PROGRESSIVE DAMAGE ANALYSIS OF STEEL-REINFORCED CONCRETE BEAMS USING HIGHER-ORDER 1D FINITE ELEMENTS

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Original Manuscript Submitted: 8/31/2022; Final Draft Received: 8/31/2022

The present work investigates progressive damage in steel-reinforced concrete structures. An elastic-perfectly plastic material response is considered for the reinforcing steel constituent, while the smeared-crack approach is applied to model the nonlinear behavior of concrete. The analysis employs one-dimensional numerical models based on higher-order finite elements derived using the Carrera unified formulation (CUF). A set of numerical assessments is presented to study the mechanical response of a steel-reinforced notched concrete beam loaded in tension. The predictions are found to be in very good agreement with reference experimental observations, thereby validating the numerical approach. It is shown that CUF allows for the explicit representation of the constituents within the composite beam, resulting in accurate solutions in a computationally efficient manner.

KEY WORDS: *Steel-reinforced concrete, damage analysis, CUF, higher-order models*

1. INTRODUCTION

Steel-reinforced concrete is one of the most common composite material systems in civil structures and commonly used to construct primary load-bearing components, such as beams, columns, and slabs. However, concrete is prone to cracking due to its brittle nature and low tensile strength. Sufficiently large cracks can expose the embedded reinforcing steel to the ambient environment, providing an avenue for corrosion (Mehta et al., 1982; Şahmaran and Yaman, 2008). Cracks can also reduce the concrete stiffness, which can significantly affect overall structural integrity (Dede and Ayvaz, 2009). The ability to predict crack formation in reinforced concrete structures and to evaluate their post-crack load-bearing capacity is an important aspect of the structural design process.

Experimental techniques to the design of composite concrete structures can involve significant resources and time. Computational techniques may therefore constitute an alternative methodology, with reduced resource overheads, to structural design and analysis. Computational methods require appropriate nonlinear material models to take into account the various failure modes that may occur in reinforced concrete structures subjected to varying load conditions. A popular class of such material models to investigate progressive damage in concrete structures is based on continuum damage mechanics (CDM) (Calayir and Karaton, 2005; de Borst, 2002; Feenstra and De Borst, 1996; Li and Li, 2001; Mai et al., 2012; Richard et al., 2010; Shahsavari et al., 2016; Underwood, 2016; Underwood et al., 2010). Also known, the smeared-crack approach, CDM models account for the presence and influence of cracks via a reduction of the material stiffness (Kachanov, 1958). CDM approaches provide an accurate representation of the softening response due to the presence of cracks, while avoiding their explicit representation via the introduction of discontinuities within the model, which can increase both the complexity and associated numerical costs (Earij et al., 2017; Hofstetter and Valentini, 2013; Park et al., 2022).

Micromechanical analysis is often used to accurately describe the nonlinear behavior of hierarchical materials, such as concrete, in which the influence of the material microstructure is taken into account (Gal et al., 2008; Sanahuja and Dormieux, 2010; Tal and Fish, 2018; Wu et al., 2010a) and can subsequently be used in homogenization schemes to model the effective behavior of bulk concrete (Contrafatto et al., 2016; Denisiewicz and Kuczma, 2014; Wu et al.,

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2010b). High-fidelity modeling and simulation is also performed using multiscale procedures so as to include the influence of multiple length scales on the global structural response (Moyeda and Fish, 2018a,b, 2019; Sciegaj et al., 2018; Sun et al., 2015), and constitute valuable tools to investigate the capabilities of advanced materials, such as ultrahigh performance fiber-reinforced concrete (UHPFRC) (Huang et al., 2019; Wang et al., 2020).

A drawback of high-fidelity computational analysis is the requirement for refined, and often 3D, numerical models in order to obtain sufficiently accurate nonlinear solutions (Cotsovos et al., 2009; Earij et al., 2017), especially when various constituents, such as the bulk concrete and reinforcing steel, are explicitly defined within the model. Homogenization techniques are a popular approach to determine the effective behavior of such multi-material systems and often employed to address the issue of analysis cost. This approach has been adopted by several researchers to model reinforced concrete structures by homogenizing the concrete and reinforcing steel into a single effective material. For instance, a homogenization scheme was proposed for reinforced concrete structures subjected to cyclic and seismic loads (Combescure et al., 2013, 2015). Recently, an inclusion-based homogenization scheme was proposed to predict the progressive damage response of steel-reinforced concrete beams (Drougkas et al., 2022).

The present work proposes a numerical method based on high-fidelity one-dimensional (1D) structural models to investigate progressive damage in reinforced concrete structures. The numerical model is based on the Carrera unified formulation (CUF), a generalized framework to derive higher-order structural theories (Carrera et al., 2014). Models based on 1D-CUF theories have been demonstrated to approach a solution accuracy equivalent to that obtained by three-dimensional 3D finite element analysis (FEA) at a fraction of the associated computational cost (De Miguel et al., 2018; Petrolo et al., 2018). CUF models have been successfully used to model fiber-reinforced composites at the constituent level and its application to failure analysis (Carrera et al., 2012, 2013a,b; Maiaru et al., 2017), investigate progressive damage and impact in fiber-reinforced composites (Nagaraj et al., 2020a,b, 2021), and more recently, used to investigate concrete damage using continuum-based approaches (Shen et al., 2022).

This paper is summarized as follows: Section 2 provides an overview of higher-order 1D models based on CUF; nonlinear material models used in the present work are described in Section 3, and numerical assessments for validation are presented in Section 4. The main conclusions are highlighted in Section 5.

2. HIGHER-ORDER 1D MODELING

The present work makes use of higher order 1D models derived using the CUF and implemented using the finite element method (FEM) (Carrera et al., 2014). CUF structural theories enrich the kinematic capabilities of 1D finite elements via the use of interpolation functions, termed expansion functions F_τ, applied over the beam's cross-sectional domain. This approach is schematically shown in Fig. 1 and results in structural models whose accuracy is equivalent to that of 3D FE models but with reduced computational overheads (De Miguel et al., 2018). Within a 1D-CUF model, the displacement field $\mathbf{u}$ is defined as follows:

$$\mathbf{u} = F_\tau(x, z)\mathbf{u}_\tau(y), \quad \tau = 1, 2, \ldots, M \tag{1}$$

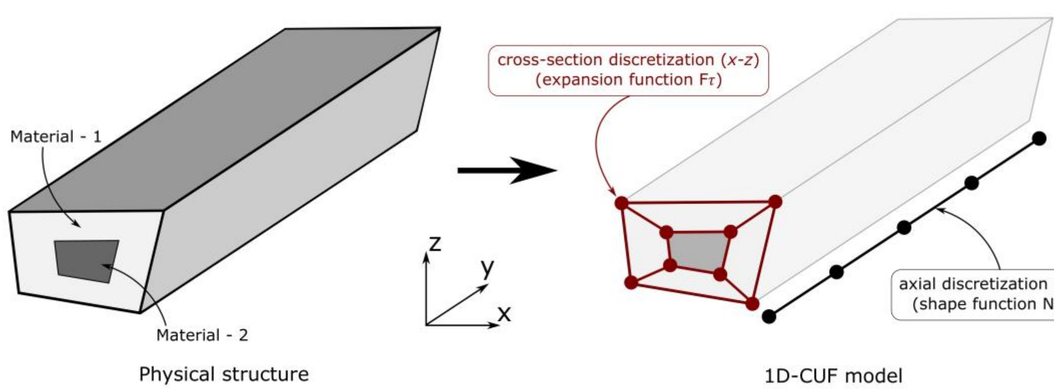

FIG. 1: Modeling of a multimaterial prismatic structure using 1D-CUF

where $\mathbf{u}_\tau$ are the generalized displacements and M is the number of terms within the expansion function. The present work employs Lagrange polynomials as the choice of F_τ, leading to a structural formulation containing only displacement degrees of freedom (DoF) (Carrera and Petrolo, 2012). The use of Lagrange polynomial-based expansion functions also results in the component wise modeling approach in which each component within a structure can be explicitly defined (Carrera et al., 2012). This is seen in Fig. 1, where multiple cross-sectional expansion elements are used to explicitly model the two structural constituents composed of Material-1 and Material-2, respectively. This modeling capability within CUF is used to perform a direct numerical simulation of steel-reinforced concrete beams in the current work.

2.1 Finite Element Formulation

The stress and strain fields can be defined in vector notation as follows:

$$\begin{aligned} \boldsymbol{\sigma} &= \{\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{xz}, \sigma_{yz}\} \\ \boldsymbol{\varepsilon} &= \{\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \varepsilon_{xy}, \varepsilon_{xz}, \varepsilon_{yz}\} \end{aligned} \tag{2}$$

Considering geometrical linearity, the displacement-strain relationship is

$$\boldsymbol{\varepsilon} = \mathbf{D}\mathbf{u} \tag{3}$$

where the differentiation operator $\mathbf{D}$ is

$$\mathbf{D} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \end{bmatrix}$$

The constitutive relation is given by

$$\boldsymbol{\sigma} = \mathbf{C}\boldsymbol{\varepsilon} \tag{4}$$

where $\mathbf{C}$ is the 6×6 material stiffness matrix, which can, in general, be nonlinear. Considering a combination of 1D finite elements with shape functions $N_i(y)$ along the beam axis and expansion functions $F_\tau(x, z)$ within the cross section (see Fig. 1), the 3D displacement field is defined as follows:

$$\mathbf{u}(x, y, z) = F_\tau(x, z)N_i(y)\mathbf{u}_{\tau i} \tag{5}$$

From the principle of virtual work

$$\delta L_{\text{int}} = \delta L_{\text{ext}} \tag{6}$$

where δL_{int} is the virtual variation of the internal strain energy and is defined as follows:

$$\delta L_{\text{int}} = \int_V \delta\boldsymbol{\varepsilon}^T : \boldsymbol{\sigma} \tag{7}$$

Combining Eqs. (4), (5), and (7), Eq. (6) is reformulated as follows:

$$\delta L_{\text{int}} = \delta\mathbf{u}_{sj}^T \mathbf{k}_{ij\tau s}\mathbf{u}_{\tau i} \tag{8}$$

where

$$\mathbf{k}_{i,j,\tau,s} = \int_l \int_A \mathbf{D}^T \left(N_i(y)F_\tau(x, z)\right)\mathbf{C}\mathbf{D}\left(N_j(y)F_s(x, z)\right) \, dA \, dl \tag{9}$$

The resulting stiffness term, $\mathbf{k}_{i,j,\tau,s}$, is a 3 × 3 matrix and termed the fundamental nucleus (FN). Its formulation, i.e., Eq. (9), remains invariant to any given combination of finite element shape functions N_i and expansion functions F_τ. Elemental stiffness matrices can be obtained by assembling the computed FN for each combination of the nodal indices $\{i, j, \tau, s\}$.

3. CONSTITUTIVE MODELING

3.1 Concrete Damage

The continuum damage mechanics approach is applied to evaluate the nonlinear response of concrete under tensile loading conditions, and the constitutive law is based on the works of Drougkas et al. (2022). After the onset of damage, the loss of stiffness, which occurs due to a reduction of material integrity, is quantified by an integrity factor, I. This factor is assigned a value of 1.0 for a pristine material and attains a value of zero for a fully damaged material. The integrity parameter for the case of tensile damage, I_t, is evaluated by considering an exponential post-peak softening response as follows:

$$I_t(\varepsilon) = \begin{cases} 1, & 0 \le \varepsilon \le \varepsilon_t^p \\ \dfrac{f_t}{\sigma_{\text{eff}}} \exp\left(-\dfrac{\varepsilon - \varepsilon_t^p}{\varepsilon_t^u}\right), & \varepsilon_t^p \le \varepsilon \end{cases} \tag{10}$$

where σ_{eff} is the effective stress and f_t is the tensile strength. The peak strain ε_t^p is evaluated as follows:

$$\varepsilon_t^p = \frac{f_t}{E_c} \tag{11}$$

where E_c is the Young's modulus of concrete. The ultimate strain ε_t^u is defined as follows:

$$\varepsilon_t^u = \frac{G_t}{f_t l_c} \tag{12}$$

where G_t is the tensile fracture energy and is regularized by scaling it with a characteristic length parameter l_c (Bažant and Oh, 1983). In the current work, the value of this parameter is computed as $l_c = V_{\text{GP}}^{1/3}$, where V_{GP} is the Gauss point volume.

3.2 Steel Yielding

The steel reinforcement is considered to exhibit an elastic and perfectly plastic stress-strain behavior. In this case, the integrity parameter under yield, I_y, is defined as follows (Drougkas et al., 2022):

$$I_y(\varepsilon) = \begin{cases} 1, & 0 \le \varepsilon \le \varepsilon_y \\ \dfrac{f_y}{\sigma_{\text{eff}}}, & \varepsilon_y \le \varepsilon \end{cases} \tag{13}$$

where f_y is the yield strength. The corresponding yield strain ε_y is evaluated as follows:

$$\varepsilon_y = \frac{f_y}{E_s} \tag{14}$$

where E_s is the Young's modulus of the steel reinforcement.

4. NUMERICAL ASSESSMENTS

Numerical assessments are performed considering a steel-reinforced notched concrete beam, as shown in Fig. 2. Three equally spaced steel rods provide longitudinal reinforcement, and the beam is loaded in tension. A notch of width 12.7 mm and depth 10 mm is present at the top and bottom of the beam at its midspan, as shown in Fig. 2, and act as stress raisers to initiate damage. Two types of concrete, i.e., normal-strength concrete (NSC) and high-strength concrete (HSC) are used in the analysis. The elastic and strength properties of the constituent materials are listed in Table 1. The lengths of the NSC and HSC beams are respectively 635 and 686 mm. The considered structural configuration is based on the works of Ouyang et al. (1997), which also provides experimental data for validation. Reference finite element results based on a homogenization scheme are available in Drougkas et al. (2022).

The reinforced concrete beam is modeled using the component-wise capability of the 1D-CUF, allowing for the explicit definition of the constituent materials. Perfect bonding is assumed between the concrete and steel components. The cross-section mesh is composed of 96 quadratic (second-order) Lagrangian expansion elements. Mesh refinement is performed by varying the number of beam elements along the longitudinal axis, leading to two sets of CUF models with five and seven quadratic beam elements, respectively. The axial and cross-sectional discretizations are schematically shown in Fig. 3. The nonlinear solution was obtained using the incremental iterative Newton-Raphson method.

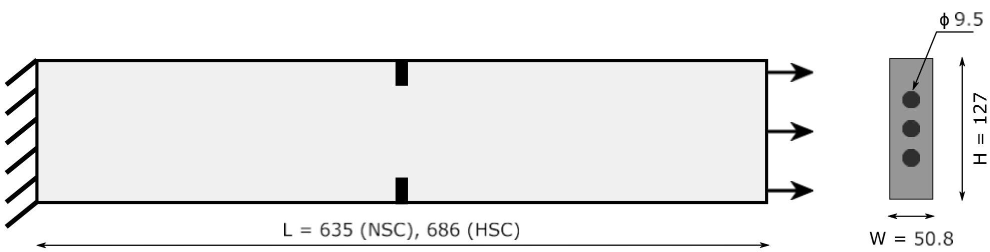

FIG. 2: Schematic representation of the steel-reinforced notched concrete beam (all dimensions in mm)

TABLE 1: Elastic and strength properties of the notched concrete beam constituent materials based on the results of Drougkas et al. (2022) and Ouyang et al. (1997)

Property	Normal-strength concrete	High-strength concrete	Steel
Young's modulus E (MPa)	27,349	36,624	191,584
Poisson's ratio v	0.175	0.175	0.280
Tensile strength f_t (MPa)	3.19	5.52	—
Yield strength f_y (MPa)	—	—	508.0

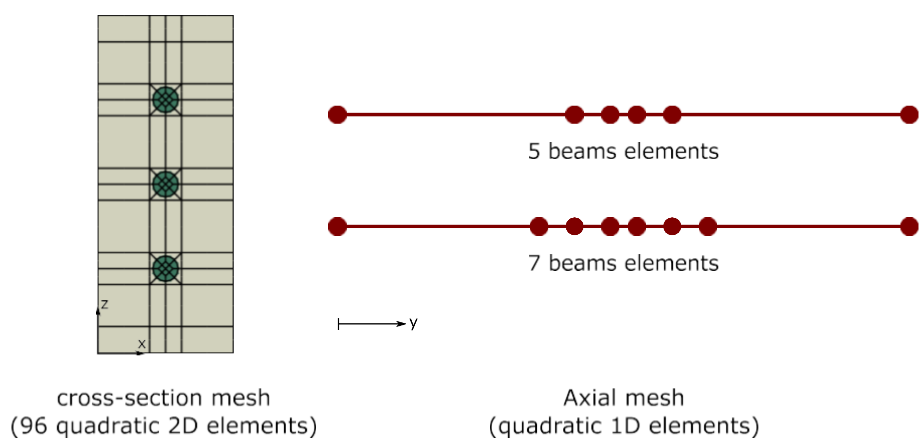

FIG. 3: Cross-sectional and axial discretizations used to develop the 1D-CUF models

The force-displacement response for the normal- and high-strength concrete beams are plotted in Figs. 4 and 5, respectively. Reference experimental data and numerical predictions have been overlaid for comparison. A summary of the CUF-based numerical models used in the analysis is presented in Table 2.

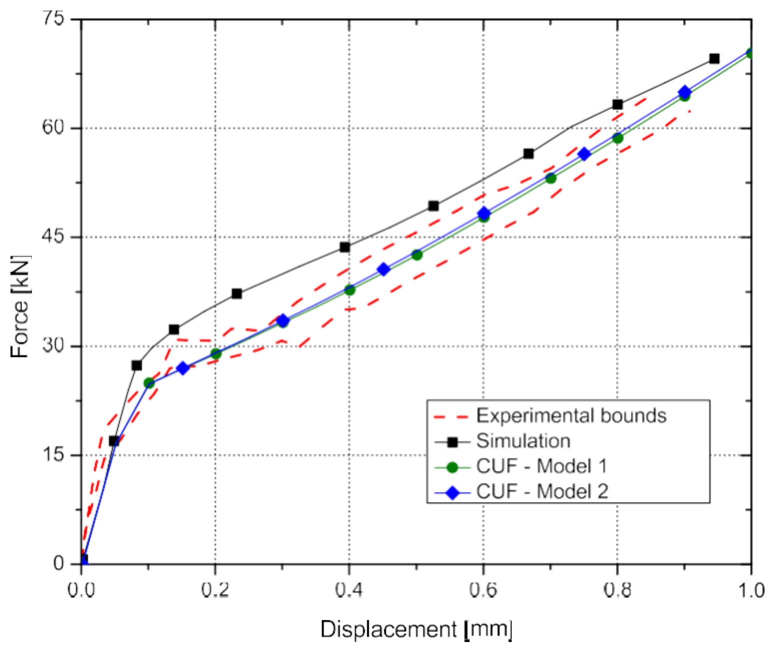

FIG. 4: Force-displacement response of the NSC notched beam in tension. Experimental data from Ouyang et al. (1997) and simulation results from Drougkas et al. (2022).

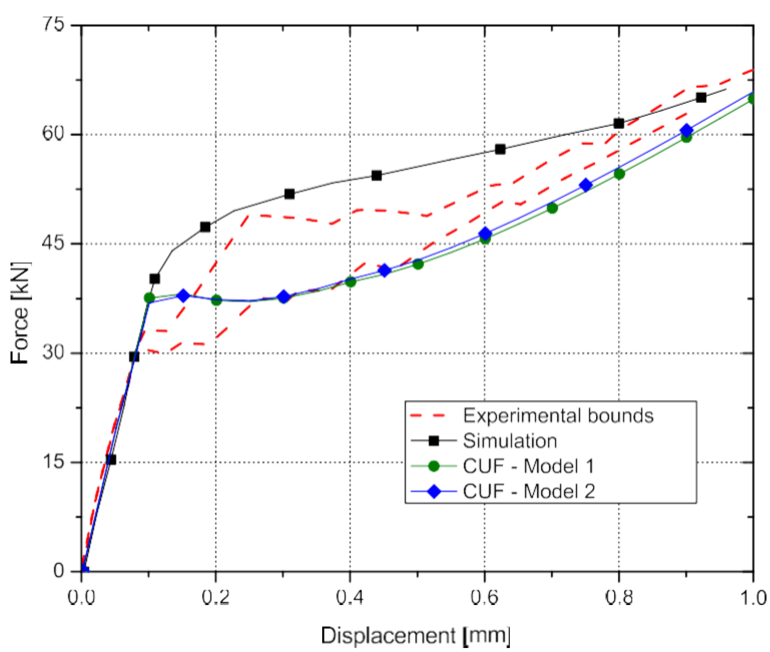

FIG. 5: Force-displacement response of the HSC notched beam in tension. Experimental data from Ouyang et al. (1997) and simulation results from Drougkas et al. (2022).

TABLE 2: Summary of the CUF models used to analyze the and NSC and HSC notched beams in tension

CUF Model	Discretization	Degrees of freedom
1	96 quadratic cross-sectional elements + 5 quadratic beam elements	13,653
2	96 quadratic cross-sectional elements + 7 quadratic beam elements	18,657

It is seen in Fig. 4 that the force-displacement response predicted by the 1D-CUF model is located within the experimental bounds of the normal-strength concrete beam in tension. This provides validation of the proposed numerical approach and a demonstration of its accuracy. The CUF models are able to better predict, compared to the FE solution, the onset of damage and the structure's load-bearing capacity in the nonlinear regime. The differences between the CUF and FEA predictions are more pronounced for the case of the high-strength concrete beam, as seen in Fig. 5. In this case, the CUF model predictions are in good general agreement with the lower experimental bounds. This is in contrast to the reference FEA which overestimates the cracking load and nonlinear response. The differences are attributed to the fact that the CUF component-wise approach explicitly models both the steel and concrete components of the reinforced beam; whereas, the reference FE approach is based on a homogenization scheme (Drougkas et al., 2022). The high-fidelity modeling capability of the proposed numerical method results in improved accuracy of the predicted nonlinear response, which is in line with experimental observations. The 3D solution is obtained using a relatively low-density discretization and hence computational size (DoF), as seen in Table 2.

5. CONCLUSION

Progressive damage in reinforced concrete structures is investigated using higher order 1D models. The numerical models are based on the Carrera unified formulation, and the component-wise capability is used to explicitly model the steel and concrete components within the structure. A smeared-crack approach is used to evaluate progressive damage in concrete, while the steel reinforcement is modeled as an elastic-perfectly plastic material. Numerical assessments have been performed for the benchmark problem of a steel-reinforced notched concrete beam loaded in tension. The nonlinear structural response predicted by the 1D-CUF model is shown to be in very good agreement with reference experiments, thus validating the proposed numerical framework for tensile damage. It is shown that this numerical approach is capable of accurately evaluating the nonlinear response of reinforced concrete structures via direct numerical simulation using a relatively low-density discretization, while avoiding the need for homogenization schemes, thereby demonstrating its computational performance.

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