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|---|---|
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| Article Sub-Title | |
| Article CopyRight | Springer-Verlag GmbH Germany, part of Springer Nature<br>(This will be the copyright line in the final PDF) |
| Journal Name | International Journal of Dynamics and Control |



| | | |
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|---|---|---|
| Schedule | Received | 28 Feb 2022 |
| | Revised | 3 Oct 2022 |
| | Accepted | 10 Oct 2022 |





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|---|---|
| Keywords (separated by '-') | Quadrotor - Distributed control - Cooperative control - Leader–follower control - Formation control |
| Footnote Information | |

International Journal of Dynamics and Control
https://doi.org/10.1007/s40435-022-01066-5



# Distributed cooperative control of multiple UAVs with uncertainty

**Shihab Ahmed[1] · Wenjie Dong[1]**

Received: 28 February 2022 / Revised: 3 October 2022 / Accepted: 10 October 2022


**Abstract**
This paper considers the formation flying of multiple quadrotors with a desired orientation and a leader. In the formation flying control, it is assumed that the desired formation is time-varying and there are the system uncertainty and the information uncertainty. In order to deal with different uncertainties, a backstepping-based approach is proposed for the controller design. In the proposed approach, different types of uncertainties are considered in different steps. By integrating adaptive/robust control results and Laplacian algebraic theory, distributed robust adaptive control laws are proposed such that the formation errors exponentially converge to zero and the attitude of each quadrotor exponentially converges to the desired value. Simulation results show the effectiveness of the proposed algorithms.

**Keywords** Quadrotor · Distributed control · Cooperative control · Leader–follower control · Formation control



## 1 Introduction

Formation flying of multiple quadrotors has been studied recently due to its wide applications in civil and military applications, such as surveillance, area exploration, target search, accident rescue tasks, and many other applications. The capacity of vertical taking-off and landing makes quadrotors superior to other unmanned aerial vehicles. Compared with a single quadrotor, the formation of multiple quadrotors can perform more difficult tasks and provides better performance. However, the underactuated nature of a single quadrotor makes the cooperative control of multiple quadrotors challenging.

Formation control of multiple quadrotors is to coordinate a group of quadrotors to achieve a desired spatial geometric pattern. In the past decades, several classical approaches have been proposed for multi-agent systems, which include the behavioral approach, the virtual structure approach, the leader-follower approach, and the graph theoretical approach. In the leader-following approach [1,2], some agents are designated as leaders and the others are designated as followers. The leaders track the predefined trajectories and the followers track the state of their neighbors according to given schemes. In the behavioral approach [3–5], the control action for each agent is defined by a weighted average of the control corresponding to each desired behavior for the agent. In the virtual structure approach [6–8], the entire formation is treated as a single rigid body. The virtual structure moves along a desired trajectory and with a desired attitude. In the graph theoretical approach [9–12], each agent is considered as a node and the communication between agents is defined by a graph. The control law is designed with the aid of the difference of neighbors' information.

Formation control of multiple unmanned aerial vehicles (UAVs) has been studied extensively. In [13,14], the dynamics of each vehicle is simplified as a linear system and the formation control is studied based on multiple linear systems. In [15–17], the formation control was studied based on the translational and rotational motions with linearized or simplified models. Noting that a UAV is a multiple-input multiple-output system with highly nonlinear and strongly coupled dynamics and has 6 degrees of freedom (6-DOF), formation control of multiple UAV was studied based on 6-DOF dynamics in [18–21]. In [19,20], formation control of multiple UAVs was studied based on the 6-DOF model with disturbances and robust distributed control laws were proposed. In [21], distributed formation control of multiple UAVs was studied based on a nonsmooth backstepping design and consensus techniques for the 6-DOF models.

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The formation controllers in the literature mentioned above ensure that the states of a UAV asymptotically converge to a desired formation as time goes to infinity. In practical applications, finite-time distributed controllers are preferred because they ensure that the states of a UAV converge to a desired formation within a finite time and the closed-loop systems have better disturbance rejection performance. In [22], formation control of multiple UAVs with nonparametric uncertainties was studied. Finite-time controllers were proposed with the aid of finite-time distributed observers. In [16], finite-time distributed controllers were proposed based on the linearized models without uncertainty with the aid of the properties of homogeneous systems. In [16,22], the attitudes of UAVs are defined by Euler angles. To make the attitude control laws nonsingular, the Euler angles are limited to some intervals.

Although there are many results on formation control of multiple UAVs, how to improve the control performance is still challenging in the presence of uncertainty and coupling among neighboring UAVs. Motivating by the research work mentioned above and the work in [23–25], in this paper we study the formation control of multiple UAVs with parametric and nonparametric uncertainties and propose new distributed control laws such that the formation errors exponentially converge to zero and the attitude of each UAV exponentially converges to a desired attitude. In order to solve the formation control problem, a multi-step *backstepping-based approach* is proposed and distributed exponential control laws are designed. The proposed approach includes six steps. In the first step an auxiliary system for each quadrotor is introduced to estimate the parametric and nonparametric uncertainties in the dynamics of each quadrotor. In the second step, distributed kinematic controllers are proposed for the translation with the aid of the graph theory and the formation problem is solved without the information whether the leader's information is available to a quadrotor or not. In the third step, distributed dynamic controllers for the translation and the desired attitude for each quadrotor are designed based on the backstepping technique. In the fourth step, the force input and the desired attitude for each quadrotor are calculated. In the fifth step, a distributed kinematic controller for the attitude control of each quadrotor is proposed with the aid of the graph theory. In the last step, a dynamic controller for the attitude of each quadrotor is proposed with the aid of the backstepping technique. In the proposed approach, the uncertainties in the dynamics and the uncertainty of the leader's information are considered separately in different steps. With the aid of this multi-step approach, distributed robust adaptive controllers are proposed such that multiple quadrotors exponentially converge to a desired formation and the Y-axis of each quadrotor exponentially converges to the desired direction. Compared to the results in literature, the contributions of this paper are as follows.

- This paper solves the formation control problem of multiple quadrotors in a more general setting. In the considered problem, the formation is time-varying and there are both parametric uncertainty and nonparametric uncertainty in the dynamics of each quadrotor. Furthermore, the communications among quadrotors are directional, which means that the information exchange between two quadrotors is one-way instead of two-way.
- A new systematic multi-step controller design approach is proposed for the formation control problem by integrating the uncertainty decomposition technique and the backstepping technique. In this approach, different types of uncertainties are dealt with in different steps. The difficulty of the controller design is greatly reduced.
- The proposed control laws ensure that a group of quadrotors exponentially converge to a desired formation with a desired orientation, which means that the proposed control systems have better performance in convergence and disturbance rejection. Moreover, the proposed controllers are distributed. No global information is required in the controllers.

The remaining part of this paper is organized as follows. In Sect. 2, the considered problem is defined and some preliminary results are presented. In Sect. 3, a multi-step approach is proposed and distributed controllers are derived. In Sect. 4, simulation results are presented. The last section concludes this paper.

## 2 Problem statement and preliminaries

### 2.1 Problem statement

Consider $m$ quadrotors. Under some assumptions, the kinematics and dynamics of $j$-th quadrotor are defined by

$$\dot{p}_j = v_j \tag{1}$$

$$\dot{v}_j = -ge_3 + \frac{1}{m_j} f_j R_j e_3 + d_{1j} \tag{2}$$

$$\dot{R}_j = RS(\omega_j) \tag{3}$$

$$J_j\dot{\omega}_j = S(J_j\omega_j)\omega_j + \tau_j + d_{2j} \tag{4}$$

where $p_j$ and $v_j$ are the position and the velocity of the mass center in the inertia frame, respectively, $g$ is the gravitational acceleration, $e_3 = [0, 0, 1]^\top$, $f_j \in \Re$ is the total thrust, $R_j = [b_{1j}, b_{2j}, b_{3j}]$ is the rotation matrix of the body frame with respect to the inertia frame, $\omega_j$ is the angular velocity of the quadrotor in its body frame, $J_j$ is the inertia moment of the quadrotor, $d_{1j}$ and $d_{2j}$ denote nonparametric uncertainty and disturbance, $S(\xi)$ for $\xi = [\xi_1, \xi_2, \xi_3]^\top$ is a skew-symmetric



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matrix defined by

$$S(\xi) = \begin{bmatrix} 0 & -\xi_3 & \xi_2 \\ \xi_3 & 0 & -\xi_1 \\ -\xi_2 & \xi_1 & 0 \end{bmatrix},$$

and $\tau_j = [\tau_{1j}, \tau_{2j}, \tau_{3j}]^\top$ is the torque input of the system.

For multiple quadrotors, there are information flows between them with the aid of sensors or wireless communication. Consider each quadrotor as a node. The communication between quadrotors is defined by a directed graph $\mathcal{G} = \{\mathcal{A}, \mathcal{E}\}$ where $\mathcal{A}$ is the node set and $\mathcal{E}$ is the edge set. If there is an edge $e_{ij}$ in $\mathcal{E}$ it means that the information of node $i$ is available to node $j$. Node $i$ is called a neighbor of node $j$ if the information of node $i$ is available to node $j$. All neighbors of node $j$ form a node set which is called the neighbor set of node $j$ and is denoted by $\mathcal{N}_j$. A directed path from node $i$ to node $j$ is a sequence of sets of edges that connect node $i$ to node $j$ by following their directions. Node $i$ is said to be reachable to node $j$ if there exists a directed path from node $i$ to node $j$. Node $i$ is said globally reachable if node $i$ is reachable for every other node in $\mathcal{A}$.

In this paper, we assume there are $m$ follower quadrotors and one leader quadrotor. The leader quadrotor is operated by a human operator and does not receive any information from the follower quadrotors. Without loss of generality, the leader quadrotor is labeled as node 0. The follower quadrotors are labeled by 1, 2, …, $m$. The communication between $m+1$ quadrotors is defined by an augmented directed graph $\mathcal{G}^a = \{\mathcal{A}^a, \mathcal{E}^a\}$ where $\mathcal{A}^a = \mathcal{A} \cup \{0\}$ and $\mathcal{E}^a$ is a union of $\mathcal{E}$ and the edges from node 0 to the followers.

For $m$ follower quadrotors and a leader quadrotor, a desired formation can be defined by $(m+1)$ vectors $h_j \in R^3$ which may be constant vectors or time-varying vectors. We say $(m+1)$ quadrotors are in the desired formation if

$$p_i - p_j = h_i - h_j$$

for any $0 \le i, j \le m$. We say $m+1$ quadrotors come into the desired formation if

$$\lim_{t\to\infty} [(p_i - h_i) - (p_j - h_j)] = 0$$

for any $0 \le i, j \le m$.

In the dynamics (1–4), the parametric uncertainty (i.e., $m_j$ and $J_j$) and nonparametric uncertainty (i.e., $d_{1j}$ and $d_{2j}$) are called the *system uncertainty*. For each quadrotor, it is unknown whether the leader quadrotor is a neighbor or not. We say there is *information uncertainty* for each quadrotor.

In this paper, we consider the following control problem.

### 2.1.1 Formation flying with a leader

For a leader quadrotor and $m$ follower quadrotors, it is assumed that $m_j$, $J_j$, $d_{1j}$, and $d_{2j}$ are unknown for $1 \le j \le m$. It is given a desired formation defined by $h_j$ for $0 \le j \le m$, the control problem is to design distributed state feedback controllers $f_j$ and $\tau_j$ using its own information and its neighbors' information such that

$$\lim_{t\to\infty} [(p_j(t) - h_j(t)) - (p_i(t) - h_i(t))] \overset{exp.}{=} 0 \tag{5}$$

$$\lim_{t\to\infty} (b_{2j}(t) - b_{2,0}(t)) \overset{exp.}{=} 0 \tag{6}$$

where $\overset{exp.}{=}$ means "exponentially converges to".

In the defined problem, (5) means that the $(m+1)$ quadrotors come into the desired formation and (6) means that the $Y$-axes of the body frames of $m+1$ quadrotors are parallel as time goes to infinity.

In order to solve the defined problem, the following assumptions are made.

**Assumption 1** The mass $m_j$ of quadrotor $j$ is an unknown constant and $\underline{m}_j \le m_j \le \bar{m}_j$ where $\underline{m}_j$ and $\bar{m}_j$ are known constants.

**Assumption 2** $d_{1j}$ and $d_{2j}$ are continuous functions of the system state and the time and are bounded.

**Assumption 3** The communication graph $\mathcal{G}^a$ is a directed graph and the node 0 is globally reachable.

**Assumption 4** $b_{2,0}(t)$ is smooth. $\dot{b}_{2,0}$ and $\ddot{b}_{2,0}$ are bounded. $b_{2,0}^\top(t) b_{3,0}(t) = 0$ for any time where $b_{3,0}(t) = \frac{\ddot{p}_0(t)+ge_3}{\|\ddot{p}_0(t)+ge_3\|_2}$.

Assumption 1 is reasonable in practice because the mass of a quadrotor is always bounded by some constant. Since $d_1$ and $d_2$ are friction and disturbance, it is reasonable to assume that they are bounded in Assumption 2.

Assumption 4 is due to the motion of the quadrotor in (1–2). In Assumption 4, $b_{3,0}$ is obtained as follows. If the quadrotor moves along the desired trajectory, by (1–2) one has

$$R_0 e_3 = \frac{f(\ddot{p}_0 + ge_3)}{m} = \frac{\ddot{p}_0 + ge_3}{\|\ddot{p}_0 + ge_3\|_2}.$$

Since $m_j$ and $f_j$ are not zero and $R_0 e_3$ is an unit vector, $b_{3,0}$ should be the third column of $R_0$.

## 2.2 Kinematics of rotation using quaternion

The attitude of $j$-th quadrotor can be defined by an unit quaternion $q_j = \left[\eta_j, \epsilon_j^\top\right]^\top$ where $\eta_j \in \Re$ and $\epsilon_j \in \Re^3$. The relation between $q_j$ and the rotation matrix $R_j$ is defined by

$$R_j = \mathcal{R}(q_j) = I + 2\eta_j S(\epsilon_j) + 2S^2(\epsilon_j).$$





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Noting that for any rotation matrix $R$, there are exactly two unit quaternions, $\pm q$, such that $R = \mathcal{R}(q) = \mathcal{R}(-q)$.

For $j$-th quadrotor, (3) can be written as

$$\dot{q}_j = \frac{1}{2} A(q_j)\omega_j \tag{7}$$

where

$$A(q_j) = \begin{bmatrix} -\epsilon_j^\top \\ \eta_j I + S(\epsilon_j) \end{bmatrix}. \tag{8}$$

## 2.3 Notations and preliminary results

Let $\mathcal{L}_\infty$ denote bounded functions and $\mathcal{L}_2$ denote square integrable functions.

For $x \in \Re$, we define the function

$$\chi(x) = \frac{x}{\sqrt{x^2 + e^{-2\kappa t}}} \tag{9}$$

where $\kappa > 0$. If $x = [x_1, \ldots, x_l]^\top \in \Re^l$, $\chi(x)$ is defined as

$$\chi(x) = [\chi(x_1), \ldots, \chi(x_l)]^\top.$$

It can be proved that

$$x^\top \chi(x) \geq \sum_{i=1}^{l} |x_i| - l e^{-\kappa t}.$$

The results in the following lemma are useful.

**Lemma 1** *Consider $m+1$ agents, where agent 0 is the leader agent and agent $j$ is a follower agent for $1 \leq j \leq m$. The state of the agent $j$ is $x_j$. The communication among agents is defined by a direct graph $\mathcal{G}^a$ and it is assumed that the state $x_0$ is globally reachable to all other agents.*

*(1) If $\dot{x}_0 = 0$ and the state of agent $j$ for $1 \leq j \leq m$ is defined by*

$$\dot{x}_j = -\sum_{i \in \mathcal{N}_j^a} a_{ji}(x_j - x_i) + u_j$$

*then the system with input $u$ and state $\tilde{x}$ has the input-to-state stability (ISS) property [26], where $u = [u_1^\top, \ldots, u_m^\top]^\top$ and $\tilde{x} = [x_1^\top - x_0^\top, \ldots, x_m^\top - x_0^\top]^\top$. Moreover, if $u$ exponentially converges to zero $\tilde{x}$ also exponentially converges to zero.*

*(2) If $\max_{t\in[0,\infty)} |\dot{x}_0(t)| \leq \nu$ and the state of agent $j$ for $1 \leq j \leq m$ is defined by*

$$\dot{x}_j = -\sum_{i \in \mathcal{N}_j^a} a_{ji}(x_j - x_i) - \nu\chi\left(\sum_{i \in \mathcal{N}_j^a} a_{ji}(x_j - x_i)\right) + u_j$$

*then the system with input $u$ and state $\tilde{x}$ has the ISS property, where $u$ and $\tilde{x}$ are defined in item 1. Moreover, if $u$ exponentially converges to zero $\tilde{x}$ also exponentially converges to zero.*

*(3) If $\max_{t\in[0,\infty)} |\dot{x}_0(t)| \leq \nu_0$ and the state of agent $j$ for $1 \leq j \leq m$ is defined by*

$$\dot{x}_j = -\sum_{i \in \mathcal{N}_j^a} a_{ji}(x_j - x_i) - \nu_j\chi\left(\sum_{i \in \mathcal{N}_j^a} a_{ji}(x_j - x_i)\right) + u_j$$

$$\dot{\nu}_j = -\sum_{i \in \mathcal{N}_j^a} a_{ji}(\nu_j - \nu_i)$$

*then $\lim_{t\to\infty}(\nu_j - \nu_0) \overset{exp.}{=} 0$ and the system with input $u$ and state $\tilde{x}$ has the ISS property, where $u$ and $\tilde{x}$ are defined in item 1. Moreover, if $u$ exponentially converges to zero $\tilde{x}$ also exponentially converges to zero.*

The lemma can be proved based on the results of properties of Laplacian matrices and ISS properties and is omitted here.

# 3 Controller design

The presence of the system uncertainty and the information uncertainty makes the distributed controller design extremely hard when the communication graph is directed. In order to solve the defined problem, we propose the following multi-step controller design approach in which different types of uncertainty are dealt with in different steps.

**Step 1:** In the dynamics of the translation (1–2), $m_j$ is an unknown constant and $d_{1j}$ is an unknown time-varying function. We use a two-layer neural network to learn $d_{1j}$. Based on the universal approximation property of neural networks [27], there is a basis matrix $\phi_{1j}$ and an optimal weighted vector $\theta_{1j}$ with appropriate dimensions such that

$$d_{1j} = \phi_{1j}\theta_{1j} + \epsilon_{1j} \tag{10}$$

where $\epsilon_{1j}$ is the approximation error vector and $\|\epsilon_{1j}\| \leq \delta_{1j}$.

In order to deal with the system uncertainty, an auxiliary system for $j$-th quadrotor ($1 \leq j \leq m$) is introduced as follows.

$$\dot{z}_{1j} = z_{2j} + L_{1j}(p_j - z_{1j}) \tag{11}$$

$$\begin{aligned}\dot{z}_{2j} = &-ge_3 + \beta_j f_j R_j e_3 + \phi_{1j}\hat{\theta}_{1j} + L_{2j}(v_j - z_{2j}) \\ &+(p_j - z_{1j}) + \delta_{1j}\chi(e^{\lambda t}(v_j - z_{2j}))\end{aligned} \tag{12}$$

where $\lambda$, $L_{1j}$ ($> \lambda$) and $L_{2j}$ ($> \lambda$) are positive constants, $\kappa > \lambda$, $\beta_j$ is an estimate of $\frac{1}{m_j}$, and $\hat{\theta}_{1j}$ is an estimate of



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$\theta_{1j}$ and will be designed later. Let $e_{1j} = e^{\lambda t}(p_j - z_{1j})$ and $e_{2j} = e^{\lambda t}(v_j - z_{2j})$, then

$$\dot{e}_{1j} = e_{2j} - (L_{1j} - \lambda)e_{1j}$$
$$\dot{e}_{2j} = \left(\frac{1}{m_j} - \beta_j\right) e^{\lambda t} f_j R_j e_3 + e^{\lambda t}\phi_{1j}(\theta_{1j} - \hat{\theta}_{1j}) + e^{\lambda t}\epsilon_{1j} - e_{1j} - (L_{2j} - \lambda)e_{2j} - \delta_{1j}e^{\lambda t}\chi(e_{2j})$$

To make $e_{1j}$ converge to zero, we choose a Lyapunov function candidate

$$V_{1j} = \frac{1}{2}e_{1j}^\top e_{1j} + \frac{1}{2}e_{2j}^\top e_{2j} + \frac{\gamma_{1j}^{-1}}{2}\left(\frac{1}{m_j} - \beta_j\right)^2 + \frac{\gamma_{2j}^{-1}}{2}(\theta_{1j} - \hat{\theta}_{1j})^\top(\theta_{1j} - \hat{\theta}_{1j})$$

where $\gamma_{1j}$ and $\gamma_{2j}$ are positive constants. The derivative of $V_{1j}$ is

$$\begin{aligned}\dot{V}_{1j} = &-e_{1j}^\top(L_{1j} - \lambda)e_{1j} + e_{2j}^\top\left(\frac{1}{m_j} - \beta_j\right) e^{\lambda t} f_j R_j e_3 \\ &+ e_{2j}^\top e^{\lambda t}\phi_{1j}(\theta_{1j} - \hat{\theta}_{1j}) + e_{2j}^\top e^{\lambda t}\epsilon_{1j} \\ &- e_{2j}^\top(L_{2j} - \lambda)e_{2j} - \gamma_{1j}^{-1}\left(\frac{1}{m_j} - \beta_j\right)\dot{\beta}_j \\ &- \gamma_{2j}^{-1}(\theta_{1j} - \hat{\theta}_{1j})^\top\dot{\hat{\theta}}_j - e_{2j}^\top\delta_{1j}e^{\lambda t}\chi(e_{2j})\end{aligned}$$

We choose

$$\dot{\beta}_j = Proj_{\Omega_j}[\gamma_{1j}e_{2j}^\top e^{\lambda t} f_j R_j e_3] \tag{13}$$
$$\dot{\hat{\theta}}_{1j} = \gamma_{2j}e^{\lambda t}\phi_{1j}^\top e_{2j} \tag{14}$$

where $Proj_{\Omega_j}$ denotes the projection to $\Omega_j = \left[\frac{1}{\bar{m}_j}, \frac{1}{\underline{m}_j}\right]$ [28], then

$$\begin{aligned}\dot{V}_{1j} = &-e_{1j}^\top(L_{1j} - \lambda)e_{1j} + e_{2j}^\top e^{\lambda t}\epsilon_{1j} - e_{2j}^\top(L_{2j} - \lambda)e_{2j} \\ &- \delta_j e_{2j}^\top e^{\lambda t}\chi(e_{2j}) \\ \le &-e_{1j}^\top L_{1j}e_{1j} - e_{2j}^\top L_{2j}e_{2j} + 3\delta_{1j}e^{-(\kappa-\lambda)t}.\end{aligned} \tag{15}$$

**Lemma 2** *For the systems in (1–2) and the auxiliary system in (11–12) with update laws in (13–14), the estimates $\beta_j$ and $\hat{\theta}_{1j}$ are bounded and*

$$\lim_{t\to\infty}(p_j - z_{1j}) \overset{exp.}{=} 0 \tag{16}$$
$$\lim_{t\to\infty}(v_j - z_{2j}) \overset{exp.}{=} 0. \tag{17}$$

***Proof*** For the Lyapunov function $V_{1j}$, we have (15). Integrating both sides of (15), we have

$$V_{1j}(t) \le V_{1j}(0) + \frac{3\delta_j}{\kappa - \lambda} - \frac{3\delta_{1j}}{\kappa - \lambda}e^{-(\kappa-\lambda)t} < \infty$$

So, $V_{1j} \in \mathcal{L}_\infty$. By the definition of $V_{1j}$, $\beta_j$, $\hat{\theta}_j$, $e_{1j}$ and $e_{2j}$ are bounded. Noting the definitions of $e_{1j}$ and $e_{2j}$, $z_{1j}$ and $z_{2j}$ exponentially converge to zero. So, (16–17) are satisfied. □

For the leader quadrotor, an auxiliary system is not required. For convenience, we define

$$z_{10} = p_0, \quad z_{20} = v_0.$$

**Step 2:** Noting that $p_{1j} - z_{1j}$ exponentially converges to zero, (5) is satisfied if

$$\lim_{t\to\infty}[(z_{1j} - h_j) - (z_{1i} - h_i)] \overset{exp.}{=} 0, \quad 0 \le i, j \le m. \tag{18}$$

We assume that $z_{2j}$ is a virtual control input and design it for the system in (11–12) such that (18) is satisfied.

Let

$$\tilde{z}_{1j} = z_{1j} - h_j - (z_{10} - h_0),$$
$$\tilde{z}_{2j} = z_{2j} - \dot{h}_j - (z_{20} - \dot{h}_0)$$

for $0 \le j \le m$, then

$$\dot{\tilde{z}}_{1j} = \tilde{z}_{2j} + L_{1j}(p_j - z_{1j}), \quad 1 \le j \le m \tag{19}$$

If $\tilde{z}_{2j}$ is a virtual input, the system in (19) can be considered as a linear system with an additional term. We choose the virtual control for $\tilde{z}_{2j}$ as

$$\begin{aligned}\alpha_{1j} &= -\sum_{i\in\mathcal{N}_j^a} a_{ji}(\tilde{z}_{1j} - \tilde{z}_{1i}) - L_{1j}(p_j - z_{1j}) \\ &= -\sum_{i\in\mathcal{N}_j^a} a_{ji}(z_{1j} - h_j - z_{1i} + h_i) - L_{1j}(p_j - z_{1j}), \quad 1 \le j \le m\end{aligned} \tag{20}$$

where $a_{ji} > 0$. With the aid of the virtual control $\alpha_{1j}$, we have

$$\dot{\tilde{z}}_{1j} = -\sum_{i\in\mathcal{N}_j^a} a_{ji}(\tilde{z}_{1j} - \tilde{z}_{1i}) + \tilde{z}_{2j} - \alpha_{1j}. \tag{21}$$

For the systems in (21), the following results can be proved with the aid of Lemma 1 and its proof is omitted.





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**Lemma 3** *For the systems in (21), under Assumption 1, if $\tilde{z}_{2j} - \alpha_{1j}$ exponentially converges to zero for $1 \le j \le m$, then $\tilde{z}_{1j}$ exponentially converges to zero for $1 \le j \le m$.*

In this step, it is unknown whether the leader's information is available to a quadrotor or not. However, with the aid of neighbors' information, the positions of all quadrotors converge to the desired position of the leader quadrotor if $\tilde{z}_{1j} = \alpha_{1j}$.

**Step 3:** Let $\bar{z}_{2j} = \tilde{z}_{2j} - \alpha_{1j}$ for $1 \le j \le m$, we have

$$\begin{aligned}\dot{\bar{z}}_{2j} =& -ge_3 + \beta_j f_j R_j e_3 + \phi_{1j}\hat{\theta}_{1j} + L_{2j}(v_j - z_{2j}) \\ &+ (p_j - z_{1j}) + \delta_{1j}\chi(e^{\lambda t}(v_j - z_{2j})) - \dot{\alpha}_{1j} - \ddot{h}_j \\ &- \dot{z}_{20} + \ddot{h}_0 \end{aligned} \tag{22}$$

where $b_{ji} > 0$.

We assume that $f_j R_j e_3$ is a virtual control input and design it such that $\bar{z}_{2j}$ is bounded and converges to zero. In order to make the system (22) be the form of the systems in item 3 of Lemma 1, the virtual control for $f_j R_j e_3$ is chosen as

$$\begin{aligned}\alpha_{2j} =& ge_3 - \phi_{1j}\hat{\theta}_{1j} - L_{2j}(v_j - z_{2j}) - (p_j - z_{1j}) - \delta_j\chi(e^{\lambda t}(v_j - z_{2j})) + \dot{\alpha}_{1j} + \ddot{h}_j \\ &- \sum_{i\in\mathcal{N}_j^a} b_{ji}(\bar{z}_{2j} - \bar{z}_{2i}) + \xi_j, \quad \bar{z}_{20} = 0 \end{aligned} \tag{23}$$

$$\dot{\xi}_j = -\sum_{i\in\mathcal{N}_j^a} b_{ji}(\xi_j - \xi_i) - \rho_{1j}\chi\left(\sum_{i\in\mathcal{N}_j^a} b_{ji}(\xi_j - \xi_i)\right) \tag{24}$$

$$\dot{\rho}_{1j} = -\sum_{i\in\mathcal{N}_j^a} b_{ji}(\rho_{1j} - \rho_{1i}) \tag{25}$$

where $\xi_0 = \dot{z}_{20} - \ddot{h}_0$ and $\rho_{1,0} = \max_{t\in[0,\infty)} |\ddot{z}_{20}(t) - \dddot{h}_0|$.

For the systems in (24–25), by Lemma 1 (item 3 with $u_j = 0$) $\rho_{1j}$ exponentially converges to $\rho_{1,0}$ and $\xi_j$ exponentially converges to $\xi_0$ for $1 \le j \le m$.

With the aid of the virtual control $\alpha_{2j}$, we have

$$\dot{\bar{z}}_{2j} = -\sum_{i\in\mathcal{N}_j^a} b_{ji}(\bar{z}_{2j} - \bar{z}_{2i}) + \xi_j - \dot{z}_{20} + \ddot{h}_0 + \beta_j f_j R_j e_3 - \alpha_{2j} \tag{26}$$

With the aid of Lemma 1, the system with input $(\xi_1 - \dot{z}_{20} + \ddot{h}_0 + \beta_1 f_1 R_1 e_3 - \alpha_{21}, \ldots, \xi_m - \dot{z}_{20} + \beta_m f_m R_m e_3 - \alpha_{2m})$ and state $(\bar{z}_{21}, \ldots, \bar{z}_{2m})$ has ISS property. Since $\xi_j - \dot{z}_{20} + \ddot{h}_0$ exponentially converges to zero for $1 \le j \le m$, $\bar{z}_{2j}$ exponentially converges to zero if $\beta_j f_j R_j e_3 - \alpha_{1j}$ exponentially converges to zero for $1 \le j \le m$.

**Step 4:** We find $f_j$ and the desired orientation $R_j^d$ for $j$-th quadrotor. Let

$$f_j R_j^d e_3 = \alpha_{2j} \tag{27}$$

where $R_j^d = [b_{1j}^d, b_{2j}^d, b_{3j}^d]$, then

$$f_j = \|\alpha_{2j}\| \tag{28}$$

$$b_{3j}^d = \frac{\alpha_{2j}}{\|\alpha_{2j}\|}. \tag{29}$$

In (29), $b_{3j}^d$ is not defined if $\alpha_{2j} = 0$. In this case, we define $b_{3j}^d$ as follows

$$b_{3j}^d = \frac{\dot{\alpha}_{2j}}{\|\dot{\alpha}_{2j}\|}.$$

To define $b_{1j}^d$ and $b_{2j}^d$, the information $b_{2,0}$ is required. However, $b_{2,0}$ is not available for all quadrotors. We propose the following distributed observer for $j$-th quadrotor.

$$\dot{r}_j = -\sum_{i\in\mathcal{N}_j^e} a_{ji}(r_j - r_i) - \rho_{2j}\chi\left(\sum_{i\in\mathcal{N}_j^e} a_{ji}(r_j - r_i)\right) \tag{30}$$

$$\dot{\rho}_{2j} = -\sum_{i\in\mathcal{N}_j^e} a_{ji}(\rho_{2j} - \rho_{2i}) \tag{31}$$

where $r_0 = b_{2,0}$ and $\rho_{20} = \max_{t\in[0,\infty)} |\dot{b}_{2,0}(t)|$.

For the systems in (30–31), under Assumption 1, by Lemma 1 $\lim_{t\to\infty}(r_j - b_{2,0}) \overset{exp.}{=} 0$ and $\lim_{t\to\infty}(\rho_{2j} - \rho_{2,0}) \overset{exp.}{=} 0$ for $1 \le j \le m$.

We choose

$$\bar{r}_j = r_j - r_j^\top b_{3j}^d b_{3j}^d \tag{32}$$

$$b_{2j}^d = \frac{\bar{r}_j}{\|\bar{r}_j\|} \tag{33}$$

$$b_{1j}^d = b_{2j}^d \times b_{3j}^d. \tag{34}$$

The desired attitude of $R_j$ is chosen as

$$R_j^d = \left[b_{1j}^d, b_{2j}^d, b_{3j}^d\right] \tag{35}$$

and the desired quaternion $q_j^d = [\eta_j^d, (\epsilon_j^d)^\top]^\top$ of $q_j$ is calculated by the equations (166–168) in [29] which are omitted here. The desired angular velocity is calculated by

$$\omega_j^d = 2A(q_j^d)^\top \frac{dq_j^d}{dt}. \tag{36}$$

**Step 5:** Since $q_j$ is not a control input, $q_j$ cannot be $q_j^d$. Let the difference between $q_j$ and $q_j^d$ be

$$\tilde{q}_j = (q_j^d)^{-1} \otimes q_j = [\tilde{\eta}_j, \tilde{\epsilon}_j^\top]^\top, \tag{37}$$





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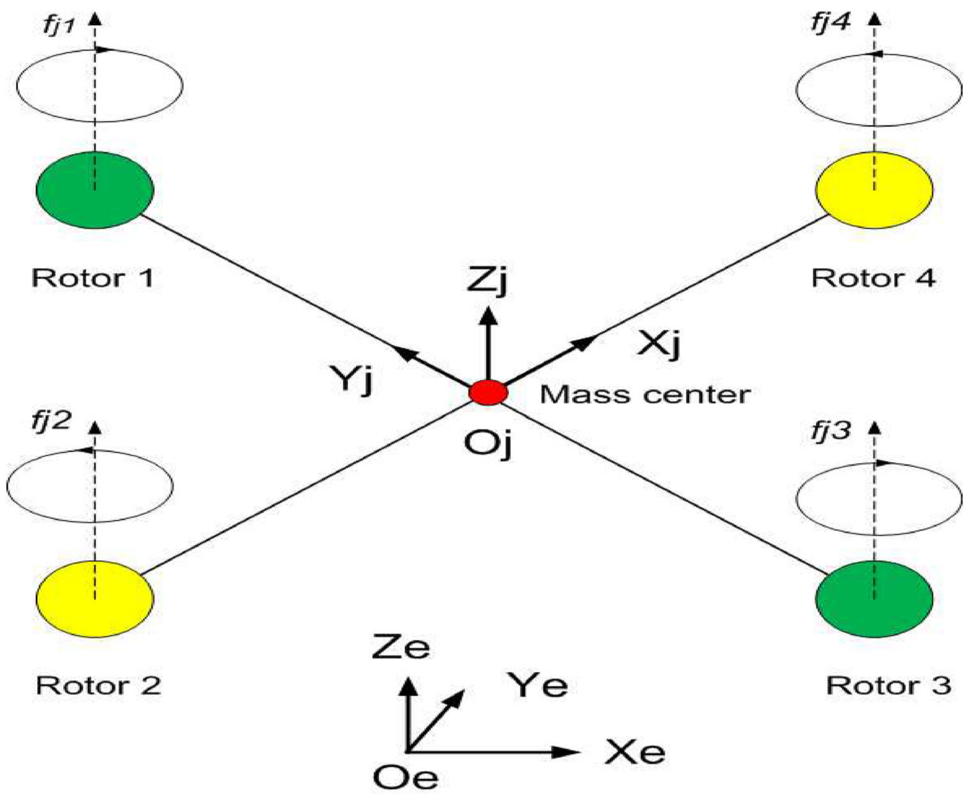


**Fig. 1** Configuration of a quadrotor

Noting that

$$
\begin{aligned}
f_j R_j e_3 - \alpha_{2j} &= f_j R_j e_3 - f_j R_j^d e_3 = f_j R_j^d (\tilde{R}_j - I) e_3 \\
&= f_j R_j^d (\tilde{\eta} I_3 + S(\tilde{\epsilon}_j)) S(e_3) \tilde{\epsilon}_j
\end{aligned}
$$

we have

$$
\begin{aligned}
\dot{\bar{z}}_{2j} = &- \sum_{i \in \mathcal{N}_j^a} b_{ji} (\bar{z}_{2j} - \bar{z}_{2i}) + \xi_j - \dot{z}_{20} + f_j R_j^d (\tilde{\eta} I_3 \\
&+ S(\tilde{\epsilon}_j)) S(e_3).
\end{aligned} \tag{38}
$$

For the systems in (38), the following results can be proved with the aid of Lemma 1 and the proof is omitted.

**Lemma 4** *For the systems in (38), under Assumption 1, if $\tilde{q}_j$ exponentially converges to an identity quaternion for $1 \leq j \leq m$, then $\bar{z}_{2j}$ exponentially converges to zero for $1 \leq j \leq m$.*

The derivative of $\tilde{q}_j$ is

$$
\dot{\tilde{q}}_j = \frac{1}{2} A(\tilde{q}_j)(\omega_j - \tilde{R}_j^\top \omega_j^d) \tag{39}
$$

where $\tilde{R}_j = (R_j^d)^\top R_j$.

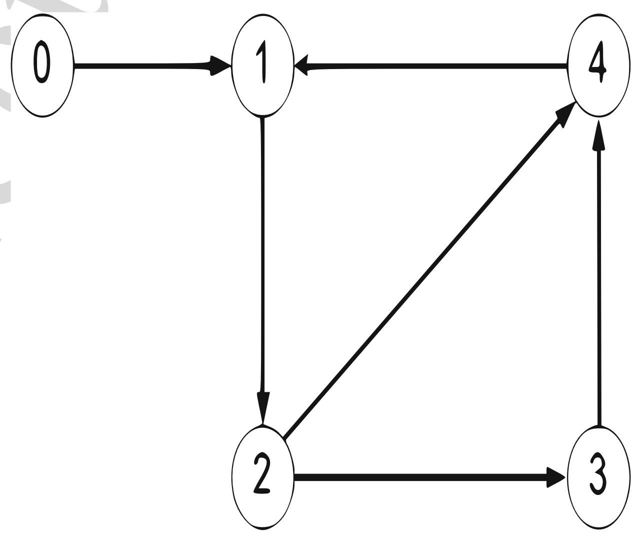


**Fig. 2** Communication graph between VTOL vehicles

We choose a Lyapunov function candidate

$$
\begin{aligned}
V_{2j} = 2(1 - \tilde{\eta}_j) = \tilde{\epsilon}_j^\top \tilde{\epsilon}_j \\
+ (1 - \tilde{\eta}_j)^2
\end{aligned} \tag{40}
$$

The derivative of $V_{2j}$ is

$$
\dot{V}_{2j} = \tilde{\epsilon}_j^\top (\omega_j - \tilde{R}_j^\top \omega_j^d)
$$





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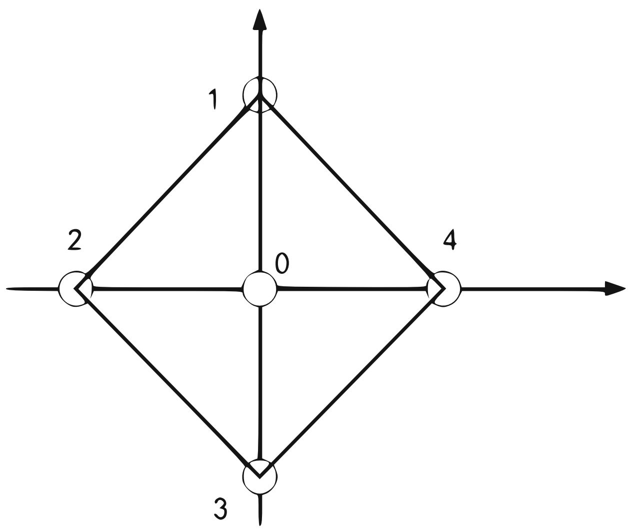


**Fig. 3** Desired formation

To make $\tilde{q}_j$ converge to an identity quaternion, a virtual controller $\mu_j$ for $\omega_j$ can be chosen as

$$\mu_j = -k_{1j}\tilde{\epsilon}_j + \tilde{R}_j^\top \omega_j^d \tag{41}$$

where $k_{1j}$ is a positive constant. Then,

$$\dot{V}_{2j} = -k_{1j}\tilde{\epsilon}_j^\top \tilde{\epsilon}_j + \tilde{\epsilon}_j^\top(\omega_j - \mu_j).$$

**Step 6:** Since $\omega_j$ is not a real control input, one cannot let $\omega_j$ be $\mu_j$. Define

$$\tilde{\omega}_j = \omega_j - \mu_j,$$

then,

$$\begin{aligned} \dot{\tilde{q}}_j &= \frac{1}{2}A(\tilde{\eta}_j, \tilde{\epsilon}_j)(-k_3\tilde{\epsilon}_j + \tilde{\omega}_j) && (42)\\ J_j\dot{\tilde{\omega}}_j &= S(J_j\omega_j)\omega_j + \tau_j - J_j\dot{\mu}_j + d_{2j}\\ &= \tau_j - (S(\omega_j)\Gamma(\omega_j) + \Gamma(\dot{\mu}_j))a_j + d_{2j} && (43) \end{aligned}$$

where $\Gamma(\omega_j) = \mathrm{diag}([\omega_j^\top, \omega_j^\top, \omega_j^\top])$ and $a_j = [J_j^1, J_j^2, J_j^3]^\top$ where $J_j^i$ is the $i$-th row of $J_j$.

A neural network is applied to approximate $d_{2j}$. Let $\phi_{2j}$ be a collection of basis vectors, there exists an optimal vector $\theta_{2j}$ such that

$$d_{2j} = \phi_{2j}\theta_{2j} + \epsilon_{2j} \tag{44}$$

where $\epsilon_{2j}$ is the approximation error vector and $\|\epsilon_{2j}\| \le \delta_{2j}$.

To design a control law such that (5–6) are satisfied, we choose a Lyapunov function candidate

$$\begin{aligned} V_{3j} &= V_{2j} + \frac{1}{2}e^{2\lambda t}\tilde{\omega}_j^\top J_j\tilde{\omega}_j + \frac{\gamma_{3j}^{-1}}{2}(a_j - \hat{a}_j)^\top(a_j - \hat{a}_j)\\ &\quad + \frac{\gamma_{4j}^{-1}}{2}(\theta_{2j} - \hat{\theta}_{2j})^\top(\theta_{2j} - \hat{\theta}_{2j}) \end{aligned}$$

where $\gamma_{3j}$ and $\gamma_{4j}$ are positive constants, $\hat{a}_j$ is an estimate of $a_j$, and $\hat{\theta}_{2j}$ is an estimate of $\theta_{2j}$. The derivative of $V_{3j}$ is

$$\begin{aligned} \dot{V}_{3j} &= -k_{1j}\tilde{\epsilon}_j^\top\tilde{\epsilon}_j + \tilde{\epsilon}_j^\top\tilde{\omega}_j + e^{2\lambda t}\tilde{\omega}_j^\top(\tau_j - (S(\omega_j)\Gamma(\omega_j)\\ &\quad + \Gamma(\dot{\mu}_j - \lambda\tilde{\omega}_j))a_j + \phi_{2j}\theta_{2j} + \epsilon_{2j})\\ &\quad -\gamma_{3j}^{-1}(a_j - \hat{a}_j)^\top\dot{\hat{a}}_j - \gamma_{4j}^{-1}(\theta_{2j} - \hat{\theta}_{2j})^\top\dot{\hat{\theta}}_{2j}. \end{aligned}$$

The control law $\tau_j$ and the update laws $\hat{a}_j$ and $\hat{\theta}_{2j}$ are chosen as follows:

$$\begin{aligned} \tau_j &= -k_{2j}\tilde{\omega}_j - e^{-2\lambda t}\tilde{\epsilon}_j + (S(\omega_j)\Gamma(\omega_j)\\ &\quad +\Gamma(\dot{\mu}_j) - \lambda\tilde{\omega}_j)\hat{a}_j - \phi_{2j}\hat{\theta}_{2j} - \delta_{2j}\chi(e^{\lambda t}\tilde{\omega}_j) && (45)\\ \dot{\hat{a}}_j &= -\gamma_{3j}e^{2\lambda t}(S(\omega_j)\Gamma(\omega_j) + \Gamma(\dot{\mu}_j) - \lambda\tilde{\omega}_j)^\top\tilde{\omega}_j && (46)\\ \dot{\hat{\theta}}_{2j} &= \gamma_{4j}e^{2\lambda t}\phi_{2j}^\top\tilde{\omega}_j && (47) \end{aligned}$$

where $k_{2j}$ is a positive constant. Then,

$$\begin{aligned} \dot{V}_{3j} &= -k_{1j}\tilde{\epsilon}_j^\top\tilde{\epsilon}_j - k_{2j}e^{2\lambda t}\tilde{\omega}_j^\top\tilde{\omega}_j - \delta_{2j}e^{2\lambda t}\tilde{\omega}_j^\top\chi(e^{\lambda t}\tilde{\omega}_j)\\ &\quad +e^{\lambda t}\tilde{\omega}_j^\top\epsilon_{2j}\\ &\le -k_{1j}\tilde{\epsilon}_j^\top\tilde{\epsilon}_j - k_{2j}e^{2\lambda t}\tilde{\omega}_j^\top\tilde{\omega}_j + 3\delta_{2j}e^{-(\kappa-\lambda)t}. && (48) \end{aligned}$$

Based on the above controller design procedure, we have the following results.

**Theorem 1** *For a leader quadrotor and m follower quadrotors in (1–4), it is given a desired formation defined by $h_j$ for $0 \le j \le m$. Under Assumptions 1–3, the distributed control inputs $(f_j, \tau_j)$ in (28) and (45) ensure that (5–6) are satisfied and $(\beta_j, \hat{a}_j, \hat{\theta}_{1j}, \hat{\theta}_{2j})$ are bounded.*

***Proof*** Integrating both sides of (48), it can be shown that $V_{3j} \in \mathcal{L}_\infty$. So, $\tilde{\epsilon}_j \in \mathcal{L}_\infty$, $e^{\lambda t}\tilde{\omega}_j \in \mathcal{L}_\infty$, $\hat{a}_j \in \mathcal{L}_\infty$, and $\hat{\theta}_j \in \mathcal{L}_\infty$. So, $\tilde{\omega}_j$ is bounded and exponentially converges to zero. Integrating both sides of (48), it can also be shown that $\tilde{\epsilon}_j \in \mathcal{L}_2$. By Lemma 1 in [30], $\tilde{\epsilon}_j$ converges to zero. So, $\tilde{q}_j$ converges to an identity quaternion for $1 \le j \le m$.

Next, we show $\tilde{q}_j$ exponentially converges to an identity quaternion for $1 \le j \le m$. With the aid of $V_{2j}$, we have

$$\begin{aligned} \dot{V}_{2j} &\le -\frac{k_{1j}}{2}\tilde{\epsilon}_j^\top\tilde{\epsilon}_j + \frac{k_{1j}}{2}\|\tilde{\omega}_j\|^2\\ &= -\frac{k_{1j}}{2}V_{2j} + \frac{k_{1j}}{8}V_{2j}^2 + \frac{k_{1j}}{2}\|\tilde{\omega}_j\|^2 \end{aligned}$$





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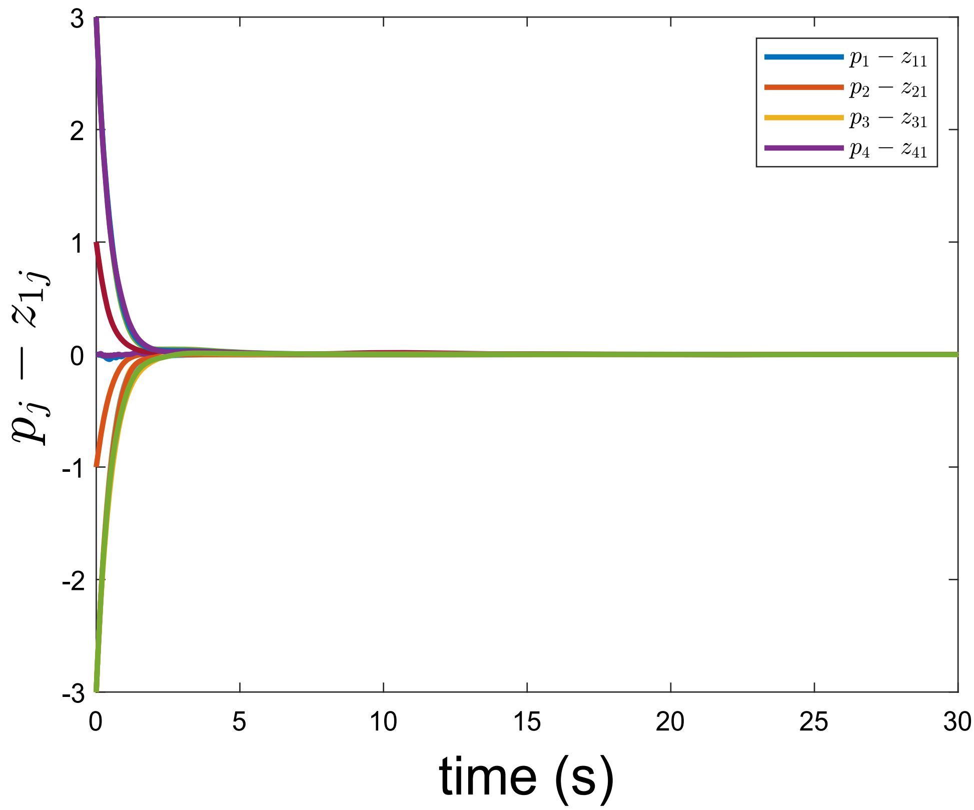


**Fig. 4** Time response of $p_j - z_{1j}$ for $1 \leq j \leq 4$

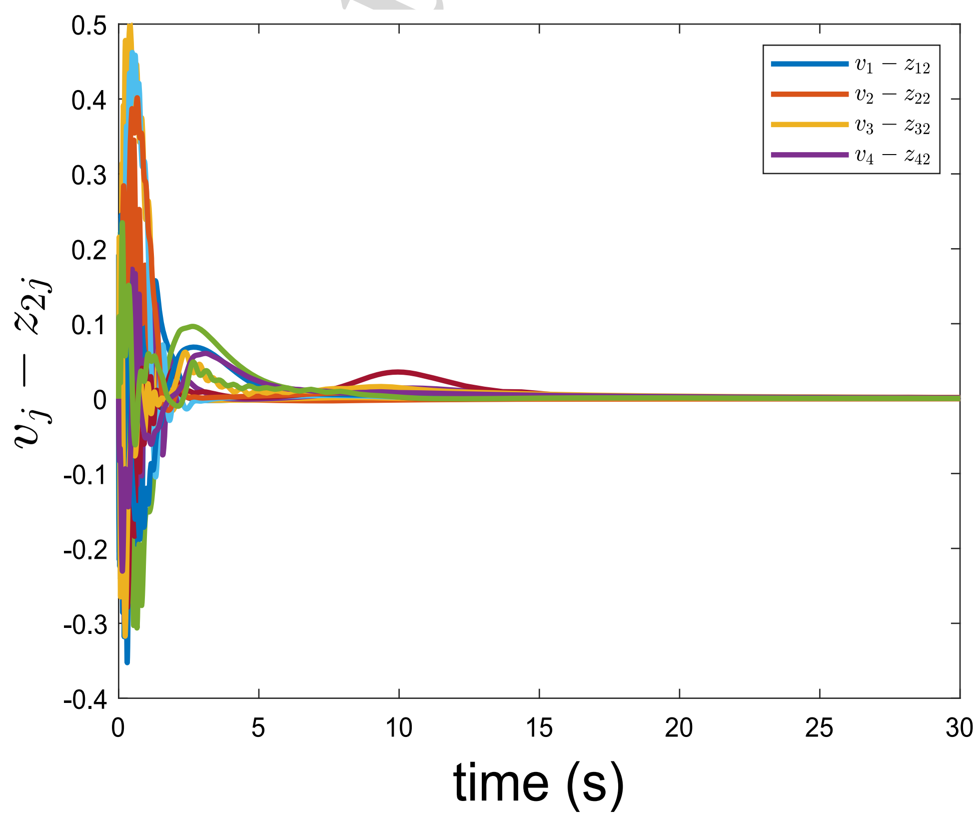


**Fig. 5** Time response of $v_j - z_{2j}$ for $1 \leq j \leq 4$



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**Fig. 6** Time response of $p_j - h_j - (p_0 - h_0)$ for $1 \leq j \leq 4$

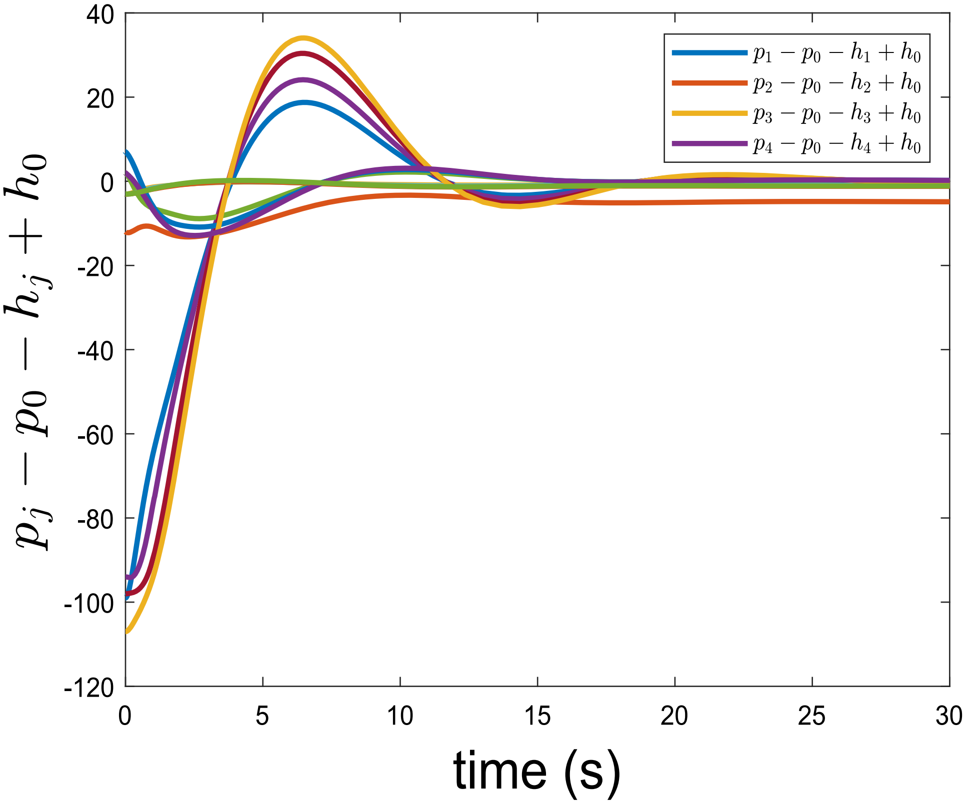


**Fig. 7** Time response of $b_{2j} - b_{20}$ for $1 \leq j \leq 4$

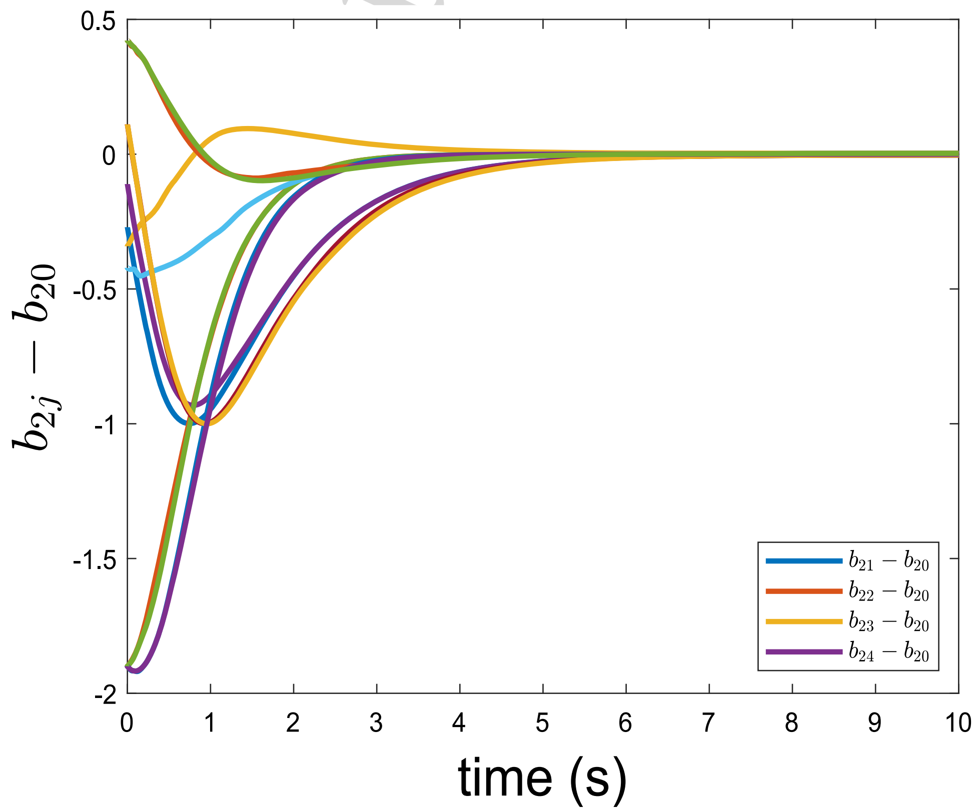




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**Fig. 8** Time response of $\beta_j$ for $1 \le j \le 4$

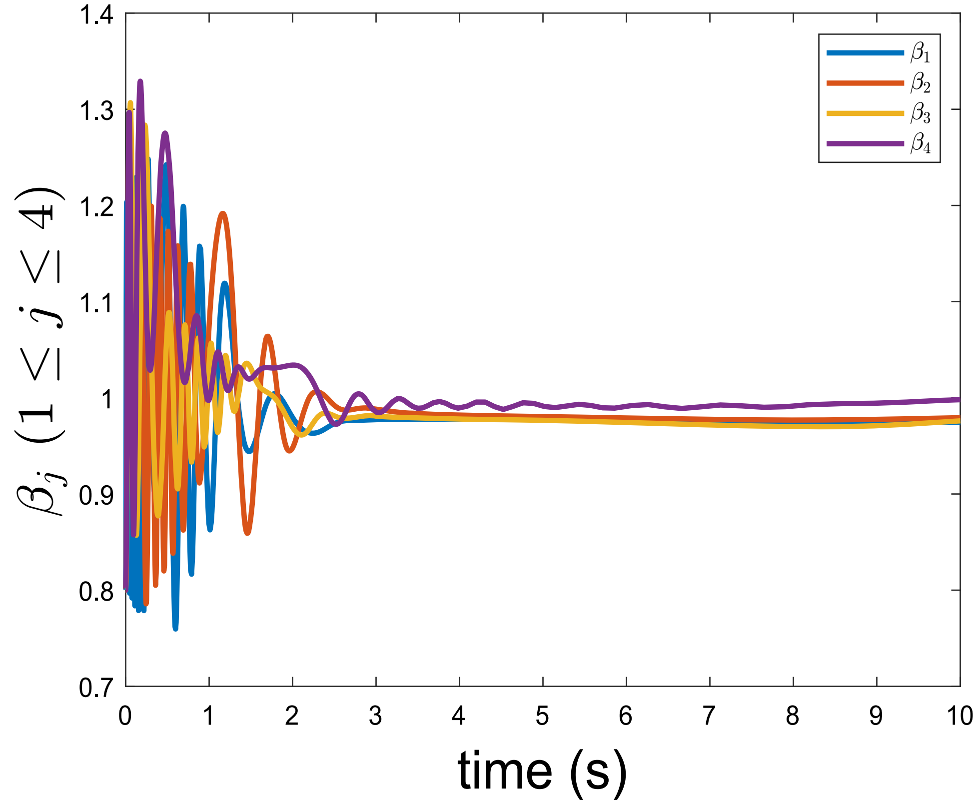
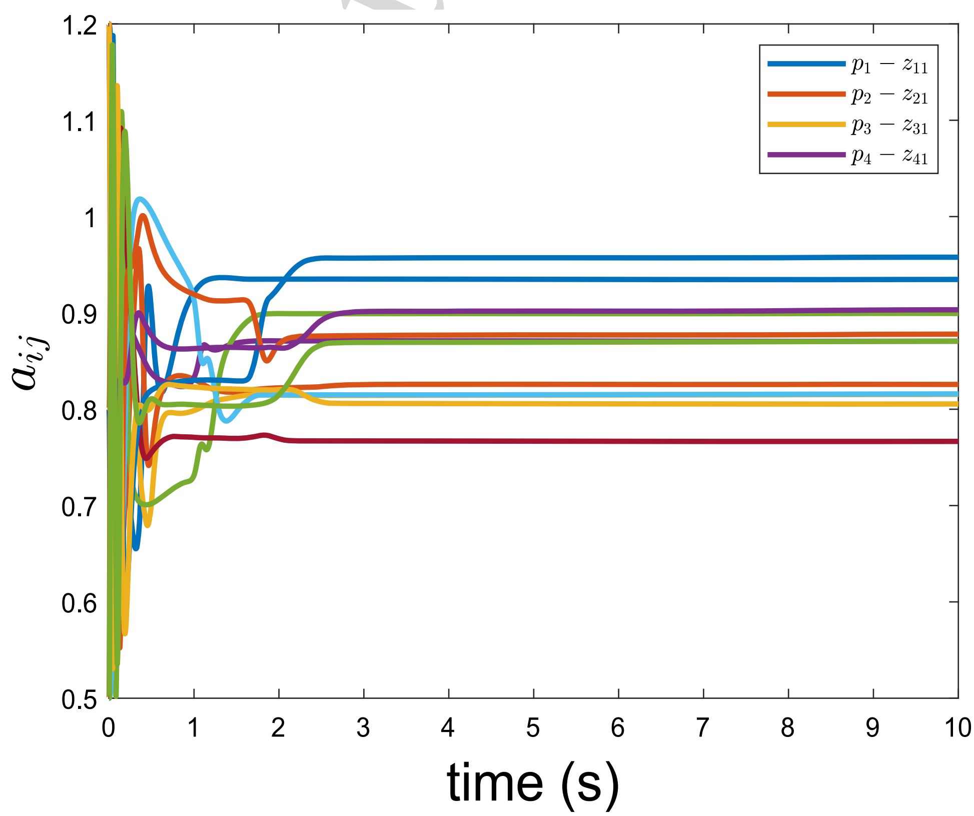



**Fig. 9** Time response of $a_{ij}$ for $1 \le j \le 4$ and $1 \le i \le 3$



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Since $\tilde{\eta}_j$ converges to one, there exists a finite time $T$ such that $V_{2j}$ is less than one. After time $T$,

$$\begin{aligned}\dot{V}_{2j} &\le -\frac{k_{1j}}{2}V_{2j} + \frac{k_{1j}}{8}V_{2j} + \frac{k_{1j}}{2}\|\tilde{\omega}_j\|^2 \\ &= -\frac{3k_{1j}}{8}V_{2j} + \frac{k_{1j}}{2}\|\tilde{\omega}_j\|^2\end{aligned}$$

Noting $\tilde{\omega}_j$ exponentially converges to zero, it can be shown that $V_{2j}$ exponentially converges to zero with the aid of the comparison lemma in [26]. So, $\tilde{q}_j$ exponentially converges to an identity quaternion for $1 \le j \le m$ after a finite time. With the aid of Lemma 4, $\bar{z}_{2j}$ exponentially converges to zero after a finite time for $1 \le j \le m$. By Lemma 3, $\tilde{z}_{1j}$ exponentially converges to zero after a finite time for $1 \le j \le m$. With the aid of the definition of $\tilde{z}_{1j}$, (18) holds. So, (5) is satisfied. (6) is satisfied because $\tilde{q}_j$ exponentially converges to an identity quaternion after a finite time. □

In the controller design procedure, the uncertainty in the dynamics of each quadrotor and the uncertain knowledge of the leader's information are dealt with in different steps. We call this design procedure the uncertainty decomposition approach.

In Theorem 1, in order to make the quadrotors come into the desired formation exponentially the control laws and the adaptive update laws contain the term $e^{\lambda t}$. If we choose a weight function carefully, it is possible to make the quadrotors come into the desired formation within a finite time. They are omitted due to space limitation. [1]

## 4 Simulation results

The proposed results can be applied to design distributed controllers for formation flying of multiple quadrotors. Considered five quadrotors. The configuration of each quadrotor is shown in Fig. 1. The dynamics of quadrotor $j$ can be written as (1–4), where the total thrust $f_j$ and the generalized moment vector $\tau_j$ are generated by the four rotors. For simplicity, we ignore the dynamics of each rotor and consider $f_j$ and $\tau_j$ as control inputs. In the simulation, it is assumed that $m_j = 1$kg and inertia tensor $J_j = \mathrm{diag}([1, 1, 1])$kg $m^2$. In the controller design, $m_j$ and $J_j$ are not exactly known. However, it is known that $m_j \in [0.8, 1.2]$kg, i.e., $\bar{m} = 1.2$kg and $\underline{m} = 0.8$kg.

In the simulation, it is assumed that the trajectory $p_0$ and $b_{2,0}$ of the leader VTOL vehicle are

$$\begin{aligned}p_0(t) &= \left[\,100\cos\tfrac{t}{20},\, 100\sin\tfrac{t}{20},\, 10 - 10\exp(-0.1t)\,\right]^\top \\ b_{2,0} &= \left[\sin\frac{\pi t}{360}, \cos\frac{\pi t}{360}, 0\right]^\top.\end{aligned}$$

The communication directed graph is shown in Fig. 2. It is can be verified that node 0 is globally reachable.

The desired formation for quadrotors is shown in Fig. 3, where $h_0 = [0, 0, 0]^\top$, $h_1 = [0, 5, 0]^\top$, $h_2 = [-5, 0, 0]^\top$, $h_3 = [0, -5, 0]^\top$, and $h_4 = [5, 0, 0]^\top$.

Distributed control laws can be designed with the aid of the procedure in the last section. The simulation was done for one group of control parameters. Figures 4, 5 show the time response of $p_j - z_{1j}$ and $v_j - z_{2j}$, respectively. It is shown that $z_{1j}$ and $z_{2j}$ are good estimates of $p_j$ and $v_j$, respectively. Figure 6 shows the time response of $p_j - h_j - (p_0 - h_0)$ for $1 \le j \le 4$. It is shown that the VTOL vehicles come into the desired formation and follow the leader quadrotor. The time response of $b_{2j} - b_{20}$ is shown in Fig. 7, which shows that the Y-axis of the body frame of each quadrotor converges to the desired direction. The estimate $\beta_j$ of $1/m_j$ for $1 \le qj \le 4$ are shown in Fig. 8. The time response of $\hat{a}_{ij}$ for $1 \le j \le 4$ and $1 \le i \le 3$ are shown in Fig. 9. They are bounded and confirm the claims in the theorem. The above simulation results show the effectiveness of the results in Theorem 1.

## 5 Conclusion

This paper considered the formation flying of multiple quadrotors with a desired attitude in the presence of parametric and nonparametric uncertainty. With the aid of the backstepping technique, a multi-step controller design approach has been proposed. With the aid of the proposed approach, distributed robust adaptive controllers were proposed such that the formation tracking errors and the attitude tracking errors exponentially converge to zero. Simulation results show the effectiveness of the proposed controllers for formation flying of five quadrotors. [2]

**Acknowledgements** The author(s) would like to acknowledge funding provided by the National Science Foundation CREST Center for Multidisciplinary Research Excellence in Cyber-Physical Infrastructure Systems (NSF Award No. 2112650) and the NSF grant no. ECCS-2037649. The opinions expressed in this paper (or thesis or report or dissertation) are solely those of the author(s), and do not necessarily represent those of the NSF.

**Author contributions** The first author did the simulation. The main results were obtained by the second author.

**Funding** This work was supported by two grants from the National Science Foundation of the United States of America.

## Declarations

**Competing interest** The authors declare no competing interests.





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SPI Journal: 40435 MS: 1066 ☐ TYPESET ☐ DISK ☐ LE ☐ CP Disp.:2022/11/23 Pages: 13 Layout: Large

Journal: 40435
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