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Suppose $p \geq 5$ is a prime number, and let $G = \mathrm{SL}_2(\mathbb{Q}_p)$. We calculate the derived functors $\mathrm{R}^n \mathcal{R}_B^G(\pi)$, where B is a Borel subgroup of G, $\mathcal{R}_B^G$ is the right adjoint of smooth parabolic induction constructed by Vignéras, and π is any smooth, absolutely irreducible, mod p representation of G.

1. Introduction

One of the most fundamental operations in the representation theory of p-adic groups is that of **parabolic induction**: given a p-adic connected reductive group G, a rational parabolic subgroup $P = MN$, and a smooth representation σ of the Levi quotient M of P (over some coefficient field C), we can construct the G-representation $\mathrm{Ind}_P^G(\sigma)$ induced from (the inflation to P of) σ. In this way, we obtain an exact functor Ind_P^G from the category of smooth M-representations over C to the category of smooth G-representations over C. The relevance of this functor comes from the fact that any smooth irreducible admissible G-representation arises as a subquotient of some $\mathrm{Ind}_P^G(\sigma)$, where σ is a so-called supercuspidal representation.

Vignéras [25] shows that the functor Ind_P^G commutes with small direct sums. Since the category of smooth M-representations is a locally small, Grothendieck abelian category, a version of the adjoint functor theorem implies that Ind_P^G possesses a (left-exact) right adjoint $\mathcal{R}_P^G$. When the characteristic of C is different from p, we can identify this functor explicitly: $\mathcal{R}_P^G$ is naturally isomorphic to the normalized Jacquet module relative to the opposite parabolic subgroup $P^- = MN^-$. For $C = \mathbb{C}$, this is [6, Main Theorem] and [7, Theorem 3], and the general $\mathrm{char}(C) \neq p$ case follows from [9, Corollary 1.3] (see also [8, Theorem 1.5] for earlier partial results).

When the characteristic of C is equal to p, the functor $\mathcal{R}_P^G$ is more mysterious. When restricted to the category of admissible representations, $\mathcal{R}_P^G$ is isomorphic to Emerton's functor $\mathrm{Ord}_{P^-}^G$ of ordinary parts (see [4, Corollary 4.13] for this

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equivalence). However, it is not clear how to describe $\mathcal{R}_P^G$ on the entire category of smooth representations.

There have also been fascinating recent advances in derived aspects of the mod p representation theory of p-adic reductive groups, which provide new methods for approaching the mod p Langlands program (see [13; 23]). In particular, the authors of [22] consider the total derived functors $\mathrm{R}\,\mathcal{R}_P^G$ over a field of characteristic p, and leave open the question of explicitly calculating these derived functors. The main result of this note is the following, which addresses some of these questions.

Theorem. *Suppose π is a smooth, absolutely irreducible representation of $G = \mathrm{SL}_2(\mathbb{Q}_p)$ over a field C of characteristic $p \geq 5$, and let B denote the upper triangular Borel subgroup of G. Then, for all $n \geq 0$, we have an isomorphism of smooth admissible T-representations*:

$$\mathrm{R}^n \mathcal{R}_B^G(\pi) \cong \mathrm{R}^n \mathrm{Ord}_{B^-}^G(\pi).$$

We make some comments regarding this result. As the category of admissible G-representations does not have enough injectives, the derived functors $\mathrm{R}^n \mathrm{Ord}_{B^-}^G$ must be computed in the category of *locally* admissible G-representations. However, it is not known whether $\mathrm{Ord}_{B^-}^G$ agrees with $\mathcal{R}_B^G$ on this category, and therefore we cannot construct a direct comparison between $\mathrm{R}^n \mathcal{R}_B^G$ and $\mathrm{R}^n \mathrm{Ord}_{B^-}^G$. To address this subtlety, we use the results of [16] to relate $\mathcal{R}_B^G$ to the analogously defined functor on pro-p-Iwahori–Hecke modules, where we can do explicit calculations to evaluate $\mathrm{R}^n \mathcal{R}_B^G(\pi)$ (and deduce *a posteriori* the isomorphism with $\mathrm{R}^n \mathrm{Ord}_{B^-}^G(\pi)$). To our knowledge, the results computing $\mathrm{R}^n \mathcal{R}_B^G(\pi)$ are the first of their kind.

We are hopeful that some of the techniques for calculating $\mathrm{R}^n \mathcal{R}_B^G(\pi)$ will generalize to other p-adic reductive groups (though some are specific to the group $\mathrm{SL}_2(\mathbb{Q}_p)$).

2. Preparation

2A. Suppose $p \geq 5$ is a prime number, and define $G := \mathrm{SL}_2(\mathbb{Q}_p)$. We let B denote the upper triangular Borel subgroup and T the diagonal maximal torus. We let $\mathfrak{Rep}^\infty(-)$ denote the category of smooth representations with coefficients in a field C of characteristic p. We will examine the (exact) functor of parabolic induction

$$\mathrm{Ind}_B^G : \mathfrak{Rep}^\infty(T) \to \mathfrak{Rep}^\infty(G)$$

and its right adjoint

$$\mathcal{R}_B^G : \mathfrak{Rep}^\infty(G) \to \mathfrak{Rep}^\infty(T),$$

constructed in [25, Section 4].

We also define two distinguished characters $\bar{\rho}, \bar{\alpha} : T \to C^\times$ which we use in the sequel:

$$\bar{\rho}\left(\begin{pmatrix} xp^a & 0 \\ 0 & x^{-1}p^{-a} \end{pmatrix}\right) = \bar{x}, \quad \bar{\alpha}\left(\begin{pmatrix} xp^a & 0 \\ 0 & x^{-1}p^{-a} \end{pmatrix}\right) = \bar{x}^2,$$

where $a \in \mathbb{Z}$, $x \in \mathbb{Z}_p^\times$, and where $\bar{x} \in \mathbb{F}_p^\times \subset C^\times$ denotes the mod p reduction of x.

2B. In order to understand the right derived functors $\mathrm{R}^n \mathcal{R}_B^G$, we use pro-$p$-Iwahori–Hecke algebras. Let I_1 denote the subgroup of $\mathrm{SL}_2(\mathbb{Z}_p)$ which is upper triangular and unipotent modulo p, and let $T_1 := T \cap I_1$. Note that $T_1 \cong 1 + p\mathbb{Z}_p \cong \mathbb{Z}_p$, so that T_1 has cohomological dimension 1. We let $\mathcal{H}$ denote the pro-p-Iwahori–Hecke algebra of G with respect to I_1, and let $\mathcal{H}_T$ denote the pro-p-Iwahori–Hecke algebra of T with respect to T_1 (see [24, Section 4] for more details and definitions). We note that $\mathcal{H}_T \cong C[T/T_1] \cong C[\mathbb{Z}] \otimes_C C[\mathbb{F}_p^\times]$, and therefore $\mathcal{H}_T$ has global dimension 1.

We have analogous functors of parabolic induction

$$\mathrm{Ind}_{\mathcal{H}_T}^{\mathcal{H}} : \mathfrak{Mod}\text{–}\mathcal{H}_T \to \mathfrak{Mod}\text{–}\mathcal{H}$$

and its right adjoint

$$\mathcal{R}_{\mathcal{H}_T}^{\mathcal{H}} : \mathfrak{Mod}\text{–}\mathcal{H} \to \mathfrak{Mod}\text{–}\mathcal{H}_T,$$

defined on categories of right modules. We refer to [18, Section 4.2] for details and definitions.

2C. Given a smooth representation π of G, the space π^{I_1} of I_1-invariants has a right action of $\mathcal{H}$, recalled in [18, proof of Lemma 4.5]. Passing to derived functors, the cohomology spaces $\mathrm{H}^i(I_1, \pi)$ also inherit a right action of $\mathcal{H}$ (described in [16, Section 2.3]). We have analogous constructions for T and $\mathcal{H}_T$.

Given a smooth character $\chi : T \to C^\times$, we have $\chi^{T_1} = \chi$, and therefore χ inherits the structure of a right $\mathcal{H}_T$-module. We will use the same notation χ to denote the T-representation and the resulting $\mathcal{H}_T$-module; the meaning should be clear from context.

2D. The goal will be to compute $\mathrm{R}^n \mathcal{R}_B^G(\pi)$ where π is an absolutely irreducible admissible G-representation. Our main tool will be [16, Theorem 3.13]: if π is an admissible G-representation, then we have an E_2 spectral sequence of $\mathcal{H}_T$-modules:

$$\mathrm{H}^i(T_1, \mathrm{R}^j \mathcal{R}_B^G(\pi)) \Rightarrow \mathcal{R}_{\mathcal{H}_T}^{\mathcal{H}}(\mathrm{H}^{i+j}(I_1, \pi)).$$

(Note that the assumption in [16, Section 3] that C be finite is not required for the construction of the above spectral sequence.) Since T_1 has cohomological dimension 1, the above spectral sequence degenerates at the E_2 page to give an isomorphism

$$\mathcal{R}_{\mathcal{H}_T}^{\mathcal{H}}(\pi^{I_1}) \cong \mathcal{R}_B^G(\pi)^{T_1} \tag{1}$$

and, for $n \geq 1$, an exact sequence

$$0 \to \mathrm{H}^1(T_1, \mathrm{R}^{n-1}\mathcal{R}_B^G(\pi)) \to \mathcal{R}_{\mathcal{H}_T}^{\mathcal{H}}(\mathrm{H}^n(I_1, \pi)) \to (\mathrm{R}^n\mathcal{R}_B^G(\pi))^{T_1} \to 0. \tag{2}$$

In particular, this implies the following.

Lemma 2.1. *If π is an admissible G-representation, then each $\mathrm{R}^n\mathcal{R}_B^G(\pi)$ is an admissible T-representation.*

Proof. Assuming π is admissible, Lemma 3.4.4 of [11] implies that $\mathrm{H}^n(I_1, \pi)$ is finite-dimensional for all $n \geq 0$. By [18, Section 4.2, Property (4)], the space $\mathcal{R}_{\mathcal{H}_T}^{\mathcal{H}}(\mathrm{H}^n(I_1, \pi))$ is also finite-dimensional, and so Theorem 6.3.2 of [19], along with (1) and (2), imply that each $\mathrm{R}^n\mathcal{R}_B^G(\pi)$ is an admissible T-representation. □

2E. In addition to the functor $\mathcal{R}_B^G$, we will also use Emerton's functors of derived ordinary parts (see [10; 11] for the relevant definitions; we note that for our applications, the assumption in op. cit. that the residue field of A is finite is not necessary). We recall that the functor $\mathrm{Ord}_{B^-}^G$ is the right adjoint to the parabolic induction functor from $\mathfrak{Rep}^{\mathrm{ladm}}(T)$, the category of locally admissible T-representations, to $\mathfrak{Rep}^{\mathrm{ladm}}(G)$, the category of locally admissible G-representations.

For a locally admissible T-representation σ and a locally admissible G-representation π, we have two spectral sequences relating extensions and parabolic induction: one in the smooth category

$$\mathrm{Ext}_T^i(\sigma, \mathrm{R}^j\mathcal{R}_B^G(\pi)) \Rightarrow \mathrm{Ext}_G^{i+j}(\mathrm{Ind}_B^G(\sigma), \pi), \tag{3}$$

(coming from an application of the Grothendieck spectral sequence) and one in the locally admissible category

$$\mathrm{Ext}_{T,\mathrm{ladm}}^i(\sigma, \mathrm{R}^j\,\mathrm{Ord}_{B^-}^G(\pi)) \Rightarrow \mathrm{Ext}_{G,\mathrm{ladm}}^{i+j}(\mathrm{Ind}_B^G(\sigma), \pi) \tag{4}$$

(see [11, (3.7.4)]). By Corollary 3.3 below, we have $\mathrm{Ext}_{H,\mathrm{ladm}}^n(\tau, \pi) \cong \mathrm{Ext}_H^n(\tau, \pi)$ if τ and π are locally admissible H-representations, where $H \in \{T, G\}$. In addition, by [16, Theorem A.4], we have $\mathrm{R}^j\,\mathrm{Ord}_{B^-}^G \simeq \mathrm{H}^j\,\mathrm{Ord}_{B^-}^G$ for the group SL_2, and the latter can be explicitly calculated by work of Emerton [11] and Hauseux [14]. Therefore, the spectral sequence (4) becomes

$$\mathrm{Ext}_T^i(\sigma, \mathrm{H}^j\,\mathrm{Ord}_{B^-}^G(\pi)) \Rightarrow \mathrm{Ext}_G^{i+j}(\mathrm{Ind}_B^G(\sigma), \pi). \tag{5}$$

3. Locally admissible representations versus smooth representations

In this section, we compare locally admissible and smooth representations.

Lemma 3.1. *Let $H \in \{T, G\}$, let $\iota : \mathfrak{Rep}^{ladm}(H) \to \mathfrak{Rep}^{\infty}(H)$ denote the fully faithful inclusion, and let $\mathfrak{J} \in \mathfrak{Rep}^{ladm}(H)$ denote an injective object. Then $\iota(\mathfrak{J}) \in \mathfrak{Rep}^{\infty}(H)$ is also an injective object.*

Proof. The proofs are virtually identical to the proof of [21, Proposition 5.16]. Since we do not fix the central character, we briefly highlight the main differences, and explain why the arguments work in our setting. Once again, the assumption in op. cit. that the coefficient field k is finite is not necessary for our purposes.

Suppose first that $H = T$. We outline the main changes to the proof in loc. cit. (using that article's notation):

- We replace c-Ind_{KZ}^{G} with $\text{c-Ind}_{T_0}^{T}$ in the proof, where $T_0 \cong \mathbb{Z}_p^\times$ denotes the maximal compact subgroup of T.
- Since $\mathbb{F}_p \subset C$, we may take the representation σ to be a character; further, by twisting, we may assume this character is trivial.
- Instead of [12, Corollary 3.8], we use [16, Theorem A.8].
- The reference to [5, Proposition 18] is no longer applicable; however, in this case, we still have a noninjective T-equivariant surjection $\psi : \text{c-Ind}_{T_0}^{T}(\mathbf{1}_{T_0}) \cong C[X^{\pm 1}] \twoheadrightarrow A$. Since this map is equivariant for the action of $C[T/T_0] \cong C[X^{\pm 1}]$ on both sides, the structure theorem for finitely generated modules over a PID implies that $A \cong C[X^{\pm 1}]/(f(X))$, where $f(X)$ is some nonzero polynomial. In particular, A is finite-dimensional and admissible.
- We may still appeal to [10, Theorem 2.3.8] to obtain the equivalence of the notions of "locally admissible" and "locally finite" representations.

The other parts of the proof apply with minor changes to give the desired result.

Suppose now that $H = G = \mathrm{SL}_2(\mathbb{Q}_p)$. Since the center Z is a finite group of order 2, and since $p \geq 5$, the action of Z on any smooth representation is semisimple. In particular, we may assume without loss of generality that Z acts by a character on $\mathfrak{J}$. The proof contained in [21] may again be adapted to the group $\mathrm{SL}_2(\mathbb{Q}_p)$, with the following changes:

- We replace c-Ind_{KZ}^{G} with $\text{c-Ind}_{\mathrm{SL}_2(\mathbb{Z}_p)}^{\mathrm{SL}_2(\mathbb{Q}_p)}$ in the proof.
- Instead of [12, Corollary 3.8], we use [16, Section A.2, proof of Step 1].
- The analog of [5, Proposition 18] holds for the group $\mathrm{SL}_2(\mathbb{Q}_p)$ (see Remark 3.2 below).
- Let
$$\mathcal{T} \in \mathrm{End}_{\mathrm{SL}_2(\mathbb{Q}_p)}(\text{c-Ind}_{\mathrm{SL}_2(\mathbb{Z}_p)}^{\mathrm{SL}_2(\mathbb{Q}_p)}(\sigma))$$
denote the spherical Hecke operator associated to the $\mathrm{SL}_2(\mathbb{Z}_p)$-bi-equivariant function with support $\mathrm{SL}_2(\mathbb{Z}_p)\left(\begin{smallmatrix} p & 0 \\ 0 & p^{-1} \end{smallmatrix}\right)\mathrm{SL}_2(\mathbb{Z}_p)$, so that the relevant spherical Hecke algebra is a polynomial algebra in $\mathcal{T}$. To see that
$$\text{c-Ind}_{\mathrm{SL}_2(\mathbb{Z}_p)}^{\mathrm{SL}_2(\mathbb{Q}_p)}(\sigma)/(\mathcal{T} - \lambda)$$

is of finite length, we appeal to [1, Theorem 3.18, Proposition 2.7, Corollary 3.26(3), proof of Proposition 4.4]. Further, each representation

$$\text{c-Ind}_{\mathrm{SL}_2(\mathbb{Z}_p)}^{\mathrm{SL}_2(\mathbb{Q}_p)}(\sigma)/(\mathcal{T}-\lambda)$$

is admissible by [1, Proposition 2.9, Corollary 3.26(3), Corollary 4.6, Proposition 4.7]. Therefore, the representation A appearing in [21, proof of Proposition 5.16] is admissible.

• In order to prove the equivalence of "locally admissible" and "locally finite," we need an analog of [10, Theorem 2.3.8] for the group $\mathrm{SL}_2(\mathbb{Q}_p)$ (more precisely, we need a version of the equivalence (1) $\Leftrightarrow$ (2) of loc. cit., Lemma 2.3.6). The validity of this claim follows from the claim that the quotients

$$\text{c-Ind}_{\mathrm{SL}_2(\mathbb{Z}_p)}^{\mathrm{SL}_2(\mathbb{Q}_p)}(\sigma)/(\mathcal{T}-\lambda)$$

are of finite length, proved in the previous bullet point. (Of independent interest: in order to verify the equivalence (1) $\Leftrightarrow$ (3) of [10, Lemma 2.3.6] for $\mathrm{SL}_2(\mathbb{Q}_p)$, one can use [1, Theorem 3.36(3)].)

The other parts of the proof apply with minor changes to give the desired result. □

Remark 3.2. We believe the counterexample contained in [1, Section 3.7.3] is incorrect.[1] In particular, in the notation of op. cit., the Hecke algebra $\mathcal{H}(G_S, I_S, \omega_{(p-1)/2})$ contains functions supported on double cosets of the form $I_S\left(\begin{smallmatrix} 0 & -p^{-\ell} \\ p^{\ell} & 0 \end{smallmatrix}\right) I_S$. Therefore, there are more Hecke operators contained in $\mathcal{H}(G_S, I_S, \omega_{(p-1)/2})$ than simply the span of the $T_{2n,2n+1}$. Moreover, the action of the Hecke operator $T_{\left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)}$ on the function f_1 is given by

$$f_1 \cdot T_{\left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)} = (-1)^{(p-1)/2} f_{-1},$$

which implies that the submodule $\bigoplus_{n\geq 1} \overline{\mathbb{F}}_p f_n$ is not Hecke-stable.

After checking details, we believe that the analog of [5, Proposition 18] for the group $\mathrm{SL}_2(\mathbb{Q}_p)$ is indeed true: any $\mathcal{H}$-stable submodule

$$W \subset (\text{c-Ind}_{\mathrm{SL}_2(\mathbb{Z}_p)}^{\mathrm{SL}_2(\mathbb{Q}_p)}(\mathrm{Sym}^r))^{I_1}$$

is of finite codimension.

Corollary 3.3. *Let $H \in \{T, G\}$, and let τ, π be two locally admissible H-representations. Then*

$$\mathrm{Ext}^i_{H,ladm}(\tau, \pi) \cong \mathrm{Ext}^i_H(\tau, \pi).$$

4. Some Ext calculations

We begin with some calculations of Ext groups which we will need below.

[1] We have discussed this with Abdellatif, and she has agreed with our assessment.

4A. *T-extensions.* Let σ, κ denote two smooth T-representations, and suppose that σ is generated by its space of T_1-invariant vectors. (As T_1 is normal in T, this is equivalent to requiring that $\sigma^{T_1} = \sigma$.) Since the category of smooth T-representations generated by their T_1-invariants is equivalent to the category of right $\mathcal{H}_T$-modules, we get an E_2 spectral sequence of C-vector spaces

$$\mathrm{Ext}^i_{\mathcal{H}_T}(\sigma^{T_1}, \mathrm{H}^j(T_1, \kappa)) \Rightarrow \mathrm{Ext}^{i+j}_T(\sigma, \kappa). \tag{6}$$

Compare [20, (33)].

Lemma 4.1. *Let $\chi, \chi' : T \to C^\times$ denote two smooth characters of T. Then*

$$\dim_C(\mathrm{Ext}^n_T(\chi, \chi')) = \tbinom{2}{n}\delta_{\chi,\chi'},$$

where $\delta_{\chi,\chi'}$ denotes the Kronecker delta function.

Proof. Since $\mathcal{H}_T$ is of global dimension 1 and T_1 has cohomological dimension 1, the spectral sequence (6) degenerates at the E_2 page. The equations

$$\mathrm{H}^1(T_1, \chi') \cong \chi' \quad \text{and} \quad \dim_C(\mathrm{Ext}^n_{\mathcal{H}_T}(\chi, \chi')) = \tbinom{1}{n}\delta_{\chi,\chi'}$$

then give the result. □

Lemma 4.2. *Suppose σ, κ are smooth T-representations, and suppose σ has finite length. Then, for $n \geq 3$, we have*

$$\mathrm{Ext}^n_T(\sigma, \kappa) = 0.$$

Proof. By induction on length, it suffices to assume σ is simple. In particular, σ is generated by its T_1-invariant vectors. Since $\mathcal{H}_T$ is of global dimension 1 and T_1 has cohomological dimension 1, the spectral sequence (6) degenerates at the E_2 page, which gives the result. □

Now let $\chi : T \to C^\times$ be a smooth character, and let σ denote a nonsplit extension of χ by χ:

$$0 \to \chi \to \sigma \to \chi \to 0. \tag{7}$$

Lemma 4.3. *We have*

$$\dim_C(\mathrm{Ext}^n_T(\sigma, \chi)) = \tbinom{2}{n}.$$

Proof. The degree $n = 0$ case follows from the fact that σ is a nonsplit extension, while the $n = 1$ case follows from a direct calculation with Yoneda extensions. Applying the functor $\mathrm{Hom}_T(-, \chi)$ to the short exact sequence (7) gives a long exact sequence of Ext groups; taking the Euler characteristic, using Lemmas 4.1 and 4.2 and the degrees already computed gives the result in degrees $n \geq 2$. (Alternatively, one can dualize and use the spectral sequence (6).) □

5. Right adjoint calculations

We are now in a position to calculate $\mathrm{R}^n \mathcal{R}_B^G$. The smooth, absolutely irreducible representations of $\mathrm{SL}_2(\mathbb{Q}_p)$ are divided into four classes (see [1, Theorems 3.42, 4.12; 15, Theorem 4]), and we discuss each in turn.

5A. ***Principal series.*** This is the most involved calculation.

For this entire subsection we let $\pi = \mathrm{Ind}_B^G(\chi)$, where $\chi : T \to C^\times$ is a smooth character. By [16, Section 5.4.2], we have

$$\mathrm{H}^n(I_1, \pi) = \begin{cases} \mathrm{Ind}_{\mathcal{H}_T}^{\mathcal{H}}(\chi) & \text{if } n = 0, \\ \text{extension of } \mathrm{Ind}_{\mathcal{H}_T}^{\mathcal{H}}(\chi^{-1}\bar{\alpha})^\vee \text{ by } \mathrm{Ind}_{\mathcal{H}_T}^{\mathcal{H}}(\chi) & \text{if } n = 1, \\ \mathrm{Ind}_{\mathcal{H}_T}^{\mathcal{H}}(\chi^{-1}\bar{\alpha})^\vee & \text{if } n = 2, \\ 0 & \text{if } n \geq 3. \end{cases} \tag{8}$$

The extension in the H^1 term is nonsplit if and only if $\chi = \bar{\rho}$, and for a right $\mathcal{H}$-module M we equip $M^\vee := \mathrm{Hom}_C(M, C)$ with the structure of a right $\mathcal{H}$-module as in [2, Section 4]. We now apply $\mathcal{R}_{\mathcal{H}_T}^{\mathcal{H}}$ to (8), and use [3, Theorem 5.20] and [2, Theorem 4.9]. This gives

$$\mathcal{R}_{\mathcal{H}_T}^{\mathcal{H}}(\mathrm{H}^n(I_1, \pi)) = \begin{cases} \chi & \text{if } n = 0, \\ \text{extension of } \chi^{-1}\bar{\alpha} \text{ by } \chi & \text{if } n = 1, \\ \chi^{-1}\bar{\alpha} & \text{if } n = 2, \\ 0 & \text{if } n \geq 3. \end{cases} \tag{9}$$

Once again, the degree 1 term is nonsplit if and only if $\chi = \bar{\rho}$. We will use the above as input into the short exact sequence (2).

Since the unit of the adjunction $(\mathrm{Ind}_B^G, \mathcal{R}_B^G)$ is an isomorphism [25, Theorem 5.3], we get

$$\mathcal{R}_B^G(\pi) = \chi.$$

Suppose next that $n \geq 3$. Then the short exact sequence (2) and equations (9) imply $(\mathrm{R}^n \mathcal{R}_B^G(\pi))^{T_1} = 0$, so that

$$\mathrm{R}^n \mathcal{R}_B^G(\pi) = 0 \quad \text{for all } n \geq 3.$$

We now calculate the remaining two degrees.

Lemma 5.1. *We have* $\mathrm{R}^2 \mathcal{R}_B^G(\pi) = 0$.

Proof. We proceed in several steps. Note first that the short exact sequence (2) for $n = 2$ and $n = 3$ along with the equations (9) give

$$\dim_C(\mathrm{R}^2 \mathcal{R}_B^G(\pi)^{T_1}) \leq 1, \quad \dim_C(\mathrm{H}^1(T_1, \mathrm{R}^2 \mathcal{R}_B^G(\pi))) = 0.$$

Step 1. *The T_1-representation $\mathrm{R}^2 \mathcal{R}_B^G(\pi)|_{T_1}$ is either 0 or isomorphic to $\mathcal{C}^\infty(T_1, C)$.*

This essentially follows from the dimension calculations above. We elaborate. Since $\mathrm{R}^2\,\mathcal{R}_B^G(\pi)$ is an admissible T-representation, the Pontryagin dual

$$\mathrm{R}^2\,\mathcal{R}_B^G(\pi)^\vee := \mathrm{Hom}_C(\mathrm{R}^2\,\mathcal{R}_B^G(\pi), C)$$

is a finitely generated module over the DVR $C[\![T_1]\!] \cong C[\![X]\!]$. Furthermore, the cosocle of $\mathrm{R}^2\,\mathcal{R}_B^G(\pi)^\vee$ is dual to the space of invariants $\mathrm{R}^2\,\mathcal{R}_B^G(\pi)^{T_1}$. By Nakayama's lemma, $\mathrm{R}^2\,\mathcal{R}_B^G(\pi)^\vee$ is generated by at most one element, which implies

$$\mathrm{R}^2\,\mathcal{R}_B^G(\pi)^\vee \cong 0 \text{ or } C[\![X]\!] \text{ or } C[\![X]\!]/X^r$$

for some $r \geq 1$. Dualizing, we obtain

$$\mathrm{R}^2\,\mathcal{R}_B^G(\pi)|_{T_1} \cong 0 \text{ or } \mathcal{C}^\infty(T_1, C) \text{ or } (C[\![X]\!]/X^r)^\vee.$$

However, the condition $\mathrm{H}^1(T_1, \mathrm{R}^2\,\mathcal{R}_B^G(\pi)) = 0$ rules out the last possibility.

Step 2. *We calculate the graded pieces of the socle filtration of* $\mathrm{R}^2\,\mathcal{R}_B^G(\pi)$.

When $\mathrm{R}^2\,\mathcal{R}_B^G(\pi) = 0$, there is nothing to prove, so assume the contrary. For the sake of brevity, we set $\kappa := \mathrm{R}^2\,\mathcal{R}_B^G(\pi)$, and for $i \geq 0$, let $\sigma^i := \mathrm{soc}_T^i(\kappa)$ denote the socle filtration of κ.

First, we have

$$\sigma^1 = \mathrm{soc}_T(\kappa) \subset \kappa^{T_1} \cong \chi^{-1}\bar{\alpha}$$

(by (2) in degree $n = 2$, assuming $\kappa \neq 0$). Thus, κ^{T_1} is semisimple as a T-representation, so that $\kappa^{T_1} \subset \mathrm{soc}_T(\kappa)$, and this implies $\sigma^1 \cong \chi^{-1}\bar{\alpha}$. Similarly, we have

$$\sigma^2/\sigma^1 = \mathrm{soc}_T(\kappa/\sigma^1) = \mathrm{soc}_T(\kappa/\chi^{-1}\bar{\alpha}) \subset (\kappa/\chi^{-1}\bar{\alpha})^{T_1}.$$

To determine the latter space (which is nonzero by Step 1), we apply the functor of T_1-invariants to the short exact sequence

$$0 \to \chi^{-1}\bar{\alpha} \to \kappa \to \kappa/\chi^{-1}\bar{\alpha} \to 0$$

to get

$$0 \to \chi^{-1}\bar{\alpha} \xrightarrow{\sim} \kappa^{T_1} \xrightarrow{0} (\kappa/\chi^{-1}\bar{\alpha})^{T_1} \to \mathrm{H}^1(T_1, \chi^{-1}\bar{\alpha}) \cong \chi^{-1}\bar{\alpha}.$$

Thus, we obtain $(\kappa/\chi^{-1}\bar{\alpha})^{T_1} \cong \chi^{-1}\bar{\alpha}$, and consequently $\sigma^2/\sigma^1 \cong \chi^{-1}\bar{\alpha}$. Continuing in this way, we see that $\dim_C(\sigma^i) = i$ and $\sigma^{i+1}/\sigma^i \cong \chi^{-1}\bar{\alpha}$ for all $i \geq 0$.

Step 3. *We have* $\mathrm{R}^2\,\mathcal{R}_B^G(\pi) = 0$.

Assume the contrary, so that $\mathrm{R}^2\,\mathcal{R}_B^G(\pi)|_{T_1} \cong \mathcal{C}^\infty(T_1, C)$. As in Step 2, we have $\mathrm{R}^2\,\mathcal{R}_B^G(\pi)^{T_1} \cong \chi^{-1}\bar{\alpha}$. By injectivity of $\mathcal{C}^\infty(T_1, C)$, the spectral sequence (6) for $\kappa = \mathrm{R}^2\,\mathcal{R}_B^G(\pi)$ collapses to give

$$\dim_C(\mathrm{Ext}_T^i(\chi^{-1}\bar{\alpha}, \mathrm{R}^2\,\mathcal{R}_B^G(\pi))) = \dim_C(\mathrm{Ext}_{\mathcal{H}_T}^i(\chi^{-1}\bar{\alpha}, \chi^{-1}\bar{\alpha})) = \binom{1}{i}. \tag{10}$$

Let $\sigma = \sigma^2$ denote the second step in the socle filtration of $\mathrm{R}^2\mathcal{R}_B^G(\pi)$, as in Step 2 above, and note that by construction we have $\dim_C(\mathrm{Hom}_T(\sigma, \mathrm{R}^2\mathcal{R}_B^G(\pi))) = 2$. Applying $\mathrm{Hom}_T(-, \mathrm{R}^2\mathcal{R}_B^G(\pi))$ to the short exact sequence

$$0 \to \chi^{-1}\bar{\alpha} \to \sigma \to \chi^{-1}\bar{\alpha} \to 0$$

gives a long exact sequence of Ext groups. Taking the Euler characteristic and using (10) implies

$$\dim_C(\mathrm{Ext}_T^1(\sigma, \mathrm{R}^2\mathcal{R}_B^G(\pi))) = \dim_C(\mathrm{Hom}_T(\sigma, \mathrm{R}^2\mathcal{R}_B^G(\pi))) = 2. \tag{11}$$

We first examine the spectral sequence (5) for σ and π as above. By Lemma 4.2 and the fact that $\mathrm{H}^j \mathrm{Ord}_{B^-}^G(\pi) = 0$ for $j \geq 2$ [11, Proposition 3.6.1], we have $E_2^{i,j} = 0$ for $i \geq 3$ or $j \geq 2$. This implies

$$\mathrm{Ext}_G^3(\mathrm{Ind}_B^G(\sigma), \pi) \cong E_\infty^{2,1} = E_2^{2,1} = \mathrm{Ext}_T^2(\sigma, \mathrm{H}^1 \mathrm{Ord}_{B^-}^G(\pi)) \cong \mathrm{Ext}_T^2(\sigma, \chi^{-1}\bar{\alpha}),$$

where the last isomorphism follows from [14, Corollary 3.3.8(ii)]. Therefore, we get

$$\dim_C(\mathrm{Ext}_G^3(\mathrm{Ind}_B^G(\sigma), \pi)) = 1 \tag{12}$$

by Lemma 4.3.

Consider now the spectral sequence (3). By Lemma 4.2 and the $\mathrm{R}^n\mathcal{R}_B^G$ already calculated, we have $E_2^{i,j} = 0$ if $i \geq 3$ or $j \geq 3$, which implies the spectral sequence degenerates at the E_3 page. In particular, we get a surjection

$$\mathrm{Ext}_G^3(\mathrm{Ind}_B^G(\sigma), \pi) \twoheadrightarrow \mathrm{Ext}_T^1(\sigma, \mathrm{R}^2\mathcal{R}_B^G(\pi)).$$

By (11) and (12), the left-hand side has dimension 1 while the right-hand side has dimension 2, and we arrive at a contradiction. □

It remains to calculate $\mathrm{R}^1\mathcal{R}_B^G(\pi)$. We proceed as follows. Using the values of $\mathrm{R}^n\mathcal{R}_B^G(\pi)$ already computed, the spectral sequence (3) yields the following exact sequence:

$$\begin{aligned} 0 \longrightarrow \mathrm{Ext}_T^1(\sigma, \mathcal{R}_B^G(\pi)) &\xrightarrow{e_1} \mathrm{Ext}_G^1(\mathrm{Ind}_B^G(\sigma), \pi) \longrightarrow \mathrm{Hom}_T(\sigma, \mathrm{R}^1\mathcal{R}_B^G(\pi)) \\ &\to \mathrm{Ext}_T^2(\sigma, \mathcal{R}_B^G(\pi)) \xrightarrow{e_2} \mathrm{Ext}_G^2(\mathrm{Ind}_B^G(\sigma), \pi) \longrightarrow \mathrm{Ext}_T^1(\sigma, \mathrm{R}^1\mathcal{R}_B^G(\pi)) \longrightarrow 0 \end{aligned} \tag{13}$$

Here, the maps e_i are the edge maps of the spectral sequence (3). Similarly, using the fact that $\mathrm{H}^j \mathrm{Ord}_{B^-}^G(\pi) = 0$ for $j \geq 2$ [11, Proposition 3.6.1] and that π is locally admissible, the spectral sequence (5) for σ locally admissible gives

$$\begin{aligned} 0 \longrightarrow \mathrm{Ext}_T^1(\sigma, \mathrm{Ord}_{B^-}^G(\pi)) &\xrightarrow{e_1^{\mathrm{ladm}}} \mathrm{Ext}_G^1(\mathrm{Ind}_B^G(\sigma), \pi) \longrightarrow \mathrm{Hom}_T(\sigma, \mathrm{H}^1\mathrm{Ord}_{B^-}^G(\pi)) \\ &\to \mathrm{Ext}_T^2(\sigma, \mathrm{Ord}_{B^-}^G(\pi)) \xrightarrow{e_2^{\mathrm{ladm}}} \mathrm{Ext}_G^2(\mathrm{Ind}_B^G(\sigma), \pi) \longrightarrow \mathrm{Ext}_T^1(\sigma, \mathrm{H}^1\mathrm{Ord}_{B^-}^G(\pi)) \longrightarrow 0 \end{aligned} \tag{14}$$

Let us now fix a locally admissible T-representation σ, and recall that $\pi = \mathrm{Ind}_B^G(\chi)$. Consider the composite map

$$\mathrm{Ext}_T^i(\sigma, \chi) \to \mathrm{Ext}_T^i(\sigma, \mathcal{R}_B^G(\mathrm{Ind}_B^G(\chi))) \xrightarrow{e_i} \mathrm{Ext}_G^i(\mathrm{Ind}_B^G(\sigma), \mathrm{Ind}_B^G(\chi)),$$

where the first map is induced by the unit $\chi \to \mathcal{R}_B^G(\mathrm{Ind}_B^G(\chi))$ of the adjunction $(\mathrm{Ind}_B^G, \mathcal{R}_B^G)$. A straightforward exercise in homological algebra shows that this composite map is, up to a sign, equal to the map obtained by applying the exact functor Ind_B^G to a Yoneda extension. A similar remark holds for the adjunction $(\mathrm{Ind}_B^G, \mathrm{Ord}_{B^-}^G)$ defined on the locally admissible categories.

Suppose further that σ has finite length; this implies that all Ext spaces appearing in the exact sequences (13) and (14) have finite dimension. By [25, Theorem 5.3] and [10, Proposition 4.3.4], the units of both the adjunctions $(\mathrm{Ind}_B^G, \mathcal{R}_B^G)$ and $(\mathrm{Ind}_B^G, \mathrm{Ord}_{B^-}^G)$ are isomorphisms, which implies that the domain of e_2 is identified with the domain of e_2^{ladm} (both of which are isomorphic to $\mathrm{Ext}_T^2(\sigma, \chi)$). Therefore, the paragraph above implies that the image of the edge map e_2 in the exact sequence (13) has the same dimension as the image of the edge map e_2^{ladm} in the exact sequence (14). In particular, by dimension counting this implies

$$\dim_C(\mathrm{Ext}_T^1(\sigma, \mathrm{R}^1\mathcal{R}_B^G(\pi))) = \dim_C(\mathrm{Ext}_T^1(\sigma, \mathrm{H}^1\,\mathrm{Ord}_{B^-}^G(\pi)));$$

the two exact sequences (13) and (14) then give

$$\begin{aligned}(15)\qquad \dim_C(\mathrm{Hom}_T(\sigma, \mathrm{R}^1\mathcal{R}_B^G(\pi))) &= \dim_C(\mathrm{Hom}_T(\sigma, \mathrm{H}^1\,\mathrm{Ord}_{B^-}^G(\pi))) \\ &= \dim_C(\mathrm{Hom}_T(\sigma, \chi^{-1}\bar{\alpha})),\end{aligned}$$

where the last equality follows from [14, Corollary 3.3.8(ii)].

Lemma 5.2. *We have* $\mathrm{R}^1\mathcal{R}_B^G(\pi) = \chi^{-1}\bar{\alpha}$.

Proof. Taking $\sigma = \chi^{-1}\bar{\alpha}$ in (15) shows that $\chi^{-1}\bar{\alpha} \hookrightarrow \mathrm{R}^1\mathcal{R}_B^G(\pi)$. Now, assume by contradiction that $\dim_C(\mathrm{R}^1\mathcal{R}_B^G(\pi)) \geq 2$, and let σ denote the second step of the socle filtration of $\mathrm{R}^1\mathcal{R}_B^G(\pi)$ (as in Step 2 of the proof of Lemma 5.1). Then σ is a nonsplit extension of $\chi^{-1}\bar{\alpha}$ by itself, and by construction, we have

$$\dim_C(\mathrm{Hom}_T(\sigma, \mathrm{R}^1\mathcal{R}_B^G(\pi))) = 2, \quad \dim_C(\mathrm{Hom}_T(\sigma, \chi^{-1}\bar{\alpha})) = 1.$$

However, this contradicts (15). □

Putting everything together gives

$$(16)\qquad \mathrm{R}^n\mathcal{R}_B^G(\mathrm{Ind}_B^G(\chi)) = \begin{cases} \chi & \text{if } n = 0, \\ \chi^{-1}\bar{\alpha} & \text{if } n = 1, \\ 0 & \text{if } n \geq 2. \end{cases}$$

5B. ***Steinberg.*** Suppose now that $\pi = \mathrm{St} := \mathrm{Ind}_B^G(\mathbf{1}_T)/\mathbf{1}_G$ is the Steinberg representation. By [16, Section 5.4.3], we have

$$\mathrm{H}^n(I_1, \mathrm{St}) = \begin{cases} \chi_{\mathrm{sign}} & \text{if } n = 0, \\ \mathrm{Ind}_{\mathcal{H}_T}^{\mathcal{H}}(\mathbf{1}_T) & \text{if } n = 1, \\ \chi_{\mathrm{triv}} & \text{if } n = 2, \\ 0 & \text{if } n \geq 3. \end{cases}$$

(For the definitions of χ_{sign} and χ_{triv}, see [18, Remarks 2.23 and 2.24(1)].) Thus, by [3, Theorem 5.20], we have

$$\mathcal{R}_{\mathcal{H}_T}^{\mathcal{H}}(\mathrm{H}^n(I_1, \mathrm{St})) = \begin{cases} \mathbf{1}_T & \text{if } n = 0, 1, \\ 0 & \text{if } n \geq 2. \end{cases}$$

By [4, Corollary 6.5], we have

$$\mathcal{R}_B^G(\mathrm{St}) = \mathbf{1}_T.$$

Using this fact, the calculation of $\mathcal{R}_{\mathcal{H}_T}^{\mathcal{H}}(\mathrm{H}^1(I_1, \mathrm{St}))$ above, and the short exact sequence (2) for $n = 1$, we get

$$0 \to \mathrm{H}^1(T_1, \mathbf{1}_T) \cong \mathbf{1}_T \to \mathbf{1}_T \to (\mathrm{R}^1 \mathcal{R}_B^G(\mathrm{St}))^{T_1} \to 0.$$

Therefore, we have $(\mathrm{R}^1 \mathcal{R}_B^G(\pi))^{T_1} = 0$, which implies

$$\mathrm{R}^1 \mathcal{R}_B^G(\mathrm{St}) = 0.$$

Finally, using the short exact sequence (2) for $n \geq 2$ and the calculation of $\mathcal{R}_{\mathcal{H}_T}^{\mathcal{H}}(\mathrm{H}^n(I_1, \mathrm{St}))$ shows that $(\mathrm{R}^n \mathcal{R}_B^G(\pi))^{T_1} = 0$. Putting everything together gives

$$\mathrm{R}^n \mathcal{R}_B^G(\mathrm{St}) = \begin{cases} \mathbf{1}_T & \text{if } n = 0, \\ 0 & \text{if } n \geq 1. \end{cases}$$

5C. ***Trivial representation.*** Suppose next that $\pi = \mathbf{1}_G$ is the trivial G-representation. By [4, Corollary 6.5], we have

$$\mathcal{R}_B^G(\mathbf{1}_G) = 0.$$

To compute higher derived functors, we use the short exact sequence

$$0 \to \mathbf{1}_G \to \mathrm{Ind}_B^G(\mathbf{1}_T) \to \mathrm{St} \to 0.$$

Applying the left-exact functor $\mathcal{R}_B^G$ to the above gives an exact sequence

$$0 \to \mathcal{R}_B^G(\mathbf{1}_G) = 0 \to \mathcal{R}_B^G(\mathrm{Ind}_B^G(\mathbf{1}_T)) \cong \mathbf{1}_T \to \mathcal{R}_B^G(\mathrm{St}) \cong \mathbf{1}_T \to 0.$$

Since $\mathrm{R}^n\mathcal{R}_B^G(\mathrm{St}) = 0$ for $n \geq 1$, the long exact sequence for higher derived functors implies $\mathrm{R}^n\mathcal{R}_B^G(\mathbf{1}_G) \cong \mathrm{R}^n\mathcal{R}_B^G(\mathrm{Ind}_B^G(\mathbf{1}_T))$ for all $n \geq 1$. Thus, using (16), we conclude

$$\mathrm{R}^n\mathcal{R}_B^G(\mathbf{1}_G) = \begin{cases} 0 & \text{if } n = 0, \\ \bar{\alpha} & \text{if } n = 1, \\ 0 & \text{if } n \geq 2. \end{cases}$$

5D. ***Supersingular representations.*** Finally, suppose that π is an absolutely irreducible supersingular G-representation. Then the $\mathcal{H}$-modules $\mathrm{H}^n(I_1, \pi)$ are supersingular for all $n \geq 0$ (when C is finite, one can use [16, Section 5.4.4]; otherwise, see [17, Corollary 8.12]). Consequently, by [3, Theorem 5.20], we have $\mathcal{R}_{\mathcal{H}_T}^{\mathcal{H}}(\mathrm{H}^n(I_1, \pi)) = 0$ for all $n \geq 0$, and (1) and (2) imply $(\mathrm{R}^n\mathcal{R}_B^G(\pi))^{T_1} = 0$ for all $n \geq 0$. Thus, we conclude

$$\mathrm{R}^n\mathcal{R}_B^G(\pi) = 0 \quad \text{for all } n \geq 0.$$

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