MATHEMATICS TEACHER
Learning & Teaching
PK 12

Mathematics Teacher: Learning and Teaching PK-12, is NCTM's newest journal that reflects the current practices of mathematics education, as well as maintains a knowledge base of practice and policy in looking at the future of the field. Content is aimed at preschool to 12th grade with peer-reviewed and invited articles. *MTLT* is published monthly.

ARTICLE TITLE:

Tackling Tangential Student Contributions

AUTHOR NAMES:

Peterson, Blake E.; Stockero, Shari L.; Leatham, Keith R.; Van Zoest, Laura R.

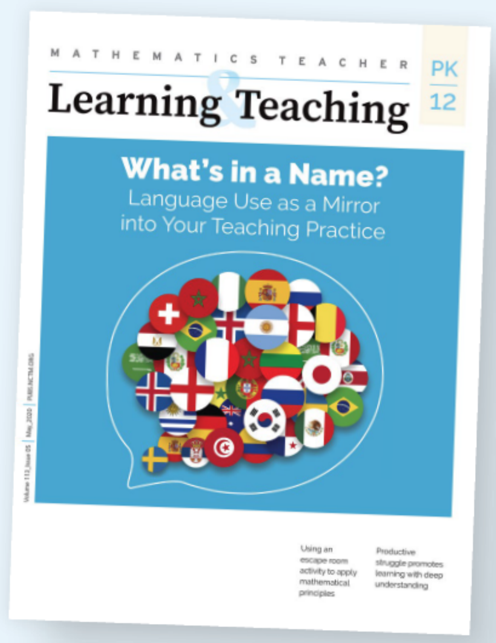
DIGITAL OBJECT IDENTIFIER:	VOLUME:	ISSUE NUMBER:
10.5951/MTLT.2021.0327	115	9

Mission Statement

The National Council of Teachers of Mathematics advocates for high-quality mathematics teaching and learning for each and every student.

Approved by the NCTM Board of Directors on July 15, 2017.

CONTACT: mtlt@nctm.org



NCTM
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Copyright © 2022 by The National Council of Teachers of Mathematics, Inc. www.nctm.org. All rights reserved. This material may not be copied or distributed electronically or in any other format without written permission from NCTM.

Brought to you by [Communal Account] | Authenticated null | Downloaded 09/08/22 03:02 AM UTC

Access digital content at
nctm.org/mtlt11509fc.

Tackling *Tangential Student* Contributions

Blake E. Peterson, Shari L. Stockero, Keith R. Leatham, and Laura R. Van Zoest

Consider the following scenario that begins with a student sharing a response to the Points on a Line task seen in figure 1a.

Angel: Yes. Point B is (0, 3) because you can put the 3 from point A in for x and the 3 from point B for the y and get $3 + 3 = 6$.

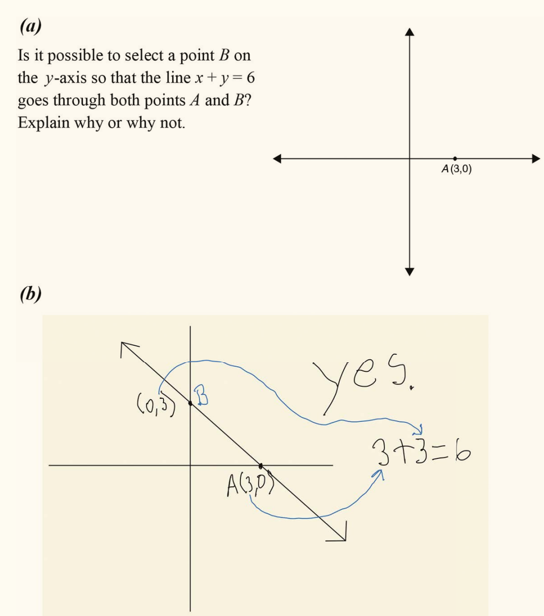
Teacher: [*Recording Angel's thinking; see figure 1b*] What do others think about how this response holds up mathematically?

[*The class begins to engage in making sense of Angel's approach.*]

Ben: If you write the equation in slope-intercept form, the y-intercept is (0, 6).

In this scenario, the teacher had chosen to engage the class in making sense of Angel's contribution, providing students with the opportunity to make sense of what it means for a point to satisfy an equation. Despite the teacher's question that explicitly asked students to engage with Angel's solution, Ben shared how he approached the task—an approach related to finding the y-intercept using the slope-intercept form of the equation.

Fig. 1



(a) Students were given the Points on a Line task, and (b) Angel's contribution was recorded on the board.

 © 2022 NCTM
Brought to you by [Communal Account] | Authenticated null | Downloaded 09/08/22 03:02 AM UTC

Do your students ever share ideas that are only peripherally related to the discussion you are having? We discuss ways to minimize and deal with such contributions.

Have you ever started into a discussion about a student contribution with your class and had a contribution with different underlying mathematics arise, as in the previous scenario? Have you ever attempted to engage students in a discussion about a classmate's thinking and felt like you were performing a juggling act? Engaging students in meaningful mathematical discussions focused on making sense of other students' mathematical contributions

Blake E. Peterson, blake@byu.edu, is a professor of mathematics education at Brigham Young University in Provo, Utah.

Shari L. Stockero, stockero@mtu.edu, is a professor of mathematics education at Michigan Technological University in Houghton.

Keith R. Leatham, kleatham@mathed.byu.edu, is a professor of mathematics education at Brigham Young University in Provo, Utah.

Laura R. Van Zoest, laura.vanzoest@wmich.edu, is a professor of mathematics education at Western Michigan University in Kalamazoo.

The authors are currently studying how to identify and productively build on teachable moments—what they call MOSTs (Mathematical Opportunities in Student Thinking).

doi:10.5951/MTLT.2021.0327

 © 2022 NCTM

Brought to you by [Communal Account] | Authenticated null | Downloaded 09/08/22 03:02 AM UTC

has been advocated by the National Council of Teachers of Mathematics (NCTM 2014). However, facilitating such discussions can be really difficult! Teachers might struggle for many reasons to engage their students in a rich discussion of a fellow student's idea. One reason is that students often say or do things that are only tangentially related to the conversation at hand. Instead, they contribute something else they had been thinking about, as seen in the opening scenario. Although the teacher may want students to contribute a variety of ideas at times during a lesson, once the teacher decides to engage students in making sense of a student contribution, staying focused on that contribution is more desirable because it honors the student who made the contribution and takes full advantage of the opportunity to learn mathematics that the contribution provided. What is a teacher to do when students share an idea with underlying mathematics that is different from that of the contribution being discussed? Could such situations be avoided to begin with? In this article, we share strategies that can help us as teachers better keep the mathematics related to the student contribution under discussion at the heart of that discussion. Although our work and the examples used throughout this article are situated within middle and high school classrooms, the suggestions are certainly applicable to the elementary classroom too. In video 1 (link online), two members of the author team share insights into how these ideas are connected to the elementary classroom.

BACKGROUND

To learn about the background and context of this article, watch video 2 (link online). In addition, before proceeding, we introduce some terms we use to talk about classroom discussions about student mathematical contributions. A common student response to the Points on a Line task (see figure 1) is the one contributed by Angel in the opening scenario, that point B is (0, 3) because you can take the x-value for the equation $x + y = 6$ from point A and the y-value from point B. To engage students with the mathematical idea that the x- and y-values need to come from a single point when determining whether a point satisfies an equation, a teacher may choose to pause the broader mathematical activity to focus on this particular student contribution. We refer to a student contribution that the teacher has chosen to pursue for some particular purpose as a *focal instance*.

Focal instances arise in a variety of ways. In our own work, as described in the video, Angel's response would be a MOST, a *Mathematical Opportunity in Student Thinking* (see Stockero et al. 2014 for details on identifying MOSTs). The goal is to position MOSTs as focal instances of whole-class sense making. Focal instances also play a critical role in work around the *Five Practices for Orchestrating Productive Mathematical Discussions*: anticipate, monitor, select, sequence, and connect (Smith and Stein 2018). When enacting these practices, any student solution that a teacher selects to include in the discussion could be considered a focal instance. A teacher could use many criteria to decide whether a student contribution should be a focal instance, but those criteria are not the focus of this article. This article focuses on teacher moves that keep the whole-class discussion centered on a contribution that the teacher has decided is a focal instance.

Productively engaging students in making sense of a focal instance requires that the instance remain at the center of the whole-class discussion. Thus, the teacher

Video 1 Discussing an Example from an Elementary Classroom

Watch the full video online.

Video 2 The Background of the Project

Watch the full video online.

 © 2022 NCTM
Brought to you by [Communal Account] | Authenticated null | Downloaded 09/08/22 03:02 AM UTC

must temporarily pause the broader mathematical activity to explicitly take up the focal instance, creating what we refer to as a *conversational bubble* around this instance. In the scenario at the beginning of this article, the teacher paused the discussion of the original question to focus on Angel's strategy of using x- and y-values from different points to satisfy an equation. As we saw in the scenario, the teacher posed the question, "How does this approach hold up mathematically?" to prompt the class to make sense of Angel's reasoning. To maintain the conversational bubble, both the teacher and students need to know that they have paused the broader mathematical activity to focus on the mathematics of the focal instance.

Even when teachers deliberately try to create a conversational bubble around a focal instance, students still make contributions that are not necessarily related to the mathematics of the focal instance. In the opening scenario, after the common focal instance had been tossed to the class for consideration, Ben contributed: "If you write the equation in slope-intercept form, the y-intercept is (0, 6)." This contribution is not related to the focal instance because the underlying mathematics is different than the mathematics of the focal instance. We refer to such mathematically unrelated instances as *tangential student contributions*.

Oftentimes a tangential student contribution is related to the original task but not to the focal instance and thus pursuing it would take the discussion out of the conversational bubble. In other instances, a tangential student contribution is about a prerequisite idea, or the relationship to the focal instance is unclear. When a tangential student contribution surfaces, you as the teacher are faced with a dilemma. What are you to do? Is it possible to honor both the student who contributed the focal instance and the student who contributed the tangential student contribution?

In this article, we share what we have learned about how teachers can both reduce the frequency of tangential student contributions and respond to such contributions if they are shared, at the same time honoring students and their ideas. Before we do so, however, we offer a word of caution. Any time a teacher is deciding whether a student contribution is related to the focal instance or is a tangential student contribution, issues of equity need to be kept in mind. Because students come from different cultural and linguistic backgrounds, it is critical that the teacher not prematurely put aside a contribution just because it is not clearly articulated or is stated in a mathematically unconventional way. The decision of whether a student contribution is related to the focal instance under discussion or is a tangential student contribution should be based on the mathematics of the contribution and not on the characteristics or demographics of the student who shared it or the form in which it was shared.

REDUCING THE FREQUENCY OF TANGENTIAL STUDENT CONTRIBUTIONS

Our work has revealed that the teacher has some control over the occurrence of many tangential student contributions. Here we describe three ways that teachers might reduce the frequency of tangential student contributions. Although no teacher action guarantees that student contributions that emerge during a discussion will be about the focal instance, teachers' actions can influence what student thinking is shared.

Asking Targeted Questions

One way to reduce the frequency of tangential student contributions relates to the questions teachers ask to engage students with the focal instance. If the goal is for students to discuss a focal instance, then the teacher's questions should be explicitly about that instance. General questions, like "What do you think?" or questions about the broader mathematical activity (e.g., Did others get different solutions?), open the door for students to share their own solution to the broader task rather than engaging with the focal instance. More targeted questions, like "How does this claim [that B is (0, 3) because $3 + 3 = 6$] hold up mathematically?" help keep the discussion centered on the focal instance and thus reduce the frequency of tangential student contributions.

Helping Students Stay in the Conversational Bubble

Teachers can also help reduce the frequency of tangential student contributions by proactively helping students stay in the conversational bubble. During classroom discussion, students often spontaneously raise their hands. Despite the fact that what they are responding to is unclear, we have frequently seen teachers invite one of those students to share. We call these invitations the "box of chocolates" approach to conducting a class discussion—you never know what you are going to get. It may feel as if the best way to honor students is to allow any student with a raised hand to share. However, doing so could actually dishonor the contributor of the focal instance because it potentially moves the conversation away from making sense of their idea. In "box of chocolates" situations, we often see students share their own solution to the task, rather than responding to the focal instance. When teachers do not know if the thinking is likely related to the focal instance, they can ask a spontaneous volunteer a targeted

Brought to you by [Communal Account] | Authenticated null | Downloaded 09/08/22 03:02 AM UTC

> The [conversational] bubble analogy was really helpful. To try and think about keep[ing] things in the bubble really worked well for me.—*Judy, middle school mathematics teacher*.

question such as, "Do you have something to share about [the focal instance]?" instead of a general question such as, "Do you have something to share?" This specificity highlights the importance of the focal instance that is being discussed, better honors the original student's thinking, and makes the surfacing of a tangential student contribution less likely. Scaffolding student participation keeps the conversation focused by providing students with guidance for meaningfully contributing to the discussion.

Establishing Classroom Norms for Whole-Class Discussion

Establishing classroom norms for whole-class discussion can also reduce the frequency of tangential student contributions. Such norms are important because despite a teacher's best efforts to ask targeted questions and to scaffold student participation, students share tangential student contributions anyway. For example, consider a situation where students were given the following task:

> The price of a necklace was first increased 50 percent and later decreased 50 percent. Is the final price the same as the original price? Why or why not?

Sonia, a middle school student, shared, "When you increase by 50 percent of the original price, you get a new value, and then, if you times that by 50 percent, it's going to be less than the original value."

The teacher made Sonia's response a focal instance by asking, "How does what Sonia said hold up mathematically?"

Edward responded, "On mine, I used the variable x to represent the original price." In this instance, even though the teacher specifically asked the class to reason about the focal instance shared by Sonia, Edward shared a tangential student contribution. Note that we are not saying that Edward's contribution might not be useful in some way, just that it does not help make sense of Sonia's thinking.

Because situations like this seem resistant to specific moves, we believe the best way to address them is by clearly communicating classroom norms over time. In the example above, the teacher could view Edward's comment as an opportunity to reinforce norms around making sense of a classmate's contribution. In fact, we see the development of norms around classroom discussion as key to helping students better keep their contributions focused on a focal instance. (For ideas on developing classroom norms, see Bennett and Morgan 2020.) Thus, we suggest that teachers work to establish norms through conscientiously using focusing moves. They can also have explicit conversations about how keeping contributions connected to the focal instance is everyone's responsibility once the teacher has established the instance as the object of consideration.

RESPONDING TO TANGENTIAL STUDENT CONTRIBUTIONS

When a tangential student contribution emerges, how can teachers respond yet maintain the conversational bubble? How can such moves be done while managing the tension a teacher might feel between remaining centered on the focal instance and honoring the student who contributed the new idea? Our work with teachers has revealed that an effective (and efficient) way to remain in the conversational bubble is to gracefully put aside the tangential student contribution and then recenter the focal instance. For example, in response to Edward's contribution in the necklace example, a teacher might say, "You've just shared another way you could think about this task, but remember that right now we're making sense of Sonia's claim"—a statement that positions Edward as a legitimate mathematical thinker but keeps the discussion centered on the focal instance. When the teachers we have worked with used a combination of putting aside and recentering moves, the discussion often stayed centered on the focal instance.

We emphasize the importance of both putting aside *and* recentering. Without a recentering move after putting aside a tangential student contribution, another tangential student contribution frequently surfaces. For example, sometimes teachers call on a spontaneous volunteer (a "box of chocolates" move) to put aside a tangential student contribution. We have also seen teachers ask general questions—such as, "What did others get?" or "What do you

> I found [recentering the focal instance] to be really helpful to me to kind of keep things in the [conversational] bubble and not let so many things continue to surface, but to keep it tied back and connected to [the focal instance] —*Trevor, middle school teacher*.

Brought to you by [Communal Account] | Authenticated null | Downloaded 09/08/22 03:02 AM UTC

think?"—which creates an opening for students to return to the broader mathematical activity or share whatever they happen to be thinking. This approach of not recentering the focal instance but continuing to gather general student comments seems to value student participation but not their mathematical thinking. Inviting students to share mostly to value their participation is better accomplished at times in a lesson other than those where a conversational bubble has been created around a focal instance.

STAYING WITH A STUDENT CONTRIBUTION THAT IS POTENTIALLY TANGENTIAL

We do not want to convey to the reader that all student thinking that is potentially tangential should be put aside as soon as possible. In some instances, a teacher needs to allow a few conversational turns between themselves and the contributing student before deciding whether to pursue or put aside a contribution—when in its current form the thinking seems unrelated to the focal instance, but a connection that is not yet evident to the teacher is possible. In such situations, the teacher might push the student to clarify the idea or ask for an explanation of how the idea helps make sense of the focal instance. If the contribution turns out to be unrelated, the teacher could gracefully put aside the tangential student contribution (e.g., "Thank you for sharing" or "I understand what you're saying"), and then recenter the focal instance (e.g., "So, where are we in making sense of [the focal instance]?") If the contribution in fact turns out to be related to the focal instance, posing a question to the class to explicitly help students connect the two instances is important: "How does [the current contribution] help us make sense of [the focal instance]?"

Another situation where a teacher may need to spend some time with a tangential student contribution is when the contribution includes mathematics that is a prerequisite for making sense of the focal instance. For example, in a task where students were comparing the two variable expressions x and $x + x$, a common focal instance was the claim that "because $x + x$ is $2x$, $x + x$ is always greater than x." When discussing this claim, sometimes the tangential student contribution that the x's can simultaneously take on different values surfaced (e.g., "I made the single x be 7 and then made each x in $x + x$ be 3. So, x is greater than $x + x$ because 7 is bigger than 6."). This misunderstanding about comparing variable expressions must be resolved before making sense of the focal instance because students need to understand that all of the x's in both expressions need to be the same value before they can meaningfully compare the relative values of x and $x + x$. As with responding to any tangential student contribution, we have found that after clearly resolving the misunderstanding, the focal instance needs to be recentered to maintain the conversational bubble. To decide if a tangential student contribution falls into this "prerequisite" category and needs to be pursued, ask yourself, "Must the students make sense of this idea before they can make sense of the focal instance?"

To gain a sense of what these ideas might look like in the classroom, check out the transcript in the appendix (link online) of class discourse around the Points on a Line task.

CONCLUSION

Having conversations about student thinking and solution strategies should be a common part of mathematics instruction (NCTM 2014). Teachers often choose to center such conversations on focal instances. We have proposed that a productive way to have a class discussion about a focal instance is to pause the broader mathematical activity and create a conversational bubble around that instance. Establishing this routine of creating conversational bubbles will take some work by both teachers and their students to create new norms about whole-class discussions—norms related to what ideas are appropriate to be shared within a conversational bubble around a focal instance. Even when a conversational bubble around a focal instance is intentionally established, however, a tangential student contribution can still arise.

Table 1 summarizes the moves we have found to be helpful and unhelpful in both reducing the frequency of and responding to tangential student contributions during class discussions. Notice that the suggested questions in the "Do" column put the focal instance at the center of the discussion, whereas the questions and moves in the "Don't" column leave students open to contribute a range of responses that may or may not be related to the focal instance.

Taking actions such as those in the "Do" column and avoiding actions in the "Don't" column of table 1 honor student mathematical thinking when facilitating a discussion about a focal instance by keeping that thinking at the center of the discussion. Being intentional about the moves we as teachers make when facilitating a discussion about a focal instance and when addressing a tangential student contribution can help us have more productive classroom discussions centered on student mathematical thinking.

Brought to you by [Communal Account] | Authenticated null | Downloaded 09/08/22 03:02 AM UTC

Table 1 Suggestions for Reducing the Frequency of and Responding to Tangential Student Contributions

	Do	Don't
To reduce the frequency of tangential student contributions	Do ask targeted questions to prompt students to make sense of the focal instance, such as "How does [the focal instance] hold up mathematically?" Do proactively help students stay in the conversational bubble by asking spontaneous volunteers a targeted question, such as "Do you have something to share about [the focal instance]?" Do work on developing norms around classroom discussion to help students keep their contributions focused on a focal instance.	Don't ask general questions, such as "What do you think?" or "What did others get?" Don't call on spontaneous volunteers using a general question, such as "Do you have something to share?"
To respond to a tangential student contribution	Do put aside the tangential student contribution and deliberately recenter the focal instance (e.g., "That's an interesting idea, but remember that right now we're making sense of [the focal instance]").	Don't attempt to transition away from the tangential student contribution by just calling on a new student or allowing another student to spontaneously contribute (Box of Chocolates moves).
To stay with a contribution that is potentially tangential	If unsure, do allow a few conversational turns to explore and better understand whether a contribution is related to the focal instance. Do quickly resolve prerequisite misconceptions and then recenter the focal instance.	Don't pursue thinking that is clearly not related to the focal instance.

REFERENCES

Bennett, Cory A., and Mick J. Morgan. 2020. "Developing Norms with Silent Discussions." *Mathematics Teacher: Learning and Teaching PK–12* 113, no. 10 (October): 841–44.

National Council of Teachers of Mathematics (NCTM). 2014. *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Smith, Margaret S., and Mary Kay Stein. 2018. *Five Practices for Orchestrating Productive Mathematics Discussion*. 2nd ed. Reston, VA: National Council of Teachers of Mathematics; Corwin Press.

Stockero, Shari L., Blake E. Peterson, Keith R. Leatham, and Laura R. Van Zoest. 2014. "The 'MOST' Productive Student Mathematical Thinking." *Mathematics Teacher* 108, no. 4 (November): 308–12.

ACKNOWLEDGMENTS

This research report is based on work supported by the National Science Foundation (NSF) under Grant Nos. DRL-1720410, DRL-1720566, and DRL-1720613. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the NSF.

We thank the teacher-researchers for opening up their practice to help us learn about building enactments, and the undergraduate and graduate students who have helped us to collect, process, and analyze building enactments.

 © 2022 NCTM

Brought to you by [Communal Account] | Authenticated null | Downloaded 09/08/22 03:02 AM UTC