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Decoupling Uniaxial Tensile Prestress and Waveguide Effects From Estimates of the Complex Shear Modulus in a Cylindrical Structure Using Transverse-Polarized Dynamic Elastography

Dynamic elastography, whether based on magnetic resonance, ultrasound, or optical modalities, attempts to reconstruct quantitative maps of the viscoelastic properties of biological tissue, properties altered by disease and injury, by noninvasively measuring mechanical wave motion in the tissue. Most reconstruction strategies that have been developed neglect boundary conditions, including quasi-static tensile or compressive loading resulting in a nonzero prestress. Significant prestress is inherent to the functional role of some biological tissues currently being studied using elastography, such as skeletal and cardiac muscle, arterial walls, and the cornea. In the present article a configuration, inspired by muscle elastography but generalizable to other applications, is analytically and experimentally studied. A hyperelastic polymer phantom cylinder is statically elongated in the axial direction while its response to transverse-polarized vibratory excitation is measured. We examine the interplay between uniaxial prestress and waveguide effects in this muscle-like tissue phantom using computational finite element simulations and magnetic resonance elastography measurements. Finite deformations caused by prestress coupled with waveguide effects lead to results that are predicted by a coordinate transformation approach that has been previously used to simplify reconstruction of anisotropic properties using elastography. Here, the approach estimates material viscoelastic properties that are independent of the nonhomogeneous prestress conditions without requiring advanced knowledge of those stress conditions.

[DOI: 10.1115/1.4056411]

Keywords: elastography, acoustoelasticity, viscoelastic

1 Introduction

Dynamic elastography methods—based on optical, ultrasonic, and magnetic resonance imaging modalities—aim to quantitatively map the shear viscoelastic properties of biological tissue, which are often altered by disease and injury. Most studies to date have focused on larger organs, such as the liver or brain, where boundary effects were assumed negligible. But, as elastography is applied to other anatomical regions where dimensions in at least one direction are smaller or of comparable length to bulk shear wavelengths—such as in slender skeletal muscles, blood vessel walls, the heart wall, and the cornea—boundary effects become non-negligible and must be considered. Researchers using optical elastography to assess the viscoelastic properties of the cornea have long recognized this, adapting models to include waveguides by treating the cornea as a plate-like structure. Here, transverse wave motion on the cornea is modeled as Rayleigh–Lamb waves [1]. Blood vessels, as well, have been modeled using cylindrical shell equations considering fluid-structure interaction [2–4]. Limited studies on cardiac elastography have also acknowledged the

frequency-dependent (i.e., wavelength-dependent) waveguide behavior of the heart wall [5].

Often, when elastography studies are done under varying nonzero quasi-static prestress conditions, observed changes in mechanical wave behavior are attributed solely to the nonlinear property of the tissue: it's been suggested that its shear and viscous constants are highly dependent on the tensile load and associated deformation. A recent article provides a summary of the literature relevant to this issue, in particular for uniaxially prestressed cylindrically-shaped structures, as well as biaxially prestressed plate-like structures [6].

In the present study, focused on uniaxially prestressed cylindrical structures, the confounding effects of finite dimensions and prestress are further explored, analytically and experimentally. Furthermore, we articulate and evaluate a strategy for decoupling prestress and waveguide effects from estimates of material shear viscoelastic properties.

2 Theory

2.1 Shear Wave Motion in a Uniaxially Prestressed Linear Viscoelastic Material. Dynamic elastography methods are based on the assumption that the measured transverse wave speed or wavelength for small amplitude (linear theory assumption) motion is directly related to the material's elastic or viscoelastic properties [7]. Assuming isotropy, homogeneity and neglecting any

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Contributed by the Applied Mechanics Division Technical Committee on Dynamics & Control of Structures & Systems (AMD-DCSS) of ASME for publication in the JOURNAL OF ENGINEERING AND SCIENCE IN MEDICAL DIAGNOSTICS AND THERAPY. Manuscript received September 4, 2022; final manuscript received November 26, 2022; published online December 23, 2022. Assoc. Editor: Mostafa Fatemi.

boundary effects or variation in density ρ in a nearly incompressible viscoelastic material, the frequency-dependent shear wave phase speed, $c[\omega]$, for harmonic excitation at circular frequency, ω , is

$$c = \omega / \text{Real}[k[\omega]] = \frac{1/\sqrt{\rho}}{\text{Real}\left[1/\sqrt{\mu[\omega]}\right]} \quad (1)$$

Here, $k[\omega]$ is the complex-valued, frequency-dependent shear wave number and $\mu[\omega]$ is the complex-valued, frequency-dependent shear modulus, comprised of the shear storage modulus, $\mu_R[\omega]$, and the shear loss modulus, $\mu_I[\omega]$, such that $\mu[\omega] = \mu_R[\omega] + j\mu_I[\omega]$ where $j = \sqrt{-1}$. The attenuation rate of the wave as it propagates is governed by the imaginary part of the wave number: $\text{Imag}[k]$. In a viscoelastic material, both the shear storage and loss moduli affect both the phase speed and attenuation rate. In a purely elastic material, $\mu_I = 0$, there is no attenuation and the phase speed is independent of frequency (nondispersive) and reduces to $c = \sqrt{\mu/\rho}$. While some studies have assumed pure elasticity (no viscosity), often their analyses can be generalized to the linear viscoelastic problem for harmonic motion by simply adding the imaginary shear loss modulus to form the complex shear modulus.

Consider the introduction of a uniaxial static prestress, $\sigma = \sigma \hat{k}$, that is parallel to the z -axis, as illustrated in Fig. 1. The z -axis is chosen here since it is the same in both the Cartesian (x, y, z) and polar (r, φ, z) coordinate systems, and is the axis of the cylinder that will be used in the present study. If the static deformation due to the prestress is assumed to be small or incremental, infinitesimal strain theory can be used to incorporate σ into the equations of motion leading to the following in Cartesian coordinates [8,9]

$$\rho u_{,tt} = \left(\kappa + \frac{4\mu}{3}\right) u_{,xx} + \mu u_{,yy} + \left(\mu + \frac{\sigma}{2}\right) u_{,zz} + \left(\kappa + \frac{\mu}{3}\right) v_{,xz} + \left(\kappa + \frac{\mu}{3} - \frac{\sigma}{2}\right) w_{,xz} \quad (2)$$

$$\rho v_{,tt} = \left(\kappa + \frac{4\mu}{3}\right) v_{,yy} + \mu v_{,xx} + \left(\mu + \frac{\sigma}{2}\right) v_{,zz} + \left(\kappa + \frac{\mu}{3}\right) u_{,xy} + \left(\kappa + \frac{\mu}{3} - \frac{\sigma}{2}\right) w_{,yz} \quad (3)$$

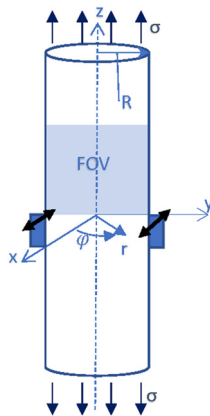


Fig. 1 Uniaxially prestressed cylinder. Shaded region (FOV) shows location of images presented in Figs. 2 and 4. This is a sagittal slice in the z - y plane. Deeper shaded blocks with transverse (x -direction) black two-way arrows denote cross section of rigid band surrounding the cylinder that delivers x -polarized harmonic actuation.

$$\rho w_{,tt} = \left(\kappa + \frac{4\mu}{3}\right) w_{,zz} + \left(\mu - \frac{\sigma}{2}\right) w_{,yy} + \left(\mu - \frac{\sigma}{2}\right) w_{,xx} + \left(\kappa + \frac{\mu}{3} + \frac{\sigma}{2}\right) u_{,xz} + \left(\kappa + \frac{\mu}{3} + \frac{\sigma}{2}\right) v_{,yz} \quad (4)$$

Here u , v , and w refer to the displacement component in the x , y , and z direction, respectively, and subscripted x , y , z , and t after a comma refer to partial derivatives with respect to that spatial or time dimension. The term κ is the bulk modulus of the material, which will affect compression wave behavior, but not shear wave behavior. In biological soft tissue or other nearly incompressible materials, κ is multiple orders of magnitude greater than μ such that the Poisson's ratio for the material approaches, but does not equal 0.5.

The prestress alters the otherwise isotropic, direction-invariant, nature of the medium. Taking the case of harmonic excitation at circular frequency, ω , solving the above equations leads to expressions for the compression (longitudinal) wave where motion polarization is parallel to the direction of propagation, and to two shear waves where motion polarization is perpendicular to the direction of wave propagation. For one of the shear waves, referred to as the slow shear wave with phase speed in the elastic case given as c_s , the motion polarization is perpendicular to the plane created by the direction of propagation and the direction of the uniaxial stress. The other shear wave propagating along the same direction, but with polarization within the plane created by the propagation direction and stress direction, is referred to as the fast shear wave with phase speed in the elastic case given as c_f . Expressions for these phase speeds squared are as follows.

$$c_s^2[\theta] = \frac{\mu}{\rho} \left(1 + \frac{\sigma}{2\mu} \cos^2[\theta]\right) \quad (5)$$

$$c_f^2[\theta] = \frac{\mu}{\rho} \left(1 + \frac{\sigma}{2\mu} \cos[2\theta]\right) \quad (6)$$

Here, θ is the angle between the direction of propagation and the axis of uniaxial stress.

The above analysis based on infinitesimal strain theory loses accuracy as the strain caused by σ becomes significant, e.g., beyond a few percent. As strain increases it may be necessary to use finite strain theory, also known as large deformation theory, to account for changed geometry, and with it use a hyperelastic model of the material properties that may introduce nonlinearity into the static analysis. If the harmonic waves imposed upon the finitely deformed medium are, themselves, of small amplitude oscillatory motion, it may still be reasonable to use a linearized analysis to describe wave motion, with linearized parameter values dependent on the degree of deformation. In the analysis of small amplitude wave motion imposed upon finitely deformed hyperelastic materials, it is common to employ a finite strain energy function ψ to describe the material's properties as static prestress or strain is applied. Many different material models and functions have been introduced and used to describe the behavior of nearly incompressible materials, like soft biological tissue [10–15]. For the present study, limiting our focus to uniaxial prestress of a nearly incompressible isotropic material and considering finite strains of no more than 20%, we will use a Gent hyperelastic model, which can be defined using the following finite strain energy function [16]

$$\psi = -\frac{\mu J_m}{2} \ln \left[1 - \frac{\bar{I}_1 - 3}{J_m}\right] + \frac{\kappa}{2} \left(\frac{J^2 - 1}{2} - \ln[J]\right) \quad (7)$$

where $\bar{I}_1 = J^{-2/3} I_1$, with $I_1 = \text{tr}[C]$, $C = F^T F$, $J = \text{Det}[F]$, and J_m is a limiting parameter for $\bar{I}_1 - 3$. Here, F is the deformation gradient, C is the right Cauchy-Green tensor, and $J = 1$ if the material is incompressible. The first term of the strain energy function

is based on the isochoric deformation of the isotropic material, and the second term only exists if the material is compressible. The shear modulus is given by $2\frac{\partial\psi}{\partial I_1}$, which in the incompressible and small strain limit is equal to μ and is consistent with infinitesimal strain theory. Consequently, Eqs. (5) and (6), governing planar shear wave phase speed should remain valid here for nearly incompressible materials. Utilization of other strain energy functions that work for larger deformations is left for future study. It is also possible to incorporate the prestress field directly into the strain energy function [17,18]. If it is assumed that the superimposed wave motion on top of the finite deformation is small, one will still recover Eqs. (5) and (6) for planar shear wave phase speed.

2.2 Waveguide Effects. The above theoretical analyses do not take into consideration the finite dimensions of the cylindrical medium shown in Fig. 1, namely, its finite radius, R . Returning to infinitesimal strain theory and neglecting axial prestress σ , for so-called flexural modes—transverse motion with displacement of the rod central axis orthogonal to the axis—we restrict ourselves to the case of $n=1$ (transverse beam or lobar modes) in Eq. 8.2.22 of Graff (1991) [19] leading to the following coupled equations for determining displacement in radial, circumferential and axial directions, respectively.

$$u_r[r, \theta, z, t] = \left(\frac{\alpha}{2} A(J_0[\alpha r] - J_2[\alpha r]) + \frac{1}{r} B_3 J_1[\beta r] + \xi(B_1 J_0[\beta r] + B_2 J_2[\beta r]) \right) \cos[\theta] e^{j(\omega t - \xi z)} \quad (8)$$

$$u_\theta[r, \theta, z, t] = \left(-\frac{1}{r} A J_1[\alpha r] + \xi(B_1 J_0[\beta r] + B_2 J_2[\beta r]) - \frac{\beta}{2} B_3 (J_0[\beta r] - J_2[\beta r]) \right) \sin[\theta] e^{j(\omega t - \xi z)} \quad (9)$$

$$u_z[r, \theta, z, t] = \left(\frac{\beta}{2} (B_2 J_0[\beta r] - B_2 J_2[\beta r] - B_1 J_1[\beta r]) - \xi A J_1[\alpha r] - \frac{2}{r} (B_1 J_0[\beta r] + B_2 J_2[\beta r]) \right) \cos[\theta] e^{j(\omega t - \xi z)} \quad (10)$$

Here, J denotes a Bessel function of the first kind of $\# = 0$ th, 1st, or 2nd order, and α and β are wavenumbers in the r (radial) direction. The wavenumber in the z (axial) direction, ξ , is determined by satisfying a complex frequency equation found in the reference (Eqs. 8.2. and 24-25) [19]. Note, ξ is in all three equations. In the present study our focus is transverse wave motion, which with respect to the polar coordinate system, will have motion polarization in the r and θ direction. Our focus will be to measure the evolution of that motion along the z direction in order to estimate ξ . In the present study, we are interested in incorporating uniaxial prestress σ and, if possible, would like to identify simpler analytical expressions than the above analysis affords.

2.3 Prestressed One-Dimensional Thin Waveguide Under Transverse Excitation. By assuming that the radius R of the cylinder is small enough such that $\beta R \ll 1$, then so-called one-dimensional beam theory approximations can be applied. The simplest of these is the Euler–Bernoulli thin beam theory, which easily allows incorporation of prestress σ . Referring again to Fig. 1, the pretensioned Euler–Bernoulli thin beam theory described in Sec. 3.3.4 of Graff [19] for x -polarized transverse wave propagation of the beam along its z -axis is the following:

$$EI u_{,zzzz} - \sigma A u_{,zz} + A \rho u_{,tt} = 0 \quad (11)$$

Here, I is the area moment of inertia about the y -axis ($I = \frac{\pi}{4} R^4$ for a circular cross section of radius R), and A is the cross-sectional area in the x - y plane. The general solution form is $u = U e^{j(\omega t - \gamma x)}$ where γ has four possible solutions

$$\gamma = \begin{matrix} + \\ - \end{matrix} \alpha, \begin{matrix} + \\ - \end{matrix} j\beta \quad (12a)$$

$$\alpha = \left\{ -\xi + \left(\xi^2 + \frac{\omega^2}{a^2} \right)^{1/2} \right\}^{1/2} \quad (12b)$$

$$\beta = \left\{ \xi + \left(\xi^2 + \frac{\omega^2}{a^2} \right)^{1/2} \right\}^{1/2} \quad (12c)$$

$$\xi = \frac{\sigma A}{2E_{||} I} \quad (12d)$$

$$a = \sqrt{\frac{EI}{\rho A}} \quad (12e)$$

We have two propagating waves (α) in the $+$ or $-z$ direction, and 2 nonpropagating (near field or evanescent) waves (β) in the $+$ or $-z$ direction. For the propagating waves, the phase speed will be: $c_{ph} = \frac{\omega}{\text{Real}[\alpha]}$. Taking the limit that $\sigma A \ll 2EI$ we see that $\alpha = \left(\frac{\omega}{a} \right)^{1/2}$ and thus for the elastic case $c_{ph} = \omega^{1/2} \left(\frac{EI}{\rho A} \right)^{1/4}$, which is the classic thin (Euler–Bernoulli) beam transverse vibration solution. On the other hand, taking the limit of tension $\sigma A \gg 2EI$ we then drop $EI u_{,zzzz}$ from Eq. (11) and reformulate the solution to find that there are two propagating solutions with $c_{ph} = \left(\frac{E}{\rho} \right)^{1/2}$. This is the classic transverse thin-string vibration solution. A case between the extremes of either neglecting σA or EI still does not match the phase speed of bulk shear waves, which is given by Eq. (5).

2.4 Prestressed One-Dimensional Thick Waveguide Under Transverse Excitation. The thin beam formulation presented in Sec. 2.3 is only a reasonable approximation when the wavelength of the propagating transverse waves is at least an order of magnitude greater than the beam's cross-sectional radius R , or equivalent radius for a noncircular cross-section. A formulation yielding a valid approximation for shorter transverse waves is based on the Timoshenko beam theory, which allows for shear deformation and accounts for rotational inertia.

Incorporating prestress into the formulation for Timoshenko beam theory given in Sec. 3.4 of Graff [19], we have the following:

$$EI u_{,zzzz} - \sigma A u_{,zz} + A \rho u_{,tt} - I \rho \left(1 + \frac{E}{\kappa \mu} \right) u_{,zztt} + \frac{\rho^2 I}{\kappa \mu} u_{,tttt} = 0 \quad (13)$$

Here, κ is the Timoshenko shear coefficient, equal to $10/9$ for a circular cross-section. Consider harmonic motion at frequency ω and the general solution form is: $u = U e^{j(\omega t - \gamma x)}$ and γ has four possible solutions:

$$\gamma^4 EI + \gamma^2 \left(\sigma A - \omega^2 I \rho \left(1 + \frac{E}{\kappa \mu} \right) \right) - \omega^2 \left(\rho A - \omega^2 \frac{\rho^2 I}{\kappa \mu} \right) = 0 \quad (14a)$$

$$\gamma = \begin{matrix} + \\ - \end{matrix} \left\{ -\xi \pm \left(\xi^2 + \frac{\omega^2}{a^2} \right)^{1/2} \right\}^{1/2} \quad (14b)$$

$$\xi = \frac{8}{D^2} \epsilon_L - \omega^2 \frac{37\rho}{60\mu} \quad (14c)$$

$$a = \sqrt{\frac{3\mu/\rho}{\frac{16}{D^2} - \omega^2 \frac{\rho}{\kappa\mu}}} \quad (14d)$$

In this case, we cannot separate the wavenumber solutions into propagating and nonpropagating pairs. Rather, there will be certain prestress-dependent frequency ranges where we have only one propagating pair and others where we have two propagating pairs.

2.5 Accounting for Uniaxial Prestress in the Three-Dimensional Equations Using Transformation Acousto-Elastography. If the dimensions of the cylinder are such that neither thin nor thick beam approximations are sufficiently accurate, one is left to somehow integrate the prestress condition introduced in Sec. 2.1 into Eqs. ((8)–(10)) of Sec. 2.2. A novel approach is proposed to accomplish this with reduced mathematical complexity. In previous studies on transversely isotropic materials not under prestress [20–23] the last author has shown that, by distorting the geometry based on direction and polarization-dependent planar phase speeds, one can then solve an equivalent isotropic problem. This approach to the transverse isotropic problem was called transformation elastography. Uniaxial prestress causes a similar, though not identical, direction and polarization dependence of the planar shear wave phase speed, as exhibited in Eqs. (5) and (6). The same approach is adapted to the acoustoelastic problem here. Note, different geometric distortions may be needed depending on whether the wave motion of interest is a slow or a fast shear wave, based on propagation direction and polarization. The wave polarization of interest here is perpendicular to the uniaxial stress direction and this is governed by slow shear waves.

Referring to Eq. (5), the phase speed in the direction of the prestress axis divided by the phase speed in the unstressed medium is $\frac{c_s(\theta=0^\circ)}{\sqrt{\mu/\rho}} = \sqrt{1 + \frac{\sigma}{2\mu}}$ and perpendicular to the stress axis is $\frac{c_s(\theta=90^\circ)}{\sqrt{\mu/\rho}} = 1$. Approximating the material as incompressible, the prestress results in a static strain of the cylindrical phantom of length L and radius R , changing its axial length to $L(1 + \frac{\sigma}{3\mu})$ and its radius to $R/\sqrt{1 + \frac{\sigma}{3\mu}}$. The equivalent isotropic cylindrical phantom of length L_e and radius R_e needs to have its dimensions adjusted so that the planar phase speed is $\sqrt{\mu/\rho}$ in all directions. This results in:

$$L_e = L \left(1 + \frac{\sigma}{3\mu} \right) / \sqrt{1 + \frac{\sigma}{2\mu}} = L(1 + \epsilon_L) / \sqrt{1 + \frac{3}{2}\epsilon_L} \quad (15)$$

$$R_e = R / \left(\sqrt{1 + \frac{\sigma}{3\mu}} \right) = \frac{R}{\sqrt{1 + \epsilon_L}} \quad (16)$$

Here, ϵ_L is the axial strain. When the formulas are expressed in terms of this, it's clear that one does not need to know σ and μ a priori in order to calculate the equivalent dimensions. One only needs know the axial strain. If the compressibility is not negligible Eq. (16) will need some adjustment.

3 Analytical/Numerical Case Study—Transformation Acousto-Elastography Validation

An analytical and numerical case study was conducted to understand the interactions between uniaxial prestress and waveguide behavior, as well as to validate the proposed transformation

Table 1 Parameter values for case studies

Parameter	Nomenclature	Value(s)
Bulk modulus	κ	2 GPa
Shear storage modulus	μ_R	42.8 kPa
Limiting parameter	J_m	50
Ratio of shear loss to storage moduli	$\eta = \mu_I / \mu_R$	0.1882
Undeformed phantom length	L	100 mm
Undeformed phantom radius	R	17.5 mm
Uniaxial tensile strain	ϵ_L	0, .05, 0.1, 0.2
Density	ρ	1070 $\frac{\text{kg}}{\text{m}^3}$
Frequency	f	600 Hz

acousto-elastography (TAE) approach proposed above. Case study geometric and material property values are provided in Table 1.

Based on these values, equations in Secs. 2.3 and 2.4 can be used to calculate the wavenumber γ and wavelength λ for transverse motion based on thin and thick beam theory, respectively. In order to numerically simulate the approach outlined in Sec. 2.5, finite element analysis (FEA) was performed using ANSYS Mechanical APDL Version 2022 R2. An axisymmetric mixed u-P formulation was used with Solid273 8-node by three circumferential plane generalized axisymmetric elements with individual element side dimensions in the axial and radial direction of 0.5 mm. Nonlinear attributes of the element were enabled and the material was defined by a hyperelastic Gent model, requiring specification of the bulk modulus, the real part of complex shear modulus, a limiting parameter, J_m , as well as the density. One end of the cylinder was fixed and the other end was incrementally displaced in the z -direction, solving the problem in steps until the desired end displacement was reached that resulted in a uniform uniaxial strain throughout the model.

Once the static analysis was done, the solution routine was exited and then reentered this time specifying a harmonic solution routine using the modified stiffness matrix acquired at the end of the static solution routine and the frequency provided in Table 1 (600 Hz). Now, harmonic x -direction displacements were applied to the nodes either on the entire end of the cylinder or just on the nodes at the outer radial edge of the end of the cylinder representing a rigid transverse x -polarized oscillating end or ring in contact with the end of the phantom (see Fig. 1). In addition to the material properties specified for the static analysis, it is also necessary to specify viscous (beta) damping, which was done by taking the ratio of the imaginary to the real part of the shear modulus in Table 1 and dividing by $2\pi f$ where f is the harmonic driving frequency.

The in-phase steady-state response over a sagittal slice is shown in Fig. 2 for selected cases in Tables 1 and 2. For the case that the entire end is transversely oscillated (upper row in Fig. 2), the response pattern is uniform and a line profile can be taken along the axial direction at any radial position to estimate the complex wavenumber γ by fitting the profile to $Ae^{-j\gamma z}$. For the case that the outer radial edge at the end has oscillated (lower row in Fig. 2), the response is more complex near the excitation source, but far enough away from the source begins to resemble the simpler case. For this case, the complex wavenumber γ is estimated by fitting $Ae^{-j\gamma z}$ to axial line profiles near the outer edge ($r = R$). Once γ is estimated, the wavelength $\lambda = 2\pi/\text{Real}[\gamma]$. The TAE-adjusted values for these, $\gamma_{N/TAE}$ and $\lambda_{N/TAE}$, are determined by first distorting the axial z dimension according to Eq. (15). Specifically, the axial location of the nodes (which have been extended already by $1 + \epsilon_L$) is then divided by $\sqrt{1 + \frac{3}{2}\epsilon_L}$. This alters both the wavenumber and wavelength.

Table 2 summarizes results obtained for 0, 5, 10, and 20% axial prestrain cases. The first four rows of the table provide the complex shear modulus value used in the thin and thick beam theoretical calculations and in the finite element simulations, the associated shear wavenumber k_{sh} , and the calculated shear

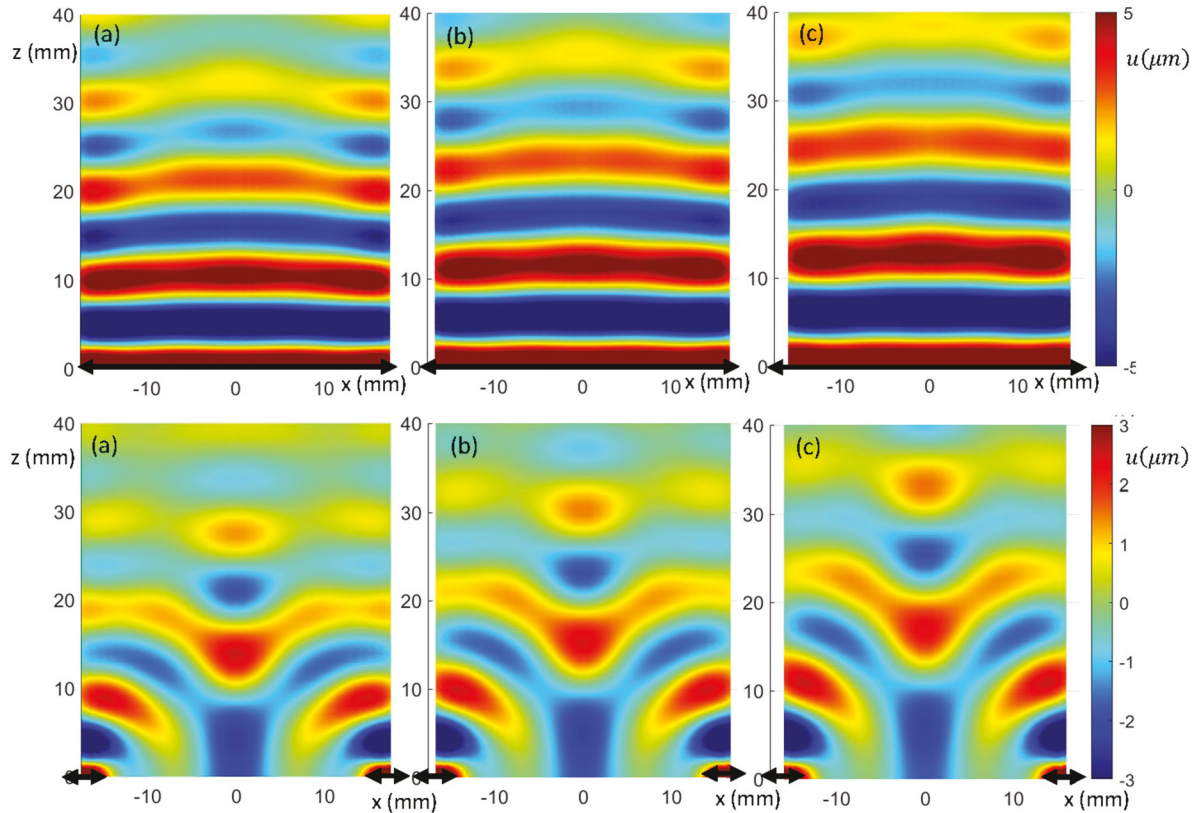


Fig. 2 Sagittal view of transverse (x)-polarized in-phase wave motion from FE simulation. ((a)–(c)) 0, 10, and 20% axial prestrain, respectively. Note reduced radius with increased prestrain. Upper set of images is with transverse oscillatory motion (indicated by arrow) applied to the entire end of the cylinder. Lower set of images, more akin to the experiment, is with transverse oscillatory motion (indicated by arrows) applied only on the outer surface at the end of the cylinder. Color indicates amplitude of x-polarized motion.

wavelengths along the axis λ_{s0° and perpendicular to the axis λ_{s90° , based on Eq. (5). Next, estimates of propagating wavenumber and wavelength based on prestressed thin beam theory (Sec. 2.3), γ_{Thin} and λ_{Thin} , are provided, followed by estimates based on prestressed thick beam theory (Sec. 2.4), γ_{Thick} and λ_{Thick} . Thin beam theory wavelengths and wavenumbers are in error by more than a factor of 2, as compared to the true values in the first two rows of Table 2. For thick beam theory, there are two pairs of propagating wavenumber and wavelength values at this frequency. For the pairs that are closer to truth, marked with an asterisk*, the corresponding complex shear modulus is calculated based on Eq. (1) and provided in the line below λ_{Thick} . Its percent difference from the true value of $42.8 + j8.05$ kPa is also provided, showing a consistent bias of about 9–10% on the high side for the shear storage modulus while being more accurate in estimating the relative value of the shear loss to shear storage modulus. It is expected that the thin or thick beam approximate theories will yield estimates with higher shear moduli, or lower shear wavenumbers, since such simplifying approximations provide greater constraints on how the material will deform, as compared to the full three-dimensional theory.

The next section of the table summarizes results of the numerical FEA simulation coupled with the TAE coordinate transformation that was introduced in Sec. 2.5. The wavenumber and wavelength prior to the coordinate transformation are first provided, γ_N and λ_N , followed by their values after the TAE coordinate transformation, $\gamma_{N/\text{TAE}}$ and $\lambda_{N/\text{TAE}}$. The complex shear modulus is subsequently calculated based on Eq. (1) and provided in the line below $\lambda_{N/\text{TAE}}$, as well as its percent difference from the true value. Consistently, this approach has outperformed thick beam theory, with percentage differences in shear storage modulus ranging from 1 to 7.5% and excellent estimates of the ratio of

shear loss to shear storage modulus. Note the importance of undergoing the TAE coordinate transformation in that the wave-number before and after transformation significantly diverges with increasing prestrain.

4 Experiment

The setup was designed and tested on an Agilent 9.4 Tesla horizontal, 30 cm bore, animal MR system. All fixture parts for the setup were designed in Solidworks (Solidworks 2021) and printed using a fused filament extrusion printer (Prusa Mk3, PRUSA REF) on Polyethylene terephthalate glycol (PETG) to limit interference with signal and keep parts light enough to be moved efficiently by the piezostack. Figure 3 shows the overall design, which prioritizes versatility in sample type, from phantoms, to muscle tissue or large structures that can be excised to fit the dimensions of the gradient coil opening.

Samples are gripped by two clamp types: a fixed clamp at the proximal end, and the tensioner clamp at the distal end. Both clamp types are slotted to allow wooden skewers to penetrate the sample, providing reasonably distributed tension, rather than surface tension that would be only available with typical clamps. The distal clamp is then attached to a reinforced nylon wire that is fed out the back of the magnet bore and attached to a pulley system where adjustable weight applies appropriate tension.

Simultaneous harmonic loading is achieved through a piezostack (P842.10, PI-USA LP) located proximally to the entry of the bore to limit interference from wiring, and attached to a Delrin counterweight to force transference of motion toward the sample. The sample is placed in a cylindrical tube, the actuator, which is attached directly to the piezo and slotted for adjustable rings that uniformly and circumferentially grip the sample, providing a fixed

Table 2 Wavenumber and wavelength estimates based on theory, numerical, and experimental studies

Prestrain ϵ_L	0%	5%	10%	20%
$\mu_R + j\mu_I$ (kPa)			42.8+j8.05	
k_{sh} (m ⁻¹)—actual			588.4-j54.9	
λ_{s0° (mm)	10.7	11.1	11.5	12.2
λ_{s90° (mm)	10.7	10.7	10.7	10.7
γ_{Thin} (m ⁻¹)	279-j13.0	277-j13.1	274-j13.2	270-j13.4
λ_{Thin} (mm)	22.5	22.7	22.9	23.3
γ_{Thick} (m ⁻¹)	565-j51.5*	564-j51.6*	563-j51.7*	561-j51.8*
λ_{Thick} (mm)	329-j32.7	329-j32.6	330-j32.6	331-j32.5
$\mu_R + j\mu_I$ (kPa)*	11.1*/19.1	11.1*/19.1	11.2*/19.1	11.2*/19.0
$\mu_R + j\mu_I$ (kPa)*	46.5+j8.56	46.6+j8.61	46.8+j8.66	47.1+j8.76
% Error	8.59+j0.43	8.92+j0.38	9.26+j0.32	9.93+j0.20
γ_N (m ⁻¹)	602-j57.6	571-j54.2	545-j51.7	498-j47.0
λ_N (mm)	10.4	11.0	11.5	12.6
$\gamma_{N/TAE}$ (m ⁻¹)	602-j57.6	592-j56.1	585-j55.4	568-j53.6
$\lambda_{N/TAE}$ (mm)	10.4	10.6	10.7	11.1
$\mu_R + j\mu_I$ (kPa)	40.8+j7.90	42.2+j8.07	43.3+j8.28	45.9+j8.74
% Error	4.48+j0.48	1.35+j0.31	1.21+j0.31	7.27+j0.23
σ (kPa)	0	6.33	12.99	27.54
σ_{FEA} (kPa)	0	6.43	12.89	26.02
σ Est Error %	0	1.56	0.78	5.84
γ_E (mm ⁻¹)	559-j60.1	545-j87.7	518-j66.9	NA
λ_E (mm)	11.2	11.5	12.1	NA
$\gamma_{E/TAE}$ (mm ⁻¹)	559-j60.1	559-j89.9	544-j70.1	NA
$\lambda_{E/TAE}$ (mm)	11.2	11.2	11.6	NA
$\mu_R + j\mu_I$ (kPa)	47.1+j10.3	45.1+j14.9	48.9+j12.9	NA
σ (kPa)	0	6.76	14.67	NA
σ_{Exp} (kPa)	0	4.33	8.55	NA
σ Est Error %	0	56.1	71.6	NA

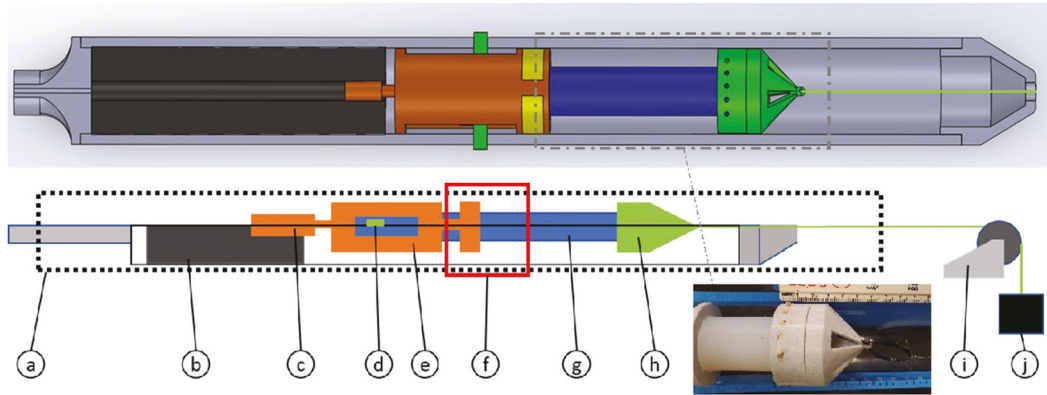


Fig. 3 Experimental setup. Diagram with top and side view, and photo from top showing cylindrical polymer phantom and claw-grip tensiometer. Color-coded view from side: (a) bore constraint, (b) piezo-counter mass, (c) piezostack, (d) fixed phantom end clamp, (e) harmonic actuator, (f) image ROI, (g) pre-stressed phantom, (h) claw-grip tensiometer, (i) pulley (outside magnet), and (j) adjustable weight.

location of the wave source that is 15 mm thick. While, schematically it appears that this setup would provide harmonic vibration that is polarized along the z -axis, the axis of the cylindrical phantom and of the tensile load, because of any small nonaxisymmetries, for example, due to gravity, the resulting excitation has both z -axis and orthogonal x -axis components.

The phantom was made with EcoflexTM 00-30 (Smooth-On, Inc., Macungie, PA) platinum-catalyzed silicone. Ecoflex silicone bases were combined in a 1 A:1B ratio and cured at room temperature with minimum shrinkage. Air bubbles produced during the mixing process were removed using a vacuum chamber (5305-1212, Thermo Scientific-Nalgene, Rochester, NY) for 5 to 10 min to minimize porosity and nonhomogeneous bulk material properties.

SLIM MRE [24] provided motion encoding in three orthogonal directions, simultaneously. Sequence parameters were TR/

TE = 1600/25 ms with 8-time steps and corresponding 180 deg offsets. A 250 mT/m MEG was used with ten cycles. The data matrix size was $64 \times 64 \times 40$ with a FOV of $48 \text{ mm} \times 48 \text{ mm} \times 30 \text{ mm}$ for an isotropic voxel size of 0.75 mm. The piezostack provided an input axial harmonic motion of $10 \mu\text{m}$ peak amplitude.

Transverse-polarized motion in the sagittal plane is shown in Fig. 4 for the cases of 0, 5, and 10% axial prestrain. The measurements reflect the fact that it is impossible to achieve a perfectly symmetric setup, as in the numerical study. Additionally, some slack in the system or uneven residual stresses may disproportionately affect the system response under 0% prestrain, and lead to the nonmonotonic trend in shear modulus estimates with respect to prestrain, given in Table 2. Images were analyzed the same way that numerical simulation data, shown in the lower half of Fig. 2,

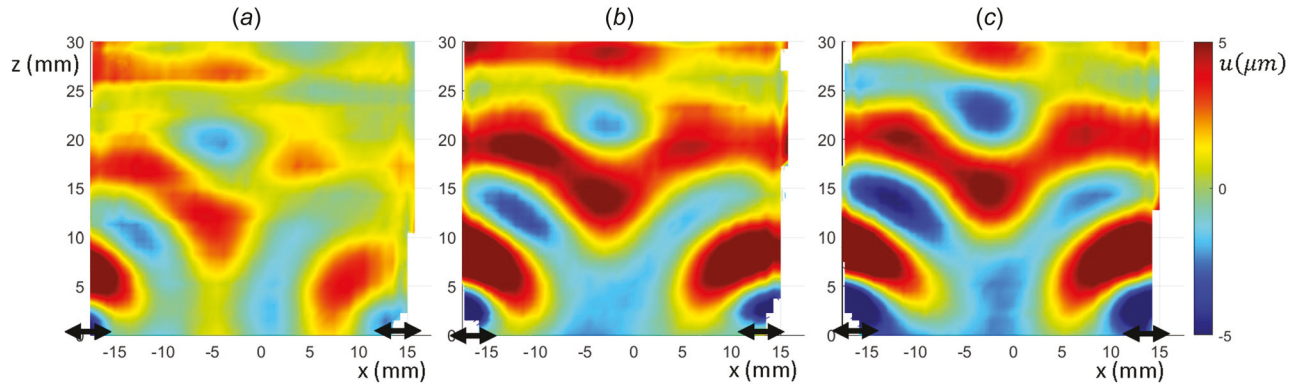


Fig. 4 Sagittal view of transverse (x)-polarized in-phase wave motion from experimental study. ((a)–(c)) 0, 5, and 10% axial prestrain, respectively.

was processed, as described in Sec. 2.5, to obtain estimates of the shear wavenumber. Results are included in Table 2 and can be compared directly to the numerical simulation results, except it was not possible with our setup to achieve 20% prestrain.

The wavenumber and wavelength prior to the coordinate transformation are first provided, γ_E and λ_E , followed by their values after the TAE coordinate transformation, $\gamma_{E/TAE}$ and $\lambda_{E/TAE}$. The complex shear modulus is subsequently calculated based on Eq. (1) and provided in the line below $\lambda_{E/TAE}$. What can be observed is that the coordinate transformation results in wavenumber values that deviate less with prestrain, since the TAE transformation is designed to remove the effect of the prestrain. In that sense, the TAE approach has succeeded. However, a direct comparison to the true value used for the theoretical and numerical studies is not as appropriate since the actual value for the complex shear modulus of the material used in the experiment is not exactly known. The theoretical value used is based on experiments conducted on samples of Ecoflex-30 harmonically vibrated at 600 Hz, but there can be some variation between samples.

5 Conclusions

The numerical finite element and experimental studies of the previous sections have identified and quantified some of the confounding effects of finite dimensions and nonzero prestress on the elastography approach to estimating material viscoelastic properties in an isotropic cylindrical structure under uniaxial normal stress aligned with the cylinder axis. Additionally, a coordinate transformation approach, transformation acousto-elastography (TAE), has been introduced that enables one to estimate material viscoelastic properties independent of the prestress condition without requiring a priori knowledge of either the viscoelastic properties or stress conditions. Rather, only the amount of deformation, or strain, from the unstressed condition is required. A priori knowledge of prestrain is more easily available than prestress, as medical images of geometry via MRI when doing MR Elastography, or via ultrasound if doing ultrasound elastography, can be acquired under varying prestrain conditions, with precise tracking of anatomical landmarks that will establish prestrain levels during a given measurement. These anatomical images provide no direct information about the corresponding prestress. Once viscoelastic properties are estimated under the known prestrain conditions, then the prestress can also be estimated.

The numerical and experimental studies show the promise of the TAE approach. In the numerical study of Sec. 3, the material shear storage μ_R and loss μ_I moduli, as well as the uniaxial normal stress σ were determined within 5% for prestrains up to 10%, and within 7.5% for a prestrain of 20%. Without the TAE adjustment to the complex wavenumber, the error in the estimate of shear modulus would reach 40% at 20% prestrain. Potential sources of error or discrepancies that affect the estimates of both shear moduli and stress are: (1) an incompressibility assumption used to

formulate the approach; (2) the assumption of linearity; and (3) ignoring more complex wave patterns that exist in the 3-dimensional structure. Since the bulk modulus is about five orders of magnitude greater than the shear moduli, we believe the assumption of incompressibility is justified. The approach could be reformulated to account for compressibility. With regard to nonlinearity, the numerical study was repeated assuming simple linear elastic material properties and not enabling the geometric nonlinearity feature in ANSYS. Results were the same, suggesting that, at least in the numerical study, nonlinearity is not a factor for the strain rates considered. The images in Fig. 2 show that, even with the uniform transverse harmonic displacement across the entire end of the cylinder, the steady-state wave pattern shows some variation as a function of radial position. This variation could result in the small variation seen in predictions based on the TAE approach.

In the experimental study, another source of error is that one must consider that the static shear modulus governing the initial deformation due to the prestress is different from the frequency-dependent shear storage modulus that will govern response to harmonic excitation. To overcome this, one possible solution is to conduct studies at multiple frequencies and then fit a rheological model that predicts the static shear storage modulus, as detailed in Sec. 4.

Logical next steps for advancing the strategy introduced here to detangle prestress from material shear stiffness estimates include consideration of more complex geometry and stress conditions, as well as anisotropic and nonuniform material properties. Finite element models based on medical images that can provide detailed geometry and deformation information under varying loading conditions and, if needed, measures of anisotropy and inhomogeneity, may provide a way to advance the TAE technique beyond simple geometries and assumptions of isotropy and homogeneity [25].

Funding Data

- National Science Foundation (NSF) (Grant No. 1852691; Funder ID: 10.13039/1000000084).
- National Institutes of Health (NIH) (Grant No. AR071162; Funder ID: 10.13039/1000000069).

Data Availability Statement

The datasets generated and supporting the findings of this article are obtainable from the corresponding author upon reasonable request.

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