# ANALYTICAL MODEL FOR COMPOSITE TRANSVERSE STRENGTH BASED ON COMPUTATIONAL MICROMECHANICS

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*The transverse strength of fiber-reinforced composites is a matrix-dominated property whose accurate prediction is crucial to designing and optimizing efficient, lightweight structures. State-of-the-art analytical models for composite strength predictions do not account for fiber distribution, orientation, and curing-induced residual stress that greatly influence damage initiation and failure propagation at the microscale. This work presents a novel methodology to develop an analytical solution for transverse composite strength based on computational micromechanics that enables the modeling of stress concentration due to representative volume elements (RVE) morphology and residual stress. Finite element simulations are used to model statistical samples of composite microstructures, generate stress-strain curves, and correlate statistical descriptors of the microscale to stress concentration factors to predict transverse strength as a function of fiber volume fraction. Tensile tests of thin plies validated this approach for carbon- and glass-reinforced composites showing promise to obtain a generalized analytical model for transverse composite strength prediction.*

**KEY WORDS:** *transverse tensile strength, analytical modeling, computational micromechanics, damage mechanics*

## 1. INTRODUCTION

Designing advanced composite parts for engineering and structural applications is extremely time consuming, because it often involves expensive and lengthy experimental campaigns or complex and time-intensive computational modeling (Fish, 2014; Fish et al., 2021; Naya et al., 2017). It is paramount to expedite the composite design and certification process to meet their ever-increasing demand. Analytical models can be used as efficient predictive tools to estimate composite response from constituent material properties, especially in problems involving many design variables for optimization (Vignoli et al., 2019, 2020), if their relative error with respect to experimental measurements is contained. State-of-the-art models employ either physics-based formulations or empirical relations to correlate the overall composite response to their constituent structures and properties. Analytical models provide significant advantage of simple implementation and computation over experiments and high-fidelity computational models for structural design and optimization (Andrianov et al., 2018).

The literature presents numerous analytical models that can estimate the elastic property of polymer matrix composites (PMCs) from their constituent properties. For instance, classical theories such as rule-of-mixture, modified rule-of-mixture (MROM) and enhanced models such as the continuous periodic fiber model (CPFM), the concentric cylindrical assembly, Mori-Tanaka, and the Bridging model (BM), have been widely employed to predict composite elastic response (Andrianov et al., 2018; Christensen, 2012; Fedotov, 2022; Hashin, 1979; Hashin and Rosen, 1964; Huang, 2019; Huang and Zhou, 2012; Hyer and White, 2009; Mal and Chatterjee, 1977; Nemat-Nasser and Hori, 2013; Reifsnider et al., 1986; Vignoli et al., 2019). However, there are only a few micromechanical strength theories that can accurately predict the composite strength from its constituent properties (Chamis, 1987, 1984; Huang, 2019; Huang and Xin, 2017; Huang and Zhou, 2012; Vignoli et al., 2020). Huang (2019) developed the BM to estimate

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the stress field inside a single-fiber repeating unit cell (RUC) with square packing. They introduced a stress concentration factor (SCF) within the RUC to compute the *in situ* matrix strength that was then used in conjunction with explicit micromechanical formulae to evaluate the transverse composite strength. Their model presented an average error of 40% when compared to transverse strengths of nine unidirectional composite laminates. Vignoli et al. (2020) developed an elasticity-based model for transverse strength prediction. They established analytical solutions to estimate the stress distribution around a single fiber in an infinite medium. Combining them with the finite element (FE) approach, adjustments functions were defined to determine the stress distribution in a more realistic square-packed RUC of several fiber volume fractions. Finally, the transverse strengths were predicted based on the matrix cavitation model. Their predictions presented an average error between 25 and 40% when compared to experimental data as well as other simplified analytical models.

These models, among others, were developed based on a simplified mathematical representation of a composite microstructure, or a single-fiber square RUC (Chamis et al., 2013; Huang, 2018; Huang and Zhou, 2012; Vignoli et al., 2019, 2020). Thus, they failed to account for inter-fiber interactions and the resulting stress concentration distribution that is observed in a multi-fiber representative volume element (RVE). Furthermore, the models did not consider the effect of process-induced *in situ* matrix property variation and residual stress generation on the transverse composite strength. It is well known that random fiber packing and residual stress generation not only influence the failure initiation in composite microstructures, but also drive failure propagation under continued loading (Andrianov et al., 2018; Beicha et al., 2016; Bouaoune et al., 2016; Elnekhaily and Talreja, 2018; Ghayoor et al., 2018; Liu and Huang, 2014; Romanov et al., 2013; Trias et al., 2006; Zhang and Yan, 2017). Existing analytical models ignore these effects and therefore present a lower correlation with transverse composite strength tests. Computational micromechanics-based simulations can fill these knowledge gaps and address the fundamental challenges associated with analytical models in order to enhance their prediction accuracy.

Over the past few decades, advances in high-performance computing, finite element methods and novel physics-based constitutive modeling have facilitated the design and development of PMCs through computational methods (Fish et al., 2021; Liu et al., 2018; Liu and Yu, 2018; LLorca et al., 2011). Computational micromechanics is an emerging field that leverages these advances to model and analyze RVEs of composite microstructures by subjecting them to various thermomechanical boundary conditions (Danzi et al., 2019; Deshpande et al., 2020; D'Mello et al., 2015, 2016, 2020; Gaikwad et al., 2021; Ghosh and Dimiduk, 2011; He et al., 2019; Hui et al., 2021a,b; Liu et al., 2013; LLorca et al., 2011; Maiaru, 2018; Maiaru et al., 2018; Mesogitis et al., 2014; Paley and Aboudi, 1992; Patil et al., 2020, 2021; Shah and Maiaru, 2018, 2021; Shah et al., 2020a, 2021; Yang et al., 2013a, 2020; Zhao et al., 2006). Such analysis can provide valuable insights into the fundamental mechanisms that influence the composite response at the constituent level, thereby overcoming the limitations associated with experimental quantification (LLorca et al., 2011).

By explicitly modeling the constituent fibers and matrix, phenomena such as stress concentration due to random fiber packing and fiber-to-fiber proximity, that induce matrix cracking under transverse tensile loading conditions and significantly influence the transverse composite strength, can be studied with relative ease and efficiency. FE-based process modeling simulations, which are informed by accurate and comprehensive material characterization and employ phenomenological and constitutive relations, can facilitate the quantification of residual stress buildup within composite RVEs during manufacturing. Virtually curing and loading RVEs in transverse tension can provide insight on the impact of process-induced residual stresses and random fiber distribution on transverse composite strength. The knowledge gained from computational micromechanical analysis of composite RVEs can be leveraged to establish a processing-microstructure-property relationship and address the aforementioned fundamental challenges to develop an accurate computational micromechanics-based analytical model for transverse strength.

The objective of this work is to present a novel methodology to develop a computational micromechanics-based analytical model for transverse tensile strength prediction. In this study, FE-based, computational micromechanical models are generated to analyze a wide array of composite microstructures for their transverse composite strengths. It is hypothesized that random fiber packing and the resulting stress concentration induce failure within the microstructures, which affects their transverse strength. Based on this hypothesis, a generalized expression for transverse strength is discussed. Critical parameters that influence the transverse composite strength are highlighted. The concept of a stress concentration factor induced by a single-fiber RUC is presented. The influence of relative fiber arrangement (distance between fiber centers and their orientation) on the SCF is then quantified through FE simulations of the two-fiber model. Leveraging the insights from the two-fiber SCF study, a statistical approach to quantify the SCF in multi-fiber microstructures is presented. Finally, a material system-specific analytical model for transverse composite strength is developed. It is shown that insights from computational models can inform the development of mathematical relations to correlate the microscale physical mechanisms, such as stress concentration to composite failure and predict transverse strength of multi-fiber composite microstructures.

This paper is organized as follows: the computational micromechanical modeling approach is detailed in Section 2, development of the analytical model is described in Section 3, and concluding remarks are discussed in Section 4.

## 2. COMPUTATIONAL MICROMECHANICS

Computational micromechanics requires efficient yet sufficiently large models to capture the physical mechanisms that induce failure. In this study, converged RVEs including 50 fibers were analyzed based on the results of a statistical size-effect study performed previously by Shah and Maiaru (2021). The modeling details and virtual manufacturing and testing procedures are discussed in Sections 2.1–2.3.

### 2.1 Modeling of Composite Microstructures

In this study, two sets of RVEs comprising a random dispersion of IM7 carbon (fiber diameter, $d_{\mathrm{f}} = 6$ μm) and E-glass fibers ($d_{\mathrm{f}} = 14$ μm) in an epoxy matrix, respectively, were generated using a random RVE generator developed by Stapleton et al. (2016). A commercial epoxy system, EPIKOTE™ Resin MGS RIMR 135 with EPIKURE™ Curing Agent MGS RIMH 1366 (henceforth referred to as RIM R135-H1366), was chosen as the matrix material due to its comprehensive thermomechanical material property data set reported in Shah et al. (2023). Several composite microstructures corresponding to various fiber volume fractions ($0.25 \leq v_{\mathrm{f}} \leq 0.75$) were considered, as illustrated in Fig. 1 and summarized in Table 1. To improve the prediction accuracy and account for statistical variations due to random fiber architecture, five distinct replicates with random fiber distribution were analyzed for each value of $v_{\mathrm{f}}$. Perfect bonding was assumed between the fibers and the matrix. Due to their nature, the constituent IM7 carbon fibers were modeled as transversely isotropic while the E-glass fibers were modeled as isotropic solids. The thermomechanical properties for both constituent fiber materials are summarized in Table 2. The matrix material was modeled as isotropic. The evolution of the matrix thermomechanical properties with the degree of cure $\phi$ and the processing temperature $T$ during virtual manufacturing was defined by experimentally determined material-specific models described in Shah et al. (2023). These properties, for a fully cured matrix ($\phi = 1$), are also listed in Table 2. Failure was admissible only in the matrix material when the maximum principal stresses exceeded its critical strength.

The virtual manufacturing and mechanical testing procedures were carried out with the commercial FE code Abaqus/STANDARD, supplemented with user-written UMATHT and UMAT subroutines, the implementations of which are detailed in Shah et al. (2023) and Shah and Maiaru (2021), and summarized in Section 2.2. Each RVE was meshed with C3D8T elements (eight-node, brick elements with temperature degrees of freedom). Flat boundary conditions (FBCs) were chosen for both virtual manufacturing and mechanical testing, as illustrated in Figs. 2(a) and 2(b), respectively (Shah and Maiaru, 2021).

### 2.2 Virtual Manufacturing

The virtual manufacturing analysis replicated the complete manufacturing of composites by accounting for (a) the kinetic progression of the cure, which was quantified by $\phi$, and (b) the evolution of the *in situ* thermomechanical matrix properties, which led to residual stress generation. The progression of cure, for a prescribed temperature profile, was defined by the Kamal-Sourour kinetic model (Kamal and Sourour, 1973),

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = (K_1 + K_2\phi^m)(1 - \phi)^n \tag{1a}$$

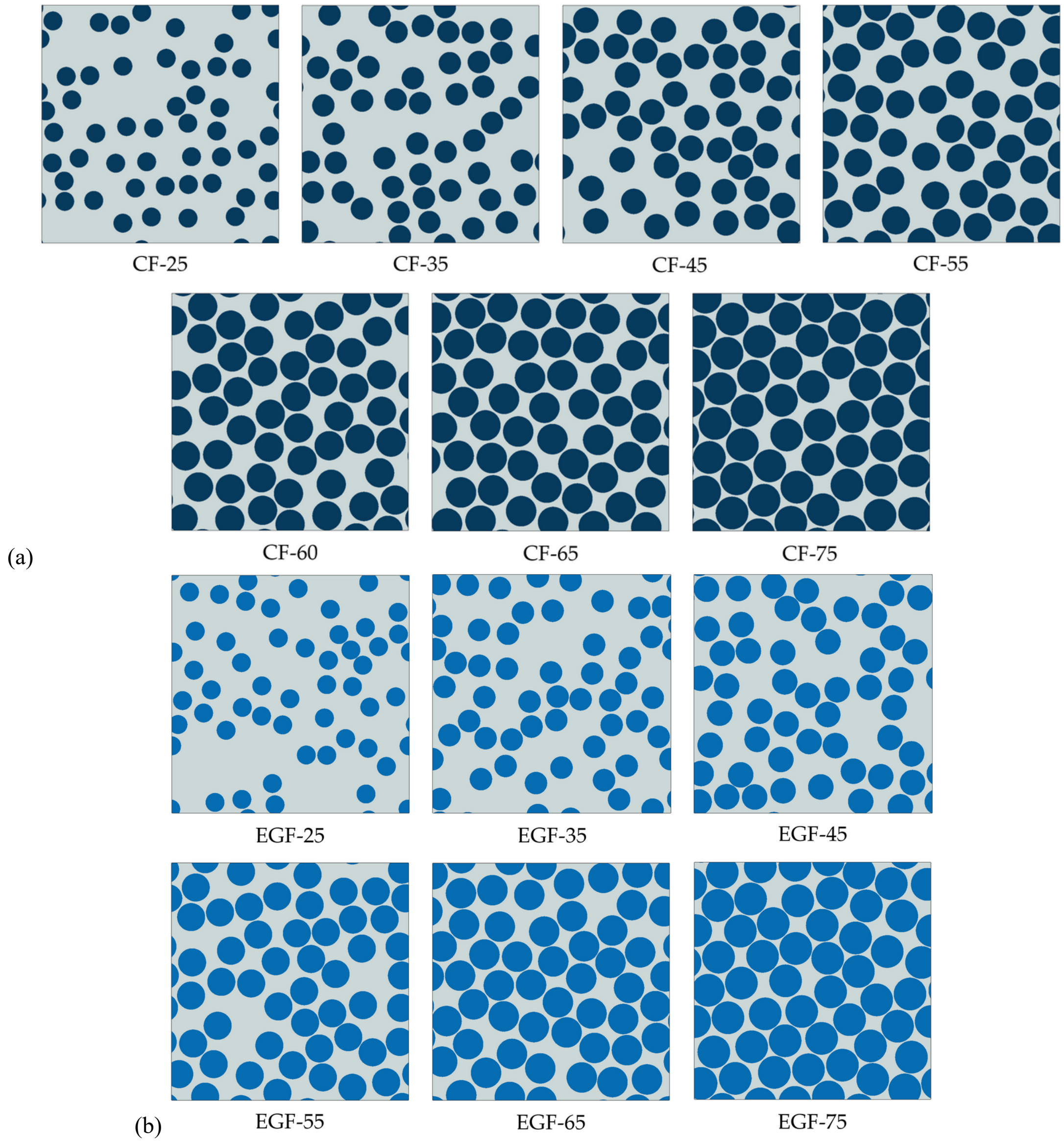



**FIG. 1:** Various realizations of (a) IM7 carbon fiber (CF) and (b) E-glass fibers (EGF) RVEs embedded in RIM R135-H1366 epoxy matrix with varying fiber volume fractions $25 \leq v_{\mathrm{f}} \leq 75\%$.

$$K_{\mathrm{i}} = A_{\mathrm{i}} \exp\left(\frac{\Delta E_{\mathrm{i}}}{RT^*}\right) \quad \mathrm{i} = 1, 2 \tag{1b}$$

where $K_1$ and $K_2$ are Arrhenius rate functions; $m = 0.4$ and $n = 1.5$ are dimensionless, real modeling parameters; $A_1 = 3.6 \times 10^9\ \mathrm{s}^{-1}$ and $A_2 = 0.01245\ \mathrm{s}^{-1}$ are pre-exponential factors; $\Delta E_1 = 85.3$ and $\Delta E_2 = 11.1$ kJ/mol are activation energies; $T^*$ is the absolute temperature (measured in degrees Kelvin); $R$ is the universal gas constant. The kinetic constants were experimentally determined as reported in Shah et al. (2023). The kinetic model was solved simultaneously with the three-dimensional Fourier heat transfer model in a coupled-temperature displacement analysis, carried out in Abaqus/STANDARD with user-subroutine UMATHT, to predict the progression of cure and the temperature distribution resulting from the exothermic heat of reaction during curing. The prescribed cure cycle and the computed degree of cure are presented in Fig. 2(c).

**TABLE 1:** Details of composite microstructures analyzed in this study

| Material | RVE ID | RVE length (μm) | No. of fibers | $v_f$ |
|---|---|---|---|---|
| IM7 carbon fiber ($d_f = 6$ μm) | CF-25 | 75.2 | 50 | 0.25 |
| | CF-35 | 63.5 | 50 | 0.35 |
| | CF-45 | 56.0 | 50 | 0.45 |
| | CF-55 | 50.7 | 50 | 0.55 |
| | CF-60 | 48.5 | 50 | 0.60 |
| | CF-65 | 46.6 | 50 | 0.65 |
| | CF-75 | 43.4 | 50 | 0.75 |
| E-glass fiber ($d_f = 14$ μm) | EGF-25 | 175.5 | 50 | 0.25 |
| | EGF-35 | 148.3 | 50 | 0.35 |
| | EGF-45 | 130.8 | 50 | 0.45 |
| | EGF-55 | 118.3 | 50 | 0.55 |
| | EGF-65 | 108.8 | 50 | 0.65 |
| | EGF-75 | 101.3 | 50 | 0.75 |

**TABLE 2:** Thermo-mechanical properties of the constituent fibers and epoxy matrix

| Property | Units | IM7 carbon | E-glass | RIM R135-H1366 |
|---|---|---|---|---|
| Density, $\rho$ | kg/m$^3$ | 1780 | 2550 | 1200 |
| Axial modulus, $E_{11}$ | MPa | 276,000 | 73,000 | 2482 |
| Transverse modulus, $E_{22} = E_{33}$ | MPa | 19,500 | 73,000 | 2482 |
| In-plane Poisson's ratio, $\nu_{12} = \nu_{13}$ | — | 0.28 | 0.22 | 0.37 |
| Out-of-plane Poisson's ratio, $\nu_{23}$ | — | 0.25 | 0.22 | 0.37 |
| In-plane shear modulus, $G_{12} = G_{13}$ | MPa | 70,000 | 30,000 | 905.8 |
| Out-of-plane shear modulus, $G_{23}$ | MPa | 7800 | 30,000 | 905.8 |
| Critical tensile strength, $\sigma_{cr}$ | MPa | — | — | 64.1 |
| Mode I fracture energy, $G_{IC}$ | J/m$^2$ | — | — | 0.001 |
| Axial CTE, $\alpha_{11}$ | K$^{-1}$ | $-0.54 \times 10^{-6}$ | $5 \times 10^{-6}$ | $73.9 \times 10^{-6}$ |
| Transverse CTE, $\alpha_{22} = \alpha_{33}$ | K$^{-1}$ | $10.08 \times 10^{-6}$ | $5 \times 10^{-6}$ | $73.9 \times 10^{-6}$ |
| Thermal conductivity, $k$ | W/m-K | 5.4 | 1.2 | 0.245 |
| Specific heat, $c_p$ | J/kg-K | 879 | 800 | 1600 |

The evolution of the *in situ* matrix properties and residual stress generation was modeled with UMAT user subroutine. For a given cure state, the instantaneous material properties were computed using experimentally determined material-specific models reported in Shah et al. (2023). For instance, the variation of the *in situ* matrix elastic modulus $E$ with $\phi$ and $T$ [see Fig. 2(c)] was defined by

$$E = A_2(\phi) + \frac{A_1(\phi) - A_2(\phi)}{1 + \exp\{[\eta - \eta_0(\phi)]/d\eta(\phi)\}} \tag{2}$$

where the dimensionless phenomenological parameters $\eta$, $\eta_0$, $d\eta$, $A_1$, and $A_2$ were determined as a function of $\phi$ and $T$ by the following expressions:

$$\eta = 1 - \frac{T^*}{T_r^*} \tag{3a}$$

$$\eta_0 = -0.860\phi + 0.955 \tag{3b}$$

$$d\eta = 0.070\phi + 0.041 \tag{3c}$$

$$A_1(\phi) = -10^{-13}\exp(34.55\phi) \tag{3d}$$

$$A_2(\phi) = 1471.3\phi + 801.39 \tag{3e}$$

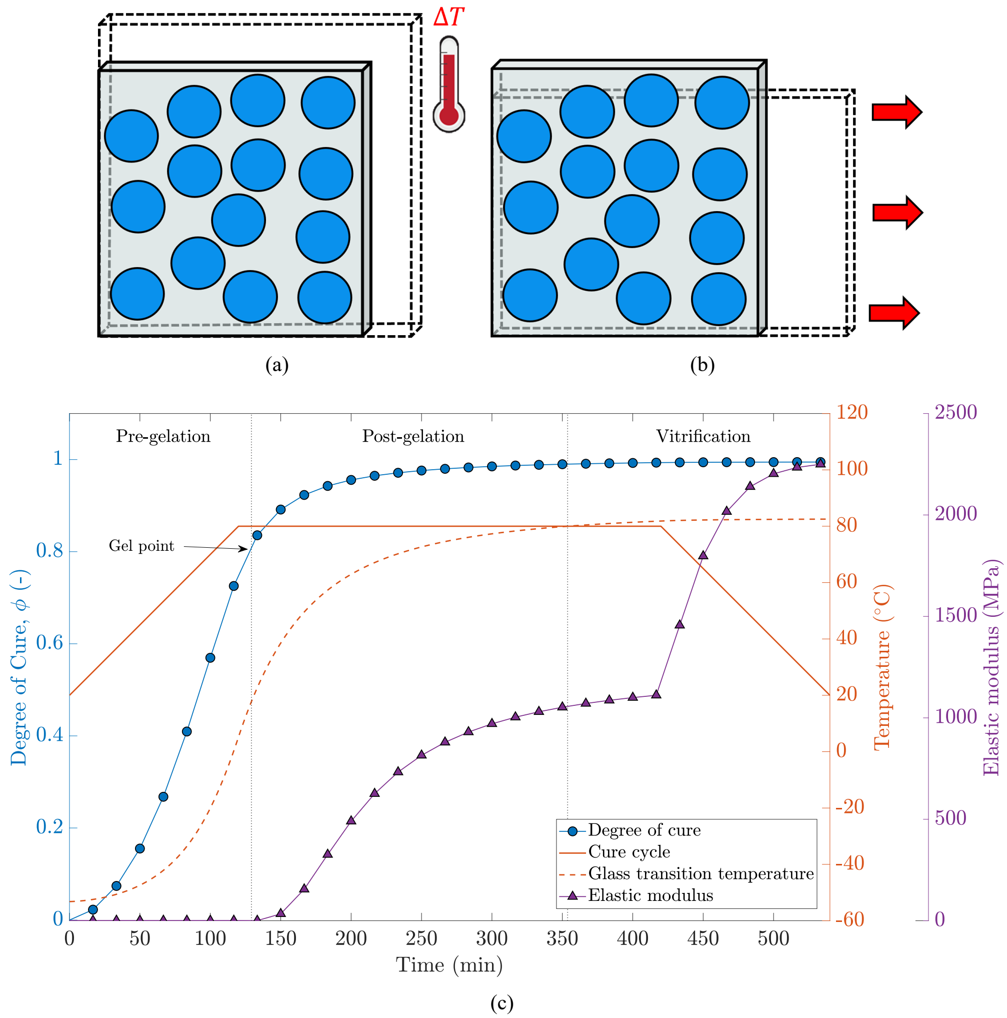



**FIG. 2:** Illustration of flat boundary conditions (FBCs) prescribed to the RVEs during: (a) virtual manufacturing and (b) virtual mechanical testing analyses; and (c) evolution of the degree of cure $\phi$, glass transition temperature $T_g$ and the elastic modulus $E$ for a prescribed cure cycle

where $T_r^*$ is the reference temperature (measured in degrees Kelvin). Similarly, the evolution of the matrix glass transition temperature $T_g^\phi$ [see Fig. 2(c)], post-gelation chemical shrinkage $\epsilon_{sh}^\phi$ and post-gelation coefficient of thermal expansion $\alpha_i^\phi$ were determined using:

$$T_g^\phi = T_g^0 + \frac{\lambda\phi}{1-(1-\lambda)\phi}\left(T_g^1 - T_g^0\right) \tag{4a}$$

$$\epsilon_{sh}^\phi = \begin{cases} 0, & \phi \leq \phi_{gel} \\ \beta(\phi - \phi_{gel}), & \phi > \phi_{gel} \end{cases} \tag{4b}$$

$$\alpha_i^\phi = \begin{cases} 7.93 \times 10^{-5}, & T \leq T_g^\phi \\ [a - b(\phi - \phi_{gel})] \times 10^{-5}, & T > T_g^\phi \end{cases} \tag{4c}$$

where $T_g^0 = -53.2$ and $T_g^1 = 85.6°C$ are glass transition temperatures of uncured and fully cured matrices; $\beta = 0.111$ is the shrinkage coefficient; $\phi_{gel} = 0.78$ is the gel point [see Fig. 2(c)]; and $\lambda = 0.2373$, $a = 21.21$, and

$b = 18.25$ are dimensionless fitting parameters. An instantaneous linear-elastic constitutive model was then employed to model the residual stress generation in the material during curing

$$\sigma_i(t) = [C_{ij}(t)]\left[\epsilon_j^{total}(t) - \left(\epsilon_j^{th}(t) + \epsilon_j^{sh}(t)\right)\delta_j\right], \quad \text{where} \begin{cases} \delta_j = 1 & \text{if } j = 1, 2, 3 \\ \delta_j = 0 & \text{if } j > 3 \end{cases} \tag{5}$$

where i and j are Voigt notation indices; $\epsilon_j^{total}(t)$, $\epsilon_j^{th}(t)$ and $\epsilon_j^{sh}(t)$ are the total, thermal and chemical shrinkage strains, respectively; $C_{ij}(t)$ is the stiffness matrix as a function of the time of cure; $\sigma_i(t)$ is the accumulated residual stress governed by the development of the *in situ* matrix elastic modulus and the chemical and thermal strains experienced by the matrix. In this study, the curing matrix was prescribed a constant strength and fracture toughness as listed in Table 2.

As the matrix material, subjected to the temperature profile shown in Fig. 2(c), cured, its elastic modulus developed as illustrated in Fig. 2(c). This, combined with the matrix chemical shrinkage and thermal mismatch between the constituent fiber and matrix, resulted in self-equilibrating residual stress generation. The contour plot of the residual stresses (maximum principal), at the end of the cure, is shown for one representative IM7 carbon and E-glass fiber microstructure in Figs. 3(a) and 3(b), respectively. These RVEs, which consisted of a random dispersion of 50 fibers with a volume fraction of 0.6 and 0.55, respectively, registered a maximum residual stress generation of 21.48 and 24.58 MPa. The same microstructures reported a volume-averaged residual stress of 6.38 and 7.49 MPa, respectively. These maximum and volume-averaged residual stress values differed considerably between RVEs due to the different constituent fiber material properties, their unique fiber packing topology, and the varying fiber volume fractions. Between the IM7 carbon fiber RVEs with $v_f = 0.6$, there was a maximum and volume-averaged, end-of-cure residual stress of $20.93 \pm 0.46$ and $6.406 \pm 0.073$ MPa, respectively. The E-glass fiber RVEs, with $v_f = 0.55$ reported a maximum and volume-averaged, end-of-cure residual stress of $24.79 \pm 1.1$ and $7.89 \pm 0.44$ MPa, respectively.

## 2.3 Virtual Mechanical Testing

The processed RVEs at the end of the previous step were subjected to transverse tensile mechanical loads by prescribing a displacement boundary condition, as illustrated in Fig. 2(b). The objective here was to compute the transverse composite response of the RVEs, namely, the transverse tensile strength $\sigma_{22}^{+}$ as a function of their fiber volume fraction.

Failure in the matrix was modeled with a previously developed progressive damage model (Shah and Maiaru, 2021), based on the crackband theory (Bažant and Oh, 1983). The maximum principal stress criterion was utilized to determine failure initiation in the matrix. A traction-separation law, governed by the fracture energy, was employed to define the post-peak softening behavior of the damaging material once the critical fracture stress was reached. The critical mode I energy release rate $G_{IC}$ was given by

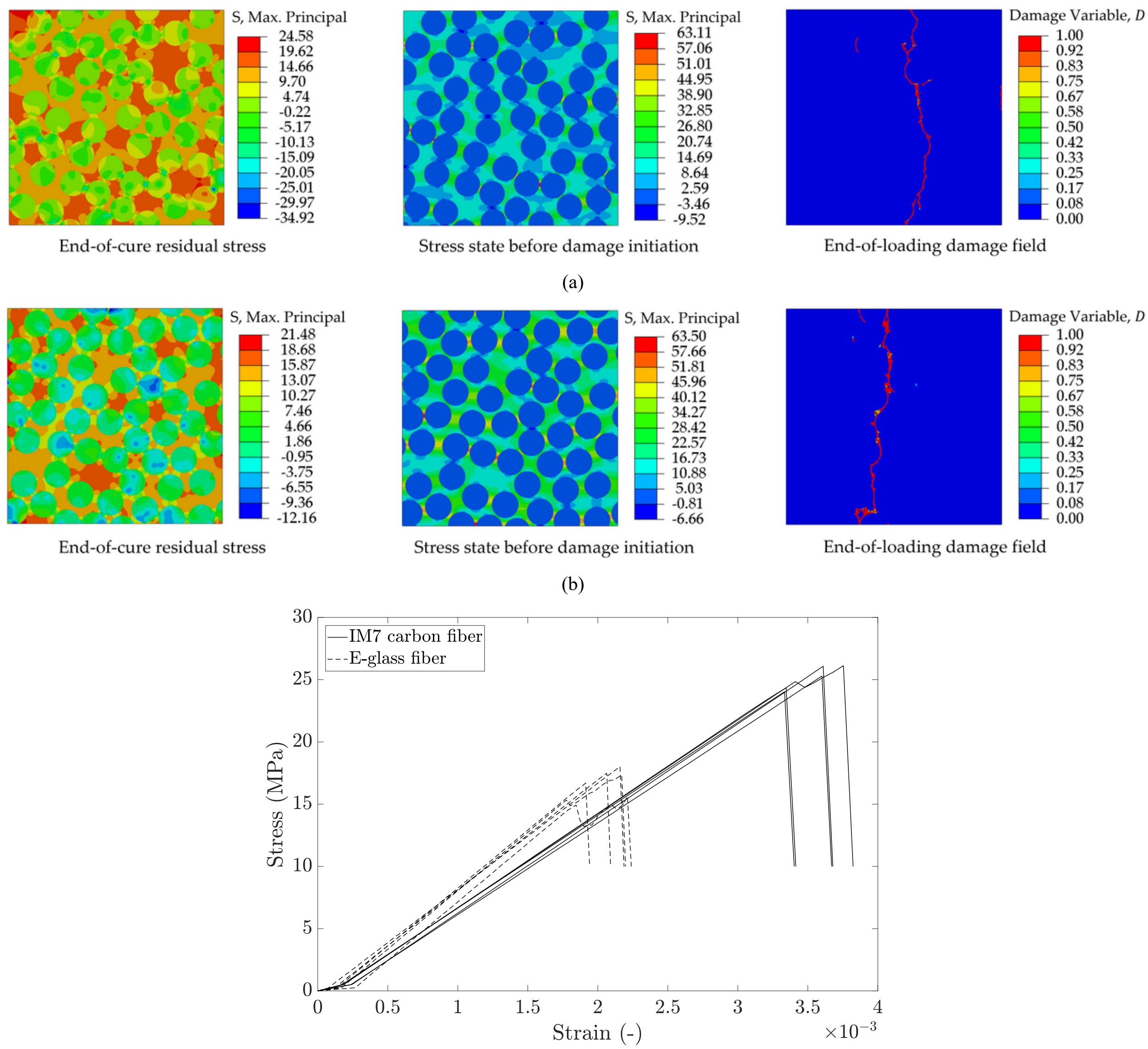
$$G_{IC} = h^{\eta} \int_0^{\bar{\epsilon}_f^{\eta}} \bar{\sigma}_{11}^{\eta}\ \bar{\epsilon}_{11}^{\eta}\ d\bar{\epsilon} \tag{6}$$

where $\bar{\sigma}_{11}^{\eta}$ and $\bar{\epsilon}_{11}^{\eta}$ are the maximum principal stress and strain values in element $\eta$, respectively; $\bar{\epsilon}_f^{\eta}$ is the value of $\bar{\epsilon}_{11}^{\eta}$, which corresponds to a zero stress state on the post-peak stress versus strain plot; and $h^{\eta}$ is the characteristic length of the element $\eta$ that preserves mesh objectivity by prescribing a normalized value of $G_{IC}$ for each element, such that $g_{IC}^{\eta} = G_{IC}/h^{\eta}$.

A scalar damage factor $D^{\eta}$ was computed to degrade the damaged element compliance components using

$$D^{\eta} = 1 - \left[\frac{\sigma_{cr}}{E_m(\bar{\epsilon}_f^{\eta} - \bar{\epsilon}_{init}^{\eta})}\left(\frac{\bar{\epsilon}_f^{\eta}}{\bar{\epsilon}_{11}^{\eta}} - 1\right)\right] \tag{7}$$

where $\bar{\epsilon}_{init}^{\eta}$ is the value of $\bar{\epsilon}_{11}^{\eta}$, when the initiation criterion ($\bar{\sigma}_{11}^{\eta} \geq \sigma_{cr}$) is satisfied; and $E_m$ is the undamaged Young's modulus of the matrix. The damage parameter could take values between 0 and 1, where $D^{\eta} = 0$ meant no damage had occurred. By contrast, a maximum damage level of 1 corresponded to a zero-stress state on the post-peak stress




(c)

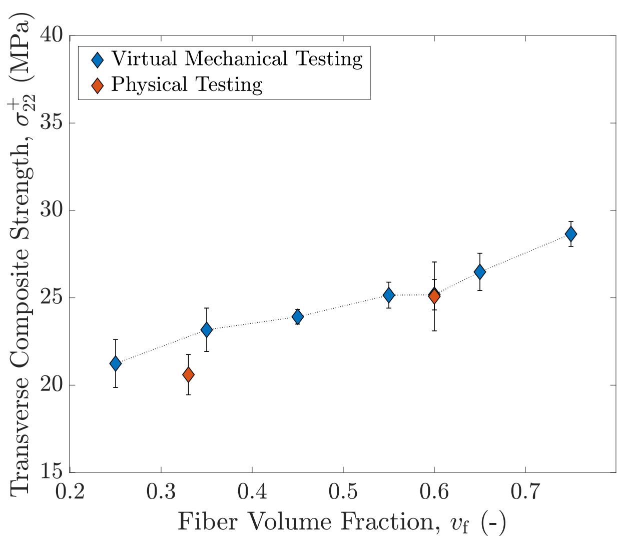
**FIG. 3:** Contour plots showing the end-of-cure residual stresses (maximum principal) from virtual manufacturing analysis, maximum principal stress before the onset of damage from virtual mechanical testing, and damage field at the end of the virtual mechanical testing in representative: (a) IM7 carbon fiber RVE ($v_\mathrm{f} = 0.60$), (b) E-glass fiber RVE ($v_\mathrm{f} = 0.55$), and (c) plot of stress versus strain from virtual mechanical testing of IM7 carbon and E-glass fiber RVEs

versus strain plot. Also, healing was inadmissible. Once the damage factor was computed, the relevant components of the compliance matrix were degraded (Pineda et al., 2013; Shah and Maiaru, 2021). The progressive damage formulation was modeled in Abaqus/STANDARD solver with user-written subroutine UMAT. The matrix strength $\sigma_{\mathrm{cr}}^{\mathrm{m}}$ and a scaled-down fracture energy $G_{\mathrm{IC}}$, corresponding to the submicron length scale, were prescribed to the material as listed in Table 2.

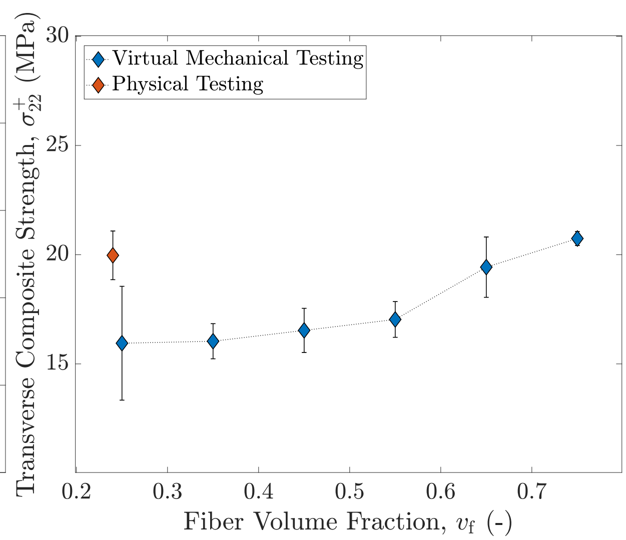
The stress versus strain plots of the five IM7 carbon fiber RVEs ($v_\mathrm{f} = 0.6$) and E-glass fiber RVEs ($v_\mathrm{f} = 0.55$), subjected to post-manufacturing transverse loads, are presented in Fig. 3(c). In Fig. 3(c), the RVEs exhibited an initial linear-elastic response, which was followed by a drop in the stress. This drop was associated with damage initiation in the matrix when the local stresses exceeded the matrix strength $\sigma_{cr}^{m}$. The contour plot of the maximum principal stress before the onset of damage in the representative microstructure is shown in Fig. 3(a) for IM7 carbon and Fig. 3(b) for E-glass fiber RVEs, respectively. Local regions with high stress concentration (corresponding to warmer colors), where failure initiation was anticipated, were clearly visible in these plots. The contour plot of the end-of-loading damage variable $D$, which was computed using Eq. (7) and used to degrade the element's stiffness, is presented in Figs. 3(a) and 3(b), respectively for IM7 carbon and E-glass fiber RVEs. It was evident from Fig. 3, that stress localized in regions of dense fiber packing, which led to failure initiation and local microcracking. These cracks eventually coalesced into a large crack that propagated through the microstructure, resulting in a two-piece failure. The peak stress in the stress versus strain plots was regarded as the transverse composite strength $\sigma_{22}^{+}$. For the representative IM7 carbon and E-glass fiber microstructures shown in Fig. 3, $\sigma_{22}^{+} = 24.03$ and $\sigma_{22}^{+} = 17.49$ MPa, respectively.

The analysis was repeated for all RVEs illustrated in Fig. 1 and their replicates. The transverse composite strength predictions from these analyses are summarized in Fig. 4 as a function of their fiber volume fraction for IM7 carbon and E-glass fiber RVEs. Each data point in these plots was obtained by averaging the numerical predictions from five distinct replicates of a RVE of a given fiber volume fraction. The relevant standard deviations from the average values were represented by the corresponding error bars. It was clear from Figs. 1 and 4 that the fiber volume fraction manifested a strong influence on the transverse strength of the composite. The transverse composite strength predictions for IM7 carbon fiber RVEs increased steadily with the fiber volume fraction from $\sigma_{22}^{+} = 21.23 \pm 1.37$ MPa for $v_f = 0.25$ to $\sigma_{22}^{+} = 28.65 \pm 0.71$ MPa for $v_f = 0.75$. Similar trends were observed for E-glass fiber RVEs where the transverse strength predictions increased from $\sigma_{22}^{+} = 15.94 \pm 2.61$ MPa for $v_f = 0.25$ to $\sigma_{22}^{+} = 20.74 \pm 0.32$ MPa for $v_f = 0.75$. Furthermore, the fiber packing manifested a strong influence on the transverse strength.

Random and densely packed fibers induce local stress concentration in composites leading to premature failure initiation and local microcracking, which typically manifests itself as a large scatter of bulk composite strength. This was evident from the large standard deviations in the predicted strength values for each volume fraction in Fig. 4. In particular, RVEs with lower fiber volume fraction ($v_f = 0.25, 0.35$) manifested a relatively larger scatter associated with their predicted strength values. This suggested that the converged RVE size of 50 fibers, which was defined based on a fiber volume fraction of $v_f = 0.55$, may not be sufficiently large and representative of composite microstructures with lower fiber volume fractions. To define an appropriate RVE size for such cases, a separate statistical size effect study is warranted, which can be computationally expensive due to the larger RVE sizes that




(a)




(b)

**FIG. 4:** Summary of the transverse composite strength predictions from the virtual testing of (a) IM7 carbon fiber ($d_f = 6$ μm) and (b) E-glass fiber RVEs ($d_f = 14$ μm) composite microstructures as a function of their fiber volume fraction

need to be analyzed. Notwithstanding that the standard deviations for these cases were found to be within 10% of the mean values, which is within the accepted limits for numerical and physical testing. Furthermore, the predicted values were directly compared to the respective experimental strength values obtained from in-house transverse testing of thin IM7 carbon and E-glass fiber composite laminates. The predicted strength values were in excellent agreement with the experimental values as presented in Fig. 4.

## 3. DEVELOPMENT OF THE ANALYTICAL MODEL

Modeling (Ghayoor et al., 2018; Liu and Huang, 2014; Shah and Maiaru, 2021) and testing (Flores et al., 2022) of composites have shown that matrix failure initiates in regions of high stress concentration as a result of dense fiber packing. This concentration of stress not only depends on the constituent fiber and matrix properties but also the random fiber distribution, residual stress generation, and the global fiber volume fraction that influences the transverse composite strength. This work proposed an analytical model that directly accounted for these parameters in

$$\sigma_{22}^{+} = \sigma_{22}^{+}\left(E^{\mathrm{m}}, E_{22}^{\mathrm{f}}, \nu^{\mathrm{m}}, \nu^{\mathrm{f}}, \sigma_{\mathrm{cr}}^{\mathrm{m}}, \mathbf{SCF}_{22}^{+}\right) \tag{8}$$

where, specifically, the transverse composite strength $\sigma_{22}^{+}$ was expressed as a function of the constituent fiber/matrix elastic properties $E^{\mathrm{m}}$, $E_{22}^{\mathrm{f}}$, $\nu^{\mathrm{m}}$, and $\nu^{\mathrm{f}}$; the *in situ* matrix strength $\sigma_{\mathrm{cr}}^{\mathrm{m}}$; and a stress concentration factor $\mathbf{SCF}_{22}^{+}$. Results from Section 2 also showed that the relative distance between the neighboring fibers and their orientation with respect to the loading direction significantly influenced stress localization between them, which eventually resulted in failure initiation. Therefore, the SCF was divided into two contributions, one that accounted for the effect of the fiber distribution (distance between the fiber centers and their orientation), $\mathbf{SCF}_{\mathrm{I}}$, and the second that accounted for the effect of the fiber volume fraction, $\mathbf{SCF}_{\mathrm{II}}$. $\mathbf{SCF}_{22}^{+}$ was defined by

$$\mathbf{SCF}_{22}^{+} = \mathbf{SCF}_{\mathrm{I}}(\delta, \theta) \times \mathbf{SCF}_{\mathrm{II}}(v_{\mathrm{f}}) \tag{9}$$

where $\delta$ is the shortest distance between a fiber pair while $\theta$ is the orientation of that shortest distance with respect to the loading direction and $v_{\mathrm{f}}$ is the fiber volume fraction. The composite transverse strength can be calculated starting from the definition of stress concentration as shown in

$$\sigma_{22}^{+} = \frac{\sigma_{\mathrm{cr}}^{\mathrm{m}}}{\mathbf{SCF}_{\mathrm{I}} \times \mathbf{SCF}_{\mathrm{II}}} \tag{10}$$

Solving for $\sigma_{22}^{+}$ required the determination of the SCFs. A novel computational approach to calculate the $\mathbf{SCF}_{\mathrm{I}}$ and $\mathbf{SCF}_{\mathrm{II}}$ for a multi-fiber RVE is proposed in Section 3.

### 3.1 Stress Concentration Factor

A stress concentration factor is defined as the ratio of the stresses in the proximity of a discontinuity within a continuum and the far-field/applied stress averaged over a boundary surface. In case of composite microstructures, a fiber inclusion within a matrix is a discontinuity that acted as a stress-riser. A simple linear-elastic analysis of a single IM7 carbon fiber embedded in an infinite RIM R135-H1366 medium yielded an SCF = 1.45. This meant the stresses in the vicinity of a fiber were 1.45 times higher than the applied stresses. A similar value of SCF = 1.54 was obtained for E-glass fibers embedded in the same RIM R135-H1366 epoxy matrix. While such results from a single-fiber RUC have been extensively used to develop analytical models for transverse composite strength, they fail to provide useful information about the nonuniform stress field in a multi-fiber RUC. In such cases, the neighboring fibers influence the stress distribution around the fiber being considered, which then affects the stress concentration surrounding it. This evaluation of a global SCF in a multi-fiber microstructure becomes increasingly challenging since the nonuniform stress field is not only influenced by several neighboring fiber interactions within a certain proximity $\delta$ but also their relative orientation with respect to the loading direction $\theta$.

To determine the SCF in multi-fiber microstructures, a unique stress concentration analysis was proposed in this study. First, the influence of introducing a neighboring fiber on the local SCF was investigated with a two-fiber model (see Section 3.2). Following this, a correlation was established between the geometrical variations in a multi-fiber microstructure and the SCF by virtue of the nearest-neighbor statistical descriptor (see Section 3.3).

### 3.2 Influence of Fiber Arrangement on SCF

The influence of fiber arrangement on the SCF was investigated through FE simulations. Several two-fiber models, as illustrated in Figs. 5 and 6, were generated in Abaqus/STANDARD with $0 \leq \theta \leq 90$ deg and $6.1 \leq \delta \leq 26$ μm for IM7 carbon fiber models while $0 \leq \theta \leq 90$ deg and $14.1 \leq \delta \leq 34$ μm for E-glass fiber models, respectively. Assuming a linear elastic material response, these models were subjected to transverse mechanical loads by prescribing a displacement boundary condition. The SCF in each case was determined using the classic definition, as follows:

$$\mathbf{SCF}_{\mathrm{I}} = \frac{\sigma_{\max}}{\sigma_{\infty}} \tag{11}$$

where $\sigma_{\max}$ is the maximum principal point stress and $\sigma_{\infty}$ is the far-field/applied principal stress. The computed $\mathbf{SCF}_{\mathrm{I}}$ for various combinations of δ and θ are shown in Figs. 5(a) and 5(b) for IM7 carbon fibers and Figs. 6(a) and 6(b) for E-glass fibers, respectively. For a given value of $\delta = 7.2$ μm in Fig. 5(a), the highest stress concentration was observed when the fiber centers were oriented parallel to the loading direction ($\theta = 0$ deg). The $\mathbf{SCF}_{\mathrm{I}}$ rapidly decreased from 2.52 to 1.45 as θ increased from 0 to 90 deg. By contrast, the $\mathbf{SCF}_{\mathrm{I}}$ gradually reduced from 3.13 to 1.56 in Fig. 5(b) as the distance between the fiber centers δ increased from 6.2 to 15 μm while $\theta = 0$ deg. A consistent trend was observed for E-glass fiber models as illustrated in Figs. 6(a) and 6(b). The influence of relative fiber arrangement on the local SCF was clearly evident from Figs. 5 and 6. By analyzing an exhaustive combination of δ and θ, an interpolation function for $\mathbf{SCF}_{\mathrm{I}}(\delta, \theta)$ was developed [see Figs. 5(c) and 6(c)], as follows:

$$\mathbf{SCF}_{\mathrm{I}}(\delta, \theta) = a_{11} + a_{12}\theta + a_{21}\delta + a_{22}\theta\delta + a_{23}\delta^2 \tag{12}$$

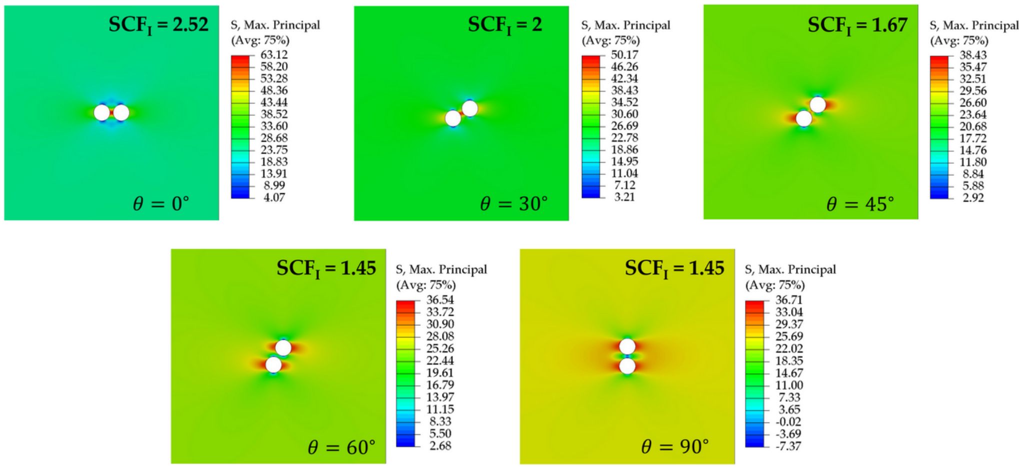
where $a_{11}$–$a_{23}$ are fitting parameters specific to the material systems considered in this study. These values for both IM7 carbon and E-glass fiber models are summarized in Table 3. Note, these values were different for the two fiber materials analyzed in this study and that their values may further vary for other material systems. This function, defined by Eq. (12) and the corresponding fitting parameters, were utilized in Section 3.3 to determine the $\mathbf{SCF}_{\mathrm{I}}$ of each multi-fiber microstructure analyzed in Section 3.2 as a function of their geometrical variations.

### 3.3 SCF in Multi-Fiber Microstructures

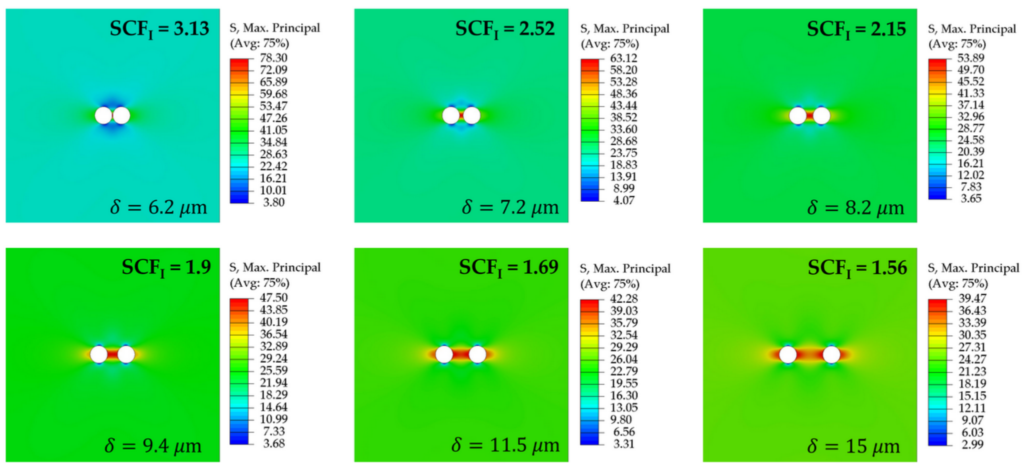
Several statistical descriptors have been reported in the literature to quantify the geometrical variations within the microstructures, such as local fiber volume fraction distribution using Voronoi tessellation, cluster analysis with Delaunay triangulation, and second-order intensity function (Maragoni et al., 2018; Melro et al., 2008; Romanov et al., 2013; Sanei et al., 2017; Schey et al., 2021; Shah et al., 2020b; Trias et al., 2006; Vaughan and McCarthy, 2010; Yang et al., 2013b). Since the goal of this study is to quantify the local geometrical variations in the multi-fiber microstructures analyzed in Section 2 and to establish a direct correlation with the global $\mathbf{SCF}_{\mathrm{I}}$, the nearest-neighbor statistical descriptor was utilized, which is based on the short-range fiber interactions. The definition of this descriptor is illustrated schematically in Fig. 7. For a given fiber center $i$ in the microstructure, the algorithm identified its closest neighbor and measured the distance between the fiber centers $\delta^i$ along with its relative orientation $\theta^i$ with respect to the loading direction. This process was repeated for all fiber centers in the microstructure. The mean shortest distance $\bar{\delta}$ and the corresponding mean orientation $\bar{\theta}$ were computed for each microstructure. The global $\mathbf{SCF}_{\mathrm{I}}$ for each of these microstructures was then computed with the help of the interpolation function $\mathbf{SCF}_{\mathrm{I}}(\bar{\delta}, \bar{\theta})$ [see Eq. (12)], where the mean shortest distance $\bar{\delta}$ and orientation $\bar{\theta}$ values from the statistical analysis were used. These values for each microstructure analyzed in this study are summarized in the Tables 4 and 5.

By substituting into Eq. (10), the neat matrix strength $\sigma_{\mathrm{cr}}^{\mathrm{m}} = 64.1$ MPa (see Table 2), composite strength predictions $\sigma_{22}^{+}$ of microstructures from Section 2 and the $\mathbf{SCF}_{\mathrm{I}}$ computed using Eq. (12), $\mathbf{SCF}_{\mathrm{II}}$ was determined for each microstructure analyzed previously (see Tables 4 and 5). The variation of $\mathbf{SCF}_{\mathrm{II}}$ for each microstructure as a function of its fiber volume fraction is presented in Fig. 8 for IM7 carbon and E-glass fiber models. Note that each data point on the plot is an average of five microstructure renditions with random fiber packing. A nonlinear least-squares regression fitting of this variation yielded an expression for $\mathbf{SCF}_{\mathrm{II}}$, as follows:

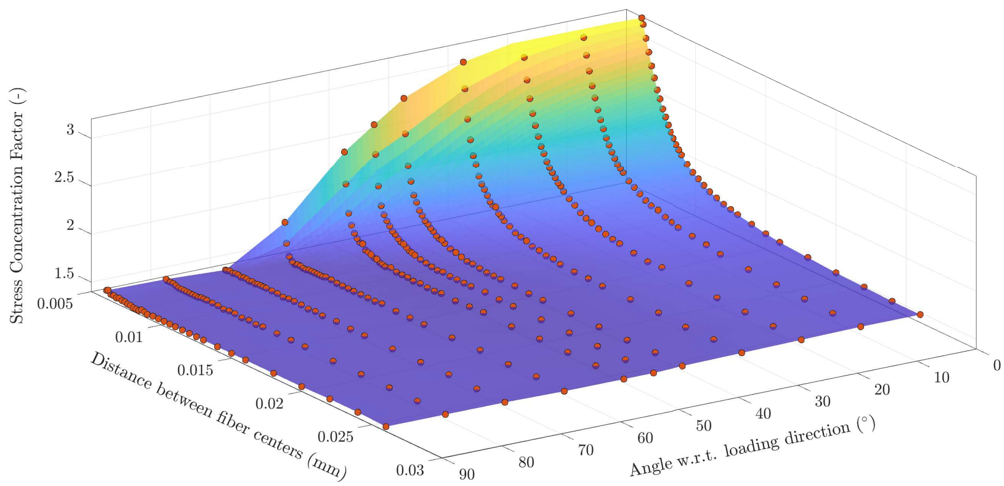
$$\mathbf{SCF}_{\mathrm{II}} = b + c\sqrt{v_{\mathrm{f}}} + dv_{\mathrm{f}} \tag{13}$$




(a)




(b)




(c)

**FIG. 5:** Variation in **SCF**$_I$ in IM7 carbon fiber microstructures with (a) fiber orientation θ when $\delta = 7.2\ \mu$m, (b) distance between the fiber centers δ when $\theta = 0$ deg (fibers not shown), and (c) interpolation function expressing **SCF**$_I$ as a function of θ and δ

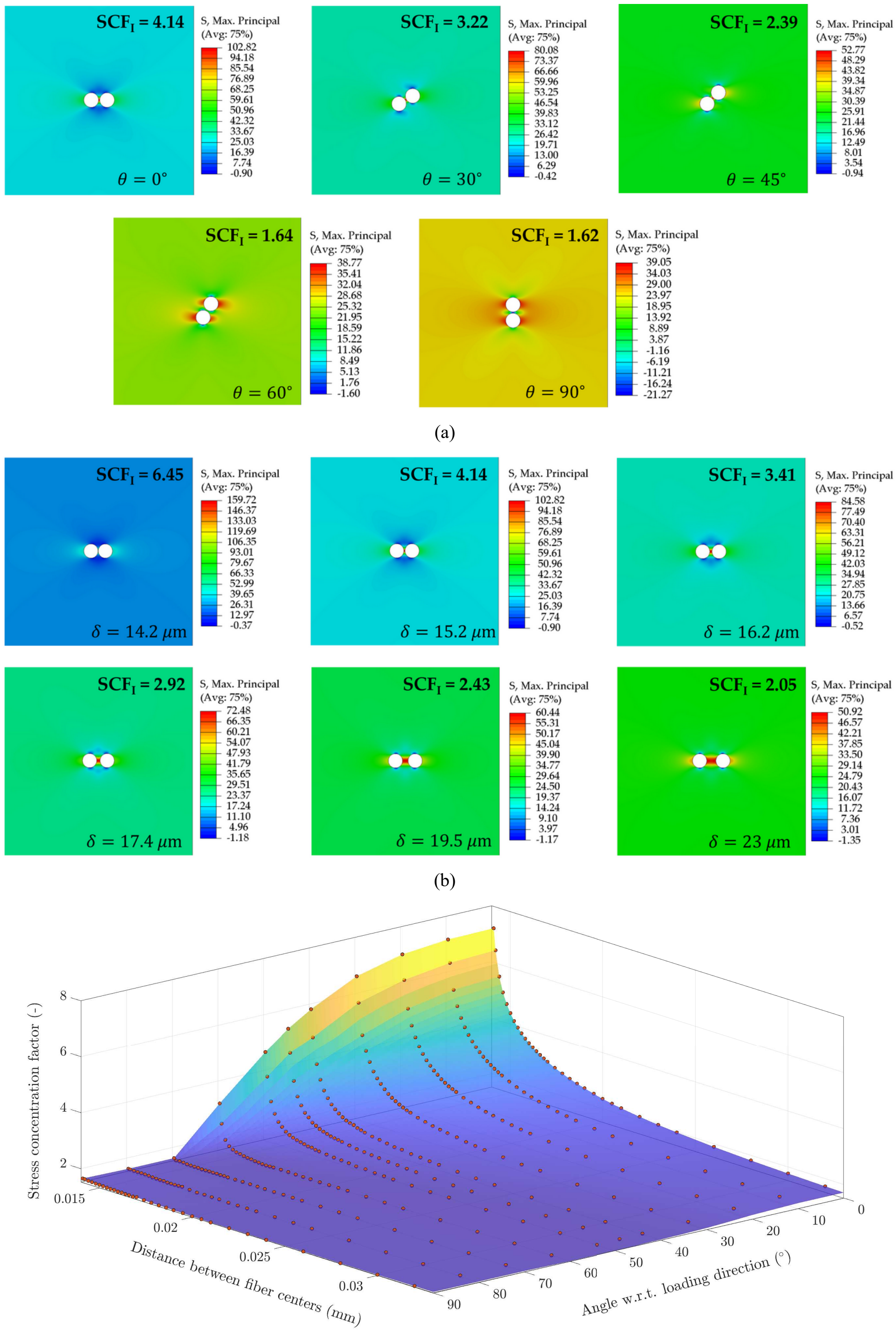



**FIG. 6:** Variation in $\mathbf{SCF}_\mathrm{I}$ in E-glass fiber microstructures with (a) fiber orientation θ when $\delta = 15.2\ \mu m$, (b) distance between the fiber centers δ when $\theta = 0$ deg (fibers not shown), and (c) interpolation function expressing $\mathbf{SCF}_\mathrm{I}$ as a function of θ and δ

**TABLE 3:** Material system-specific fitting parameters used for the analytical model predictions

| | Parameter | IM7 carbon | E-glass |
|---|---|---|---|
| **SCF** | Single fiber RUC | 1.45 | 1.54 |
| $a_{11}$ | | 3.698 | 13.3 |
| $a_{12}$ | | $-1.872 \times 10^{-2}$ | $-7.035 \times 10^{-2}$ |
| $a_{21}$ | Eq. (12) | $-2.123 \times 10^{-1}$ | $-7.86 \times 10^{-1}$ |
| $a_{22}$ | | $-9.742 \times 10^{-4}$ | $2.508 \times 10^{-3}$ |
| $a_{23}$ | | $-4.645 \times 10^{-3}$ | $1.278 \times 10^{-2}$ |
| $b$ | | $2.99 \times 10^{-1}$ | $1.336 \times 10^{-1}$ |
| $c$ | Eq. (13) | 5.916 | 8.449 |
| $d$ | | –5.882 | –8.811 |

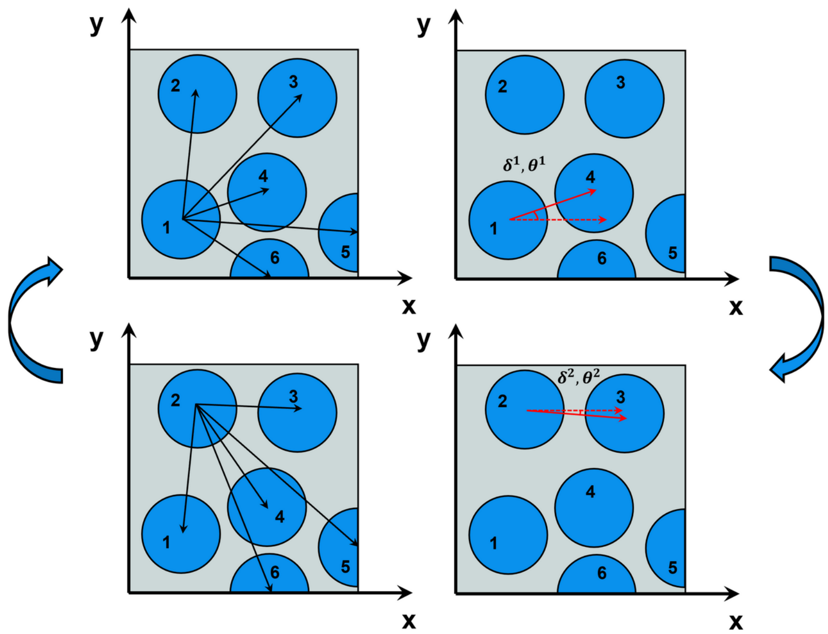



**FIG. 7:** Schematic showing the working of the nearest neighbor distribution statistical descriptor

where $b - d$ are fitting parameters specific to the material systems and microstructures considered in this study. These values for both IM7 carbon and E-glass fiber models are summarized in Table 3. Equations (12) and (13) were substituted in Eq. (10) to obtain the final expression for the transverse composite strength. The analytical model, thus developed, accurately predicted the transverse strength of IM7 carbon and E-glass fiber composite microstructures for a wide range of fiber volume fractions and random fiber distributions. For instance, in Fig. 9(a), the model predicted close to the neat matrix strength of $\sigma_{\mathrm{cr}}^{\mathrm{m}} = 64.1$ MPa, where $v_{\mathrm{f}} = 0$, is representative of the neat matrix.

Subsequently, a sudden drop in strength predictions was observed with the introduction of fibers due to stress concentration. This drop was followed by a steady increase in the composite strength with the fiber volume fraction. A good correlation was established between the numerical model predictions, experimental measurements, and the analytical model predictions, as demonstrated in Fig. 9(a). Consistent trends were obtained form E-glass fiber/RIM R135-H1366 microstructures, as seen in Fig. 9(b).

It should be noted that Eqs. (10), (12), and (13) are material system specific and cannot be employed to predict the transverse composite response of a different constituent material system. This limitation is associated with the

**TABLE 4:** Analysis summary for IM7 carbon fiber RVEs

| RVE ID | $v_f$ | $\sigma_{22}^{+}$ (MPa) | $\delta$ (mm) | $\theta$ (deg) | $SCF_I$ | $SCF_{II}$ |
|---|---|---|---|---|---|---|
| CF-25-1 | 0.25 | 18.68 | $8.194 \pm 1.46$ | $41 \pm 32$ | 1.61 | 2.14 |
| CF-25-2 | 0.25 | 22.48 | $8.214 \pm 1.82$ | $39 \pm 30$ | 1.64 | 1.74 |
| CF-25-3 | 0.25 | 21.27 | $7.835 \pm 1.53$ | $47 \pm 27$ | 1.55 | 1.94 |
| CF-25-4 | 0.25 | 22.36 | $7.563 \pm 1.24$ | $40 \pm 26$ | 1.69 | 1.69 |
| CF-25-5 | 0.25 | 21.39 | $7.835 \pm 1.61$ | $40 \pm 25$ | 1.66 | 1.81 |
| CF-35-1 | 0.35 | 24.36 | $7.34 \pm 1.2$ | $41 \pm 28$ | 1.72 | 1.53 |
| CF-35-2 | 0.35 | 22.97 | $7.213 \pm 0.92$ | $47 \pm 24$ | 1.63 | 1.71 |
| CF-35-3 | 0.35 | 21.93 | $7.078 \pm 0.91$ | $47 \pm 24$ | 1.63 | 1.79 |
| CF-35-4 | 0.35 | 24.82 | $7.219 \pm 0.85$ | $40 \pm 26$ | 1.77 | 1.46 |
| CF-35-5 | 0.35 | 21.75 | $7.479 \pm 1.34$ | $46 \pm 28$ | 1.61 | 1.84 |
| CF-45-1 | 0.45 | 23.38 | $6.997 \pm 0.86$ | $51 \pm 26$ | 1.58 | 1.74 |
| CF-45-2 | 0.45 | 24.66 | $6.71 \pm 0.41$ | $52 \pm 24$ | 1.62 | 1.60 |
| CF-45-3 | 0.45 | 23.79 | $6.749 \pm 0.44$ | $51 \pm 24$ | 1.63 | 1.65 |
| CF-45-4 | 0.45 | 23.91 | $6.942 \pm 0.81$ | $46 \pm 27$ | 1.69 | 1.59 |
| CF-45-5 | 0.45 | 23.82 | $6.751 \pm 0.39$ | $42 \pm 24$ | 1.86 | 1.45 |
| CF-55-1 | 0.55 | 25.96 | $6.582 \pm 0.25$ | $46 \pm 27$ | 1.81 | 1.37 |
| CF-55-2 | 0.55 | 24.51 | $6.606 \pm 0.34$ | $50 \pm 27$ | 1.70 | 1.54 |
| CF-55-3 | 0.55 | 25.59 | $6.559 \pm 0.28$ | $50 \pm 23$ | 1.73 | 1.44 |
| CF-55-4 | 0.55 | 24.03 | $6.592 \pm 0.32$ | $52 \pm 27$ | 1.64 | 1.63 |
| CF-55-5 | 0.55 | 25.66 | $6.572 \pm 0.32$ | $50 \pm 23$ | 1.71 | 1.46 |
| CF-60-1 | 0.6 | 26.08 | $6.512 \pm 0.26$ | $43 \pm 26$ | 2.01 | 1.22 |
| CF-60-2 | 0.6 | 26.12 | $6.478 \pm 0.19$ | $48 \pm 26$ | 1.81 | 1.36 |
| CF-60-3 | 0.6 | 24.34 | $6.465 \pm 0.19$ | $45 \pm 27$ | 1.95 | 1.35 |
| CF-60-4 | 0.6 | 24.03 | $6.501 \pm 0.28$ | $40 \pm 24$ | 2.09 | 1.28 |
| CF-60-5 | 0.6 | 25.29 | $6.532 \pm 0.3$ | $41 \pm 25$ | 2.04 | 1.24 |
| CF-65-1 | 0.65 | 26.85 | $6.457 \pm 0.17$ | $42 \pm 25$ | 2.09 | 1.14 |
| CF-65-2 | 0.65 | 27.82 | $6.403 \pm 0.09$ | $46 \pm 29$ | 1.98 | 1.16 |
| CF-65-3 | 0.65 | 24.60 | $6.436 \pm 0.24$ | $46 \pm 28$ | 1.96 | 1.33 |
| CF-65-4 | 0.65 | 26.24 | $6.396 \pm 0.15$ | $49 \pm 27$ | 1.85 | 1.32 |
| CF-65-5 | 0.65 | 26.89 | $6.428 \pm 0.15$ | $52 \pm 25$ | 1.72 | 1.39 |
| CF-75-1 | 0.75 | 29.54 | $6.309 \pm 0.01$ | $44 \pm 25$ | 2.21 | 0.98 |
| CF-75-2 | 0.75 | 27.65 | $6.301 \pm 0.01$ | $39 \pm 30$ | 2.39 | 0.97 |
| CF-75-3 | 0.75 | 28.09 | $6.302 \pm 0.01$ | $40 \pm 23$ | 2.33 | 0.98 |
| CF-75-4 | 0.75 | 28.68 | $6.315 \pm 0.03$ | $54 \pm 19$ | 1.74 | 1.29 |
| CF-75-5 | 0.75 | 29.29 | $6.302 \pm 0.01$ | $40 \pm 24$ | 2.33 | 0.94 |

underlying assumption that the influence of the constituent fiber/matrix elastic properties on the SCF was implicitly accounted for in the data fitting process. As summarized in Table 3, a different set of fitting parameters were obtained for E-glass fiber microstructures. Thus, to better quantify the influence of constituent material properties on the SCF and to develop a generalized analytical mode, independent of any fitting parameters, explicit correlations between the constituent properties and the SCF must be defined. Additionally, the effect of process-induced residual stresses

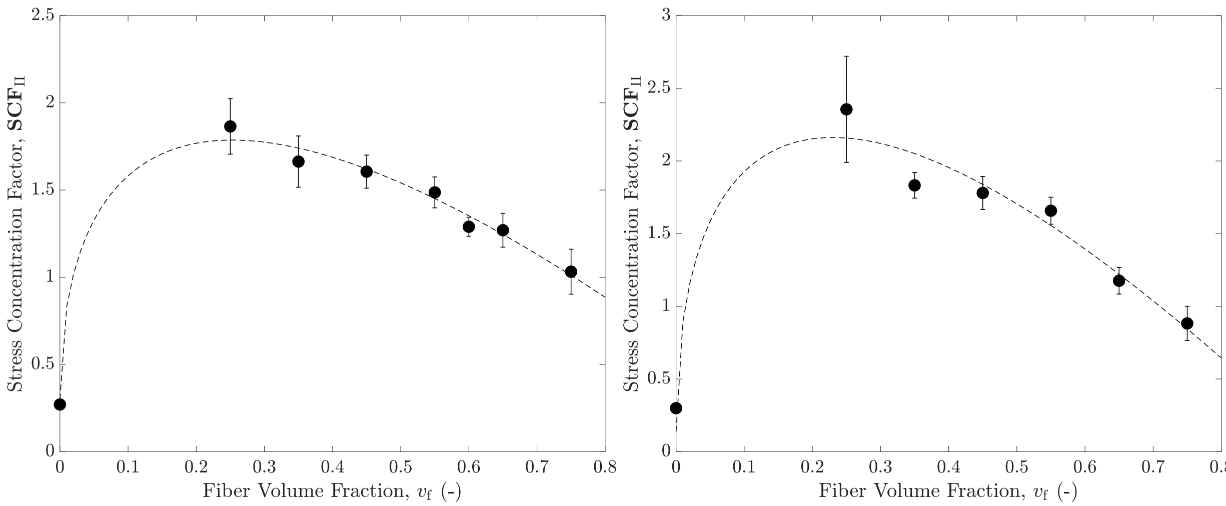
**TABLE 5:** Analysis summary for E-glass fiber RVEs

| RVE ID | $v_f$ | $\sigma_{22}^{+}$ (MPa) | $\delta$ (mm) | $\theta$ deg | $SCF_I$ | $SCF_{II}$ |
|---|---|---|---|---|---|---|
| EGF-25-1 | 0.25 | 18.51 | 18.945 ± 4.01 | 50 ± 26 | 1.62 | 2.14 |
| EGF-25-2 | 0.25 | 18.44 | 18.051 ± 4.11 | 51 ± 28 | 1.65 | 2.11 |
| EGF-25-3 | 0.25 | 11.78 | 18.037 ± 4.3 | 45 ± 23 | 1.77 | 3.08 |
| EGF-25-4 | 0.25 | 14.19 | 17.664 ± 3.83 | 39 ± 25 | 1.97 | 2.29 |
| EGF-25-5 | 0.25 | 16.82 | 17.522 ± 3.44 | 47 ± 25 | 1.77 | 2.16 |
| EGF-35-1 | 0.35 | 14.89 | 16.473 ± 1.94 | 45 ± 30 | 2.19 | 1.97 |
| EGF-35-2 | 0.35 | 15.80 | 16.36 ± 2.64 | 45 ± 26 | 2.30 | 1.76 |
| EGF-35-3 | 0.35 | 16.53 | 16.68 ± 2.72 | 51 ± 26 | 2.25 | 1.72 |
| EGF-35-4 | 0.35 | 17.27 | 16.717 ± 2.1 | 48 ± 25 | 2.04 | 1.82 |
| EGF-35-5 | 0.35 | 15.70 | 16.321 ± 1.7 | 40 ± 24 | 2.16 | 1.89 |
| EGF-45-1 | 0.45 | 18.33 | 15.395 ± 1.46 | 46 ± 27 | 2.19 | 1.60 |
| EGF-45-2 | 0.45 | 16.11 | 15.566 ± 1.95 | 43 ± 28 | 2.30 | 1.73 |
| EGF-45-3 | 0.45 | 16.02 | 15.911 ± 1.61 | 41 ± 25 | 2.25 | 1.78 |
| EGF-45-4 | 0.45 | 16.85 | 15.835 ± 2.3 | 46 ± 28 | 2.04 | 1.87 |
| EGF-45-5 | 0.45 | 15.37 | 15.786 ± 2.47 | 44 ± 22 | 2.16 | 1.93 |
| EGF-55-1 | 0.55 | 17.34 | 15.229 ± 1.21 | 49 ± 28 | 2.09 | 1.77 |
| EGF-55-2 | 0.55 | 18.02 | 15.177 ± 0.92 | 46 ± 25 | 2.34 | 1.52 |
| EGF-55-3 | 0.55 | 15.64 | 15.117 ± 0.84 | 42 ± 24 | 2.59 | 1.58 |
| EGF-55-4 | 0.55 | 17.49 | 15.178 ± 0.85 | 49 ± 23 | 2.12 | 1.73 |
| EGF-55-5 | 0.55 | 16.69 | 15.172 ± 1.04 | 47 ± 26 | 2.28 | 1.69 |
| EGF-65-1 | 0.65 | 18.57 | 14.65 ± 0.44 | 43 ± 27 | 3.09 | 1.12 |
| EGF-65-2 | 0.65 | 18.69 | 14.803 ± 0.52 | 47 ± 26 | 2.59 | 1.33 |
| EGF-65-3 | 0.65 | 20.31 | 14.674 ± 0.44 | 45 ± 27 | 2.93 | 1.08 |
| EGF-65-4 | 0.65 | 21.69 | 14.775 ± 0.39 | 47 ± 26 | 2.64 | 1.12 |
| EGF-65-5 | 0.65 | 17.89 | 14.763 ± 0.6 | 43 ± 25 | 2.90 | 1.24 |
| EGF-75-1 | 0.75 | 20.75 | 14.39 ± 0.17 | 43 ± 24 | 3.74 | 0.83 |
| EGF-75-2 | 0.75 | 21.32 | 14.439 ± 0.3 | 36 ± 23 | 4.08 | 0.74 |
| EGF-75-3 | 0.75 | 20.37 | 14.397 ± 0.11 | 46 ± 25 | 3.40 | 0.92 |
| EGF-75-4 | 0.75 | 20.69 | 14.38 ± 0.15 | 52 ± 21 | 2.85 | 1.09 |
| EGF-75-5 | 0.75 | 20.57 | 14.454 ± 0.21 | 41 ± 26 | 3.71 | 0.84 |

on the transverse strength predictions was implicitly accounted for during the data fitting process, where composite strength of the virtually manufactured microstructures were used to inform the analytical model development.

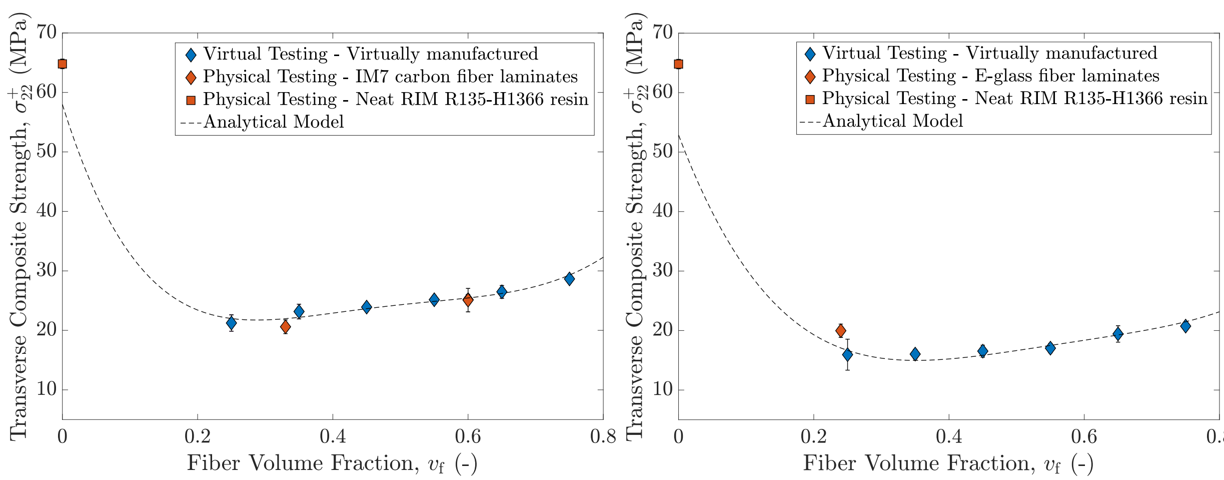
To explicitly determine this effect and to account for it in the improved analytical model, further investigation into the development of residual stress and the influence of random fiber distribution is warranted. It has been shown that fiber can influence how stress is generated during processing and redistributed during subsequent loading, which drives failure initiation and propagation in composites. A good understanding of these physical phenomena is necessary before its influence on the transverse composite strength can be explicitly modeled. Once its influence is quantified, a dedicated parameter can be introduced to capture the effect of process-induced residual stress on the transverse composite strength prediction.

Finally, the present model was developed using the nearest-neighbor distribution (based on short-range interaction) statistical descriptor to quantify the random fiber distribution and determine failure initiation in composite microstructures. However, it was observed that post-initiation, failure propagation through the microstructure not




(a) (b)

**FIG. 8:** Variation in $\mathbf{SCF_{II}}$ with the fiber volume fraction of (a) IM7 carbon fiber ($d_f = 6$ μm) and (b) E-glass fiber RVEs ($d_f = 14$ μm)




(a) (b)

**FIG. 9:** Comparison of the analytical model predictions for transverse composite strength with the numerical model predictions and experimental data for (a) IM7 carbon ($d_f = 6$ μm) and (b) E-glass fiber RVEs ($d_f = 14$ μm)

only depended on the fiber closeness near the initiation site but also on the local fiber distribution (local fiber volume fraction) that influenced residual stress generation during manufacturing and stress redistribution during subsequent mechanical loading. To quantify such geometrical variations and to fully capture their effect, more sophisticated statistical descriptors must be assessed. Despite these limitations, the proof of concept to develop a computational micromechanics-based analytical model for transverse composite strength prediction is promising and warrants further investigation.

## 4. CONCLUSIONS

This work presented a novel methodology to develop an analytical model for transverse composite strength for carbon and glass-reinforced epoxy resin using a numerical and statistical approach. Computational micromechanical analysis, involving virtual manufacturing and testing, were carried out on composite microstructures with varying fiber volume fractions to predict their transverse composite strength. A novel stress concentration factor for multi-fiber microstructures were defined based on the nearest neighbor statistical descriptor. A material system-specific analytical model was developed for transverse strength predictions based on the computational modeling results. It was shown that computational micromechanics can provide insight on the physical mechanisms that influenced the transverse composite strength, which are otherwise extremely challenging to quantify experimentally. Accounting for such physical mechanisms, a mathematical relation was established, which linked microscale failure in composites to these physical mechanisms, to effectively predict the transverse composite strength.

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No potential competing interest was reported by the authors.

## REFERENCES

Andrianov, I.V., Awrejcewicz, J., and Danishevskyy, V.V., *Asymptotical Mechanics of Composites*, Cham, Switzerland: Springer, 2018.

Bažant, Z.P. and Oh, B.H., Crack Band Theory for Fracture of Concrete, *Mater. Construct.*, vol. **16**, no. 3, pp. 155–177, 1983.

Beicha, D., Kanit, T., Brunet, Y., Imad, A., Moumen, A.E., and Khelfaoui, Y., Effective Transverse Elastic Properties of Unidirectional Fiber Reinforced Composites, *Mech. Mater.*, vol. **102**, pp. 47–53, 2016.

Bouaoune, L., Brunet, Y., El Moumen, A., Kanit, T., and Mazouz, H., Random versus Periodic Microstructures for Elasticity of Fibers Reinforced Composites, *Compos. B. Eng.*, vol. **103**, pp. 68–73, 2016.

Chamis, C., Simplified Composite Micromechanics for Predicting Microstresses, *J. Reinforced Plast. Compos.*, vol. **6**, no. 3, pp. 268–289, 1987.

Chamis, C.C., Simplified Composite Micromechanics Equations for Strength, Fracture Toughness, Impact Resistance and Environmental Effects, Tech. Rep. No. NASA TM-83696, Cleveland, OH, 1984.

Chamis, C.C., Abdi, F., Garg, M., Minnetyan, L., Baid, H., Huang, D., Housner, J., and Talagani, F., Micromechanics-Based Progressive Failure Analysis Prediction for WWFE-III Composite Coupon Test Cases, *J. Compos. Mater.*, vol. **47**, nos. 20-21, pp. 2695–2712, 2013.

Christensen, R.M., *Mechanics of Composite Materials*, New York: Dover Publication, 2012.

Danzi, F., Fanteria, D., Panettieri, E., and Mancino, M.C., A Numerical Micro-Mechanical Study on Damage Induced by the Curing Process in Carbon/Epoxy Unidirectional Material, *Compos. Struct.*, vol. **210**, pp. 755–766, 2019.

Deshpande, P.P., Shah, S., Patil, S., Kashmari, K., Olaya, M., Odegard, G.M., and Maiaru, M., Multiscale Modelling of the Cure Process in Thermoset Polymers Using ICME, *Proc. of ASC, 35th Technical Conf.*, ASC, Dayton, OH, pp. 701–708, 2020.

D'Mello, R.J., Maiaru, M., and Waas, A.M., Effect of the Curing Process on the Transverse Tensile Strength of Fiber-Reinforced Polymer Matrix Lamina Using Micromechanics Computations, *Integr. Mater. Manuf. Innov.*, vol. **4**, no. 1, pp. 119–136, 2015.

D'Mello, R.J., Maiaru, M., and Waas, A.M., Virtual Manufacturing of Composite Aerostructures, *Aeronaut. J.*, vol. **120**, no. 1223, pp. 61–81, 2016.

D'Mello, R.J., Waas, A.M., Maiaru, M., and Koon, R., Integrated Computational Modeling for Efficient Material and Process Design for Composite Aerospace Structures, *AIAA Scitech 2020 Forum*, AIAA, Reston, VA, 2020.

Elnekhaily, S.A. and Talreja, R., Damage Initiation in Unidirectional Fiber Composites with Different Degrees of Nonuniform Fiber Distribution, *Compos. Sci. Technol.*, vol. **155**, pp. 22–32, 2018.

Fedotov, A.F., Mori-Tanaka Experimental-Analytical Model for Predicting Engineering Elastic Moduli of Composite Materials, *Compos. B*, vol. **232**, p. 109635, 2022.

Fish, J., *Practical Multiscaling*, Hoboken, NJ: Wiley, 2014.

Fish, J., Wagner, G.J., and Keten, S., Mesoscopic and Multiscale Modelling in Materials, *Nat. Mater.*, vol. **20**, no. 6, pp. 774–786, 2021.

Flores, M., Sharits, A., Wheeler, R., Sesar, N., and Mollenhauer, D., Experimental Analysis of Polymer Matrix Composite Microstructures under Transverse Compression Loading, *Compos. A*, vol. **156**, p. 106859, 2022.

Gaikwad, P.S., Krieg, A.S., Deshpande, P.P., Patil, S.U., King, J.A., Maiaru, M., and Odegard, G.M., Understanding the Origin of the Low Cure Shrinkage of Polybenzoxazine Resin by Computational Simulation, *ACS Appl. Polym. Mater.*, vol. **3**, no. 12, pp. 6407–6415, 2021.

Ghayoor, H., Hoa, S.V., and Marsden, C.C., A Micromechanical Study of Stress Concentrations in Composites, *Compos. B*, vol. **132**, pp. 115–124, 2018.

Ghosh, S. and Dimiduk, D. (Eds.), *Computational Methods for Microstructure-Property Relationships*, Boston, MA: Springer, 2011.

Hashin, Z., Analysis of Properties of Fiber Composites with Anisotropic Constituents, *J. Appl. Mech.*, vol. **46**, no. 3, pp. 543–550, 1979.

Hashin, Z. and Rosen, B.W., The Elastic Moduli of Fiber-Reinforced Materials, *J. Appl. Mech.*, vol. **31**, no. 2, pp. 223–232, 1964.

He, C., Ge, J., Qi, D., Gao, J., Chen, Y., Liang, J., and Fang, D., A Multiscale Elasto-Plastic Damage Model for the Nonlinear Behavior of 3D Braided Composites, *Compos. Sci. Technol.*, vol. **171**, pp. 21–33, 2019.

Huang, Z.M., On Micromechanics Approach to Stiffness and Strength of Unidirectional Composites, *J. Reinf. Plastics Composites*, vol. **38**, no. 4, pp. 167–196, 2019.

Huang, Z.M., Micromechanical Failure Analysis of Unidirectional Composites, in *Failure Analysis*, Vol. 4, Rijeka, Croatia: IntechOpen, 2018.

Huang, Z.M. and Xin, L.M., *In Situ* Strengths of Matrix in a Composite, *Acta Mech. Sinica*, vol. **33**, no. 1, pp. 120–131, 2017.

Huang, Z.M. and Zhou, Y.X., *Strength of Fibrous Composites*, Berlin: Springer-Verlag, 2012.

Hui, X., Xu, Y., Wang, J., and Zhang, W., Microscale Viscoplastic Analysis of Unidirectional CFRP Composites under the Influence of Curing Process, *Compos. Struct.*, vol. **266**, p. 113786, 2021a.

Hui, X., Xu, Y., and Zhang, W., An Integrated Modeling of the Curing Process and Transverse Tensile Damage of Unidirectional CFRP Composites, *Compos. Struct.*, vol. **263**, p. 113681, 2021b.

Hyer, M.W. and White, S.R., *Stress Analysis of Fiber-Reinforced Composite Materials*, Philadelphia, PA: DEStech Publications, Inc, 2009.

Kamal, M.R. and Sourour, S., Kinetics and Thermal Characterization of Thermoset Cure, *Polym. Eng. Sci.*, vol. **13**, no. 1, pp. 59–64, 1973.

Liu, L. and Huang, Z.M., Stress Concentration Factor in Matrix of a Composite Reinforced with Transversely Isotropic Fibers, *J. Compos. Mater.*, vol. **48**, no. 1, pp. 81–98, 2014.

Liu, X., Furrer, D., Kosters, J., and Holmes, H., Vision 2040: A Roadmap for Integrated, Multiscale Modeling and Simulation of Materials and Systems, Tech. Rep. No. NASA/CR–2018-219771, 2018.

Liu, X. and Yu, W., Multiscale Modeling of Viscoelastic Behaviors of Textile Composites Using Mechanics of Structure Genome, *2018 AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conf.*, AIAA, Kissimmee, FL, 2018.

Liu, Y., Steven Greene, M., Chen, W., Dikin, D.A., and Liu, W.K., Computational Microstructure Characterization and Reconstruction for Stochastic Multiscale Material Design, *Comput.-Aided Des.*, vol. **45**, no. 1, pp. 65–76, 2013.

LLorca, J., González, C., Molina-Aldareguía, J.M., Segurado, J., Seltzer, R., Sket, F., Rodríguez, M., Sádaba, S., Muñoz, R., and Canal, L.P., Multiscale Modeling of Composite Materials: A Roadmap towards Virtual Testing, *Adv. Mater.*, vol. **23**, no. 44, pp. 5130–5147, 2011.

Maiaru, M., D'Mello, R.J., and Waas, A.M., Characterization of Intralaminar Strengths of Virtually Cured Polymer Matrix Composites, *Compos. B*, vol. **149**, pp. 285–295, 2018.

Maiaru, M., Effect of Uncertainty in Matrix Fracture Properties on the Transverse Strength of Fiber Reinforced Polymer Matrix Composites, *2018 AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, AIAA, Kissimmee, FL, 2018.

Mal, A.K. and Chatterjee, A.K., The Elastic Moduli of a Fiber-Reinforced Composite, *J. Appl. Mech.*, vol. **44**, no. 1, pp. 61–67, 1977.

Maragoni, L., Carraro, P.A., and Quaresimin, M., Development, Validation and Analysis of an Efficient Micro-Scale Representative Volume Element for Unidirectional Composites, *Compos. A*, vol. **110**, pp. 268–283, 2018.

Melro, A.R., Camanho, P.P., and Pinho, S.T., Generation of Random Distribution of Fibres in Long-Fibre Reinforced Composites, *Compos. Sci. Technol.*, vol. **68**, no. 9, pp. 2092–2102, 2008.

Mesogitis, T.S., Skordos, A.A., and Long, A.C., Uncertainty in the Manufacturing of Fibrous Thermosetting Composites: A Review, *Compos. A*, vol. **57**, pp. 67–75, 2014.

Naya, F., González, C., Lopes, C.S., Van der Veen, S., and Pons, F., Computational Micromechanics of the Transverse and Shear Behavior of Unidirectional Fiber Reinforced Polymers Including Environmental Effects, *Compos. A*, vol. **92**, pp. 146–157, 2017.

Nemat-Nasser, S. and Hori, M., *Micromechanics: Overall Properties of Heterogeneous Materials*, New York: Elsevier, 2013.

Paley, M. and Aboudi, J., Micromechanical Analysis of Composites by the Generalized Cells Model, *Mech. Mater.*, vol. **14**, no. 2, pp. 127–139, 1992.

Patil, S., Shah, S., Deshpande, P., Olaya, M., Kashmari, K., Odegard, G.M., and Maiaru, M., Multi-Scale Approach to Predict Cure-Induced Residual Stresses in an Epoxy System, *Proc. of ASC, 35th Technical Conf.*, ASC, Dayton, OH, pp. 709–716, 2020.

Patil, S.U., Shah, S.P., Olaya, M., Deshpande, P.P., Maiaru, M., and Odegard, G.M., Reactive Molecular Dynamics Simulation of Epoxy for the Full Cross-Linking Process, *ACS Appl. Polym. Mater.*, vol. **3**, no. 11, pp. 5788–5797, 2021.

Pineda, E.J., Bednarcyk, B.A., Waas, A.M., and Arnold, S.M., Progressive Failure of a Unidirectional Fiber-Reinforced Composite Using the Method of Cells: Discretization Objective Computational Results, *Int. J. Solids Struct.*, vol. **50**, no. 9, pp. 1203–1216, 2013.

Reifsnider, K., Sendeckyj, G., Wang, S., Feng, W., Johnson, W., Stinchcomb, W., Pagano, N., Caruso, J., and Chamis, C., Assessment of Simplified Composite Micromechanics Using Three-Dimensional Finite-Element Analysis, *J. Compos. Technol. Res.*, vol. **8**, no. 3, p. 77, 1986.

Romanov, V., Lomov, S.V., Swolfs, Y., Orlova, S., Gorbatikh, L., and Verpoest, I., Statistical Analysis of Real and Simulated Fibre Arrangements in Unidirectional Composites, *Compos. Sci. Technol.*, vol. **87**, pp. 126–134, 2013.

Sanei, S.H.R., Barsotti, E.J., Leonhardt, D., and Fertig, R.S., Characterization, Synthetic Generation, and Statistical Equivalence of Composite Microstructures, *J. Compos. Mater.*, vol. **51**, no. 13, pp. 1817–1829, 2017.

Schey, M.J., Beke, T., Appel, L., Zabler, S., Shah, S.P., Hu, J., Liu, F., Maiaru, M., and Stapleton, S., Identification and Quantification of 3D Fiber Clusters in Fiber-Reinforced Composite Materials, *JOM*, vol. **73**, pp. 2129–2142, 2021.

Shah, S. and Maiaru, M., Microscale Analysis of Virtually Cured Polymer Matrix Composites Accounting for Uncertainty in Matrix Properties during Manufacturing, in *Proc. American Society for Composites, 33rd Tech. Conf.*, Lancaster, PA: DEStech Publications, Inc., 2018.

Shah, S.P. and Maiaru, M., Effect of Manufacturing on the Transverse Response of Polymer Matrix Composites, *Polymers*, vol. **13**, no. 15, p. 2491, 2021.

Shah, S., Patil, S., Deshpande, P., Krieg, A., Kashmari, K., Al Mahmud, H., King, J., Odegard, G.M., and Maiaru, M., Multiscale Modeling for Virtual Manufacturing of Thermoset Composites, *AIAA Scitech 2020 Forum*, AIAA, Reston, VA, 2020a.

Shah, S., Plaka, E., Schey, M., Hu, J., Liu, F., Beke, T., Stapleton, S.E., and Maiaru, M., *In-Situ* Characterization of Polymer Matrix Composites and Progressive Damage Analysis of Virtually Reconstructed Microstructures, *Proc. of ASC, 35th Tech. Conf.*, ASC, Dayton, OH, pp. 932–942, 2020b.

Shah, S.P., Patil, S.U., Hansen, C.J., Odegard, G.M., and Maiaru, M., Process Modeling and Characterization of Thermoset Composites for Residual Stress Prediction, *Mech. Adv. Mater. Struct.*, vol. **30**, pp. 486–497, 2023.

Stapleton, S.E., Appel, L., Simon, J.W., and Reese, S., Representative Volume Element for Parallel Fiber Bundles: Model and Size Convergence, *Compos. A*, vol. **87**, pp. 170–185, 2016.

Trias, D., Costa, J., Turon, A., and Hurtado, J.E., Determination of the Critical Size of a Statistical Representative Volume Element (SRVE) for Carbon Reinforced Polymers, *Acta Mater.*, vol. **54**, no. 13, pp. 3471–3484, 2006.

Vaughan, T.J. and McCarthy, C.T., A Combined Experimental-Numerical Approach for Generating Statistically Equivalent Fibre Distributions for High Strength Laminated Composite Materials, *Compos. Sci. Technol.*, vol. **70**, no. 2, pp. 291–297, 2010.

Vignoli, L.L., Savi, M.A., Pacheco, P.M.C.L., and Kalamkarov, A.L., Comparative Analysis of Micromechanical Models for the Elastic Composite Laminae, *Compos. B*, vol. **174**, p. 106961, 2019.

Vignoli, L.L., Savi, M.A., Pacheco, P.M.C.L., and Kalamkarov, A.L., Micromechanical Analysis of Transversal Strength of Composite Laminae, *Compos. Struct.*, vol. **250**, p. 112546, 2020.

Yang, L., Yan, Y., Ma, J., and Liu, B., Effects of Inter-Fiber Spacing and Thermal Residual Stress on Transverse Failure of Fiber-Reinforced Polymermatrix Composites, *Comput. Mater. Sci.*, vol. **68**, pp. 255–262, 2013a.

Yang, L., Yan, Y., Ran, Z., and Liu, Y., A New Method for Generating Random Fibre Distributions for Fibre Reinforced Composites, *Compos. Sci. Technol.*, vol. **76**, pp. 14–20, 2013b.

Yuan, Z., Aitharaju, V., and Fish, J., A Coupled Thermo-Chemo-Mechanical Reduced-Order Multiscale Model for Predicting Process-Induced Distortions, Residual Stresses, and Strength, *Int. J. Numer. Methods Eng.*, vol. **121**, no. 7, pp. 1440–1455, 2020.

Zhang, T. and Yan, Y., A Comparison between Random Model and Periodic Model for Fiber-Reinforced Composites Based on a New Method for Generating Fiber Distributions, *Polym. Compos.*, vol. **38**, no. 1, pp. 77–86, 2017.

Zhao, L.G., Warrior, N.A., and Long, A.C., A Micromechanical Study of Residual Stress and Its Effect on Transverse Failure in Polymer-Matrix Composites, *Int. J. Solids Struct.*, vol. **43**, no. 18, pp. 5449–5467, 2006.