Distributed Nonlinear Placement for Multicluster Systems: A Time-Varying Nash Equilibrium-Seeking Approach

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***Abstract*—In this article, a class of distributed nonlinear placement problems is considered for a multicluster system. The task is to determine the positions of the agents in each cluster subject to the constraints on agent positions and the network topology. In particular, the agents in each cluster are placed to form the desired shape and minimize the sum of squares of the Euclidean lengths of the links amongst the center of each cluster and its corresponding cluster members. The problem is converted into a time-varying noncooperative game and then a distributed Nash equilibrium-seeking algorithm is designed based on a distributed observer method. A new iterative approach is employed to prove the convergence with the aid of the Lyapunov stability theorem. The effectiveness of the distributed algorithm is validated by numerical examples.**

***Index Terms*—Distributed algorithm, Nash equilibrium (NE) seeking, noncooperative game, nonlinear placement.**

I. Introduction

NONLINEAR placement is a typical geometric problem and has attracted extensive attention due to its diverse applications. The objective of nonlinear placement is to place some points with additional constraints such that the sum of squares of the Euclidean lengths of the links amongst the points is minimized. The applications of nonlinear placement include transportation cost control, wire placement in integrated circuits, and sensor placement in networks [1], [2]. For example, in the transportation cost control problem, given several possible locations of plants or warehouses of a company and the routes over which goods must be shipped, the task is to find specific locations that minimize the total transportation cost. On the other hand, in the wire placement problem for integrated circuits, given the positions of modules or cells and the wires that connect pairs of cells, the objective is to place the cells such that the total length of wires used to interconnect the cells is minimized. All these problems can be modeled as a nonlinear placement problem and the solutions can be found by a convex optimization method, as shown in [1].

The study on multiagent system has gained increasing attention in the literature on consensus control [3], [4]; distributed optimization [5], [6]; formation control [7]–[9]; etc. Specifically, recent years have witnessed a rapid growth in the study of multicluster systems. The main reasons lie in the following facts. First, multicluster systems allow the coexistence of cooperation and competition. In particular, cooperation and competition normally coexist in natural and engineering systems, as well as in many practical applications of multiagent systems. For example, cooperation and competition appear simultaneously in interactive living systems [10], [11]. In the noncooperative game for multicluster systems, the clusters behave as self-interested "virtual players" while the agents in the same cluster coordinate to minimize their total cost [12], [13]. Second, the multicluster structure is beneficial to the implementation of distributed control for multiagent systems with a large number of agents. For example, formation control is a widely studied topic within the realm of multiagent systems. A typical position-based formation control is to guarantee a prescribed desired formation shape and/or track some prescribed desired reference trajectories [8], [9]. Generally speaking, the prescribed formation shape and reference trajectories are designed by a centralized method, which is often quite difficult for a large number of agents. A natural idea is to cut apart the network into some clusters, where each cluster only includes a few agents with a prescribed designed formation shape or some reference trajectories. Moreover, the shape connecting the centers of the clusters can be designed automatically by using the distance between the clusters.

Therefore, formation control of a multicluster system is modeled as a distributed nonlinear placement problem in this

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article. The shape connecting the centers of the clusters is automatically guaranteed by a distributed method. In particular, the shape depends on the placement of the followers such that the sum of squares of the Euclidean lengths of the links amongst the center of each cluster and its corresponding cluster members is minimized, subject to additional constraints on agent positions and the network topology. The problem is solved in two steps. First, the problem is converted into a time-varying Nash equilibrium (NE)-seeking problem. Then, a distributed seeking algorithm is designed based on a distributed observer method, and the results are proved by using the Lyapunov stability theorem.

The main contributions lie in the following facts.

1) We solve the distributed nonlinear placement problem for multicluster systems. It can be viewed as a special position-based formation control problem since the formation shape is automatically guaranteed by a distributed method. To the best of our knowledge, there are no results that are capable of solving the distributed nonlinear placement problem for multicluster systems with limited information exchange. For example, the methods in [1] and [2] are inapplicable to our problem since they are centralized.
2) The most promising feature of our algorithm is that it is capable of handling time-varying NEs for the multicluster systems, which renders the NE-seeking algorithm design and the convergence analysis more challenging. In contrast to most of the distributed NE-seeking algorithms in [12]–[17], where the cost functions are required to be time invariant, our algorithm is capable of tackling time-varying cost functions.
3) In contrast to [18], where a time-varying NE-seeking algorithm is designed by using a nonmodel-based method, our algorithm has a zero convergent error and the multicluster models are not considered in [18].

The remainder of this article is organized as follows. In Sections II and III, preliminaries and problem formulation are presented. Section IV presents the distributed algorithm for the considered nonlinear placement problem. The proposed approach is validated by numerical examples in Section V. Finally, Section VI concludes the article.

II. Preliminaries

A. Notation

Let $\mathbb{R}^n$ and $\mathbb{C}^n$ denote the n-dimensional Euclidean space and complex space, respectively. For $x_i \in \mathbb{R}^{n_i}$, $i = 1, \ldots, m$, $\text{col}(x_1, \ldots, x_m) = [x_1^{\mathrm{T}}, \ldots, x_m^{\mathrm{T}}]^{\mathrm{T}}$, where x_i^{T} is the transpose of x_i. Let $I_m \in \mathbb{R}^{m\times m}$ denote the identity matrix and $\otimes$ denote the Kronecker product. Let $\mathbf{1}_m$ denote the m-dimensional vector with all entries being 1. Let $f(x,t) : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$ be a continuously differentiable function with respect to x and t, and $\nabla_x f(x,t)$ denote the first partial derivative of the function $f(x,t)$ with respect to vector x. Let $\|x\|_1$ and $\|x\|$ denote, respectively, the 1-norm and 2-norm of vector x.

B. Graph Theory

In this article, a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is used to describe the information exchange of a network system, where $\mathcal{V} = \{1, 2, \ldots, N\}$ is the node set and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the edge set. Here, $(j, i) \in \mathcal{E}$ represents that node i can obtain information from node j. For an undirected graph, $(j, i) \in \mathcal{E}$ implies $(i, j) \in \mathcal{E}$. Let $\mathcal{N}_i = \{j : (j, i) \in \mathcal{E}\}$ denote the set of neighbors of node i. A path is a sequence of edges of the form $(i, j), (j, k), \ldots$. A graph is connected if there is a path between each pair of nodes.

For graph $\mathcal{G}$, the adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ is defined as $a_{ii} = 0$, $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{N\times N}$ is defined as $l_{ii} = \sum_{j=1}^{N} a_{ij}$, and $l_{ij} = -a_{ij}$ for any $i \neq j$. Let $[a_{ij}]^{\max}$ denote the maximum of a_{ij}, $i, j \in \mathcal{V}$, that is, $[a_{ij}]^{\max} = \max_{i,j\in\mathcal{V}} a_{ij}$.

Lemma 1: Let $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ and $L \in \mathbb{R}^{N\times N}$ be, respectively, the adjacency matrix and the Laplacian matrix of an undirected and connected graph $\mathcal{G}$. Assume that $[a_{ij}]^{\max} < (2l_{ii}/[N-1])$, $i \in \mathcal{V}$. For $L_a = L + a\mathbf{1}_N\mathbf{1}_N^{\mathrm{T}}$ with $[a_{ij}]^{\max} \le a < (2l_{ii}/[N-1])$, L_a is positive definite and $L_a^{-1}\mathbf{1}_N = (1/aN)\mathbf{1}_N$.

Proof: Let $\lambda_i, i = 1, \ldots, N$, be the eigenvalues of L_a. It follows from the Gershgorin theorem ([19]) that λ_i lies in the set:

$$\bigcup_{i=1}^{n}\left\{z \in \mathbb{C}^2 : |z - (l_{ii} + a)| \le \sum_{j=1}^{N} |a - a_{ij}|\right\}.$$

Because $[a_{ij}]^{\max} \le a < (2l_{ii}/[N-1])$, it follows that:

$$l_{ii} + a > \sum_{j=1}^{N} |a - a_{ij}|$$

indicating that L_a is positive definite. It follows from $L_a\mathbf{1}_N = (L + a\mathbf{1}_N\mathbf{1}_N^{\mathrm{T}})\mathbf{1}_N = aN\mathbf{1}_N$ that $L_a^{-1}\mathbf{1}_N = (1/aN)\mathbf{1}_N$, and the proof is completed. ■

III. Distributed Nonlinear Placement Problem

Consider a nonlinear placement problem for a multicluster system composed of $n = \sum_{k=0}^{\varrho} n_k$ agents. There are n_0 ($n_0 \ge 2$) leaders and $\sum_{k=1}^{\varrho} n_k$ followers in the multicluster system. All the followers are divided by ϱ clusters (each cluster has n_k agents with $k = 1, \ldots, \varrho$). The positions of the leaders $q_r^0(t) \in \mathbb{R}^m$, $r = 1, \ldots, n_0$, are time varying. The dynamics of each follower i in cluster k is described by the following single-integrator system:

$$\dot{q}_i^k = \tau_i^k,\ \ i = 1, \ldots, n_k,\ \ k = 1, \ldots, \varrho \tag{1}$$

where $q_i^k \in \mathbb{R}^m$ denotes the generalized position and $\tau_i^k \in \mathbb{R}^m$ denotes the control force. The network topology constraint is given as follows.

1) The followers in cluster k (includes n_k agents with $k = 1, \ldots, \varrho$) are specified by a communication network graph $\mathcal{G}^k = (\mathcal{V}^k, \mathcal{E}^k)$, where $\mathcal{V}^k = \{q_1^k, \ldots, q_{n_k}^k\}$ is the node set and $\mathcal{E}^k \subset \mathcal{V}^k \times \mathcal{V}^k$ is the edge set. Let $A^k = [a_{ij}^k] \in \mathbb{R}^{n_k\times n_k}$ denote the adjacency matrix of the graph $\mathcal{G}^k$.
2) The information flows amongst the followers and the rth leader q_r^0 are specified by graph $\mathcal{G}^{f+r} = (\mathcal{V}^{f+r}, \mathcal{E}^{f+r})$

where $\mathcal{V}^{f+r} = \mathcal{V}^f \bigcup \{q_r^0\}$ and $\mathcal{E}^{f+r} \subset \mathcal{V}^{f+r} \times \mathcal{V}^{f+r}$. For graph $\mathcal{G}^{f+r}$, $\mathcal{N}_{q_i^k}^{f+r}$ denotes the set of neighbors of agent q_i^k. Similarly, $\mathcal{G}^{f+0} = (\mathcal{V}^{f+0}, \mathcal{E}^{f+0})$ and $\mathcal{N}_{q_i^k}^{f+0}$ are used to specified all the followers (without any leaders) and the set of neighbors of agent q_i^k in graph $\mathcal{G}^{f+0}$, respectively.

3) The information flows amongst the followers and leaders are specified by graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \ldots, \sum_{k=0}^{\varrho} n_k\}$ is the node set and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the edge set. For graph $\mathcal{G}$, $\mathcal{N}_{q_i^k}$ denotes the set of neighbors of agent q_i^k.

The objective is to design the following distributed control law:

$$\dot{v}_i^k = F_i^k\left(q_i^k, q_j^l, v_i^k, v_j^l\right)$$
$$\tau_i^k = H_i^k\left(q_i^k, q_j^l, v_i^k, v_j^l\right),\ i = 1, \ldots, n_k,\ k = 1, \ldots, \varrho \quad (2)$$

where $v_i^k \in \mathbb{R}^{m_k}$ and m_k is a positive integer, F_i^k and H_i^k are some smooth functions, and j and l are chosen such that q_i^k can obtain information from q_j^l in graph $\mathcal{G}$.

Distributed Nonlinear Placement Problem: Consider the closed-loop system composed of (1) and (2) with any initial conditions $q_i^k(0)$ and $v_i^k(0)$. The objective is achieved if v_i^k is bounded while q_i^k converges to the global optimal solution $\tilde{q}_i^k$, that is, $\lim_{t\to\infty}(q_i^k(t) - \tilde{q}_i^k(t)) = 0$, $i = 1, \ldots, n_k$ and $k = 1, \ldots, \varrho$, where $\tilde{q}_i^k(t)$ is the solution to

$$\text{minimize} \ \sum_{s=1}^{k-1} \omega_{sk} \left\| \mathcal{Z}^s - \mathcal{Z}^k \right\|^2 + \sum_{r=1}^{n_0} \omega_{rk}^0 \left\| q_r^0(t) - \mathcal{Z}^k \right\|^2 \quad (3a)$$

$$\text{subject to} \ \ q_i^k - q_j^k = d_{ij}^k, i, j = 1, \ldots, n_k. \quad (3b)$$

Here, $\mathcal{Z}^k = (1/n_k) \sum_{i=1}^{n_k} q_i^k$ is the center of cluster k, d_{ij}^k is the desired constant displacement constraint vector, $d_{ii}^k = 0$, and $\omega_{sk} \geq 0$ (or $\omega_{rk}^0 \geq 0$) is the weight of cluster s [or leader $q_r^0(t)$] and cluster k.

Remark 1: We assume that the parameters n_k, ω_{sk}, and ω_{rk}^0 are known to all agents in cluster k. A weight $\omega_{sk} = 0$ (or $\omega_{rk}^0 = 0$) implies that the positions of cluster s [or leader $q_r^0(t)$] and cluster k are irrelevant. Note that the nonlinear placement problem (3) includes two targets.

1) *Nonlinear Placement for Clusters in (3a):* Define the length $\varepsilon_{s,k}$ (or $\varepsilon_{r,k}^0$) of the link that connects the centers of cluster s (or leader q_r^0) and cluster k, that is, $\varepsilon_{s,k} = \|\mathcal{Z}^s - \mathcal{Z}^k\|$ (or $\varepsilon_{r,k}^0 = \|q_r^0 - \mathcal{Z}^k\|$). The followers are placed to satisfy that the measure of the total interconnection length of the links, that is, $\sum_{r=1}^{n_0} \omega_{rk}^0 (\varepsilon_{r,k}^0)^2 + \sum_{s=1}^{k-1} \omega_{sk} (\varepsilon_{s,k})^2$, is minimized.
2) *Inner Cluster Formation in (3b):* The followers q_i^k and q_j^k in cluster k, $k = 1, \ldots, \varrho$, are maintained according to the desired distance vector d_{ij}^k.

Remark 2: It is worth mentioning that if $n_i = 1, i = 1, \ldots, \varrho$, the placement problem, (3) is reduced to a nonlinear placement problem which is studied in [1, Sec. 8.7]. Even in such a case, the placement problem considered in this article is distributed and the results in [1] are centralized. Moreover, if $\varrho = 1$, the considered placement problem is reduced to a formation control problem that is studied in [20]. In such a case, the center of the formation in this article is placed to minimize the distance from the leaders, and this condition is not required in [20].

Remark 3: The considered problem is similar to the formation problem of multileader networks (e.g., [21]). Note that the multiformation problem can be converted into the group/cluster consensus problem, multiconsensus problem, or multitracking problem since a desired geometric formation is given. In contrast, the formation shape is automatically guaranteed by a distributed method in this article.

Remark 4: In [1], function (3a) is given as $\sum_{s=1}^{k-1} \omega_{sk} h(\|\mathcal{Z}^s - \mathcal{Z}^k\|) + \sum_{r=1}^{n_0} \omega_{rk}^0 h(\|q_r^0(t) - \mathcal{Z}^k\|)$, where $h(\cdot)$ is defined as an monotonically increasing (on $\mathbb{R}^+$) and convex function, for example, $h(z) = z$, $h(z) = z^2$ and $h(z) = z^4$. In this article, we only consider $h(z) = z^2$ for notational simplicity.

Assumption 1: Graphs $\mathcal{G}^k,\ k = 1, \ldots, \varrho$ and $\mathcal{G}^{f+r},\ r = 0, 1, \ldots, n_0$ are undirected and connected.

Assumption 2: For cluster $k,\ k = 1, \ldots, \varrho$ and its desired distance vector d_{ij}^k, the formation problem is solvable, for example, there exists at least some vectors $\check{q}_i^k,\ i = 1, \ldots, n_k$ such that $\check{q}_i^k - \check{q}_j^k = d_{ij}^k$.

Assumption 3: For the leaders, $\ddot{q}_j^0(t), \dot{q}_j^0(t), j = 1, \ldots, n_0$ are bounded. For cluster k, $k = 1, \ldots, \varrho$, the weight set $\Omega_k = \{\omega_{1k}, \ldots, \omega_{k-1\ k}, \omega_{1k}^0, \ldots, \omega_{n_0k}^0\}$ has at least two positive entries.

Remark 5: Assumption 1 is a common assumption for both distributed formation and distributed game problems. Assumption 2 is necessary to ensure that the formation problem in (3b) is solvable. Note that under Assumption 2, $\check{q}_i^k + d(t)$ with any vector $d(t)$ is also a solution to (3b). Hence, the center of each cluster is not fixed under Assumption 2. In fact, the centers are placed according to the solution to (3a). Assumption 3 implies that the states of leaders are bounded, which is necessary to ensure that the nonlinear displacement problem (3) is solvable. Note that problem (3) has a unique nontrivial solution unless the set Ω_k has at least two positive entries. If the weights of w_{sk} and w_{sk}^0 are all equal to 0, then the solution isn't unique. Moreover, if Ω_k has only one positive entry, then the solution is trivial, that is, $\mathcal{Z}^k = \mathcal{Z}^s$ or $\mathcal{Z}^k = q_r^0(t)$.

Lemma 2: Consider the nonlinear placement problem (3). Assume that Assumptions 2 and 3 hold. Then, there exists a unique solution $\tilde{q}(t) = \text{col}(\tilde{q}_1^1(t), \ldots, \tilde{q}_{n_1}^1(t), \tilde{q}_1^2(t), \ldots, \tilde{q}_{n_\varrho}^{\varrho}(t))$ to problem (3).

Proof: Note that function (3a) is a strictly convex function with respect to $\mathcal{Z}^k$, that is, function $\|\cdot\|^2$, composed of an affine mapping on $\mathcal{Z}^k$. Hence, for cluster k, there exists a unique solution $\mathcal{Z}^k(t)$ for (3a) by using Assumption 3. Moreover, for cluster k with the center trajectory $\mathcal{Z}^k(t)$, Assumption 2 implies that there exists a unique $\tilde{q}(t)$ such that $\tilde{q}_i^k - \tilde{q}_j^k = d_{ij}^k$. ■

IV. Main Results

A. Equivalence to Distributed NE Seeking

In this section, we show that the placement problem (3) can be converted into a distributed NE-seeking problem. In particular, the ith agent (player) in cluster k aims to minimize its cost function

$$J_i^k(q,t) = \left\| g_k \sum_{\rho=1}^{n_k}\left(q_\rho^k - \varpi_k \sum_{s=1}^{k-1}\sum_{j=1}^{n_s}\frac{\omega_{sk}}{n_s}q_j^s - \varpi_k \sum_{r=1}^{n_0}\omega_{rk}^0 q_r^0(t)\right)\right\|^2 + \left(\sum_{j=1}^{n_k} a_{ij}^k\right)^{-1}\left\|\sum_{j=1}^{n_k} a_{ij}^k\left(q_i^k - q_j^k - d_{ij}^k\right)\right\|^2 \quad (4)$$

where $q = \mathrm{col}(q_1^1,\ldots,q_{n_1}^1,q_1^2,\ldots,q_{n_2}^2,\ldots,q_1^\varrho,\ldots,q_{n_\varrho}^\varrho)$, $g_k > 0$, $k = 1,\ldots,\varrho$, $i = 1,\ldots,n_k$, and $\varpi_k = (\sum_{s=1}^{k-1}\omega_{sk} + \sum_{r=1}^{n_0}\omega_{rk}^0)^{-1}$. A profile of state $\tilde{q}(t) = \mathrm{col}\ (\tilde{q}_1^1(t),\ldots,\tilde{q}_{n_1}^1(t),\tilde{q}_1^2(t),\ldots,\tilde{q}_{n_2}^2(t),\ldots,\tilde{q}_1^\varrho(t),\ldots,\tilde{q}_{n_\varrho}^\varrho(t))$ is said to constitute an NE [22] for game (4) if

$$J_i^k\left(\tilde{q}_1^1(t),\ldots,\tilde{q}_{i-1}^k(t),\tilde{q}_i^k(t),\tilde{q}_{i+1}^k(t),\ldots,\tilde{q}_{n_\varrho}^\varrho(t),t\right) \le J_i^k\left(\tilde{q}_1^1(t),\ldots,\tilde{q}_{i-1}^k(t),q_i^k(t),\tilde{q}_{i+1}^k(t),\ldots,\tilde{q}_{n_\varrho}^\varrho(t),t\right) \quad \forall q_i^k(t). \quad (5)$$

In fact, an NE means that no player can further reduce its associated cost function by unilaterally changing its own state. Note that the ith player in cluster k only has direct access to the states of the players that are its neighbors.

Remark 6: Note that the function (4) contains two terms. The first term is designed to fix the center of cluster k, which corresponds to the nonlinear placement target (3a). The second term is designed for the inner cluster formation target (3b). See the proof of Proposition 1 for more details. It is not hard to see that the function (4) is not unique and any function is available if it has a unique solution to problem (3). Here, we only consider quadratic function (4) for notational simplicity.

Distributed NE-Seeking Problem: Consider the $\sum_{k=1}^{\varrho} n_k$-player noncooperative game (1) and (4). Design distributed control law (2) to seek the NE of game (4), that is, for any initial condition $q_i^k(0)$ and $v_i^k(0)$, v_i^k is bounded and $\lim_{t\to\infty}(q(t)-\tilde{q}(t)) = 0$, where $\tilde{q}(t)$ is defined in (5).

Lemma 3: Consider the noncooperative game (4). Assume that Assumption 1 holds. If $[a_{ij}^k]^{\max} \le g_k < ([2\sum_{j=1}^{n_k} a_{ij}^k]/[n_k-1])\ \forall\ i,j = 1,\ldots,n_k, k = 1,\ldots,\varrho$, then there exists a unique time-varying NE for game (4).

Proof: Note that the cost function (4) a is quadratic function (i.e., $\|\cdot\|^2$) composed of an affine mapping on q_i^k. It follows that the cost function $J_i^k(q,t)$ is strictly convex and radially unbounded in q_i^k for $\mathrm{col}(q_1^1,\ldots,q_{n_1}^1,q_1^2,\ldots,q_{n_2}^2,\ldots,q_{i-1}^k,q_{i+1}^k,\ldots,q_{n_1^\varrho},\ldots,q_{n_\varrho}^\varrho)$. Moreover, it is not hard to prove that the pseudogradient of function (4), that is, $F : q \to \mathrm{col}(\nabla_{q_1^1}J_1^1(q,t),\ldots,\nabla_{q_{n_\varrho}^\varrho}J_{n_\varrho}^\varrho(q,t))$, is strictly monotonic. In particular, define the auxiliary variable $\chi^k = \mathrm{col}(\chi_1^k,\ldots,\chi_{n_k}^k)$ with χ_i^k being

$$\begin{aligned}\chi_i^k &= \frac{1}{2}\nabla_{q_i^k}J_i^k(q,t)\\ &= g_k\sum_{\rho=1}^{n_k}\left(q_\rho^k - \varpi_k\sum_{s=1}^{k-1}\sum_{j=1}^{n_s}\frac{\omega_{sk}}{n_s}q_j^s - \varpi_k\sum_{r=1}^{n_0}\omega_{rk}^0 q_r^0(t)\right)\\ &\quad + \sum_{j=1}^{n_k} a_{ij}^k\left(q_i^k - q_j^k - d_{ij}^k\right)\\ &= g_k\sum_{\rho=1}^{n_k} q_\rho^k + \sum_{j=1}^{n_k} a_{ij}^k\left(q_i^k - q_j^k\right) + \Theta_k - \sum_{j=1}^{n_k} a_{ij}^k d_{ij}^k\end{aligned}$$

where

$$\Theta_k = -g_k n_k \varpi_k\left(\sum_{s=1}^{k-1}\sum_{j=1}^{n_s}\frac{\omega_{sk}}{n_s}q_j^s + \sum_{r=1}^{n_0}\omega_{rk}^0 q_r^0(t)\right).$$

Define $L_{g_k}^k = L^k + g_k\mathbf{1}_{n_k}\mathbf{1}_{n_k}^{\mathrm{T}}$, where $L^k = [l_{ij}^k] \in \mathbb{R}^{n_k\times n_k}$ denotes the Laplacian matrix of graph $\mathcal{G}^k$, $k = 1,\ldots,\varrho$. It follows that:

$$\chi^k = \left(L_{g_k}^k \otimes I_m\right)q^k + \mathbf{1}_{n_k}\otimes\Theta_k - D_k \quad (6)$$

where $D_k = \mathrm{col}(\sum_{j=1}^{n_k} a_{1j}^k d_{1j}^k,\ldots,\sum_{j=1}^{n_k} a_{n_k j}^k d_{n_k j}^k)$ and $q^k = \mathrm{col}(q_1^k,\ldots,q_{n_k}^k)$. Note that Θ_k and D_k are independent of $q^s = \mathrm{col}(q_1^s,\ldots,q_{n_s}^s)$, $s = k, k+1,\ldots,\varrho$. Therefore, the Jacobian matrix of the pseudogradient map F is given by

$$J_F = 2\begin{pmatrix} L_{g_1}^1\otimes I_m & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ * & L_{g_2}^2\otimes I_m & \mathbf{0} & \mathbf{0}\\ \vdots & \vdots & \ddots & \vdots\\ * & * & * & L_{g_\varrho}^\varrho\otimes I_m\end{pmatrix}$$

where "$*$" implies the hidden matrices and $\mathbf{0}$ denotes the matrix with appropriate dimension and all entries being 0. Since Assumption 1 holds and $[a_{ij}^k]^{\max} \le g_k < ([2\sum_{j=1}^{n_k} a_{ij}^k]/[n_k-1])\ \forall\ i = 1,\ldots,n_k$, it follows from Lemma 1 that $L_{g_k}^k\otimes I_m$ is positive definite. According to [Proposition 2.5 and Th. 2.3] [23], the pseudogradient F is strictly monotonic (since J_F is positive definite) and there exists a unique NE for game (4). ■

Proposition 1: Assume that Assumptions 1–3 hold. Consider the NE $\tilde{q}(t)$ in (5). If $[a_{ij}^k]^{\max} \le g_k < ([2\sum_{j=1}^{n_k} a_{ij}^k]/[n_k-1])\ \forall\ i,j = 1,\ldots,n_k, k = 1,\ldots,\varrho$, then $\tilde{q}(t)$ is also the unique solution to the nonlinear placement problem (3).

Proof: According to Lemmas 2 and 3, both problems (3) and (4) have only one unique solution. It follows from (4) that:

$$J_i^k(q,t) \ge 0 \quad \forall\ i = 1,\ldots,n_k, k = 1,\ldots,\varrho.$$

Next, we show that zero is the minimum of $J_i^k(q,t)$ for all i and k. Assume that there exists $\tilde{q}$ such that $J_i^k(\tilde{q},t) = 0$, $k = 1,\ldots,\varrho$ and $i = 1,\ldots,n_k$. It follows that:

$$\sum_{\rho=1}^{n_k}\left(\tilde{q}_\rho^k - \varpi_k\sum_{s=1}^{k-1}\sum_{j=1}^{n_s}\frac{\omega_{sk}}{n_s}\tilde{q}_j^s - \varpi_k\sum_{r=1}^{n_0}\omega_{rk}^0 q_r^0(t)\right) = 0 \quad (7a)$$

$$\sum_{j=1}^{n_k} a_{ij}^k\left(\tilde{q}_i^k - \tilde{q}_j^k - d_{ij}^k\right) = 0. \tag{7b}$$

Based on Assumptions 1 and 2, (7b) implies $\tilde{q}_i^k - \tilde{q}_j^k = d_{ij}^k$ since graph $\mathcal{G}^k, k = 1, \ldots, \varrho$ is undirected and connected. Consequently, the solution to (7b) can be given as $\tilde{q}_i^k = \breve{q}_i^k + \Delta_k$, where $\breve{q}_i^k, k = 1, \ldots, \varrho$ and $i = 1, \ldots, n_k$, is an arbitrary solution to the equation $\breve{q}_i^k - \breve{q}_j^k = d_{ij}^k, k = 1, \ldots, \varrho, i = 1, \ldots, n_k$, and Δ_k is a constant vector to be determined. It is straightforward to compute Δ_k according to (7a) and the solution is unique. Hence, (7a) and (7b) have a unique solution and zero is the minimum of $J_i^k(q, t)$ for all i and k.

It follows from the definition of ϖ_k and (7a) that:

$$\left(\sum_{s=1}^{k-1}\omega_{sk} + \sum_{r=1}^{n_0}\omega_{rk}^0\right)\frac{1}{n_k}\sum_{i=1}^{n_k}\tilde{q}_i^k = \sum_{s=1}^{k-1}\frac{\omega_{sk}}{n_s}\sum_{i=1}^{n_s}\tilde{q}_i^s + \sum_{r=1}^{n_0}\omega_{rk}^0 q_r^0(t)$$

which gives

$$\sum_{s=1}^{k-1}\omega_{sk}\left(\frac{1}{n_s}\sum_{i=1}^{n_s}\tilde{q}_i^s - \frac{1}{n_k}\sum_{i=1}^{n_k}\tilde{q}_i^k\right) + \sum_{r=1}^{n_0}\omega_{rk}^0\left(q_r^0(t) - \frac{1}{n_k}\sum_{i=1}^{n_k}\tilde{q}_i^k\right) = 0.$$

Note that the gradient of the objective function in (3a) can be written as the left-hand side of the above equation, which is equal to 0. Combining $\tilde{q}_i^k - \tilde{q}_j^k = d_{ij}^k$ from (7b), the optimal solution of (3) is achieved. It can be concluded that $\tilde{q}(t)$ is also the solution to the nonlinear placement problem (3).

Remark 7: Note that Proposition 1 illustrates that the distributed nonlinear placement problem for the multicluster systems can be converted into a time-varying noncooperative game problem. It is hard to deal with the problem (3) by the methods of [1, Sec. 8.7]. The main challenges include three aspects: 1) the problem is distributed other than centralized; 2) the existence of the displacement constraint vector d_{ij}; and 3) the leaders' states are time varying.

Remark 8: Note that the parameter g_k should be the same for all agents in cluster k. We assume it is possible because every cluster only includes a few agents and it can be viewed as an initialization step. Moreover, if there exist $a_{ij}^k \in \{0, 1\}$ and $\sum_{j=1}^{n_k} a_{ij}^k > ([n_k - 1]/2)$, we can choose $g_k = 1$ for simplicity.

B. Centralized NE-Seeking Algorithm

In this section, we first give a centralized solution to the NE-seeking problem. Consider the controller in (1) as

$$\tau_i^k = \phi_i^k\Big(q_1^0, \ldots, q_{n_0}^0, q_1^1, \ldots, q_{n_{k-1}}^{k-1}, \dot{q}_1^0, \ldots, \dot{q}_{n_0}^0, \dot{q}_1^1, \ldots, \dot{q}_{n_{k-1}}^{k-1}\Big) - g_k\sum_{\rho=1}^{n_k} q_\rho^k - \sum_{j=1}^{n_k} a_{ij}^k\left(q_i^k - q_j^k - d_{ij}^k\right) \tag{8}$$

where

$$\begin{aligned}\phi_i^k&\Big(q_1^0, \ldots, q_{n_0}^0, q_1^1, \ldots, q_{n_{k-1}}^{k-1}, \dot{q}_1^0, \ldots, \dot{q}_{n_0}^0, \dot{q}_1^1, \ldots, \dot{q}_{n_{k-1}}^{k-1}\Big)\\ &= g_k n_k \varpi_k\left(\sum_{s=1}^{k-1}\sum_{j=1}^{n_s}\frac{\omega_{sk}}{n_s}q_j^s + \sum_{r=1}^{n_0}\omega_{rk}^0 q_r^0(t)\right)\\ &\quad + \varpi_k\left(\sum_{s=1}^{k-1}\sum_{j=1}^{n_s}\frac{\omega_{sk}}{n_s}\dot{q}_j^s + \sum_{r=1}^{n_0}\omega_{rk}^0 \dot{q}_r^0(t)\right).\end{aligned} \tag{9}$$

■

Lemma 4: Consider system (1) with controller (8). Suppose that Assumptions 1–3 hold. If $[a_{ij}^k]^{\max} \leq g_k < ([2\sum_{j=1}^{n_k} a_{ij}^k]/[n_k - 1])\ \forall\, i, j = 1, \ldots, n_k, k = 1, \ldots, \varrho$, it follows that $\lim_{t\to\infty}(q(t) - \tilde{q}(t)) = 0$, where $\tilde{q}(t)$ is the NE of game (4), which is also the solution to problem (3).

Proof: It follows from (1) and (8) that:

$$\dot{q}_i^k = -\chi_i^k - \frac{1}{n_k g_k}\frac{d}{dt}\Theta_k. \tag{10}$$

This is done by using the definition of χ_i^k and Θ_k given in the proof of Lemma 3. Since $[a_{ij}^k]^{\max} \leq g_k < ([2\sum_{j=1}^{n_k} a_{ij}^k]/[n_k - 1])\ \forall\ i = 1, \ldots, n_k$, it follows from Lemma 1 that $(L_{g_k}^k)^{-1}\mathbf{1}_{n_k} = (1/n_k g_k)\mathbf{1}_{n_k}$. Equation (10) can be written in a compact form as

$$\dot{q}^k = -\chi^k - \left(L_{g_k}^k\right)^{-1}\mathbf{1}_{n_k} \otimes \frac{d}{dt}\Theta_k.$$

Taking the derivative of (6) and using the above equation lead to

$$\begin{aligned}\dot{\chi}^k &= \left(L_{g_k}^k \otimes I_m\right)\dot{q}^k + \mathbf{1}_{n_k} \otimes \frac{d}{dt}\Theta_k\\ &= -\left(L_{g_k}^k \otimes I_m\right)\chi^k.\end{aligned} \tag{11}$$

According to Lemma 1 again, it follows that $L_{g_k}^k \otimes I_m$ is positive definite. Hence, $\lim_{t\to\infty}\chi^k(t) = 0$, which implies $\lim_{t\to\infty}\nabla_{q_i^k}J_i^k(q, t) = 0$. It thus follows that $\lim_{t\to\infty}(q(t) - \tilde{q}(t)) = 0$, where $\tilde{q}(t)$ is the NE to (4) [17]. The proof is completed based on the results of Proposition 1. ■

C. Distributed NE-Seeking Algorithm

Note that the variables $q_j^l, \dot{q}_j^l,\ l = 0, 1, \ldots, k - 1$ and $j = 1, \ldots, n_l$, in function ϕ_i^k may be unavailable to agent q_i^k because of local information exchange. Motivated by the centralized algorithm (8), we will focus on solving the distributed NE-seeking problem (4) in this section. Consider the distributed controller for (1) as

$$\begin{aligned}\tau_i^k &= \phi_i^k\Big(\hat{q}_{i,1}^{k,0}, \ldots, \hat{q}_{i,n_0}^{k,0}, \hat{q}_{i,1}^{k,1}, \ldots, \hat{q}_{i,n_{k-1}}^{k,k-1}, \hat{\theta}_{i,1}^{k,0}, \ldots, \hat{\theta}_{i,n_0}^{k,0},\\ &\qquad \hat{\theta}_{i,1}^{k,1}, \ldots, \hat{\theta}_{i,n_{k-1}}^{k,k-1}\Big) - g_k n_k \eta_i^k\\ &\quad - \sum_{j=1}^{n_k} a_{ij}^k\left(q_i^k - q_j^k - d_{ij}^k\right)\\ \dot{\hat{\theta}}_{i,j}^{k,l} &= -\alpha_j^l \sum_{rs\in\mathcal{N}_{ij}^{kl}}\left(\hat{\theta}_{i,j}^{k,l} - \hat{\theta}_{r,j}^{s,l}\right) - \beta_j^l \mathrm{sgn}\sum_{rs\in\mathcal{N}_{ij}^{kl}}\left(\hat{\theta}_{i,j}^{k,l} - \hat{\theta}_{r,j}^{s,l}\right)\end{aligned}$$

$$\begin{aligned}\dot{\hat{q}}_{i,j}^{k,l} &= -\gamma_j^l \sum_{rs\in\mathcal{N}_{ij}^{kl}} \left(\hat{q}_{i,j}^{k,l} - \hat{q}_{r,j}^{s,l}\right) - \delta_j^l \mathrm{sgn} \sum_{rs\in\mathcal{N}_{ij}^{kl}} \left(\hat{q}_{i,j}^{k,l} - \hat{q}_{r,j}^{s,l}\right)\\ \dot{\xi}_i^k &= \varsigma_k \sum_{j=1}^{n_k} a_{ij}^k \mathrm{sgn}\left(\eta_j^k - \eta_i^k\right)\\ \eta_i^k &= \xi_i^k + q_i^k, \quad \xi_i^k(0) = 0\end{aligned} \tag{12}$$

where $k = 1, \ldots, \varrho$, $i = 1, \ldots, n_k$, $l = 0, 1, \ldots, \varrho - 1$, $j = 1, \ldots, n_l$, $\hat{\theta}_{j,j}^{l,l} = \dot{q}_j^l$, $\hat{q}_{j,j}^{l,l} = q_j^l$, $\mathcal{N}_{ij}^{kl} = \begin{cases}\mathcal{N}_{q_i^k}^{f+j}, & l = 0\\ \mathcal{N}_{q_i^k}^{f}, & l \neq 0\end{cases}$, α_j^l and γ_j^l are nonnegative constants, and β_j^l, δ_j^l, and ς_k are some positive constants to be determined. $rs \in \mathcal{N}_{ij}^{kl}$ implies that q_i^k can obtain information from q_r^s in graph $\mathcal{G}^{f+j}$ for $l = 0$ and $\mathcal{G}^f$ for $l \neq 0$, respectively.

Remark 9: The algorithm (12) is designed based on a distributed observer method. In particular, $\hat{q}_{ij}^{kl}$, $\hat{\theta}_{ij}^{kl}$, and η_i^k are used by agent i of cluster k to estimate q_j^l, $\dot{q}_j^l$, and $(1/n_k)\sum_{\rho=1}^{n_k} q_\rho^k$, respectively. Note that $k = 1, \ldots, \varrho$ implies that each leader does not need to estimate other agents' information. For agent i of cluster k, when the observed target is the jth leader (i.e., $l = 0$), it follows from $\mathcal{N}_{ij}^{kl} = \mathcal{N}_{q_i^k}^{f+j}$ that only the jth leader's information is used. Similarly, when the observed target is a follower (i.e., $l \neq 0$), it follows from $\mathcal{N}_{ij}^{kl} = \mathcal{N}_{q_i^k}^{f}$ that all leaders' information is unused.

Remark 10: Note that the algorithm (12) is divided into two phases. First, two auxiliary observers are designed (see Remark 9 for more details). Second, the centralized NE-seeking algorithm (8) is composed of the two auxiliary observers. The design is reasonable since algorithm (8) ensures $\lim_{t\to\infty} \nabla_{q_i^k} J_i^k(q, t) = 0$. Moreover, the design and analytical processes are different from most of the distributed NE-seeking algorithms in [12]–[17] since the cost function (4) is time varying.

Remark 11: The algorithm (12) is discontinuous since the signum function is used. However, we do not use the nonsmooth analysis in the proofs hereafter for two reasons. One is that the signum function is measurable and locally essentially bounded, and the other is that the Lyapunov function candidate, which we will adopt in the proof of Theorem 1 is continuously differentiable. Moreover, the agents' state trajectories are continuous.

Lemma 5: Consider the dynamics of $\hat{\theta}_{ij}^{kl}$ and $\hat{q}_{ij}^{kl}$ in (12). Assume that Assumption 1 holds. If $\ddot{q}_j^l(t)$ is bounded, then there exists $\beta_j^l \geq \bar{\beta}_j^l$ with $\bar{\beta}_j^l$ being a positive constant, such that $\lim_{t\to\infty}(\hat{\theta}_{ij}^{kl}(t) - \dot{q}_j^l(t)) = 0$. Also, if $\dot{q}_j^l$ is bounded, then there exists $\delta_j^l \geq \bar{\delta}_j^l$ with $\bar{\delta}_j^l$ being a positive constant, such that $\lim_{t\to\infty}(\hat{q}_{ij}^{kl}(t) - q_j^l(t)) = 0$.

Proof: Consider the case $l = 0$ that the observed target is the jth leader and $\mathcal{N}_{ij}^{kl} = \mathcal{N}_{q_i^k}^{f+j}$. Thus, the dynamics of $\hat{\theta}_{ij}^{kl}$ in (12) can be written as

$$\dot{\hat{\theta}}_{i,j}^{k,0} = -\alpha_j^0 \sum_{rs\in\mathcal{N}_{q_i^k}^{f+j}} \left(\hat{\theta}_{i,j}^{k,0} - \hat{\theta}_{r,j}^{s,0}\right) - \beta_j^0 \mathrm{sgn} \sum_{rs\in\mathcal{N}_{q_i^k}^{f+j}} \left(\hat{\theta}_{i,j}^{k,0} - \hat{\theta}_{r,j}^{s,0}\right).$$

Note that the above dynamics has the same form with [24, eq. (2)]. Moreover, Assumption 1 and the boundedness of $\ddot{q}_j^l(t)$ imply that the conditions of [24, Th. 3.1] hold. Therefore, it follows from in [24, Th. 3.1] that there exists $\beta_j^l \geq \bar{\beta}_j^l \geq \|\ddot{q}_j^l(t)\|_\infty$ with $\bar{\beta}_j^l$ being a positive constant, such that $\lim_{t\to\infty}(\hat{\theta}_{ij}^{kl}(t) - \dot{q}_j^l(t)) = 0$. The proofs for $\hat{q}_{ij}^{kl}$ and the other case $l \neq 0$ are similar and are hence omitted. ■

Theorem 1: Assume that Assumptions 1–3 hold. Consider the closed-loop system composed of (1) and (12). If $[a_{ij}^k]^{\max} \leq g_k < ([2\sum_{j=1}^{n_k} a_{ij}^k]/[n_k - 1])$ $\forall\ i, j = 1, \ldots, n_k$ and $k = 1, \ldots, \varrho$, then there exist $\beta_j^l \geq \bar{\beta}_j^l$, $\delta_j^l \geq \bar{\delta}_j^l$, and $\varsigma_k \geq \bar{\varsigma}_k$ with $\bar{\beta}_j^l$, $\bar{\delta}_j^l$ and $\bar{\varsigma}_k$ being some positive constants, such that $\lim_{t\to\infty}(q(t) - \tilde{q}(t)) = 0$, where $\tilde{q}(t)$ is the NE of game (4), which is also the solution to problem (3).

Proof: Now, we prove the results using an iterative approach. First, consider $k = 1$. Define $e_{i,j}^{1,0} = \hat{\theta}_{i,j}^{1,0} - \dot{q}_j^0(t)$, $\sigma_{i,j}^{1,0} = \hat{q}_{i,j}^{1,0} - q_j^0(t)$, $j = 1, \ldots, n_0$. Based on (8) and (9), the q_i^1 subsystem in (12) can be rewritten as

$$\begin{aligned}\dot{q}_i^1 = &\ \phi_i^1\left(q_1^0, \ldots, q_{n_0}^0, \dot{q}_0^1(t), \ldots, \dot{q}_{n_0}^0\right)\\ &- g_1 \sum_{\rho=1}^{n_1} q_\rho^1 - \sum_{j=1}^{n_1} a_{ij}^1\left(q_i^1 - q_j^1 - d_{ij}^1\right)\\ &+ \phi_i^1\left(e_{i,1}^{1,0}, \ldots, e_{i,n_0}^{1,0}, \sigma_{i,1}^{1,0}, \ldots, \sigma_{i,n_0}^{1,0}\right)\\ &+ g_1 n_1\left(\frac{1}{n_1}\sum_{\rho=1}^{n_1} q_\rho^1 - \eta_i^1\right).\end{aligned} \tag{13}$$

Define the state observer error Φ_1 for the $\hat{\theta}_{i,j}^{1,0}$ and $\hat{q}_{i,j}^{1,0}$ subsystems and the average tracking error Ψ_1 for ξ_i^1 subsystem as follows:

$$\begin{aligned}\Phi_1 &= \mathrm{col}\Big(\phi_1^1\left(e_{i,1}^{1,0}, \ldots, e_{i,n_0}^{1,0}, \sigma_{i,1}^{1,0}, \ldots, \sigma_{i,n_0}^{1,0}\right), \ldots,\\ &\qquad \phi_{n_1}^1\left(e_{i,1}^{1,0}, \ldots, e_{i,n_0}^{1,0}, \sigma_{i,1}^{1,0}, \ldots, \sigma_{i,n_0}^{1,0}\right)\Big)\\ \Psi_1 &= \mathrm{col}\left(g_1 n_1\left(\frac{1}{n_1}\sum_{\rho=1}^{n_1} q_\rho^1 - \eta_1^1\right), \ldots,\right.\\ &\qquad \left. g_1 n_1\left(\frac{1}{n_1}\sum_{\rho=1}^{n_1} q_\rho^1 - \eta_{n_1}^1\right)\right).\end{aligned}$$

Based on the same arguments of (11), system (13) and ξ_i^1 subsystem in (12) can be rewritten as

$$\begin{aligned}\dot{\chi}^1 &= -\left(L_{g_1}^1 \otimes I_m\right)\chi^1 + \left(L_{g_1}^1 \otimes I_m\right)(\Phi_1 + \Psi_1)\\ \dot{\xi}_i^1 &= \varsigma_1 \sum_{j=1}^{n_1} a_{ij}^1 \mathrm{sgn}\left(\eta_j^1 - \eta_i^1\right)\\ \eta_i^1 &= \xi_i^1 + q_i^1, \quad \xi_i^1(0) = 0, \ i = 1, \ldots, n_1\end{aligned} \tag{14}$$

where

$$\chi^1 = \left(L_{g_1}^1 \otimes I_m\right)q^1 + \mathbf{1}_{n_1} \otimes \Theta_1 - D_1$$

$$\Theta_1 = -g_1 n_1 \varpi_1 \sum_{r=1}^{n_0} \omega_{rk}^0 q_r^0(t)$$

$$D_1 = \mathrm{col}\left(\sum_{j=1}^{n_1} a_{1j}^1 d_{1j}^1, \ldots, \sum_{j=1}^{n_1} a_{n_1 j}^1 d_{n_1 j}^1\right)$$

and $q^1 = \mathrm{col}(q_1^1, \ldots, q_{n_1}^1)$. Note that Assumption 3 requires that $\ddot{q}_j^0(t)$ and $\dot{q}_j^0(t)$ are bounded. According to Lemma 5, there exist $\beta_j^0 \geq \bar{\beta}_j^0 \geq \|\ddot{q}_j^0(t)\|_\infty$, $\delta_j^0 \geq \bar{\delta}_j^0 \geq \|\dot{q}_j^0(t)\|_\infty$ with $\bar{\beta}_j^0$ and $\bar{\delta}_j^0$ being some positive constants, such that

$$\lim_{t\to\infty} e_{i,j}^{1,0}(t) = 0, \quad \lim_{t\to\infty} \sigma_{i,r}^{1,0}(t) = 0.$$

It follows from (9) that ϕ_i^k is a nonnegative weighted sum function of its variables. Then, we have that

$$\lim_{t\to\infty} \Phi_1(t) = 0.$$

Now, we can study the stability of system (14). Note that Assumption 1 requires $\mathcal{G}^1$ being connected. It follows from (12) that $\sum_{i=1}^{n_1} \dot{\xi}_i^1 = 0$. Consequently, we have $\sum_{i=1}^{n_1} \xi_i^1(t) = \sum_{i=1}^{n_1} \xi_i^1(0) = 0$, which implies $\sum_{i=1}^{n_1} \eta_i^1(t) = \sum_{i=1}^{n_1} q_i^1(t)$. Let $Z^1 = \mathrm{col}(z_1^1, \ldots, z_{n_1}^1)$ with $z_j^1 = \eta_j^1 - (1/n_1)\sum_{\rho=1}^{n_1} q_\rho^1, j = 1, \ldots, n_1$. It follows that:

$$\sum_{i=1}^{n_1}\left(z_i^1\right)^{\mathrm{T}} \sum_{j=1}^{n_1} a_{ij}^1 \mathrm{sgn}\left(z_j^1 - z_i^1\right) = -\frac{1}{2}\sum_{i=1}^{n_1} a_{ij}^1 \left\|z_j^1 - z_i^1\right\|_1 \tag{15}$$

$$\begin{aligned}\sum_{i=1}^{n_1}\left\|z_i^1\right\|_1 &\leq \frac{1}{n_1-1}\sum_{i=1}^{n_1}\sum_{j=1}^{n_1} a_{ij}^1 \left\|z_i^1 - z_j^1\right\|_1 \\ &\leq \frac{n_1-1}{2}\sum_{i=1}^{n_1} a_{ij}^1 \left\|z_j^1 - z_i^1\right\|_1.\end{aligned} \tag{16}$$

Using (14), we have that

$$\begin{aligned}\dot{z}_i^1 &= \dot{\eta}_i^1 - \frac{1}{n_1}\sum_{\rho=1}^{n_1} \dot{q}_\rho^1 \\ &= \varsigma_1 \sum_{j=1}^{n_1} a_{ij}^1 \mathrm{sgn}\left(z_j^1 - z_i^1\right) + \dot{q}_i^1 - \frac{1}{n_1}\sum_{\rho=1}^{n_1} \dot{q}_\rho^1.\end{aligned} \tag{17}$$

It follows from $\chi^1 = (L_{g_1}^1 \otimes I_m) q^1 + \mathbf{1}_{n_1} \otimes \Theta_1 - D_1$ that $\dot{q}^1 = (L_{g_1}^1 \otimes I_m)^{-1}(\dot{\chi}^1 - \mathbf{1}_{n_1} \otimes \Theta_1)$. System (17) can be rewritten as

$$\begin{aligned}\dot{Z}^1 = \varsigma_1 \mathrm{col}&\left(\sum_{j=1}^{n_1} a_{1j}^1 \mathrm{sgn}\left(\eta_j^1 - \eta_1^1\right), \ldots, \sum_{j=1}^{n_1} a_{n_1 j}^1 \mathrm{sgn}\left(\eta_j^1 - \eta_{n_1}^1\right)\right) \\ &+ M^1\left(L_{g_1}^1 \otimes I_m\right)^{-1}\left(\dot{\chi}^1 - \mathbf{1}_{n_1} \otimes \dot{\Theta}_1\right)\end{aligned}$$

where $M^1 = (I_{mn_1} - (1/n_1)\mathbf{1}_{n_1}\mathbf{1}_{n_1}^{\mathrm{T}} \otimes I_m)$. Note that $\Psi_1 = -g_1 n_1 Z^1$. Combining the χ^1 subsystem in (14) and the Z^1 system, we have

$$\begin{aligned}\dot{\chi}^1 &= -\left(L_{g_1}^1 \otimes I_m\right)\left(\chi^1 - \Phi_1 + g_1 n_1 Z^1\right) \\ \dot{Z}^1 &= \varsigma_1 \mathrm{col}\left(\sum_{j=1}^{n_1} a_{1j}^1 \mathrm{sgn}\left(\eta_j^1 - \eta_1^1\right), \ldots, \sum_{j=1}^{n_1} a_{n_1 j}^1 \mathrm{sgn}\left(\eta_j^1 - \eta_{n_1}^1\right)\right) \\ &\quad - g_1 n_1 M^1 Z^1 - M^1 \chi^1 \\ &\quad + M^1\left(\Phi_1 - \left(L_{g_1}^1 \otimes I_m\right)^{-1}\left(\mathbf{1}_{n_1} \otimes \dot{\Theta}_1\right)\right).\end{aligned}$$

Define the Lyapunov function candidates as

$$V^1 = \frac{1}{2}\left(Z^1\right)^{\mathrm{T}} Z^1 + \frac{1}{2 g_1 n_1}\left(\chi^1\right)^{\mathrm{T}}\left(L_{g_1}^1 \otimes I_m\right)^{-1}\chi^1.$$

It follows from (15) that:

$$\begin{aligned}\dot{V}^1 = &-\frac{\varsigma_1}{2}\sum_{i=1}^{n_1} a_{ij}^1 \left\|z_j^1 - z_i^1\right\|_1 - g_1 n_1 (Z^1)^{\mathrm{T}} M^1 Z^1 \\ &+ \left(Z^1\right)^{\mathrm{T}} M^1\left(\Phi_1 + \left(L_{g_1}^1 \otimes I_m\right)^{-1}\left(\mathbf{1}_{n_1} \otimes \dot{\Theta}_1\right)\right) \\ &- \frac{1}{g_1 n_1}\left(\chi^1\right)^{\mathrm{T}}\chi^1 + \frac{1}{g_1 n_1}\left(\chi^1\right)^{\mathrm{T}}\Phi_1 \\ &- \left(Z^1\right)^{\mathrm{T}} M^1 \chi^1 - \left(Z^1\right)^{\mathrm{T}}\chi^1.\end{aligned} \tag{18}$$

Note that $\lim_{t\to\infty}\Phi_1(t) = 0$ and Assumption 3 imply that $\dot{\Theta}_1 = -g_1 n_1 \varpi_1 \sum_{r=1}^{n_0} \omega_{rk}^0 \dot{q}_r^0(t)$ is bounded. Thus, there exists $\varsigma^* > 0$ such that

$$\left|\left(Z^1\right)^{\mathrm{T}} M^1\left(\Phi_1 + \left(L_{g_1}^1 \otimes I_m\right)^{-1}\left(\mathbf{1}_{n_1} \otimes \dot{\Theta}_1\right)\right)\right| \leq \varsigma^* \sum_{i=1}^{n_1}\left\|z_i^1\right\|_1.$$

Using (16), there exists $\varsigma_1 > \bar{\varsigma}_1 = (n_1 - 1)(\varsigma^* + 1)$ such that

$$\begin{aligned}\dot{V}^1 \leq &-\sum_{i=1}^{n_1}\left\|z_i^1\right\|_1 - g_1 n_1\left(Z^1\right)^{\mathrm{T}} M^1 Z^1 - \frac{1}{g_1 n_1}\left(\chi^1\right)^{\mathrm{T}}\chi^1 \\ &- \left(Z^1\right)^{\mathrm{T}}\left(M^1 + I_{mn_1}\right)\chi^1 + \frac{1}{g_1 n_1}\left\|\chi^1\right\|\|\Phi_1\| \\ \leq &-g_1 n_1\left(Z^1\right)^{\mathrm{T}} Z^1 - \frac{1}{g_1 n_1}\left(\chi^1\right)^{\mathrm{T}}\chi^1 - 2\left(Z^1\right)^{\mathrm{T}}\chi^1 \\ &+ \frac{1}{g_1 n_1}\left\|\chi^1\right\|\left\|M^1\Phi_1\right\| - \sum_{i=1}^{n_1}\left\|z_i^1\right\|_1 \\ = &-\left\|\left(\sqrt{g_1 n_1} Z^1 + \frac{1}{\sqrt{g_1 n_1}}\chi^1\right)\right\|^2 - \sum_{i=1}^{n_1}\left\|z_i^1\right\|_1 \\ &+ \frac{1}{g_1 n_1}\left\|\chi^1\right\|\left\|M^1\Phi_1\right\|.\end{aligned}$$

The second inequality is obtained by $(Z^1)^{\mathrm{T}} M^1 = (Z^1)^{\mathrm{T}}$, which comes from $(\mathbf{1}_{n_1}^{\mathrm{T}} \otimes I_m) Z^1 = 0$. Using the comparison principle in [25, Ch. 3.4], it follows from $\lim_{t\to\infty}\Phi_1(t) = 0$ that $\lim_{t\to\infty} V^1(t) = 0$. Therefore, $\lim_{t\to\infty}\nabla_{q_i^1} J_i^1(q,t) = \lim_{t\to\infty} 2\chi^1(t) = 0$. It follows that $\lim_{t\to\infty}(q_i^1(t) - \tilde{q}_i^1(t)) = 0$, $\|\dot{q}_i^1(t)\| \leq Q^1$, and $\|\ddot{q}_i^1(t)\| \leq Q^1$ with constant Q^1.

Second, assume that $\lim_{t\to\infty}(q_i^\nu(t) - \tilde{q}_i^\nu(t)) = 0$, $\|\dot{q}_i^\nu(t)\| \leq Q^\nu$, and $\|\ddot{q}_i^\nu(t)\| \leq Q^\nu$ with constant Q^ν holding for $\nu = 0, \ldots, k-1$. Next, we will show that $\chi^k(t) \to 0$. Let $e_{i,j}^{k,l} = \hat{\theta}_{i,j}^{k,l} - \dot{q}_j^l(t)$ and $\sigma_{i,j}^{k,l} = \hat{q}_{i,j}^{k,l} - q_j^l(t)$. Based on (8) and (9), the q_i^k subsystem in (12) can be rewritten as

$$\begin{aligned}\dot{q}_i^k = &\,\phi_i^k\left(q_1^0, \ldots, q_{n_0}^0, q_1^1, \ldots, q_{n_{k-1}}^{k-1}, \dot{q}_1^0, \ldots, \dot{q}_{n_0}^0, \dot{q}_1^1, \ldots, \dot{q}_{n_{k-1}}^{k-1}\right) \\ &- g_k \sum_{\rho=1}^{n_k} q_\rho^k - \sum_{j=1}^{n_k} a_{ij}^k\left(q_i^k - q_j^k - d_{ij}^k\right)\end{aligned}$$

$$+ \phi_i^k\Big(e_1^0, \dots, e_{n_0}^0, e_1^1, \dots, e_{n_{k-1}}^{k-1}, \sigma_1^0, \dots, \sigma_{n_0}^0, \sigma_1^1, \dots, \sigma_{n_{k-1}}^{k-1}\Big) + g_k n_k \left(\frac{1}{n_k}\sum_{\rho=1}^{n_k} q_\rho^k - \eta_i^k\right). \tag{19}$$

Define the state observer error Φ_k for the $\hat{\theta}_{i,j}^{k,l}$ and $\hat{q}_{i,j}^{k,l}$ subsystems, and the average tracking error Ψ_k for the ξ_i^k subsystem as follows:

$$\Phi_1 = \mathrm{col}\Big(\phi_1^1\Big(e_{i,1}^{1,0}, \dots, e_{i,n_0}^{1,0}, \sigma_{i,1}^{1,0}, \dots, \sigma_{i,n_0}^{1,0}\Big), \dots, \phi_{n_1}^1\Big(e_{i,1}^{1,0}, \dots, e_{i,n_0}^{1,0}, \sigma_{i,1}^{1,0}, \dots, \sigma_{i,n_0}^{1,0}\Big)\Big)$$

$$\Psi_1 = \mathrm{col}\left(g_k n_k\left(\frac{1}{n_k}\sum_{\rho=1}^{n_k} q_\rho^k - \eta_1^k\right), \dots, g_k n_k\left(\frac{1}{n_k}\sum_{\rho=1}^{n_k} q_\rho^k - \eta_{n_k}^k\right)\right).$$

Based on the same arguments of (11), the system (19) and the ξ_i^k subsystem in (12) can be written as

$$\begin{aligned}
\dot{\chi}^k &= -\Big(L_{g_k}^k \otimes I_m\Big)\chi^k + \Big(L_{g_k}^k \otimes I_m\Big)(\Phi_k + \Psi_k)\\
\dot{\xi}_i^k &= \varsigma_k \sum_{j=1}^{n_k} a_{ij}^k \mathrm{sgn}\Big(\eta_j^k - \eta_i^k\Big)\\
\eta_i^k &= \xi_i^k + q_i^k, \quad \xi_i^k(0) = 0, \ i = 1, \dots, n_k
\end{aligned} \tag{20}$$

where

$$\begin{aligned}
\chi^k &= \Big(L_{g_k}^k \otimes I_m\Big)q^k + \mathbf{1}_{n_k} \otimes \Theta_k - D_k\\
\Theta_k &= -g_k n_k \varpi_k \left(\sum_{s=1}^{k-1}\sum_{j=1}^{n_s} \frac{\omega_{sk}}{n_s} q_j^s + \sum_{r=1}^{n_0} \omega_{rk}^0 q_r^0(t)\right)\\
D_k &= \mathrm{col}\left(\sum_{j=1}^{n_k} a_{1j}^k d_{1j}^k, \dots, \sum_{j=1}^{n_k} a_{n_k j}^k d_{n_k j}^k\right)
\end{aligned}$$

and $q^k = \mathrm{col}(q_1^k, \dots, q_{n_k}^k)$. Note that (20) has the same form with (14). Using the same argument of (14), we have that $\lim_{t\to\infty}(q_i^k(t) - \tilde{q}_i^k(t)) = 0$, $\|\dot{q}_i^k(t)\| \le Q^k$, and $\|\ddot{q}_i^k(t)\| \le Q^k$ with constant Q^k.

The process is repeated ϱ times to obtain $\lim_{t\to\infty}(q_i^\varrho(t) - \tilde{q}_i^\varrho(t)) = 0$, and the proof is completed based on the results in Proposition 1. ■

Remark 12: Note that all the agents' states are bounded since the leaders' states are bounded and the followers' states are constrained in the bounding box of the leaders (see the requirements of a nonlinear placement problem in [1, Sec. 8.7]). The lower bounds of $\bar{\beta}_j^l$, $\bar{\delta}_j^l$, and $\bar{\varsigma}_k$ are given in the proof. In particular, we require $\bar{\beta}_j^l \ge \|\ddot{q}_j^l(t)\|_\infty$ and $\bar{\delta}_j^l \ge \|\dot{q}_j^l(t)\|_\infty$, which come from the conditions of [24, Th. 3.1]. The parameter $\bar{\varsigma}_k$ depends on Φ_k and Θ_k, which comes from the conditions of the average tracking algorithm in [26].

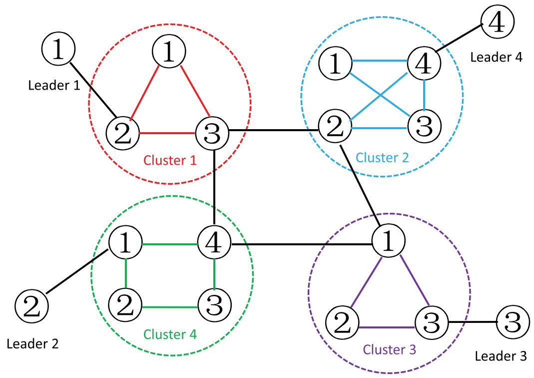


Fig. 1. Network topology among agents.

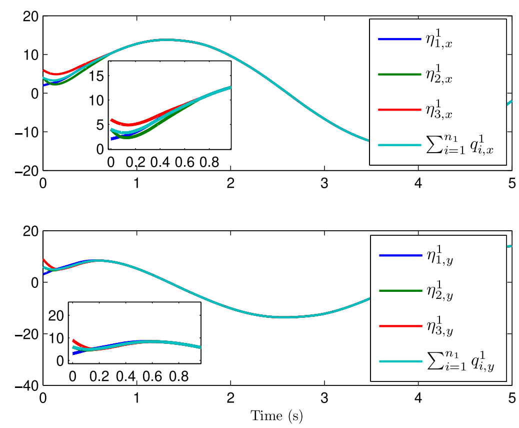


Fig. 2. Average tracking states η_i^k & $\sum_{i=1}^{n_k} q_i^k$.

V. Examples

Consider a formation control problem of a multicluster system composed of four leaders and ten followers, and assume that the problem can be modeled as the distributed nonlinear placement problem (3). All the followers are divided into four clusters (including 3, 3, 4, and 4 followers, respectively). The communication network graphs $\mathcal{G}^k$ and $\mathcal{G}$ are given in Fig. 1. Let the state of system (1) be $q_i^k = [q_{i,x}^k, q_{i,y}^k]^{\mathrm{T}}$. The positions of the leaders are $q_i^0(t) = (5+5i)[\sin\omega t; \cos\omega t]$ with $\omega = 4\pi \times 10^{-4}$, $i = 1, 2, 3, 4$. Some elements of the weight are $\omega_{11}^0 = \omega_{31}^0 = 1$, $\omega_{22}^0 = \omega_{42}^0 = 1$, $\omega_{13}^0 = \omega_{23}^0 = \omega_{13} = 1$, and $\omega_{14} = \omega_{34} = 1$, and others are 0. The desired distance vectors are $d_{12}^1 = d_{42}^2 = [5; 5]^{\mathrm{T}}$, $d_{13}^1 = d_{13}^2 = d_{13}^3 = [-5; 5]^{\mathrm{T}}$, $d_{43}^2 = d_{43}^4 = [0; 5]^{\mathrm{T}}$, and $d_{32}^3 = d_{32}^4 = d_{41}^4 = [5; 0]^{\mathrm{T}}$. Assume that all the agents' initial positions are $q_i^k(0) = [10i, 15i]$. It is not hard to show that Assumptions 1 and 2, and $[a_{ij}^k]^{\max} \le g_k < ([2\sum_{j=1}^{n_k} a_{ij}^k]/[n_k - 1]) \ \forall\, i = 1, \dots, n_k$ hold.

Let $g_k = 1$, $\alpha_j^l = 2 \times 10^{-3}$, $\beta_j^l = 50$, $\gamma_j^l = 5 \times 10^{-5}$, $\delta_j^l = 50$, and $\varsigma_k = 3$. The effectiveness of the control algorithm (12) in maintaining a formation shape is demonstrated in Figs. 2–6 (only some results are presented due to space limitation). In particular, Figs. 2 and 3 show that the average tracking errors and states observer errors asymptotically converge. It follows from Fig. 4 that the nonlinear placement for cluster k is satisfied, that is, the total interconnection length of the links connecting the centers of cluster 1 and leaders q_1^0

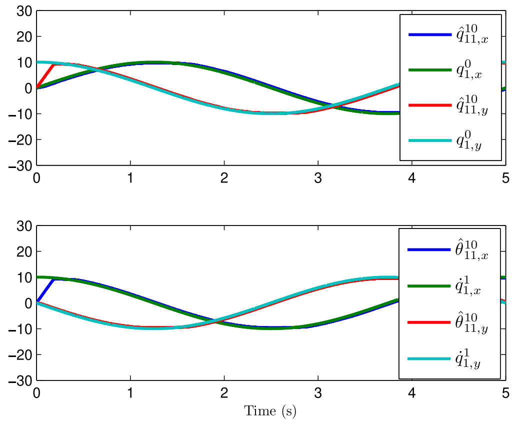


Fig. 3. State observers $\hat{q}^{kl}_{ij}$ & q^l_j and $\hat{\theta}^{kl}_{ij}$ & $\dot{q}^l_j$.

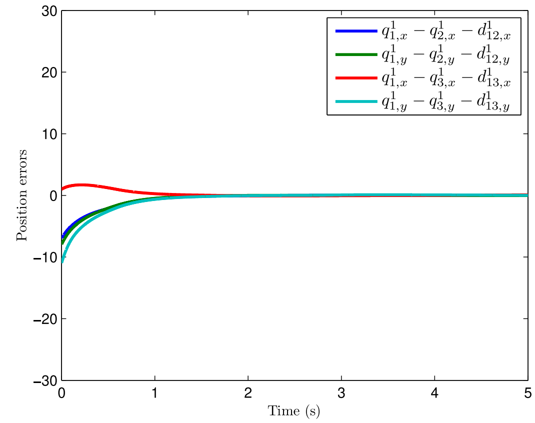


Fig. 6. Formation errors $q^k_i - q^k_j - d^k_{ij}$.

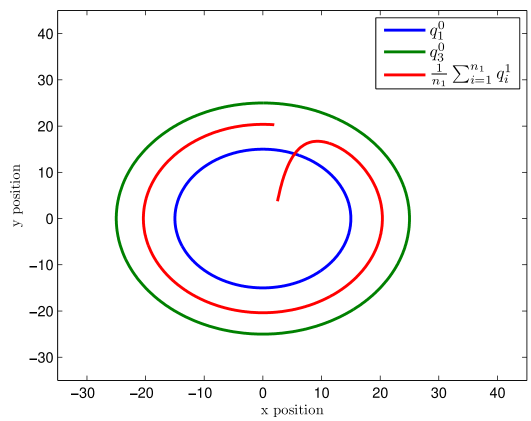


Fig. 4. Positions of leaders q^0_r and centers of clusters $(1/n_k)\sum_{i=1}^{n_k} q^k_i$.

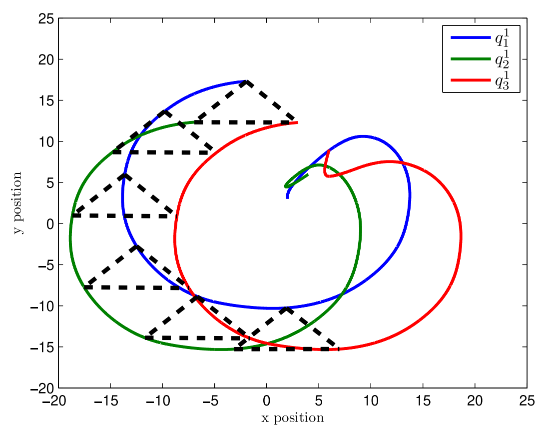


Fig. 5. Clusters maintaining a formation $(q^k_{i,x}, q^k_{i,y})$.

and q^0_3 is minimized. Moreover, Fig. 5 shows that the followers in each cluster are maintained according to some desired distance vector d^k_{ij}. The formation errors of cluster 1 are given in Fig. 6. It can be observed that the formation errors asymptotically converge, which indicates that the system actually converges to the NE.

VI. Conclusion

In this article, we proposed a distributed nonlinear placement algorithm, which was designed based on a distributed observer-based method. The communication among all the agents is an undirected connected topology. The main results were proved by an iterative approach. Future study directions include several open and interesting questions. For example, the communication amongst the agents is directed and time varying or the dynamics for each agent is nonlinear and heterogeneous. Moreover, it is interesting to extend the designed algorithm to the nonlinear placement problem with a more complex function $h(\cdot)$.

References

[1] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.

[2] S. S. Dhillon and K. Chakrabarty, "Sensor placement for effective coverage and surveillance in distributed sensor networks," in *Proc. IEEE Conf. Wireless Commun. Netw.*, vol. 3, Mar. 2003, pp. 1609–1614.

[3] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1520–1533, Sep. 2004.

[4] W. Ren and R. W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Trans. Autom. Control*, vol. 50, no. 5, pp. 655–661, May 2005.

[5] B. Johansson, M. Rabi, and M. Johansson, "A randomized incremental subgradient method for distributed optimization in networked systems," *SIAM J. Optim.*, vol. 20, no. 3, pp. 1157–1170, 2009.

[6] A. Nedić, A. Ozdaglar, and P. A. Parrilo, "Constrained consensus and optimization in multi-agent networks," *IEEE Trans. Autom. Control*, vol. 55, no. 4, pp. 922–938, Apr. 2010.

[7] W. Ren and Y. Cao, *Distributed Coordination of Multi-Agent Networks*. London, U.K.: Springer-Verlag, 2011.

[8] K.-K. Oh, M.-C. Park, and H.-S. Ahn, "A survey of multi-agent formation control," *Automatica*, vol. 53, pp. 424–440, Mar. 2015.

[9] X. Yang, C.-C. Hua, J. Yan, and X.-P. Guan. "Adaptive formation control of cooperative teleoperators with intermittent communications," *IEEE Trans. Cybern.*, vol. 49, no. 7, pp. 2514–2523, Jul. 2019.

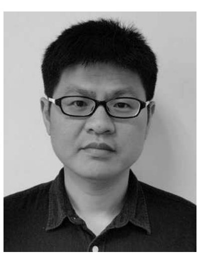
[10] J. Hu and W. X. Zheng, "Emergent collective behaviors on coopetition networks," *Phys. Lett.*, vol. 378, pp. 1787–1796, May 2014.

[11] O. Shehory and S. Kraus, "Methods for task allocation via agent coalition formation," *Artif. Intell.*, vol. 101, nos. 1–2, pp. 165–200, 1998.

[12] X. Zeng, J. Chen, S. Liang, and Y. Hong, "Generalized Nash equilibrium seeking strategy for distributed nonsmooth multi-cluster game," *Automatica*, vol. 103, pp. 20–26, May 2019.

[13] M. Ye, G. Hu, and S. Xu, "An extremum seeking-based approach for Nash equilibrium seeking in N-cluster noncooperative games," *Automatica*, vol. 114, Apr. 2020, Art. no. 108815.

[14] Y. Lou, Y. Hong, L. Xie, G. Shi, and K. H. Johansson, "Nash equilibrium computation in subnetwork zero-sum games with switching communications," *IEEE Trans. Autom. Control*, vol. 61, no. 10, pp. 2920–2935, Oct. 2016.
[15] S. Liang, P. Yi, and Y. Hong, "Distributed Nash equilibrium seeking for aggregative games with coupled constraints," *Automatica*, vol. 85, pp. 179–185, Nov. 2017.
[16] D. Gadjov and L. Pavel, "A passivity-based approach to Nash equilibrium seeking over networks," *IEEE Trans. Autom. Control*, vol. 64, no. 3, pp. 1077–1092, Mar. 2019.
[17] Y. Zhang, S. Liang, X. Wang, and H. Ji, "Distributed nash equilibrium seeking for aggregative games with nonlinear dynamics under external disturbances," *IEEE Trans. Cybern.*, vol. 50, no. 12, pp. 4876–4885, Dec. 2020, doi: 10.1109/TCYB.2019.2929394.
[18] M. Ye and G. Hu, "Distributed seeking of time-varying nash equilibrium for non-cooperative games," *IEEE Trans. Autom. Control*, vol. 60, no. 11, pp. 3000–3005, Nov. 2015.
[19] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 1985.
[20] D. Gu, "A differential game approach to formation control," *IEEE Trans. Control Syst. Technol.*, vol. 16, no. 1, pp. 85–93, Jan. 2008.
[21] T. Han, Z. Guan, M. Chi, B. Hu, T. Li, and X.-H. Zhang, "Multiformation control of nonlinear leader-following multi-agent systems," *ISA Trans.*, vol. 69, pp. 140–147, Jul. 2017.
[22] J. Nash, "Non-cooperative games," *Ann. Math.*, vol. 54, no. 2, pp. 286–295, 1951.
[23] A. Nagurney and D. Zhang, *Projected Dynamical Systems and Variational Inequalities With Applications*. Dordrecht, The Netherlands: Kluwer Academic Publ., 1996.
[24] Y. Cao and W. Ren, "Distributed coordinated tracking with reduced interaction via a variable structure approach," *IEEE Trans. Autom. Control*, vol. 57, no. 1, pp. 33–48, Jan. 2012.
[25] H. K. Khalil, *Nonlinear Systems*, 3rd ed. Englewood Cliffs, NJ, USA: Prentice Hall, 2002.
[26] F. Chen, Y. Cao, and W. Ren, "Distributed average tracking of multiple time-varying reference signals with bounded derivatives," *IEEE Trans. Autom. Control*, vol. 57, no. 12, pp. 3169–3174, Dec. 2012.



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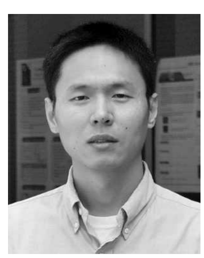

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