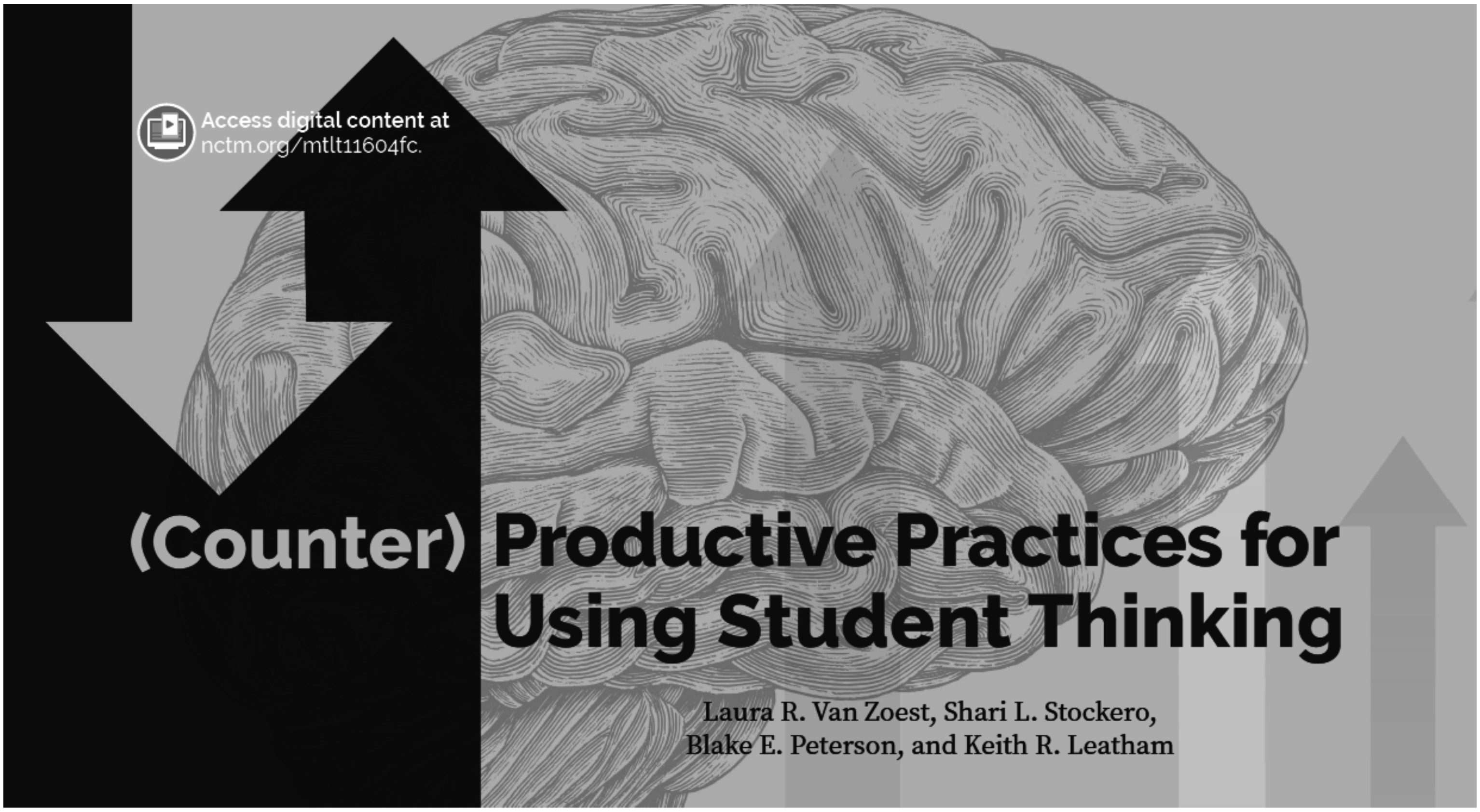

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(Counter) Productive Practices for Using Student Thinking

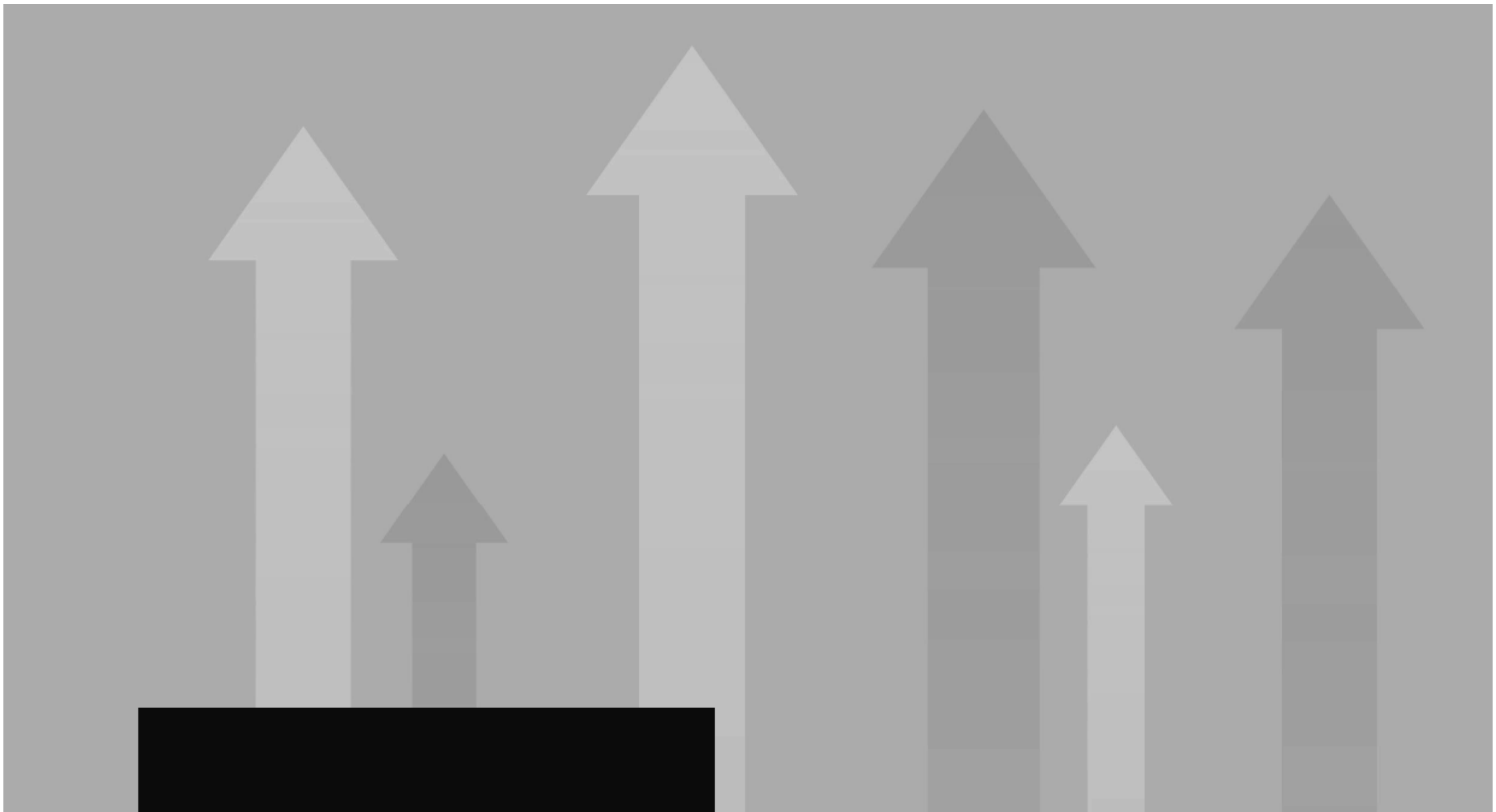
Laura R. Van Zoest, Shari L. Stockero, Blake E. Peterson, and Keith R. Leatham

The importance of mathematics instruction that meaningfully incorporates student thinking is widely agreed on (e.g., National Council of Teachers of Mathematics [NCTM], 2014). Many teachers regularly use teaching practices that have been found to support such instruction, including asking particular types of questions to support student engagement in making sense of mathematical ideas (Kazemi & Hintz, 2014), and the five practices for orchestrating a productive discussion around student ideas (Smith & Stein, 2018).

In our work, we study what it looks like to build on a high-leverage student mathematical contribution—one that provides an in-the-moment opportunity to engage the class in joint sense making about the contribution to better understand the important mathematics within it (see, for example, Stockero et al., 2014; Leatham et al., 2022). We have noticed that some “go-to” teacher practices work well in some situations but can actually be counterproductive in others. In this article, we discuss three of these practices that we have seen regularly in our work—collecting information from the class, asking a student to clarify their contribution, and asking students to revoice a peer’s contribution—providing examples of both productive and counterproductive uses of each practice. Understanding these distinctions helps teachers become more intentional about the practices that they engage in to facilitate whole-class discussion that builds on students’ contributions. To illuminate the distinctions, we use excerpts based on classroom discussions we have seen in video recordings provided to us by middle and high school teachers who used the problems in Figure 1 as part of our research project. Although our work has been based in secondary classrooms, the practices occur at all levels. At the end of the article, we include a video where we discuss these ideas within an elementary school classroom episode.

COLLECTING INFORMATION FROM THE CLASS

Collecting student ideas and solution strategies is at the core of a student-centered classroom. Common ways that teachers collect are by asking such questions as “What do you think?” and “Does anyone have a different solution?” In the following sections, we identify several ways that collecting can be productive, followed by some that are counterproductive.



Learn why collecting, clarifying, and revoicing—often great teaching moves—do not always work.

Productive Collecting

Collecting input from the class when *launching a task* can help set the stage for students to productively work on the task (Jackson et al., 2012). This can be done by asking such questions as “What do you know about [the context of the task]?”, “What might someone need to understand to get started?”, and “What are you wondering?” Another way to support students’ engagement with a task is to collect a list of “noticings” about a problem that

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students can draw from if they get stuck (Lucenta & Kelemanik, 2022).

If students are working on a task and *multiple approaches can be compared* to better understand a mathematical idea or connection among ideas, collecting students' written solutions on the board might be helpful so the class can look across the collection and draw conclusions. For example, asking early elementary school students to come up with different ways to add numbers to make 15 and writing those expressions on the board gives students the opportunity to compare and notice connections among their expressions.

Collecting can also be useful for expanding the content of a discussion ("Does anyone have a different way of thinking about this?") or for expanding participation in the discussion by engaging more students in the conversation ("Let's hear from someone who hasn't contributed yet."). In these situations, the goal is to *broaden the discussion*.

What all of these examples of productive collecting have in common is the desire to find out what students are thinking in a general way.

Counterproductive Collecting

We have found that collecting more student responses to the initial prompt is counterproductive when a high-leverage student contribution that would allow students to engage in a rich mathematical discussion is already publicly available to the class. In these cases, additional collecting often diminishes the sense-making opportunity provided by the high-leverage contribution. For example, when discussing the Variables problem (see Figure 1a), the teacher first projected Tony's (incorrect) claim on the board (see Figure 2). Tony's claim is a high-leverage contribution because it provides an in-the-moment opportunity to engage the class in joint sense making about the contribution to better understand an important mathematical idea—that all values in the domain of the variable must be considered to determine relative values of variable expressions.

The teacher then turned Tony's claim over to the class: "I want to know what you all think about this proposal. Does Tony's claim hold up mathematically, or does it not?" The class expressed both agreement and disagreement. The teacher called on Andrew, a student who was expressing disagreement, and the following interaction ensued:

Andrew: Um, I'm saying no 'cause if *x* was a negative number, then the negative and the negative would be smaller than just a normal *x*, so . . .

Figure 2 Tony's (Incorrect) Claim About the Variables Problem (see Figure 1a)

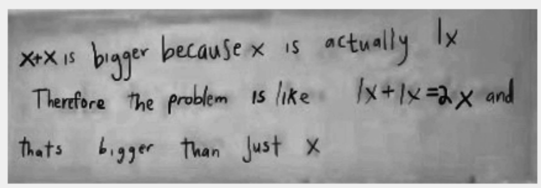
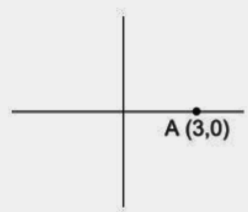


Figure 1 Problems That Provide a Context for the Examples

(a) Variables
Which is larger, x or $x + x$? Explain your reasoning

(b) Percent discount
The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?

(c) Points on a line
Is it possible to select a Point B on the y-axis so that the line $x + y = 6$ goes through both Points A and B? Explain why or why not.



(d) Bike ride
On Blake's morning bike ride, he averaged 3 miles per hour (mph) riding a trail up a hill and 15 mph returning back down that same trail. What was his average speed for his whole ride?

Note. Developed as part of National Science Foundation Grant Nos. DRL-1720410, DRL-1720566, and DRL-1720613.

Teacher: So, you're saying you have to think about x being a negative number. Ming?

Ming: But if it was a negative, then you'd have a negative plus negative, not um, just x.

Teacher: So, you're thinking that if x were negative, it would say something different than it says right now? Other thoughts? Joya?

Joya: Well, there's one x; it's just x, so if you add another x, then it's—there's two x's now. So, which, basically showing that there's one of something. 'Cause if there's nothing, it's gonna be zero x.

Teacher: Interesting; alright, a couple more. Tammy.

This interaction contains much that is productive: The teacher is listening and encouraging students to participate, and the students are providing ideas that are related to the task at hand. What is counterproductive is the gathering of new ideas from Ming, Joya, and Tammy, rather than focusing the students on making sense of Tony's contribution—a contribution that the teacher had decided was productive to discuss. Even though Tony's claim is being displayed, it is not the focus of this discussion. Imagine if instead the teacher had written down Andrew's counterclaim and asked the students to think about the connection between the two contributions. The teacher might have done this by extending their response to Andrew, "So you're saying you have to think about x being a negative number," with a question such as "What do the rest of you think that [*pointing to the new claim*] has to do with Tony's claim?" Ming and Joya could then have shared their contributions as part of a focused mathematical discussion.

Counterproductive collecting can also occur in the context of applying Smith and Stein's (2018) five practices. This counterproductive collecting happens when, after the teacher has *monitored* the students at work, they have *selected* a student contribution for the class to engage with as part of their plan for *sequencing* and *connecting*. For example, when working on the Percent Discount problem (see Figure 1b), a teacher knew from their monitoring that several students were convinced that the initial and final price would be the same and that other students realized that they would not be the same. The teacher had *anticipated* that this would be the case, and their goal was to use these discrepancies to help students better understand percentages. Things were going according to plan when the teacher elicited and established this initial (incorrect) claim from Jordan: "If you start with a price, add 50% of the price and subtract 50% of the price, you end up with the price you started with."

At this point, continuing to collect a variety of other initial ideas would have been counterproductive because Jordan's contribution provided something for students to engage with and make sense of. Instead, the teacher made the productive move of inviting the class to consider Jordan's claim, which elicited this counterclaim from Samara: "I don't agree with Jordan's claim, because if you take 50% of the original price of 100 and add it to the 100 that is the original price, you'll have 150; and then if you take 50% of that, you'll have 75."

Samara's counterclaim provided an opportunity to identify the problematic nature of Jordan's claim and to better understand that when taking percentages, it matters what one is taking a percentage of. Thus, a productive teacher move at this point in the discussion would be to ask the class to engage with the claims that were already on the table; for example, by recording Samara's claim on the board next to Jordan's claim and asking, "How is what Samara said different from Jordan's claim?" Instead, however, the teacher made a counterproductive move by asking, "Did anybody have a different way of thinking about Jordan's claim?" This move diffused the momentum of the counterclaim and did not take advantage of the opportunity for students to make sense of these claims to better understand an important idea about percentages. In an interview, the teacher said that they collected here because they were concerned about the students taking up the counterclaim and resolving the problem too quickly, because the counterclaim "gave it away." What we have found is that just because the teacher can see the resolution of the problem in a student contribution does not mean that the students can. In fact, rather than short circuiting their learning, a "correct" counterclaim provides the opportunity for students to contrast the original contribution and the counterclaim so they can decide which is correct and why.

We also discovered through interviewing teachers that counterproductive collecting often stems from a desire to involve as many students as possible in discussions. Although asking for more contributions is a tempting way to get additional students involved, we have found that doing so after you have already established a high-leverage student contribution as the focus of the class discussion undermines the potential of that contribution to support student learning. Instead, teachers can deepen the learning by shifting their focus to keeping students engaged with making sense of that contribution to help the class better understand important mathematical ideas.

ASKING CLARIFYING QUESTIONS

Asking clarifying questions is a teacher practice that can be productive if something in the contribution is actually unclear. However, it can be counterproductive if used out of habit rather than because it is needed.

Productive Clarifying

Interacting with an individual student to clarify their contribution is productive when students use *vague language*, such as a pronoun that does not have a clear referent (Peterson et al., 2020), or when an important part of their *reasoning is implicit* and needs to be made explicit. In such cases, a move to clarify the contribution is necessary to ensure that everyone in the class has a clear sense of the idea that the teacher wants the class to discuss.

Consider, for example, a situation where students have individually worked on the Percent Discount problem (see Figure 1b). The teacher asks Dean to share his solution and he replies: “Uh, so I think that the original, it will be the original price. And the reason for this is because if we do it, as it says, it increases by 50%. So, 50% of 10 is $5. We would add $5. So, $15 would be the price; and then if it decreases by 50%, we would subtract 5, and then that would be 10.”

The teacher then clarifies Dean’s claim by asking, “So you’re saying yes, the final price is the same as the original price?”, to which Dean responds, “Yeah.” In this case, a clarifying move was productive because the fact that Dean is claiming that the final price is the same as the original price is only implicit in his response. It is important that other students in the class know exactly what Dean is claiming before they engage in making sense of his idea. Recording a student contribution on the board, as this teacher later did (see Figure 3), is an important way to anchor a whole-class discussion about that contribution (for more information, see Freeburn et al., 2022; Garcia & Shaughnessy, 2021).

Unproductive Clarifying

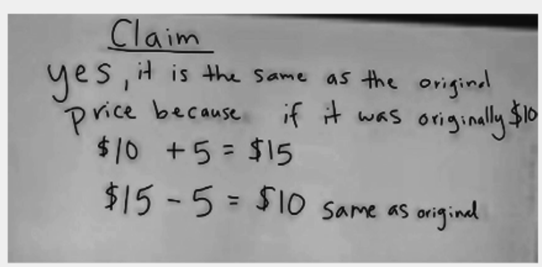
As the class discussion unfolds, however, the teacher continues to interact with Dean (and importantly, no other students in the class) to clarify his contribution in unproductive ways:

Teacher: Because, and you said, you chose, $10?
Dean: Yes, as my original price, $10.
Teacher: OK [*beginning to record Dean’s contribution on board; see* Figure 3], then it would increase by?
Dean: Fifty percent, which would be $5.
Teacher: [*Continuing to record*] And that gives you?
Dean: $15.
Teacher: [*Continuing to record*] OK. And then?
Dean: I would subtract it by 5 because it would decrease it by 50%.
Teacher: So, then you would do $15 [*continuing to record*] minus $5?
Dean: Yeah.
Teacher: Which would give you?
Dean: Ten.
Teacher: [Continuing to record] Ten. Right?
Dean: Yes.
Teacher: [*Continuing to write Dean’s contribution on board*] And you wrote, “same as original,” I notice on your paper.
Dean: Yes, yes, I did.

In this exchange, the teacher asks Dean a series of eight questions about his contribution. The problem here is that Dean had already clearly said everything that the teacher wanted to record on the board to support the subsequent discussion, except for the claim that the original price was the same as the final price (which the teacher productively asked about). Thus, nothing else needed to be clarified.

We see several problems with going back to the same student repeatedly when doing so is unnecessary. First, getting lost in the details of a high-leverage contribution diminishes the opportunity to engage the class in making sense of the contribution as a whole—potentially losing sight of the

Figure 3 Public Record of Dean’s Claim for the Percent Discount Problem (Figure 1b) at the End of the Teacher’s Questioning



forest for the trees. Second, not engaging other students increases the likelihood that they will lose interest in the mathematics underlying the student contribution. Third, this type of interaction may send a message to the contributing student that they did not clearly articulate their idea when, in fact, they did. We see this last problem as a particular concern when working with English language learners and other students who are still learning to share their ideas in productive ways.

Although we did not see it arise in the exchange above, another potential problem with the teacher asking unnecessary clarifying questions of the contributing student is that the teacher can actually probe too much and bring out the big mathematical idea from that student. Having the same student make sense of their own idea when interacting with the teacher takes the mathematical opportunity provided by the contribution away from other students. For example, in the exchange above, had the teacher asked Dean a question such as, "So 50% of 15 is 5?", Dean might have realized the error with his reasoning, removing the opportunity for other students to consider the idea that when taking percentages, which number you are taking the percentage of matters.

In general, once a teacher has decided to make a high-leverage student contribution the focus of a whole-class discussion (see Stockero et al. [2014] for information on this decision-making process), we have found that it is most productive to ask the contributing student only the questions that are necessary to make the student's idea "clear enough" for other students to engage with it. Getting the student contribution clear enough that students know what they need to think about, without going so far as to diminish the sense-making work, positions the class to engage in joint sense making about the important mathematics underlying the contribution.

ASKING A STUDENT TO REVOICE A PEER'S CONTRIBUTION

Asking a student to revoice their peer's contribution is a common teaching practice that can be productive when it is used to enhance student engagement and get a sense of whether students are tracking the classroom discussion, but it is counterproductive if it detracts from opportunities for sense making.

Productive Student Revoicing

Asking students to revoice a peer's contribution can be productive when you want to *assess whether students understand* what another student has said. Sometimes productive revoicing can occur in the midst of a discussion to make sure that students understand an important student question that you want the class to consider. For example, in the Points on a Line problem (see Figure 1c), a student asked, "Are coordinates A and B, *x* and *y*? Like are Points A and B, like the *x* and *y* in the equation; so, like, is A *x*, and is B *y*?" The teacher then asked, "Can someone revoice his question? What is he asking? It's an important one." In this case, the teacher used revoicing as a move to ensure that students understood an important question that got at the heart of the mathematical issue under discussion—what it means for a point to satisfy an equation. Another student's response, "He's saying that, like, the A point *x* and the B point *y*, like, is that how you add them together to get 6, or like . . . ," gives the teacher a sense of whether other students in the class know what question they will be trying to answer.

Revoicing can also be productive when you want to ensure that students are *taking away the big ideas* from a class discussion. For instance, in a discussion of the Points on a Line problem, Jasmine summarized the main issue that had surfaced in the discussion: "Like A equals A, *x*, A, *y*; and B has its own *x*- and its own *y*-coordinate. So, this one [*pointing to A on the graph*] is (3, 0), and then this one [*pointing to B on the graph*], they're saying it's (0, 3). What the line is saying in this equation [*pointing at "$x + y = 6$" on the board*], it's saying that *x* plus *y* needs to equal 6. So, if you do that, it's 3 plus 0 equals 6, and that's false. And like 0 plus 3 equals 6, and that's also false. So, they're getting, I think—or multiple people are getting confused that, like, this and this, you can't add these two 3's together because it's not in the same coordinate. 'Cause the line could go through (3, 3). It's 3 plus the 3 equals 6, and it'll go through 6 plus 0 equals 6. But it's not gonna go through (3, 0) or (0, 3)."

Because this summary was lengthy and possibly not understood by everyone in the class, the teacher productively used a revoicing move to check for student understanding: "Can someone revoice what she's saying in their own words? How would you say it?"

Bradley replied, "So, an *x*- and *y*-coordinate at any point must add up to 6 on this line, and 3 plus 0 do[es] not." As with the first example, Bradley's revoicing gives the teacher a sense that other students in the class understand the issue at hand.

Counterproductive Student Revoicing

We have found that revoicing is often counterproductive when initially trying to establish a high-leverage student contribution as the focus of a class discussion. Even if the contribution is not entirely clear, going back to the student who contributed the idea (as discussed above) is better than asking another student to revoice. This is because we have found that the revoicing student often does not accurately restate the idea that the teacher is trying to get on the table. We suspect that this is the case because even though which aspects of the student contribution matter are obvious to the teacher, if students are in the process of learning those aspects, which parts of the contribution are important to revoice are unlikely to be obvious to them. Additionally, we have seen that students often do not revoice the contribution at all, but instead contribute their own ideas. In both cases, the response to a teacher requesting that another student revoice is likely to diminish students' opportunity to engage with the high-leverage contribution. Consider the following exchange from the Bike Ride problem (see Figure 1d):

Loret: OK, so why I think it's 9 is because if you do, I guess that would be 4, 5, 6 [*writing out numbers 3 through 15*] and you find the number in the middle. So, what I did is I just kind of cross them out as I go [*crossing out all the numbers except for 9*]. Nine is the number that's in the middle in between 3 and 15. So, that is how we find the average, so that's why I said 9.

Teacher: Could someone revoice how Loret thought about this? Yeah, how'd she think about this?

Lila: So, how I was taught to find the average is you add the two numbers together, so 3 plus 15 is 18. And then you divide it by the amount of numbers given, so 18 divided by 2 is 9.

In this case, the responding student moved the discussion away from Loret's idea to instead introduce the standard calculation for finding an average—a calculation that Loret never used.

In general, asking students to revoice a peer's contribution is something that needs to be done with caution. It can be productive when assessing students' understanding of what has been said. However, it can be unproductive when the teacher is trying to get a particular high-leverage contribution on the table for the class to discuss because having another student revoice the contribution might lose the mathematical opportunity the original contribution provided.

CONCLUSION

Table 1 summarizes the three (counter)productive teaching practices we have discussed—collecting information from the class, asking a student to clarify their contribution, and asking students to revoice their peer's contribution—with criteria for when each practice is productive or counterproductive. What is common among all the counterproductive uses of these teaching practices is that they diminish opportunities to engage the whole class with a high-leverage contribution that is already available for discussion.

Table 1 When to Use and Avoid Using Three (Counter)Productive Teaching Practices

Teaching Practice	Is Productive When	Is Counterproductive When
Collecting information from the class	You want to elicit students' thinking in a general way.	You already have a student contribution that you want the class to engage with to make sense of an important mathematical idea.
Asking clarifying questions	Something in a student contribution needs to be clarified for students to engage in making sense of it.	A student contribution is "clear enough" for the class to engage in making sense of it.
Asking students to revoice	You want to assess whether students understand the focus of the discussion.	You are trying to establish a contribution that you want students to discuss.

We invite you to view Video 1 now, and then use Table 1 to assess your current use of these teaching practices. For example, you might videotape a lesson and look for instances where you engage in these teaching practices and then use the ideas discussed in this article to decide if the practice was used in productive or counterproductive ways. In addition, consider other teaching practices that you routinely use. In what situations might they actually be counterproductive? Intentionally reflecting on whether common teaching practices are equally effective in all situations can help you make minor adjustments to your practice that can have a major effect on students' opportunities for sense making.

Video 1: Discussing an Elementary School Classroom Episode

Watch the full video online

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