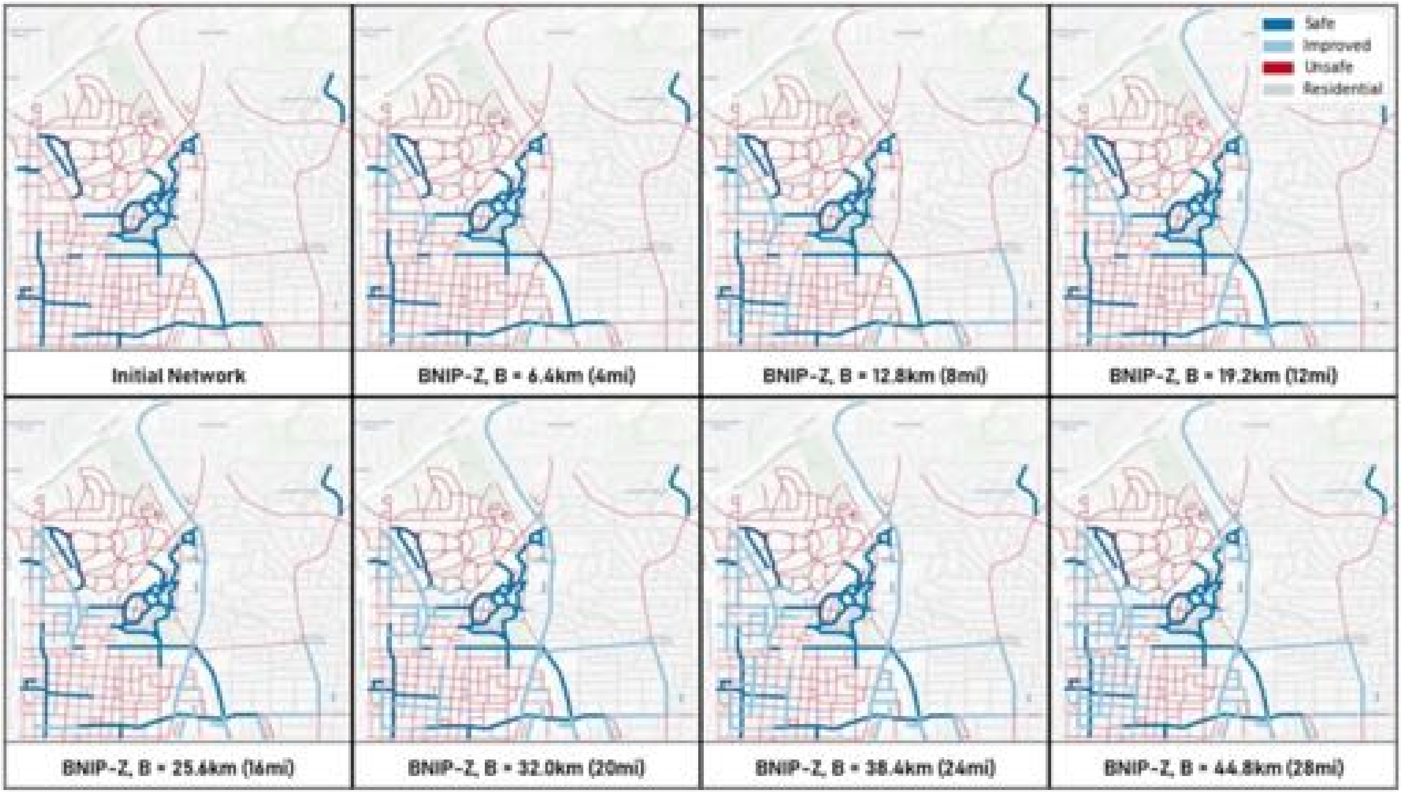

Fig. 29. Optimal bicycle network improvement plans using BNIP-Z, $L_k = 1.5s_k$. (©OpenStreetMap contributors.)

Data Availability Statement

Some or all data, models, or code generated or used during the study are available in a repository or online in accordance with funder data retention policies (Atlanta Regional Commission 2017).

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References

Atlanta Regional Commission. 2017. "Activity based modeling." Accessed March 6, 2020. https://atlantaregional.org/transportation-mobility/modeling/modeling/.

Atlanta Regional Commission. 2020. "Bike-pedestrian plan: Walk, bike, thrive!" Accessed December 5, 2020. https://atlantaregional.org/plans-reports/bike-pedestrian-plan-walk-bike-thrive/.

Bao, J., T. He, S. Ruan, Y. Li, and Y. Zheng. 2017. "Planning bike lanes based on sharing-bikes' trajectories." In *Proc., 23rd ACM SIGKDD Int. Conf. on Knowledge Discovery and Data Mining*, 1377–1386. New York: Association for Computing Machinery.

Benders, J. F. 1962. "Partitioning procedures for solving mixed-variables programming problems." *Numerische Mathematik* 4 (1): 238–252. https://doi.org/10.1007/BF01386316.

Bottoms, K. L. 2018. "City of Atlanta 2018 annual bicycle report." Accessed December 10, 2020. https://www.atlantaga.gov/home/showdocument?id=40599.

Bryant, M. 2020. "Cycling 'explosion': Coronavirus fuels surge in US bike ridership." Accessed December 8, 2020. https://www.theguardian.com/us-news/2020/may/13/coronavirus-cycling-bikes-american-boom.

Cervero, R., S. Denman, and Y. Jin. 2019. "Network design, built and natural environments, and bicycle commuting: Evidence from British cities and towns." *Transp. Policy* 74 (Feb): 153–164. https://doi.org/10.1016/j.tranpol.2018.09.007.

CIVITAS Initiative. 2020. "Smart choices for cities: Cycling in the city." Accessed December 10, 2020. https://ec.europa.eu/transport/sites/transport/files/cycling-guidance/smart_choices_for_the_city_cycling_in_the_city_0.pdf.

Croxton, K. L., B. Gendron, and T. L. Magnanti. 2003. "A comparison of mixed-integer programming models for nonconvex piecewise linear cost minimization problems." *Manage. Sci.* 49 (9): 1268–1273. https://doi.org/10.1287/mnsc.49.9.1268.16570.

Dill, J., and T. Carr. 2003. "Bicycle commuting and facilities in major U.S. cities: If you build them, commuters will use them." *Transp. Res. Rec.* 1828 (1): 116–123. https://doi.org/10.3141/1828-14.

Dill, J., and N. McNeil. 2016. "Revisiting the four types of cyclists: Findings from a national survey." *Transp. Res. Rec.* 2587 (1): 90–99. https://doi.org/10.3141/2587-11.

Duthie, J., and A. Unnikrishnan. 2014. "Optimization framework for bicycle network design." *J. Transp. Eng.* 140 (7): 04014028. https://doi.org/10.1061/(ASCE)TE.1943-5436.0000690.

Gadsby, A., M. Hagenzieker, and K. Watkins. 2021. "An international comparison of the self-reported causes of cyclist stress using quasi-naturalistic cycling." *J. Transp. Geogr.* 91 (Feb): 102932. https://doi.org/10.1016/j.jtrangeo.2020.102932.

Gössling, S. 2020. "Integrating e-scooters in urban transportation: Problems, policies, and the prospect of system change." *Transp. Res. Part D Transp. Environ.* 79 (Feb): 102230. https://doi.org/10.1016/j.trd.2020.102230.

Handy, S., B. van Wee, and M. Kroesen. 2014. "Promoting cycling for transport: Research needs and challenges." *Transp. Rev.* 34 (1): 4–24. https://doi.org/10.1080/01441647.2013.860204.

Heinen, E., B. van Wee, and K. Maat. 2010. "Commuting by bicycle: An overview of the literature." *Transp. Rev.* 30 (1): 59–96. https://doi.org/10.1080/01441640903187001.

Hsu, T.-P., and Y. T. Lin. 2011. "A model for planning a bicycle network with multi-criteria suitability evaluation using GIS." *WIT Trans. Ecol. Environ.* 148 (Oct): 243–252. https://doi.org/10.2495/rav110231.

Hull, A., and C. O'Holleran. 2014. "Bicycle infrastructure: Can good design encourage cycling?" *Urban Plann. Transp. Res.* 2 (1): 369–406. https://doi.org/10.1080/21650020.2014.955210.

Kelley, J. E., Jr. 1960. "The cutting-plane method for solving convex programs." *J. Soc. Ind. Appl. Math.* 8 (4): 703–712. https://doi.org/10.1137/0108053.

Kleinbaum, D. G., and M. Klein. 2010. *Logistic regression*. New York: Springer.

Kroesen, M. 2017. "To what extent do e-bikes substitute travel by other modes? Evidence from the Netherlands." *Transp. Res. Part D Transp. Environ.* 53 (Jun): 377–387. https://doi.org/10.1016/j.trd.2017.04.036.

Lanzendorf, M., and A. Busch-Geertsema. 2014. "The cycling boom in large German cities—Empirical evidence for successful cycling campaigns." *Transp. Policy* 36 (Nov): 26–33. https://doi.org/10.1016/j.tranpol.2014.07.003.

Lin, J.-J., and R.-Y. Liao. 2016. "Sustainability SI: Bikeway network design model for recreational bicycling in scenic areas." *Networks Spatial Econ.* 16 (1): 9–31. https://doi.org/10.1007/s11067-014-9245-7.

Lin, J.-J., and C.-J. Yu. 2013. "A bikeway network design model for urban areas." *Transportation* 40 (1): 45–68. https://doi.org/10.1007/s11116-012-9409-6.

Liu, S., Z.-J. M. Shen, and X. Ji. 2020. "Urban bike lane planning with bike trajectories: Models, algorithms, and a real-world case study." Preprint, submitted August 21, 2020. https://arxiv.org/abs/2008.09645.

Magnanti, T. L., and R. T. Wong. 1981. "Accelerating benders decomposition: Algorithmic enhancement and model selection criteria." *Oper. Res.* 29 (3): 464–484. https://doi.org/10.1287/opre.29.3.464.

Mauttone, A., G. Mercadante, M. Rabaza, and F. Toledo. 2017. "Bicycle network design: Model and solution algorithm." *Transp. Res. Procedia* 27 (Jan): 969–976. https://doi.org/10.1016/j.trpro.2017.12.119.

McDaniel, D., and M. Devine. 1977. "A modified Benders' partitioning algorithm for mixed integer programming." *Manage. Sci.* 24 (3): 312–319. https://doi.org/10.1287/mnsc.24.3.312.

Midtown Alliance. 2019. "2019 midtown community survey." Accessed December 5, 2020. https://www.midtownatl.com/about/programs-and-projects/2019survey.

Northrop, K. 2011. "Bicycle commuter trends in the US—Graphic representation." Accessed December 8, 2020. https://www.webikeeugene.org/2011/03/07/bicycle-commuter-trends-in-the-us-graphic-representation/.

Ogilvie, D., M. Egan, V. Hamilton, and M. Petticrew. 2004. "Promoting walking and cycling as an alternative to using cars: Systematic review." *BMJ* 329 (7469): 763. https://doi.org/10.1136/bmj.38216.714560.55.

OpenStreetMap. 2020. "Planet dump." Accessed March 21, 2020. https://planet.osm.org.

Orozco, L. G. N., F. Battiston, G. Iniguez, and M. Szell. 2020. "Data-driven strategies for optimal bicycle network growth." *R. Soc.* 7 (12): 201130. https://doi.org/10.1098/rsos.201130.

Ospina, J. P., V. Botero-Fernandez, J. C. Duque, M. Brussel, and A. Grigolon. 2020. "Understanding cycling travel distance: The case of Medellin city (Colombia)." *Transp. Res. Part D Transp. Environ.* 86 (Sep): 102423. https://doi.org/10.1016/j.trd.2020.102423.

Reed, T. 2019. "Global traffic scorecard." Accessed December 10, 2020. https://trid.trb.org/view/1456836.

Ryu, S., A. Chen, J. Su, and K. Choi. 2018. "Two-stage bicycle traffic assignment model." *J. Transp. Eng. Part A Syst.* 144 (2): 04017079. https://doi.org/10.1061/JTEPBS.0000108.

Saelens, B. E., J. F. Sallis, and L. D. Frank. 2003. "Environmental correlates of walking and cycling: Findings from the transportation, urban design, and planning literatures." *Ann. Behav. Med.* 25 (2): 80–91. https://doi.org/10.1207/S15324796ABM2502_03.

Walljasper, J. 2016. "10 ways bicycle-friendly streets are good for people who don't ride bikes." Accessed December 8, 2020. https://www.aarp.org/livable-communities/getting-around/info-2016/why-bicycling-infrastructure-is-good-for-people-who-dont-ride-bikes.html.

Wang, Y., M. Douglas, and B. Hazen. 2021. "Diffusion of public bicycle systems: Investigating influences of users' perceived risk and switching intention." *Transp. Res. Part A Policy Pract.* 143 (Jan): 1–13. https://doi.org/10.1016/j.tra.2020.11.002.

Yang, L., S. Sahlqvist, A. McMinn, S. J. Griffin, and D. Ogilvie. 2010. "Interventions to promote cycling: Systematic review." *BMJ* 341 (Oct): c5293. https://doi.org/10.1136/bmj.c5293.

Zhao, X., X. Yan, and P. Van Hentenryck. 2019. "Modeling heterogeneity in mode-switching behavior under a mobility-on-demand transit system: An interpretable machine learning approach." Preprint, submitted February 8, 2019. https://arxiv.org/abs/1902.02904.

Zhu, S., and F. Zhu. 2020. "Multi-objective bike-way network design problem with space-time accessibility constraint." *Transportation* 47 (5): 2479–2503. https://doi.org/10.1007/s11116-019-10025-7.