

CompOSE Reference Manual

Version 3.01

CompStar Online Supernovæ Equations of State

“harmonising the concert of nuclear physics and astrophysics”

<https://compose.obspm.fr>

CompOSE Core Team

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March 8, 2022

Contents

I. Introduction	6
1. What CompOSE can do and what not	7
2. How to read this document	8
2.1. Recent Changes	9
3. Definitions and notation	11
3.1. Units and conventions	11
3.2. Physical conditions	11
3.3. Particle number densities and particle fractions	13
3.4. Particle indexing	15
3.5. Thermodynamic potentials and basic quantities	19
3.6. Thermodynamic coefficients	23
3.7. Microscopic quantities	24
3.8. Transport properties	26
3.8.1. Viscosities	26
3.8.2. Conductivities	27
3.9. Contribution of photons	27
3.10. Properties of compact stars	27
II. For Contributors	30
4. How to prepare EoS tables	32
4.1. Tabulation of quantities	32
4.1.1. Parameters and parameter ranges	32
4.1.2. Thermodynamic consistency	34
4.2. Structure of tables	35
4.2.1. Types of EoS tables	35
4.2.2. Table with thermodynamic quantities (required)	37
4.2.3. Table with the composition of matter (optional)	38
4.2.4. Table with information on microscopic quantities (optional)	39
4.2.5. Table with information on transport quantities (optional)	40
4.2.6. Table with information on properties of compact start (optional)	40
4.2.7. Tables with parameters	40
4.2.8. Identification of tables	41

4.2.9. Precision of tabulated quantities	41
4.3. Data sheet	42
5. Extending CompOSE	44
 III. For Users	 45
6. Models for the equation of state	46
6.1. Nuclear matter properties	46
6.2. Remarks of caution	49
7. Online services and data handling	51
7.1. Access to CompOSE	51
7.2. EoS data sheets	51
7.3. Options for using EoS data	51
7.4. Different families of EoS data	52
7.5. Handling and transforming EoS data	55
7.5.1. Downloading and compiling	56
7.5.2. Direct use of <code>compose</code>	56
7.5.3. HDF5 table	58
7.5.4. Additional output	58
7.5.5. Error messages	58
7.6. Web tools	59
 IV. Appendix	 67
A. Technical details	68
A.1. Interpolation	68
A.1.1. Interpolation in one dimension	69
A.1.2. Interpolation in two dimensions	71
A.2. Matter in beta equilibrium	73
B. Structure of input files	74
B.1. <code>eos.init</code>	74
B.2. <code>eos.quantities</code>	74
B.3. <code>eos.parameters</code>	78
C. Structure of the <code>compose.f90</code> program	81
D. Organization of the CompOSE team	87
E. Citing CompOSE	88
F. Acknowledgments	89

Abstract

CompOSE (CompStar Online Supernovae Equations of State) is an online repository of equations of state (EoS) for use in nuclear physics and astrophysics, e.g., in the description of compact stars or the simulation of core-collapse supernovae and neutron-star mergers, see <http://compose.obspm.fr>. The main services, offered via the website, are: a collection of data tables in a flexible and easily extendable data format for different EoS types and related physical quantities with extensive documentation and referencing; software for download to extract and to interpolate these data and to calculate additional quantities; webtools to generate EoS tables that are customized to the needs of the users and to illustrate dependencies of various EoS quantities in graphical form. This manual is an update of previous versions that are available on the CompOSE website, at [arXiv:1307.5715](https://arxiv.org/abs/1307.5715) [[astro-ph.SR](#)], and that was originally published in the journal "Physics of Particles and Nuclei" with [doi:10.1134/S1063779615040061](https://doi.org/10.1134/S1063779615040061). It contains a detailed description of the service, containing a general introduction as well as instructions for potential contributors and for users. Short versions of the manual for EoS users and providers will also be available as separate publications.

Preliminary Remark

This CompOSE reference manual, version 3.01, is an updated and extended version of previous manuals that appeared as a preprint [arXiv:1307.5715](https://arxiv.org/abs/1307.5715) [[astro-ph.SR](#)] on [arXiv.org](https://arxiv.org/). and as a refereed article in the journal "Physics of Particles and Nuclei" [1]. The content of this publication was adapted by permission from Springer Nature Customer Service Centre GmbH: Springer Nature, Physics of Particles and Nuclei, Vol. 46, No. 4, 633-664 ("CompOSE CompStar online supernova equations of state harmonising the concert of nuclear physics and astrophysics compose.obspm.fr", S. Typel, M. Oertel, and T. Klähn), Copyright Pleiades Publishing, Ltd. (2015). Previous versions are available on the CompOSE website compose.obspm.fr.

Part I.

Introduction

1. What CompOSE can do and what not

The online service CompOSE provides information and data tables for different equations of state (EoS) ready for further use in astrophysical applications, nuclear physics and beyond. See the review [2] for a general introduction. CompOSE has three major purposes:

- CompOSE is a repository of EoS tables in a common format for direct usage with information on a large number of thermodynamic properties, on the chemical composition of dense matter and, if available, on microphysical quantities of the constituents.
- CompOSE allows to interpolate the provided tables using different schemes to obtain the relevant quantities, selected by the user, for grids that are tailored to specific applications.
- CompOSE can provide information on additional thermodynamic quantities, which are not stored in the original data tables, and on further quantities, which characterize an EoS, such as nuclear matter parameters and compact star properties.

The format of the files, as well as the calculational mesh, is mainly determined according to the needs of scientific groups performing extensive numerical simulations of astrophysical objects. We try to provide the tables in a large parameter space to cover most applications.

We cannot offer an online service for all features of CompOSE, e.g. to run all codes online. This is mostly due to limitations in storage and computation times, but also gives better control on avoiding unphysical input parameters. However, we offer several computational tools that allow the user to extract the data from the tables that are relevant for her/him. These tools can be downloaded from the CompOSE website

<https://compose.obspm.fr> .

Appendix E describes how to cite CompOSE and the EoS tables it contains.

CompOSE is designed in a modular way, thus allowing to extend the service over time. More and more models and parameter sets will be provided in time. It is foreseen that additional features will be added in the future, too.

2. How to read this document

While reading this document please always keep in mind: this document was written by physicists for physicists. It is divided into three major parts.

The first one is relevant both to providers and users of equations of state, as it serves as a basis for the discussions in the following parts of the manual and the web site. Both, contributors and users, should first of all have a look at the introductory chapter 3 where we will discuss general conventions and the notation used throughout CompOSE. In addition, we give definitions and details on the system of units that is used within CompOSE and on physical constants that should be used in order to standardise the generation of new equations of state.

The second part concerns those persons who wish to contribute to an extension of the CompOSE data base by the active development of an EoS. CompOSE allows them to make their favourite EoS available for a broad range of astrophysical and nuclear physics applications. In part II, detailed instructions, minimal requirements and recommendations are specified for the preparation of EoS tables that can be incorporated in future versions of the CompOSE data base. If you plan to contribute your EoS, you should contact the CompOSE core team (develop.compose@obspm.fr), see appendix D. A summary of possible future extensions of the CompOSE data base are summarized in chapter 5.

The third part concerns the users of EoS data, that are provided by CompOSE, who want to test various equations of state in their simulations of core-collapse supernovae, neutron-star mergers and other scenarios. In general, they can safely skip the second part and go directly to the third one. This part gives a brief introduction on nuclear matter properties relevant for the construction of an EoS, as well as a classification of different types of EoS models in chapter 6. The models can be distinguished either by using different techniques to treat the many-body system of strongly interacting particles or by assuming a different particle composition. The main aim is to give the relevant information for the interpretation of the data sheets, provided with each available EoS table on the web site. You will find remarks on the range of applicability of the various models, i.e. the range of parameters where the code is tested and/or the made approximations are still valid. Characteristic parameters of each model will be specified. For more detailed information about the physics behind each model, we refer to the original references.

In addition, the third part of the manual explains in chapter 7 how to proceed in order to download an EoS table and the computational tools. The latter allow the generation of tables with a mesh different from the original one via interpolation, in order to adapt the table to the need of the user. In addition, several thermodynamic quantities can be calculated which are not contained in the original tables. The use of the online service

and web interface is described in chapter 7 too.

2.1. Recent Changes

Version 1.00 of this manual was published in [1] and is still available on the CompOSE web site. Version 2.00 contains some major changes, corrections and additions, in particular concerning the structure of the manual and the use of the provided software for handling data. The main changes from version 1.00 to 2.00 in the manual are:

- the numerical values of physical constants, recommended for the preparation of EoS tables, in table 3.1 were updated;
- the discussion of constraints on nuclear matter properties in section 6.1 has been modified;
- sections 6.2 (Overview of models) and 6.3 (Phase transitions) have been removed since a comprehensive review [2] is available;
- the title 'User registration' of section 7.1 has been changed to 'Access to CompOSE' with an adjusted description of the online service;
- the description of how to use the `compose` program in section 7.5.2 was adapted to the new version and the description of the files `eos.parameters` (with a change of the structure) and `eos.quantities` was moved to appendix B;
- subsection 7.4.3 was moved to appendix C and the corresponding tables with the (sub-)routines and functions were updated;
- tables 7.1, 7.4, and 7.8 were amended and a new table 7.3 was introduced;
- new appendices B and C were introduced;
- the appendices D and F were updated;
- the bibliography has been updated.

The main changes to the `compose` program are

- most vectors and matrices are allocated dynamically with the appropriate size for the particular EoS used for generating a user-defined EoS table;
- the program `compose` runs by default in a standard version that guides the user through the selection of output data and the tabulation scheme;
- the input files `eos.parameters` (with modified structure) and `eos.quantities` don't have to be provided by the user because they are created automatically by running the `compose` program;

- an option for generating tables for given entropy per baryon instead of the temperature has been added;
- the number of output quantities has been extended.

Major changes in the current version 3.01 are:

- update of references to physical constants and nuclear properties in section 3.1;
- update of physical constants in table 3.1;
- distinguishing of particle and antiparticle densities as well as net number densities for fermions with extended indexing scheme, modified new tables 3.2 and 3.3;
- discussion of conversion from nuclear to astrophysical units in section 3.5;
- additional discussion of calculation of speed of sound in section 3.6;
- addition of new section 3.8 on transport properties;
- addition of new section 3.10 on properties of neutron stars;
- extended classification of EoS tables in section 4.2;
- introduction of new tables with transport properties and information on neutron stars in section 4.2;
- discussion of different families of tabulated data and EoS models in section 7.4;
- extended description of using EoS tables with software from CompOSE and LORENE;
- availability of web tools to calculate EoS tables and visualise the results in section 7.6;
- extension of table 7.2;
- update of CompOSE team in appendix D;
- information on citing CompOSE in appendix E.

Examples on using the `compose` program are given in a new “quick guide for users” with some concrete examples.

3. Definitions and notation

The equations of state in the CompOSE data base are provided under some common assumptions on the physical conditions of the matter that are specified in this chapter. In order to fix the notation in the present document, the definition of all relevant quantities is given.

3.1. Units and conventions

We use natural units $\hbar = c = k_B = 1$ throughout this document as is customary in nuclear physics. Energies and temperatures are measured in MeV, lengths in fm. Units are given in parentheses [...] for all quantities when they are defined within this chapter. For conversion among your favourite units, the use of the CODATA values [3] (<http://physics.nist.gov/cuu/Constants/index.html>, www.codata.org) is recommended, see Tab. 3.1¹. They should also be used in the preparation of new EoS tables to be incorporated in the CompOSE data base.

In astrophysical applications of equations of state it is often customary to change from nuclear units to units that are traditionally used in that field, in particular for temperature, pressure and energy densities. See section 3.5 for examples.

For experimental binding energies of nuclei, the values of the 2016 Atomic mass evaluation [4, 5] (AME2016) or the more recent 2020 Atomic mass evaluation [6, 7] (AME2020, <http://amdc.impcas.ac.cn>) and corresponding updates are recommended. Ground state spin assignments should be taken from the Nubase2016 [8] or the Nubase2020 [9] evaluation of nuclear properties (<http://amdc.impcas.ac.cn>) and corresponding updates.

3.2. Physical conditions

Predictions for the properties of dense matter can differ considerably depending on the employed model, the considered constituents and interactions. In the CompOSE database, the equation of state is considered to describe dense matter in thermodynamic equilibrium, a short denotation of the fulfillment of several equilibrium conditions. Each of them corresponds to the existence of an intensive thermodynamic variable that applies to the full system under consideration. First of all there are the thermal and mechanic

¹The compilation on physical constants and review of particle properties [10] by the Particle Data Group, pdg.lbl.gov, contains the CODATA values together with some additional constants concerning the properties and interaction of elementary particles.

Table 3.1.: Recommended values for physical constants from the 2018 CODATA evaluation and the compilation of the Particle Data Group [10].

Quantity	Symbol	Value	Unit
speed of light in vacuum	c	299792458	m s^{-1}
Planck's constant	$\hbar$	$1.054571817 \times 10^{-34}$	J s
		$6.582119569 \times 10^{-22}$	MeV s
		197.3269804	$\text{MeV fm } c^{-1}$
Boltzmann's constant	k_B	1.380649×10^{-23}	J K^{-1}
		$8.617333262 \times 10^{-11}$	MeV K^{-1}
gravitational constant	G	6.67430×10^{-11}	$\text{m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
		6.70883×10^{-39}	$\text{GeV}^{-2} c^{-4}$
square of elementary charge	e^2	1.439964547	MeV fm
fine structure constant	$\alpha = e^2/(\hbar c)$	1/137.035999084	—
neutron mass	m_n	939.56542052	$\text{MeV } c^{-2}$
proton mass	m_p	938.27208816	$\text{MeV } c^{-2}$
electron mass	m_e	0.51099895000	$\text{MeV } c^{-2}$
muon mass	m_μ	105.6583745	$\text{MeV } c^{-2}$
unified atomic mass unit	u	931.49410242	$\text{MeV } c^{-2}$
solar mass	$M_\odot$	1.98841×10^{30}	kg
		1.11542×10^{60}	$\text{MeV } c^{-2}$

equilibrium that allow to define the temperature T and the pressure p . Chemical equilibrium means equilibrium with regard to strong, electromagnetic and weak reactions. For the latter, one has to distinguish several cases, i.e., reactions within a generation of particles, e.g., electronic leptons and muonic leptons, and reactions across flavor generations, e.g., strangeness changing reaction. The equilibrium of each type of reaction is linked to the corresponding chemical potential. These are the baryonic (μ_b), charge (μ_q), electron lepton (μ_{le}), muon lepton ($\mu_{l\mu}$), and strangeness (μ_s) chemical potentials.

It is assumed that all the constituents are in chemical equilibrium regardless of the time scales and reaction rates for the relevant conversion reactions mediated by strong and electromagnetic interactions. This condition leads to relations between the chemical potentials of all particles.

In contrast, an equilibrium with respect to weak interaction reactions, in particular β -equilibrium, is not supposed in general. Similarly, the chemical potentials of the massive leptons of all generations, i.e. electrons and muons (tauons are irrelevant for the considered conditions) cannot be assumed to be equal. There is an option available such that EoS tables can be provided in particular cases which also take β -equilibrium into account, reducing the number of independent parameters. Assumptions on the relation between the electron and muon chemical potentials are discussed in the description of

each model separately. In almost all cases they are assumed to be identical.

For EoS models with strangeness bearing particles, e.g. hyperons or kaons, it is assumed that the strangeness chemical potential vanishes. This means that we assume the strangeness changing weak interactions to be in equilibrium. Note that this is not the case in heavy-ion collisions, where, on the contrary, strangeness is conserved, i.e., there is no net strangeness.

Except for the tables of pure hadronic or quark matter without massive leptons, also local charge neutrality is assumed to hold. Neutrinos and their contribution to thermodynamic properties are never included in the present tables. They are usually treated independently from the EoS in astrophysical simulations because a thermodynamic equilibrium can not be assumed in general.

Photons are usually included in equations of state for astrophysical applications. They only contribute at nonzero temperatures. The treatment of photons is discussed in section 3.9.

3.3. Particle number densities and particle fractions

Particle number densities of all particles i are given by

$$n_i = \frac{N_i}{V} \quad [\text{fm}^{-3}] \quad (3.1)$$

where N_i [dimensionless] is the particle number inside the volume V [fm^3]. Note that for particles with half-integer spin, n_i represents the net particle density, i.e. it is the difference between the particle and antiparticle density. E.g., for electrons we have $n_e = n_{e^-} - n_{e^+}$. For particles with integer spin, e.g. mesons, particle and antiparticle densities are distinguished and given separately. For particles with half-integer spin, it will be possible to give the particle and antiparticle densities separately in an upcoming version of the `CompOSE` program.

From the individual particle number densities several new *composite* number densities can be deduced that are convenient to characterize the state of the system. The baryon number density n_b is given by

$$n_b = \frac{B}{V} = \sum_i B_i n_i \quad [\text{fm}^{-3}] \quad (3.2)$$

with the baryon number B_i [dimensionless] for a particle i and the total baryon number

$$B = \sum_i B_i \quad [\text{dimensionless}] \quad (3.3)$$

inside the volume V . The baryon number of a nucleus i is just the mass number A_i [dimensionless] and for a quark i one has $B_i = 1/3$. **Warning:** In many astrophysical applications, a mass density $\varrho = m n_b$ [MeV fm^{-3}] is introduced as a parameter with m representing, e.g., the neutron mass or the atomic mass unit u . But, ϱ is not identical to the total mass (= total internal energy) density. Hence, ϱV is not a conserved quantity.

The strangeness number density n_s

$$n_s = \frac{S}{V} = \sum_i S_i n_i \quad [\text{fm}^{-3}] \quad (3.4)$$

with the total strangeness

$$S = \sum_i S_i \quad [\text{dimensionless}] \quad (3.5)$$

inside the volume V with strangeness numbers S_i [dimensionless], as well as the lepton number densities n_{le} , $n_{l\mu}$ and number of leptons N_{le} , $N_{l\mu}$

$$n_{le} = \frac{N_{le}}{V} = \sum_i L_i^e n_i \quad [\text{fm}^{-3}] \quad (3.6)$$

$$n_{l\mu} = \frac{N_{l\mu}}{V} = \sum_i L_i^\mu n_i \quad [\text{fm}^{-3}] \quad (3.7)$$

with lepton numbers L_i^e , L_i^μ [dimensionless], respectively, defined similarly to the baryon number density. Since neutrinos are not included in the CompOSE EoS, $n_{le} = n_e$ and $n_{l\mu} = n_\mu$. We also consider the charge density of strongly interacting particles n_q

$$n_q = \frac{Q}{V} = \sum_i 'Q_i n_i \quad [\text{fm}^{-3}] \quad (3.8)$$

with electric charge numbers Q_i [dimensionless] and the total charge

$$Q = \sum_i 'Q_i \quad [\text{dimensionless}] \quad (3.9)$$

inside the volume V . The prime at the sum symbols indicates that the summation runs over all particles (including quarks) except leptons. For a nucleus ${}^{A_i}Z_i$, the baryon number B_i and the charge number Q_i are simply given by the mass number A_i and atomic number Z_i , respectively. Tables 3.2 and 3.4 summarize baryon, strangeness, charge and lepton numbers of the most relevant particles that appear in the definition of the composite number densities.

Corresponding to the particle number densities n_i , the particle number fractions

$$Y_i = \frac{n_i}{n_b} \quad [\text{dimensionless}] \quad (3.10)$$

are defined. Due to this definition with n_b , we have the normalization condition

$$\sum_i B_i Y_i = 1 \quad (3.11)$$

with a summation over all particles. Note that Y_i is not necessarily identical to the particle mass number fraction

$$X_i = B_i Y_i \quad [\text{dimensionless}] \quad (3.12)$$

that is often introduced. We prefer to use Y_i and not X_i since the latter quantity is zero for all particles with baryon number $B_i = 0$, e.g. mesons.

Because of the imposed physical conditions (see 3.2), the state of the system is uniquely characterized by the three quantities temperature T [MeV], baryon number density n_b [fm^{-3}] and charge density of strongly interaction particles n_q [fm^{-3}]. Charge neutrality implies $n_q = n_{le} + n_{l\mu}$ and the strangeness number density n_s is fixed by the condition $\mu_s = 0$. Instead of n_q , it is more convenient to use the charge fraction of strongly interacting particles

$$Y_q = \frac{n_q}{n_b} \quad [\text{dimensionless}] \quad (3.13)$$

as the third independent quantity. The choice of Y_q instead of the electron fraction

$$Y_e = \frac{n_e}{n_b} \quad [\text{dimensionless}] \quad (3.14)$$

as a parameter is motivated by the following facts. In pure hadronic (quark) equations of state (i.e. without leptons) only Y_q and not Y_e is defined. In EoS models with the condition of charge neutrality that contain electrons as the only considered charged lepton, the charge fraction of strongly interacting particles Y_q is identical to the electronic charge fraction Y_e . In models with electrons and muons, charge neutrality requires

$$Y_q = Y_e + Y_\mu = Y_l \quad (3.15)$$

with the muon fraction

$$Y_\mu = \frac{n_\mu}{n_b} \quad [\text{dimensionless}] \quad (3.16)$$

and the total lepton fraction Y_l [dimensionless]. In this case, the balance between the electron and muon densities depends on the assumed relation of the electron and muon chemical potentials.

3.4. Particle indexing

The composition of dense matter can rapidly change with temperature T [MeV], baryon number density n_b [fm^{-3}] and the charge fraction of strongly interacting particles Y_q [dimensionless]. We introduce an indexing scheme for the particles that allows to identify them uniquely in order to store only the most abundant particles in the EoS tables. In tables 3.2, 3.3, and 3.4 an overview of indices for most of the relevant particles is presented. Indices of missing particles can be added on request. In the case of fermions, indices of quantities for particles and antiparticles can be given separately as well as net quantities, i.e., net particle number densities as differences of particle and antiparticle densities. Thus in table 3.3 indices for net quantities are given without quantum numbers.

In addition to the various particles that can appear in dense matter, there is the possibility of strong two- or even three-particle correlations such as pairing of nucleons or quarks and phenomena such as superfluidity can arise. Hence, an indexing scheme

for identifying these channels is introduced, too. In table 3.5 the notation is given for these correlations. Indices of missing particles can be added on request. In the case of fermions, quantities can be given for particles and antiparticles separately as well as for the corresponding net quantities, e.g., number densities as differences of particle and antiparticle densities.

Table 3.2.: Baryon (B_i), strangeness (S_i), charge (Q_i), lepton (L_i^e , L_i^μ) numbers and indices I_i of nuclei and the most relevant fermions i in dense matter. (Not yet implemented in the CompOSE program.)

particle class	symbol of particle i	B_i	S_i	Q_i	L_i^e	L_i^μ	particle index I_i
nuclei ($A > 1$)	AZ	A	0	Z	0	0	$1000000 + 1000 \cdot A + Z$
	${}^A\bar{Z}$	$-A$	0	$-Z$	0	0	$2000000 + 1000 \cdot A + Z$
leptons	e^-	0	0	-1	+1	0	2
	e^+	0	0	+1	-1	0	4
	μ^-	0	0	-1	0	+1	3
	μ^+	0	0	+1	0	-1	5
baryons	n	+1	0	0	0	0	12
	$\bar{n}$	-1	0	0	0	0	14
	p	+1	0	+1	0	0	13
	$\bar{p}$	-1	0	-1	0	0	15
	Δ^-	+1	0	-1	0	0	24
	$\bar{\Delta}^-$	-1	0	+1	0	0	28
	Δ^0	+1	0	0	0	0	25
	$\bar{\Delta}^0$	-1	0	0	0	0	29
	Δ^+	+1	0	+1	0	0	26
	$\bar{\Delta}^+$	-1	0	-1	0	0	30
	Δ^{++}	+1	0	+2	0	0	27
	$\bar{\Delta}^{++}$	-1	0	-2	0	0	31
	Λ	+1	-1	0	0	0	101
	$\bar{\Lambda}$	-1	+1	0	0	0	102
	Σ^-	+1	-1	-1	0	0	113
	$\bar{\Sigma}^-$	-1	+1	+1	0	0	116
	Σ^0	+1	-1	0	0	0	114
	$\bar{\Sigma}^0$	-1	+1	0	0	0	117
	Σ^+	+1	-1	+1	0	0	115
	$\bar{\Sigma}^+$	-1	+1	+1	0	0	118
	Ξ^-	+1	-2	-1	0	0	122
	$\bar{\Xi}^-$	-1	+2	+1	0	0	124
	Ξ^0	+1	-2	0	0	0	123
	$\bar{\Xi}^0$	-1	+2	0	0	0	125
quarks	u	1/3	0	+2/3	0	0	503
	$\bar{u}$	-1/3	0	-2/3	0	0	506
	d	1/3	0	-1/3	0	0	504
	$\bar{d}$	-1/3	0	+1/3	0	0	507
	s	1/3	-1	-1/3	0	0	505
	$\bar{s}$	-1/3	+1	+1/3	0	0	508

Table 3.3.: Indices I_i of nuclei and the most relevant fermions i in dense matter for net quantities.

particle class	symbol of particle i	particle index I_i
nuclei ($A > 1$)	$A^Z/\bar{A}^Z$	$1000 \cdot A + Z$
leptons	e^-/e^+	0
	μ^-/μ^+	1
baryons	$n/\bar{n}$	10
	$p/\bar{p}$	11
	$\Delta^-/\bar{\Delta}^-$	20
	$\Delta^0/\bar{\Delta}^0$	21
	$\Delta^+/\bar{\Delta}^+$	22
	$\Delta^{++}/\bar{\Delta}^{++}$	23
	$\Lambda/\bar{\Lambda}$	100
	$\Sigma^-/\bar{\Sigma}^-$	110
	$\Sigma^0/\bar{\Sigma}^0$	111
	$\Sigma^+/\bar{\Sigma}^+$	112
	$\Xi^-/\bar{\Xi}^-$	120
	$\Xi^0/\bar{\Xi}^0$	121
quarks	$u/\bar{u}$	500
	$d/\bar{d}$	501
	$s/\bar{s}$	502

Table 3.4.: Baryon (B_i), strangeness (S_i), charge (Q_i), lepton (L_i^e, L_i^μ) numbers and indices I_i of the most relevant bosons i in dense matter.

particle class	symbol of particle i	B_i	S_i	Q_i	L_i^e	L_i^μ	particle index I_i
mesons	ω	0	0	0	0	0	200
	σ	0	0	0	0	0	210
	η	0	0	0	0	0	220
	η'	0	0	0	0	0	230
	ρ^-	0	0	-1	0	0	300
	ρ^0	0	0	0	0	0	301
	ρ^+	0	0	+1	0	0	302
	δ^-	0	0	-1	0	0	310
	δ^0	0	0	0	0	0	311
	δ^+	0	0	+1	0	0	312
	π^-	0	0	-1	0	0	320
	π^0	0	0	0	0	0	321
	π^+	0	0	+1	0	0	322
	ϕ	0	0	0	0	0	400
	σ_s	0	0	0	0	0	410
	K^-	0	-1	-1	0	0	420
	K^0	0	+1	0	0	0	421
	$\bar{K}^0$	0	-1	0	0	0	422
	K^+	0	+1	+1	0	0	423
photon	γ	0	0	0	0	0	600

3.5. Thermodynamic potentials and basic quantities

All thermodynamic properties of a system are completely determined if a thermodynamic potential is known as a function of its natural variables. In the general case, this can be formulated as follows. The properties can be derived from the thermodynamic potential $\Xi = \Xi(\{x_i\}, \{\xi_j\})$ depending on n natural variables $x_i, i = 1, \dots, n_1$ and $\xi_j, j = 1, \dots, n_2$ with $n_1 + n_2 = n$. The quantities x_i and ξ_i represent extensive and intensive variables, respectively. The relations

$$\xi_i = \left. \frac{\partial \Xi}{\partial x_i} \right|_{x_k, k \neq i; \xi_j} = \xi_i(\{x_i\}, \{\xi_j\}) \quad (3.17)$$

and

$$x_j = - \left. \frac{\partial \Xi}{\partial \xi_j} \right|_{\xi_k, k \neq j; x_i} = x_j(\{x_i\}, \{\xi_j\}) \quad (3.18)$$

are the thermodynamic equations of state and define the variables ξ_i that are conjugate to x_i as first partial derivatives of Ξ and vice versa. Due to Euler's theorem on homogeneous

Table 3.5.: Baryon (B_i), strangeness (S_i), charge (Q_i) numbers and indices I_i of the most relevant two-particle correlations i in dense matter. The lepton numbers L_i^e and L_i^μ are always zero for these correlations.

correlation class	particles	channel	B_i	S_i	Q_i	index I_i of correlation
two-body	nn	1S_0	2	0	0	700
	np	1S_0	2	0	1	701
	pp	1S_0	2	0	2	702
	np	3S_1	2	0	1	703

functions, the thermodynamic potential is given by the sum

$$\Xi(\{x_i\}, \{\xi_j\}) = \sum_{i=1}^{n_1} \xi_i x_i \quad (3.19)$$

running over all $i = 1, \dots, n_1$ extensive variables that are primary variables of the thermodynamic potential Ξ . Thus, the knowledge of all relevant equations of state or first derivatives is sufficient to recover the thermodynamic potential Ξ completely. For the mixed second partial derivatives of a thermodynamic potential Ξ , the result is independent of the order of differentiation and the Maxwell relations

$$\left. \frac{\partial \xi_l}{\partial x_k} \right|_{x_j, j \neq k} = \left. \frac{\partial \xi_k}{\partial x_l} \right|_{x_j, j \neq l} \quad (3.20)$$

are obtained. For a more detailed discussion of these and further aspects we refer the reader to standard text books on thermodynamics.

In most models for the EoS of dense matter, the temperature T [MeV], the volume V [fm³] and the individual particle numbers N_i [dimensionless] are selected as natural variables. This case corresponds to the (Helmholtz) free energy $F = F(T, V, N_i)$ [MeV] as the relevant thermodynamic potential that contains all information. Note that we assume that the free energy includes contributions by the rest masses of the particles. Keeping the volume V fixed, it is convenient to define the free energy density

$$f(T, n_i) = \frac{F}{V} \quad [\text{MeV fm}^{-3}] \quad (3.21)$$

and the entropy density

$$s(T, n_i) = - \left. \frac{\partial f}{\partial T} \right|_{n_i} \quad [\text{fm}^{-3}] \quad (3.22)$$

with the entropy

$$S(T, V, N_i) = V s(T, n_i) = - \left. \frac{\partial F}{\partial T} \right|_{V, N_i} \quad [\text{dimensionless}] . \quad (3.23)$$

The chemical potential of a particle i is given by

$$\mu_i = \left. \frac{\partial F}{\partial N_i} \right|_{T,V,N_j,j \neq i} = \left. \frac{\partial f}{\partial n_i} \right|_{T,n_j,j \neq i} \quad [\text{MeV}] \quad (3.24)$$

including the rest mass m_i [MeV]. The pressure is obtained from

$$p = - \left. \frac{\partial F}{\partial V} \right|_{T,N_i} = n_b^2 \left. \frac{\partial(f/n_b)}{\partial n_b} \right|_{T,Y_q} = \sum_i \mu_i n_i - f \quad [\text{MeV fm}^{-3}]. \quad (3.25)$$

Each of the composite densities $n_b, n_s, n_{le}, n_{l\mu}, n_q$ is accompanied by a corresponding chemical potential, i.e. we have the baryon number chemical potential μ_b [MeV], the strangeness number chemical potential μ_s [MeV], the electron lepton number chemical potential μ_{le} [MeV], the muon lepton number chemical potential $\mu_{l\mu}$ [MeV], and the charge chemical potential μ_q [MeV]. The chemical potential of a particle i is then given by

$$\mu_i = B_i \mu_b + Q_i \mu_q + S_i \mu_s + L_i^e \mu_{le} + L_i^\mu \mu_{l\mu} \quad [\text{MeV}], \quad (3.26)$$

e.g. $\mu_n = \mu_b$, $\mu_p = \mu_b + \mu_q$ and $\mu_e = \mu_{le} - \mu_q$. Note again that we use the relativistic definition of chemical potentials including rest masses. It has to be mentioned that n_b, n_s, n_{le} and $n_{l\mu}$ are conjugate to μ_b, μ_s, μ_{le} and $\mu_{l\mu}$, respectively. However, this is not true for n_q and μ_q if leptons are included in the EoS. In general, it is assumed that $\mu_s = 0$ since strangeness changing weak interaction reactions are in equilibrium. Nevertheless, the strangeness density n_s can be non-zero if particles with strangeness are considered in the EoS.

In addition to the free energy density, several other thermodynamic potentials can be defined by applying Legendre transformations, e.g. the internal energy

$$E = E(S, V, N_i) = F + TS \quad [\text{MeV}], \quad (3.27)$$

the free enthalpy (Gibbs potential)

$$G = G(T, p, N_i) = F + pV = \sum_i \mu_i N_i \quad [\text{MeV}], \quad (3.28)$$

the enthalpy

$$H = H(S, p, N_i) = E + pV \quad [\text{MeV}], \quad (3.29)$$

and the grand canonical potential

$$\Omega = \Omega(T, V, \mu_i) = F - \sum_i \mu_i N_i = -pV \quad [\text{MeV}] \quad (3.30)$$

with the corresponding densities $e = E/V$, $g = G/V$, $h = H/V$ and $\omega = \Omega/V$ [MeV fm⁻³], respectively.

It is convenient to define the free energy per baryon

$$\mathcal{F} = \frac{F}{N_b} = \frac{f}{n_b} \quad [\text{MeV}], \quad (3.31)$$

the internal energy per baryon

$$\mathcal{E} = \frac{E}{N_b} = \frac{e}{n_b} \quad [\text{MeV}] , \quad (3.32)$$

the enthalpy per baryon

$$\mathcal{H} = \frac{H}{N_b} = \frac{h}{n_b} \quad [\text{MeV}] , \quad (3.33)$$

and the free enthalpy per baryon

$$\mathcal{G} = \frac{G}{N_b} = \frac{g}{n_b} \quad [\text{MeV}] \quad (3.34)$$

by dividing the corresponding thermodynamic potential by the total number of baryons

$$N_b = n_b V \quad [\text{dimensionless}] . \quad (3.35)$$

All equations of state can be obtained from the thermodynamic potentials $E(S, V, N_i)$, $F(T, V, N_i)$, $G(T, p, N_i)$, $H(S, p, N_i)$ or $\Omega(T, V, \mu_i)$ by appropriate partial derivatives, in particular

$$T = \left. \frac{\partial E}{\partial S} \right|_{V, N_i} = \left. \frac{\partial H}{\partial S} \right|_{p, N_i} , \quad (3.36)$$

$$S = - \left. \frac{\partial F}{\partial T} \right|_{V, N_i} = - \left. \frac{\partial G}{\partial T} \right|_{p, N_i} = - \left. \frac{\partial \Omega}{\partial T} \right|_{V, \mu_i} , \quad (3.37)$$

$$-p = \left. \frac{\partial E}{\partial V} \right|_{S, N_i} = \left. \frac{\partial F}{\partial V} \right|_{T, N_i} , \quad (3.38)$$

$$V = \left. \frac{\partial G}{\partial p} \right|_{T, N_i} = \left. \frac{\partial H}{\partial p} \right|_{S, N_i} = \left. \frac{\partial \Omega}{\partial p} \right|_{T, \mu_i} , \quad (3.39)$$

$$\mu_i = \left. \frac{\partial E}{\partial N_i} \right|_{S, V, N_{j \neq i}} = \left. \frac{\partial F}{\partial N_i} \right|_{T, V, N_{j \neq i}} = \left. \frac{\partial G}{\partial N_i} \right|_{T, p, N_{j \neq i}} = \left. \frac{\partial H}{\partial N_i} \right|_{S, p, N_{j \neq i}} , \quad (3.40)$$

$$N_i = - \left. \frac{\partial \Omega}{\partial \mu_i} \right|_{S, V, \mu_{j \neq i}} . \quad (3.41)$$

The nuclear physics units given above can be converted easily to units that are frequently used in astrophysics. The most relevant cases are the following:

1. The temperature T in MeV can be expressed in K (Kelvin) by a division with the Boltzmann constant k_B as given in table 3.1. E.g., a temperature of 1 MeV corresponds to about $1.16 \cdot 10^{10}$ K and, conversely, 10^{10} K are about 0.862 MeV.
2. Any energy density, e.g, the internal energy density, in MeV fm^{-3} can be expressed in the CGS unit erg/cm^{-3} ($1 \text{ erg} = 10^{-7} \text{ J}$) by a multiplication with the factor $1.602176634 \cdot 10^{33}$.
3. The pressure p in MeV fm^{-3} can be expressed in the CGS unit dyn/cm^2 ($1 \text{ dyn} = 1 \text{ erg/cm}$) by a multiplication with the same numerical factor $1.602176634 \cdot 10^{33}$ as for the energy density.

3.6. Thermodynamic coefficients

In many applications tabulated values of the thermodynamic potentials and their first derivatives are not sufficient and additional quantities that depend on second derivatives of the thermodynamic potentials are needed. In general, the relevant thermodynamic potential can depend on a large number of independent variables such as temperature, volume and particle densities or chemical potentials. In applications, however, this number often reduces due to physical constraints. E.g., the condition of weak chemical (β) equilibrium between certain particle species reduces the number of independent chemical potentials. The condition of charge neutrality relates the particle numbers of charged constituents. In the following, we consider only systems that are completely determined by the temperature T , the volume V , the total number of baryons $N_b = n_b V$ and the charge number $N_q = Y_q N_b = Y_q n_b V$, or in the case of an equation of state where leptons are present, the number of charged leptons, which are equal to N_q due to charge neutrality. Then we have from the free energy per baryon $\mathcal{F}(T, n_b, Y_q)$ [MeV] the specific heat capacity at constant volume

$$c_V = \frac{T}{N_b} \frac{dS}{dT} \Big|_{V, N_b, N_q} = -T \frac{\partial^2 \mathcal{F}}{\partial T^2} \Big|_{n_b, Y_q} \quad [\text{dimensionless}] , \quad (3.42)$$

the tension coefficient at constant volume

$$\beta_V = \frac{dp}{dT} \Big|_{V, N_b, N_q} = \frac{dS}{dV} \Big|_{T, N_b, N_q} = n_b^2 \frac{\partial^2 \mathcal{F}}{\partial T \partial n_b} \Big|_{Y_q} \quad [\text{fm}^{-3}] , \quad (3.43)$$

the isothermal compressibility

$$\begin{aligned} \kappa_T &= -\frac{1}{V} \frac{dV}{dp} \Big|_{T, N_b, N_q} = \left(n_b \frac{\partial p}{\partial n_b} \Big|_{T, Y_q} \right)^{-1} \\ &= \left(n_b^2 \frac{\partial^2 (\mathcal{F} n_b)}{\partial n_b^2} \Big|_{T, Y_q} \right)^{-1} \quad [\text{MeV}^{-1} \text{fm}^3] , \end{aligned} \quad (3.44)$$

the expansion coefficient at constant pressure

$$\alpha_p = \frac{1}{V} \frac{dV}{dT} \Big|_{p, N_b, N_q} = \kappa_T \beta_V \quad [\text{MeV}^{-1}] , \quad (3.45)$$

the specific heat capacity at constant pressure

$$c_p = \frac{T}{N_b} \frac{dS}{dT} \Big|_{p, N_b, N_q} = c_V + \frac{T}{n_b} \alpha_p \beta_V \quad [\text{dimensionless}] , \quad (3.46)$$

the thermodynamic adiabatic index

$$\Gamma = \frac{c_p}{c_V} \quad [\text{dimensionless}] \quad (3.47)$$

and the adiabatic compressibility

$$\kappa_S = -\frac{1}{V} \frac{dV}{dp} \Big|_{S, N_b, N_q} = \frac{\kappa_T}{\Gamma} \quad [\text{MeV}^{-1} \text{fm}^3]. \quad (3.48)$$

Different definitions of the adiabatic index are sometimes employed in astrophysical contexts. E.g., the form

$$\tilde{\Gamma} = \frac{\partial \ln p}{\partial \ln n_b} \Big|_S = \frac{1}{p \kappa_S} \quad [\text{dimensionless}] \quad (3.49)$$

is used in applications with polytrope equations of state. A tilde is introduced here to distinguish it from (3.47). A further definition is given by

$$\gamma = \frac{\partial \ln p}{\partial \ln e} \Big|_S \quad [\text{dimensionless}] \quad (3.50)$$

with the internal energy density e . The square of the speed of sound (isoscalar longitudinal compression wave) in the medium for a one-fluid flow at finite temperatures is given by the relativistic definition

$$c_s^2 = \frac{dp}{de} \Big|_{S, N_b, N_q} = \frac{1}{h \kappa_S} = \frac{p}{h} \tilde{\Gamma} = \frac{p}{e} \gamma \quad [c^2] \quad (3.51)$$

with the enthalpy density $h = e + p$ [MeV fm^{-3}]. Alternatively, the expression

$$c_s^2 = \frac{n_b}{h} \frac{\partial p}{\partial n_b} \Big|_{Y_q, e} + \frac{\partial p}{\partial e} \Big|_{n_b, n_q} \quad [c^2] \quad (3.52)$$

can be used for constant hadronic charge fraction Y_q . In the application to cold compact stars, i.e., matter at zero temperature in β equilibrium, the EoS depends only on a single variable, e.g., the baryon density n_b or the baryon chemical potential μ_b . Then the square of the speed of sound can be calculated from

$$c_s^2 = \frac{dp}{de} \Big|_{T=0, \beta \text{ eq.}} = \frac{n_b}{\mu_b} \frac{d\mu_b}{dn_b} \Big|_{T=0, \beta \text{ eq.}} \quad [c^2] \quad (3.53)$$

in the unit of the squared speed of light. All of the above quantities can be calculated with the help of the second derivatives of $\mathcal{F}$ or f with respect to the parameters T , n_b , and n_q .

3.7. Microscopic quantities

Most EoS models are based on microscopic models that can give information on microscopic particle properties in addition to the thermodynamic and compositional quantities. Only few models are just simple parametrizations of thermodynamic quantities without recurrence to the underlying microphysics. The CompOSE database allows to

store these microscopic values, too, such that they can be used in applications. In the present version, we consider a small set of quantities that are discussed in the following. It can be easily extended if required.

In particular, we consider the effective masses of the particles in which case one has to distinguish between different definitions. The effective Landau mass

$$m_i^L \quad [\text{MeV}] \quad (3.54)$$

of a particle i is related to the single-particle density of states and defined as

$$\frac{1}{m_i^L} = \frac{1}{p_i^F} \left. \frac{dE_i}{dp} \right|_{p=p_i^F}, \quad (3.55)$$

where p_i^F denotes the respective Fermi-momentum and E_i is the single-particle energy. In non-relativistic Skyrme-Hartree-Fock models it appears in the kinetic energy contribution

$$T_i = \frac{p_i^2}{2m_i^L} \quad [\text{MeV}] \quad (3.56)$$

to the total single-particle energy. In addition to the kinetic single-particle energy T_i , a single-particle potential

$$U_i \quad [\text{MeV}] \quad (3.57)$$

contributes to the single-particle energy

$$E_i = T_i + U_i + m_i \quad (3.58)$$

in nonrelativistic models. The effective Dirac mass

$$m_i^D = m_i - S_i \quad [\text{MeV}] \quad (3.59)$$

is found in the single-particle Hamiltonian of relativistic models, such as relativistic mean-field approaches. This quantity does not reflect directly the density of single-particle states. At zero temperature the corresponding effective Landau mass can be obtained from

$$m_i^L = \sqrt{(m_i^D)^2 + (p_i^F)^2}. \quad (3.60)$$

The effective Dirac mass depends on the scalar self-energy

$$S_i = \Sigma_i \quad [\text{MeV}] \quad (3.61)$$

that enters the relativistic single-particle Hamiltonian together with the vector self-energy

$$V_i = \Sigma_i^0 \quad [\text{MeV}] \quad (3.62)$$

that is the time-component of the general four-vector self-energy Σ_i^μ .

In certain parameter regions of an EoS, the phenomena of superconductivity or superfluidity can be found. In this case, it is worthwhile to know the size of the pairing gaps

$$\Delta_i \quad [\text{MeV}], \quad (3.63)$$

e.g. in the $i = nn(^1S_0)$ channel in case of neutron pairing. In the general case, it can be energy and momentum dependent and cannot be easily tabulated in the present format. In many cases, it is given for specific conditions defined in the EoS data sheet.

3.8. Transport properties

Besides the thermodynamic quantities and coefficients as described in sections (3.5) and (3.6), there are transport coefficients that are required in astrophysical simulations of dynamically evolving systems in non-ideal hydrodynamics that consider dissipative effects. There is no general agreement on the choice of units for the transport coefficients. These have to be specified in the data sheet of each EoS.

3.8.1. Viscosities

Corrections to the equations of ideal hydrodynamics [21] can be introduced by considering the energy-momentum tensor (in relativistic notation neglecting heat flow)

$$T^{\mu\nu} = T_{\text{ideal}}^{\mu\nu} + \Pi^{\mu\nu} \quad [\text{MeV fm}^{-3}] \quad (3.64)$$

with the ideal contribution

$$T_{\text{ideal}}^{\mu\nu} = (e + p)u^\mu u^\nu - pg^{\mu\nu} = eu^\mu u^\nu - p\Delta^{\mu\nu} \quad [\text{MeV fm}^{-3}] \quad (3.65)$$

where

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \quad [\text{dimensionless}] \quad (3.66)$$

and the correction $\Pi^{\mu\nu}$. The ideal part contains the energy density e , the pressure p , the four-velocity u^μ and the metric tensor $g^{\mu\nu}$. The correction can be written in lowest order as

$$\Pi^{\mu\nu} = \pi^{\mu\nu} - \Phi\Delta^{\mu\nu} \quad [\text{MeV fm}^{-3}] \quad (3.67)$$

with a symmetric traceless contribution $\pi^{\mu\nu}$ and a traceful part. These can be expressed in first order as

$$\pi^{\mu\nu} = \eta \left(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3} \Delta^{\mu\nu} \nabla_\alpha u^\alpha \right) \quad [\text{MeV fm}^{-3}] \quad (3.68)$$

and

$$\Phi = \zeta \nabla_\alpha u^\alpha \quad [\text{MeV fm}^{-3}] \quad (3.69)$$

with the coefficients η [MeV fm^{-2}] and ζ [MeV fm^{-2}] that are called the shear and bulk viscosity, respectively.

3.8.2. Conductivities

The thermal conductivity κ [fm^{-2}] relates the heat flux density, i.e., the transported heat energy per area and time,

$$\vec{q} = -\kappa \vec{\nabla} T \quad [\text{MeV fm}^{-3}] \quad (3.70)$$

to the gradient of the temperature T . The electrical conductivity σ [fm^{-1}] connects the electrical current density

$$\vec{j} = \sigma \vec{E} \quad [e \text{ fm}^{-3}] \quad (3.71)$$

and the electrical field strength $\vec{E} = -\vec{\nabla}\varphi$ [$e \text{ fm}^{-2}$] with the electric potential φ [$e \text{ fm}^{-1}$] (e is the elementary charge). In general, the thermal and electrical conductivities can be tensors if the direction of the induced currents is not in the direction of the field gradients.

3.9. Contribution of photons

If photons are considered in an EoS, they are treated as a gas of massless bosons with simple analytical expressions for the relevant basic thermodynamic quantities. The contribution of the photon can be simply added to the thermodynamic quantities of the other constituents. We have the photon free energy density

$$f_\gamma(T) = -\frac{\pi^2}{45} T^4 \quad [\text{MeV fm}^{-3}] \quad (3.72)$$

the photon entropy density

$$s_\gamma(T) = \frac{4\pi^2}{45} T^3 \quad [\text{fm}^{-3}] \quad (3.73)$$

and the photon pressure

$$p_\gamma(T) = \frac{\pi^2}{45} T^4 \quad [\text{MeV fm}^{-3}] \quad (3.74)$$

depending only on the temperature. The chemical potential of the photon vanishes

$$\mu_\gamma = 0 \quad [\text{MeV}] \quad (3.75)$$

since it is its own antiparticle. From these quantities, all other relevant photon thermodynamic quantities can be derived.

3.10. Properties of compact stars

If an EoS model can provide the thermodynamic properties of charge-neutral matter in β equilibrium at zero temperature, e.g., a compact-star EoS, it is possible to calculate the radius R , mass M , and tidal deformability Λ of a sequence of compact stars. For

non-rotating, spherically symmetric stars, the mass-radius relation is obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) equation [11, 12]

$$\frac{dp}{dr} = -G \frac{M(r)e(r)}{r^2} \left[1 + \frac{p(r)}{e(r)} \right] \left[1 + \frac{4\pi r^3 p(r)}{M(r)} \right] \left[1 - \frac{2GM(r)}{r} \right]^{-1} \quad (3.76)$$

with the pressure p and the mass inside the radius r [km]

$$M(r) = 4\pi \int_0^r dr' (r')^2 e(r') \quad [\text{M}_\odot] \quad (3.77)$$

that depends on the (internal) energy density e . Equation (3.76) is a first-order differential equation that can be solved by assuming a particular central density n_c with corresponding pressure and energy density at the center of the star and then integrating outward until the pressure vanishes (or is sufficiently small) which defines the surface of the star at a radius R [km]. Thus the radius R and mass $M(R)$ of a particular compact star are obtained. Varying the central density n_c , the mass-radius relation of a sequence of stars is found.

A compact star will change its shape under the influence of an external gravitational field. The major effect will be a quadrupole deformation that can be quantified with the tidal deformability [13, 14]. This quantity is defined as

$$\Lambda = \frac{2}{3} k_2 C^{-5} \quad [\text{dimensionless}] \quad (3.78)$$

with the compactness

$$C = \frac{GM(R)}{R} \quad [\text{dimensionless}] \quad (3.79)$$

and the $l = 2$ Love number k_2 . It can be found as

$$\begin{aligned} k_2 = & \frac{8}{5} C^5 (1 - 2C)^2 [2 + 2C(y - 1) - y] \\ & \times \{ 2C [6 - 3y + 3C(5y - 8)] + 4C^3 [13 - 11y + C(3y - 2) + 2C^2(1 + y)] \\ & + 3(1 - 2C)^2 [2 - y + 2C(y - 1)] \ln(1 - 2C) \}^{-1} \quad [\text{dimensionless}] \end{aligned} \quad (3.80)$$

when the logarithmic derivative

$$y = \frac{d \ln H}{d \ln R}, \quad [\text{dimensionless}] \quad (3.81)$$

of a function $H(R)$ is known for a compact star configuration with radius R . The quantity H for a particular star is obtained by solving the second-order differential equation

$$\begin{aligned} 0 = & \frac{d^2 H}{dr^2} + \frac{dH}{dr} \left(\frac{2}{r} + \frac{d\Phi}{dr} - \frac{d\lambda}{dr} \right) \\ & + H \left[-\frac{6}{r^2} e^{2\lambda} - 2 \left(\frac{d\Phi}{dr} \right)^2 + 2 \frac{d\Phi^2}{dr^2} + \frac{3}{r} \frac{d\lambda}{dr} + \frac{7}{r} \frac{d\Phi}{dr} + 2 \frac{d\Phi}{dr} \frac{d\lambda}{dr} + \frac{f}{r} \left(\frac{d\Phi}{dr} + \frac{d\lambda}{dr} \right) \right] \end{aligned} \quad (3.82)$$

simultaneously with (3.76) for dp/dr using the potential

$$e^{2\lambda} = \left[1 - \frac{2GM(r)}{r} \right]^{-1} \quad [\text{dimensionless}] \quad (3.83)$$

of the Schwarzschild metric and the auxiliary quantities

$$\Phi = -\frac{1}{\varepsilon(r) + p(r)} \frac{dp}{dr} \quad [\text{km}^{-1}], \quad (3.84)$$

$$f = \frac{de}{dp} = 1/c_s^2 \quad [\text{dimensionless}] \quad (3.85)$$

that are obtained from the compact-star EoS. The boundary condition for $H(r)$ is given by the condition that $H(r) \rightarrow a_0 r^2$ for $r \rightarrow 0$ with an arbitrary constant a_0 which has no influence on the value of the tidal deformability.

Publicly available numerical codes exist to solve Einstein equations, which can in particular be used to obtain stationary models of rotating axisymmetric stars for a given EoS, see, e.g., the RNS code [15, 16, 17] and the LORENE library [18]. The latter has an interface to directly employ EoS data from CompOSE, see subsection 7.5.2. It can compute initial data for binary neutron stars, too.

Another interesting quantity for neutron stars with respect to cooling is the threshold density for the operation of the direct Urca process. For a neutron-star EoS with only electrons as leptonic degrees of freedom, the threshold charge fraction is given by

$$Y_q^{\text{dUrca}} = \frac{1}{9} \quad (3.86)$$

or

$$Y_q^{\text{dUrca}} = \left\{ 1 + \left[1 + \left(\frac{Y_e}{Y_e + Y_\mu} \right)^{1/3} \right]^3 \right\}^{-1} \quad (3.87)$$

if both electrons and muons are accounted for [19, 20]. The direct Urca process can only operate if Y_q is larger than these threshold values. Otherwise the neutrino generating direct Urca reactions such as for example

$$n \rightarrow p + e^- + \bar{\nu}_e \quad (3.88)$$

are kinematically blocked; energy and momentum conservation cannot be fulfilled simultaneously. From the solution of the TOV equation and the resulting NS masses as function on the central density – together with the range of baryon densities allowing for direct Urca processes – threshold NS masses for the processes can be obtained. Since in general Y_q increases with n_b in the core, this gives a lower limit on the NS mass for the direct Urca process to operate for a given EoS.

Part II.

For Contributors

The success of the CompOSE data base depends on the support of nuclear physicists providing tables with their favourite EoS. Some well known EoS used extensively in astrophysical applications are already incorporated in the CompOSE data base and are ready for use. However, a larger collection of EoS from different models is highly desirable. In order to be suitable for a simple usage, contributors should follow the guidelines specified in this part of the manual.

4. How to prepare EoS tables

In general, tables with EoS data contain a wealth of information on the thermodynamic properties, the composition of dense matter and on the microphysical properties of the constituents. In order to minimize the memory size, only the essential thermodynamic quantities should be stored in the tables that are used in the CompOSE data base. These quantities are selected such that they are sufficient to calculate a particular thermodynamic potential, here the free energy F [MeV], and its first derivatives with respect to the parameters that correspond to the natural variables of F , i.e. temperature T [MeV], volume V [fm³] and particle numbers N_i [dimensionless]. However, it is worthwhile to have redundant information in order to check the consistency of the EoS tables or to simplify the determination of additional quantities. Further thermodynamic quantities relevant for astrophysical applications can, if not directly available, be generated with the help of thermodynamic identities and, for quantities depending on higher derivatives, by using appropriate interpolation schemes. The organization of the data concerning composition etc. depends on the particular EoS. Thus it is required that up to three separate tables are provided with unique formats that contain the thermodynamic quantities (required), the information on the composition of matter (depending on the choice of the constituents of the model, optional) and the microscopic properties of the particles (optional), respectively. Details of the employed discretization mesh of the parameters have to be supplied in additional tables.

Additional information that is not contained in the EoS tables has to be provided by the contributor. It will appear in the data sheet that accompanies each EoS and is available on the CompOSE web site. Predictions for properties of cold, spherical compact stars will be made available in a separate file if it is possible to calculate these quantities in the model. See the new “quick guide for providers” for a summary.

4.1. Tabulation of quantities

4.1.1. Parameters and parameter ranges

Because of the imposed physical conditions, see section 3.2, the state of the system is uniquely characterized by only three quantities, see section 3.5, that are used as parameters in the EoS tables with the properties of dense matter:

1. **temperature** T [MeV],
2. **baryon number density** n_b [fm⁻³],
3. **charge fraction of strongly interacting particles** Y_q [dimensionless].

In order to be useful in many applications, the parameter ranges should be chosen as wide as possible and the mesh spacing as fine as possible, i.e. the number of grid points as large as possible within reason. The recommended parameter values for the definition of the grid are given in the tables in the end of this chapter. In some cases it might be useful to adapt the resolution of the tables, in particular for the baryon density, to the physical situation and occurring phenomena. E.g. a rapid change in some quantities or a phase transition can occur. For this purpose, several tables for different blocks in density with different resolution can be supplied for a single EoS model.

The recommended discretization scheme for a general purpose EoS table depends on the parameters:

1. temperature

In this case, two standards are suggested:

a. linear mesh in $\ln T$, i.e.

$$T(i_T) = T^{\text{ref}}(f_T)^{i_T-1} \quad f_T = 10^{1/M_T} \quad i_T = N_T^{\min}, N_T^{\min} + 1, \dots, N_T^{\max} \quad (4.1)$$

with parameters $T^{\text{ref}} > 0$ MeV, $N_T^{\min} < N_T^{\max}$ and $M_T > 1$ points per decade in temperature;

b. a mesh that is linear in T at low temperatures and linear in $\ln T$ at high temperatures, i.e.

$$T(i_T) = T^{\text{ref}} \frac{\sinh(f_T i_T)}{\sinh(f_T)} \quad i_T = N_T^{\min}, N_T^{\min} + 1, \dots, N_T^{\max} \quad (4.2)$$

with parameters $T^{\text{ref}} > 0$ MeV, $N_T^{\min} < N_T^{\max}$ and $f_T > 0$. The resolution parameter $M_T > 0$ determines $f_T = \ln(10)/M_T$;

In both cases the index $i_T = 0$ is reserved for $T = 0$ MeV;

2. baryon number density

linear mesh in $\ln n_b$, i.e.

$$n_b(i_{n_b}) = n_b^{\text{ref}}(f_{n_b})^{i_{n_b}-1} \quad f_{n_b} = 10^{1/M_{n_b}} \quad i_{n_b} = N_{n_b}^{\min}, N_{n_b}^{\min} + 1, \dots, N_{n_b}^{\max} \quad (4.3)$$

with parameters $n_b^{\text{ref}} > 0$ fm⁻³, $N_{n_b}^{\min} < N_{n_b}^{\max}$ and $M_{n_b} > 1$ points per decade in the baryon number density;

3. charge fraction of strongly interacting particles

linear mesh in Y_q , i.e.

$$Y_q(i_{Y_q}) = \frac{i_{Y_q}}{M_{Y_q}} \quad i_{Y_q} = N_{Y_q}^{\min}, N_{Y_q}^{\min} + 1, \dots, N_{Y_q}^{\max} \quad (4.4)$$

with parameters $N_{Y_q}^{\min} < N_{Y_q}^{\max}$ and $M_{Y_q} > N_{Y_q}^{\max}$ with a resolution in Y_q of $1/M_{Y_q}$.

Each grid point in the table is identified with the triple (i_T, i_{n_b}, i_{Y_q}) of indices. The recommended values for the parameters are given in section 4.2.1. Actually used meshes for particular EoS are specified in the online documentation for each individual EoS table.

4.1.2. Thermodynamic consistency

The equations of state included in the CompOSE data base are required to fullfill some basic thermodynamic consistency relations. Of course, the consistency can only hold up to some numerical level. Measures of the thermodynamic consistency are included in the characterisation of the EoS tables on the web site.

Because the free energy density is considered in the present case as the basic thermodynamic potential, the homogeneity condition, cf. eq. (3.19),

$$f(T, n_b, Y_q) = -p + \mu_b n_b + \mu_q (n_q - n_{le} - n_{l\mu}) + \mu_{le} n_{le} + \mu_{l\mu} n_{l\mu} \quad (4.5)$$

with the pressure p , the chemical potentials $\mu_b, \mu_q, \mu_{le}, \mu_{l\mu}$ and corresponding densities $n_b, n_q, n_{le}, n_{l\mu}$ should hold. Note that n_q contains only the contribution of strongly interacting particles to the charge density, that a fixed relation between μ_{le} and $\mu_{l\mu}$ is assumed and that $n_{le} + n_{l\mu} = n_q = Y_q n_b$ due to local charge neutrality if leptons are considered in the equation of state. Additionally, in eq. (4.5) it was assumed that the strangeness chemical potential μ_s is zero.

It is convenient to define the effective lepton density as

$$n_l = n_{le} + n_{l\mu} \quad [\text{fm}^{-3}] \quad (4.6)$$

and the effective lepton chemical potential

$$\mu_l = \frac{\mu_{le} n_{le} + \mu_{l\mu} n_{l\mu}}{n_l} \quad [\text{MeV}] \quad (4.7)$$

such that

$$f(T, n_b, Y_q) = -p + (\mu_b + Y_q \mu_q) n_b \quad (4.8)$$

in case of an EoS without leptons and

$$f(T, n_b, Y_q) = -p + (\mu_b + Y_l \mu_l) n_b \quad (4.9)$$

in the case with leptons and charge neutrality ($Y_l = Y_q$), respectively. The tabulated values for the entropy density, pressure and chemical potentials should be given by the first partial derivatives as

$$s = - \left. \frac{\partial f}{\partial T} \right|_{n_b, Y_q}, \quad (4.10)$$

$$p = n_b^2 \left. \frac{\partial(f/n_b)}{\partial n_b} \right|_{T, Y_q}, \quad (4.11)$$

$$\mu_b + Y_i \mu_i = \left. \frac{\partial f}{\partial n_b} \right|_{T, Y_q}, \quad (4.12)$$

$$\mu_i = \frac{1}{n_b} \left. \frac{\partial f}{\partial Y_i} \right|_{T, n_b} \quad (4.13)$$

where $i = l(q)$ if leptons are (not) included in the EoS. These relations will be used to derive the first derivatives of f from the tabulated thermodynamic quantities s , p , μ_b , μ_q , and μ_l in some particular interpolation schemes.

For a mixed second partial derivative, the Maxwell relations are

$$-n_b^2 \left. \frac{\partial(s/n_b)}{\partial n_b} \right|_{T, Y_q} = \left. \frac{\partial p}{\partial T} \right|_{n_b, Y_q}, \quad (4.14)$$

$$-\left. \frac{\partial s}{\partial n_x} \right|_{T, n_y, y \neq x} = \left. \frac{\partial \mu_x}{\partial T} \right|_{n_y}, \quad (4.15)$$

$$\left. \frac{\partial p}{\partial n_x} \right|_{T, n_y, y \neq x} = n_b \left. \frac{\partial \mu_x}{\partial n_b} \right|_{T, n_y, y \neq b}, \quad (4.16)$$

$$\left. \frac{\partial \mu_x}{\partial n_y} \right|_{T, n_z, z \neq y} = \left. \frac{\partial \mu_y}{\partial n_x} \right|_{T, n_z, z \neq x} \quad (4.17)$$

where $x, y, z \in \{b, q, le, l\mu\}$ should hold.

The knowlegde of three functions $p = p(T, n_b, Y_q)$, $\mu_b = \mu_b(T, n_b, Y_q)$, and $\mu_q = \mu_q(T, n_b, Y_q)$ or $\mu_l = \mu_l(T, n_b, Y_q)$ is sufficient to recover the free energy density $f = f(T, n_b, Y_q)$ for the particular physical conditions of section 3.2 ($\mu_s = 0$ and $n_q = n_l$). Further quantities can be derived by partial derivatives with respect to the parameters T , n_b and Y_q .

Thus it would be convenient to store only these three quantities in the EoS table for the thermodynamic properties in order to reduce the memory size. However, we require that the pressure, the entropy, the three chemical potentials and also the free and internal energy densities should be provided independently in the EoS tables for checking purposes. It is also allowed to store more than these seven basic quantities.

4.2. Structure of tables

The CompOSE data base contains for each EoS model data tables that specify the used *parameter grid* and tables that contain the *thermodynamic properties*, the *compositional information* and the *microscopic information*, respectively. Properties of a sequence of cold, spherical *compact stars* will be collected in a separate file if cacculable in the EoS model.

4.2.1. Types of EoS tables

In general, three types of EoS input tables should be available for each EoS model in order to address different applications:

1. Three-dimensional tables

These tables depend on all three indepedent parameters T , n_b and Y_q . There is only one case:

- **General purpose EoS table**

The recommended parameter values for the definition of the grid are given in table 4.1. This choice of discretization corresponds to a table with $N_T^{\max} \times N_{n_b}^{\max} \times N_{Y_q}^{\max} = 1119720(1462860)$ data points for the case b(a) in the T grid. The table includes neither points with $T = 0$ MeV nor with $Y_q = 0$.

2. Two-dimensional tables

These tables depend on two of the three independent parameters T , n_b and Y_q . Four cases are offered here:

- **Zero-temperature EoS table**

The recommended parameter values for the definition of the grid are given in table 4.2. This choice of discretization corresponds to a table with $N_{n_b}^{\max} \times (N_{Y_q}^{\max} + 1) = 18361$ data points. Only $T = 0$ MeV is considered. Data points for $Y_q = 0$, i.e. pure neutron matter, are included in this EoS table.

- **Symmetric matter EoS table**

The recommended parameter values for the definition of the grid are given in table 4.3. This choice of discretization corresponds to a table with $N_T^{\max} \times N_{n_b}^{\max} = 18963(24381)$ data points for the case b(a) in the T grid. Only $Y_q = 0.5$ is considered. $T = 0$ MeV, i.e. the zero-temperature case, is (not) included in this EoS table for case b(a).

- **Neutron matter EoS table**

The recommended parameter values for the definition of the grid are given in table 4.3. This choice of discretization corresponds to a table with $N_T^{\max} \times N_{n_b}^{\max} = 18963(24381)$ data points for case b(a). Only $Y_q = 0$ is considered. $T = 0$ MeV, i.e. the zero-temperature case, is (not) included in this EoS table for case b(a).

- **EoS table of β -equilibrated matter**

The recommended parameter values for the definition of the grid are given in table 4.3. This choice of discretization corresponds to a table with $N_T^{\max} \times N_{n_b}^{\max} = 18963(24381)$ data points for the case b(a) in the T grid. The charge fraction of strongly interacting particles Y_q is determined by the conditions of charge neutrality and β equilibrium with leptons. $T = 0$ MeV, i.e. the zero-temperature case, is (not) included in this EoS table for case b(a).

3. One-dimensional tables

These tables depend only on the parameter n_b . Three cases are offered here:

- **EoS table of cold symmetric matter**

The recommended parameter values for the definition of the grid are given in table 4.4. This choice of discretization corresponds to a table with $N_{n_b}^{\max} = 301$ data points. Only $Y_q = 0.5$ and $T = 0$ MeV is considered.

- **EoS table of cold neutron matter**

The recommended parameter values for the definition of the grid are given in table 4.4. This choice of discretization corresponds to a table with $N_{n_b}^{\max} = 301$ data points. Only $Y_q = 0.0$ and $T = 0$ is considered.

- **EoS table of cold β -equilibrated matter**

The recommended parameter values for the definition of the grid are given in table 4.4. This choice of discretization corresponds to a table with $N_{n_b}^{\max} = 301$ data points. Only $T = 0$ MeV is considered and the charge fraction of strongly interacting particles Y_q is determined by the conditions of β -equilibrium and local charge neutrality with leptons.

Every set of EoS tables contains up to three tables: the first that specifies the thermodynamic state of the system (always required), the second that contains information on the composition of the matter (optional), the third that gives microscopic information (optional).

The format of the thermodynamic, the compositional and the microscopic table, respectively, has to follow the following prescriptions.

4.2.2. Table with thermodynamic quantities (required)

The first row of this table contains three entries: the mass of the neutron m_n and that of the proton m_p in MeV of the particular EoS model and then an integer I_l that indicates whether the EoS contains leptons or not. If electrons (and/or muons) are considered in the EoS, one sets $I_l = 1$. For other values of I_l , it is assumed that there are no electrons (and/or muons) and the lepton chemical potential μ_l should be set to zero. The neutron mass m_n is used for scaling certain quantities as specified below. The following rows in this table contain information on the particular grid point in the table and the relevant thermodynamic quantities. The actually tabulated quantities are given by

1. **pressure divided by the baryon number density** $Q_1 = p/n_b$ [MeV],
2. **entropy per baryon (or entropy density per baryon density)** $Q_2 = s/n_b$ [dimensionless],
3. **scaled and shifted baryon chemical potential** $Q_3 = \mu_b/m_n - 1$ [dimensionless],
4. **scaled charge chemical potential** $Q_4 = \mu_q/m_n$ [dimensionless],
5. **scaled effective lepton chemical potential** $Q_5 = \mu_l/m_n$ [dimensionless],
6. **scaled free energy per baryon** $Q_6 = f/(n_b m_n) - 1$ [dimensionless],
7. **scaled internal energy per baryon** $Q_7 = e/(n_b m_n) - 1$ [dimensionless].

For an ideal gas Q_1 is just the temperature T . The traditional scaling of the chemical potentials and the energies per baryon with the neutron mass m_n has been introduced.

with N_{pairs} pairs that combine the particle index I_i as defined in tables 3.2, 3.3, 3.4 and 3.5 and the corresponding particle fraction Y_i . (Note the definition of particle fractions in equation 3.10.) In addition, there are N_{quad} quadruples that contain an index I_i that specifies a group of nuclei $\mathcal{M}_{I_i}$ with average mass number

$$A_{I_i}^{\text{av}} = \frac{\sum_{j \in \mathcal{M}_{I_i}} A_j Y_j}{\sum_{j \in \mathcal{M}_{I_i}} Y_j}, \quad (4.20)$$

average charge number $Z_{I_i}^{\text{av}}$

$$Z_{I_i}^{\text{av}} = \frac{\sum_{j \in \mathcal{M}_{I_i}} Z_j Y_j}{\sum_{j \in \mathcal{M}_{I_i}} Y_j}, \quad (4.21)$$

and combined fraction

$$Y_{I_i} = \frac{\sum_{j \in \mathcal{M}_{I_i}} A_j Y_j}{A_{I_i}^{\text{av}}} = \sum_{j \in \mathcal{M}_{I_i}} Y_j. \quad (4.22)$$

The average neutron number

$$N_{I_i}^{\text{av}} = \frac{\sum_{j \in \mathcal{M}_{I_i}} N_j Y_j}{\sum_{j \in \mathcal{M}_{I_i}} Y_j}, \quad (4.23)$$

is not stored because it is easily found from $N_{I_i}^{\text{av}} = A_{I_i}^{\text{av}} - Z_{I_i}^{\text{av}}$. In case that there are no pairs (quadruples) to be stored, the number N_{pairs} (N_{quad}) is set to zero. The average mass and charge numbers correspond to those of a representative heavy nucleus if there is only one group of nuclei except the lightest that are considered explicitly. However, it is also possible to define several subsets of nuclei with corresponding average mass numbers, charge numbers and fractions. The correlation between the index I_i and the set of nuclei $\mathcal{M}_{I_i}$ is defined for each EoS model separately and given in the data sheet on the CompOSE web pages. Since the composition of matter can vary rapidly with a change of the parameters, it is not required that the fractions of all particles or particle sets have to be given in each row. It is sufficient to list only those that are relevant, e.g. with nonzero Y_i . Others can be omitted.

4.2.4. Table with information on microscopic quantities (optional)

Besides the information on thermodynamic and compositional quantities, most EoS models can provide additional information on microscopic quantities of an individual particle i , e.g.

1. **Landau effective mass divided by the particle mass** m_i^L/m_i [dimensionless],
2. **Dirac effective mass divided by the particle mass** m_i^D/m_i [dimensionless],
3. **non-relativistic single-particle potential** U_i [MeV],
4. **relativistic vector self-energy** V_i [MeV],

5. **relativistic scalar self-energy** S_i [MeV],

6. **pairing gap** Δ_i [MeV].

These quantities are stored in an additional table if available. In this case, each line has the format

$$i_T \ i_{n_b} \ i_{Y_q} \ N_{\text{qty}} \ \underbrace{K_1 \ q_{K_1} \ K_2 \ q_{K_2} \ \dots \ K_{N_{\text{qty}}} \ q_{K_{N_{\text{qty}}}}}_{N_{\text{qty}} \text{ pairs}} \quad (4.24)$$

where N_{qty} is the number of stored quantities q_{K_i} . The composite index K_i identifies uniquely both the particle or correlation and the physical quantity. It is formed as

$$K_i = 1000 \ I_i + J_i \quad (4.25)$$

with the particle or correlation index I_i from tables 3.2, 3.3, 3.4 or 3.5, respectively, and the quantity index J_i from table 7.5.

4.2.5. Table with information on transport quantities (optional)

The structure of this table will be defined after the CompOSE code has been updated to handle transport quantities.

4.2.6. Table with information on properties of compact start (optional)

If an EoS model can predict the properties of cold, spherical compact stars, e.g., the mass-radius relation and corresponding tidal deformabilities, these data will be provided in a separate file. The first line of this file identifies the given quantities and their units, followed by their actual values in the subsequent lines. The exact structure depends on the available data. Most relevant are

- **radius** R [km],
- **mass** M [$M_\odot$],
- **tidal deformability** Λ [dimensionless],
- **central density** n_c [fm^{-3}].

It is required that the radius R and the mass M using the units km and solar masses, $M_\odot$, are given in the first and second column of the file, respectively.

4.2.7. Tables with parameters

The mesh points for the general purpose EoS are stored in three individual data files that are used as an input in addition to the three tables containing the thermodynamic, compositional and microphysical information. It is assumed that the discretization scheme for the relevant parameters in the zero-temperature and in the neutron-matter table is identical to the grid for the general purpose table. Parameter values should be given

with at least eight significant digits. For the parameter temperature, the corresponding table contains a single entry in each row. In the first row, $N_T^{\min}$ is given, and $N_T^{\max}$ in the second row. Then the parameter values $T(N_T^{\min})$, $T(N_T^{\min} + 1)$, $\dots$, $T(N_T^{\max})$, are given in every following row. In total, the table contains $N_T^{\max} - N_T^{\min} + 3$ rows. The same storage scheme applies for the parameters n_b and Y_q .

4.2.8. Identification of tables

Each Eos table is identified by a unique name and a unique extension that correspond to the model and type of table, respectively. Using the generic name `eos` for a particular EoS model, there are always three parameter tables

- `eos.t` (required)
- `eos.nb` (required)
- `eos.yq` (required)

that define the discretization mesh for the three parameters T , n_b and Y_q as discussed in the previous section. All three tables are required even for EoS tables with less than three dimensions.

The thermodynamic, compositional and microscopic quantities are stored in three tables with the generic names

- `eos.thermo` (required)
- `eos.compo` (optional)
- `eos.micro` (optional)

where only the first one is required for every EoS model since information on the composition or microscopic details are not always available.

If the EoS model allows to calculate properties of cold, spherical compact stars, these data will be stored in a file with the name

- `eos.mr` (optional).

4.2.9. Precision of tabulated quantities

EoS quantities of a particular model have a limited precision in practical calculations. Machine precision is hardly ever reached. The format of the tabulated data should reflect this fact and only the significant number of digits should be given to avoid excessive memory size. Usually eight significant digits are sufficient since the error in the interpolation can be considerably larger.

4.3. Data sheet

Information on an EoS that is not provided directly in the tables but relevant for the characterisation of the particular model can be found in a data sheet that comes along with the tables on the CompOSE web pages. Some quantities are extracted automatically from the tabulated data, others need to be specified by the contributor. E.g., characteristic nuclear matter parameters (see section 6.1) are obtained by using the program `compose.f90` (see section 7.5) if an EoS table for pure nuclear matter without electrons or muons is available. Similarly, information on the parameter grid, type and dimension of the EoS table, considered particles and additionally stored quantities are extracted by the program. The EoS of stellar matter in β -equilibrium is generated by using `compose.f90` if the EoS table for matter with electrons or muons is provided by the contributor. From this extracted EoS table, characteristic properties of neutron stars, e.g. maximum masses or the threshold of the direct Urca process, are calculated and given in the data sheet. It also contains information on the considered constituent particles in the model. The contributor has to supply a short characterisation of the EoS model and relevant references. He/she also needs to define the meaning of the index I_{phase} for the phases considered in the EoS.

Table 4.1.: Parameter values for the recommended discretization in T , n_b and Y_q for a general purpose EoS table.

quantity	reference value(s)	minimum index	maximum index	resolution parameter
T , case a	$T^{\text{ref}} = 0.1 \text{ MeV}$	$N_T^{\text{min}} = 1$	$N_T^{\text{max}} = 81$	$M_T = 25$
T , case b	$T^{\text{ref}} = 0.1 \text{ MeV}$	$N_T^{\text{min}} = 1$	$N_T^{\text{max}} = 62$	$M_T = 25$
n_b	$n_b^{\text{ref}} = 10^{-12} \text{ fm}^{-3}$	$N_{n_b}^{\text{min}} = 1$	$N_{n_b}^{\text{max}} = 301$	$M_{n_b} = 25$
Y_q	N/A	$N_{Y_q}^{\text{min}} = 1$	$N_{Y_q}^{\text{max}} = 60$	$M_{Y_q} = 100$

Table 4.2.: Parameter values for the recommended discretization in n_b and Y_q for a zero-temperature EoS table.

quantity	reference value	minimum index	maximum index	resolution parameter
n_b	$n_b^{\text{ref}} = 10^{-12} \text{ fm}^{-3}$	$N_{n_b}^{\text{min}} = 1$	$N_{n_b}^{\text{max}} = 301$	$M_{n_b} = 25$
Y_q	N/A	$N_{Y_q}^{\text{min}} = 0$	$N_{Y_q}^{\text{max}} = 60$	$M_{Y_q} = 100$

Table 4.3.: Parameter values for the recommended discretization in T and n_b for EoS tables of symmetric matter, neutron matter or β -equilibrated matter.

quantity	reference value	minimum index	maximum index	resolution parameter
T , case a	$T^{\text{ref}} = 0.1 \text{ MeV}$	$N_T^{\text{min}} = 1$	$N_T^{\text{max}} = 81$	$M_T = 25$
T , case b	$T^{\text{ref}} = 0.1 \text{ MeV}$	$N_T^{\text{min}} = 0$	$N_T^{\text{max}} = 62$	$M_T = 25$
n_b	$n_b^{\text{ref}} = 10^{-12} \text{ fm}^{-3}$	$N_{n_b}^{\text{min}} = 1$	$N_{n_b}^{\text{max}} = 301$	$M_{n_b} = 25$

Table 4.4.: Parameter values for the recommended discretization in n_b for one-dimensional EoS tables.

quantity	reference value	minimum index	maximum index	resolution parameter
n_b	$n_b^{\text{ref}} = 10^{-12} \text{ fm}^{-3}$	$N_{n_b}^{\text{min}} = 1$	$N_{n_b}^{\text{max}} = 301$	$M_{n_b} = 25$

5. Extending CompOSE

CompOSE is meant to be under constant development. The main aim is to enlarge the data base by adding EoS tables of more and more models. The most simple way is to convert tables of existing model calculations to the generic CompOSE scheme of tabulation in the data files. A more involved task is to develop a new EoS model and to provide the results in the appropriate format. In either case, we ask you to contact the CompOSE core team by sending an email to

`develop.compose@obspm.fr`.

All questions will be clarified by them on how your results can be incorporated in the CompOSE data base and made accessible to the public.

For the future, it is foreseen that the functionality of the CompOSE database will be extended in order to make even more results of EoS model calculations available that are not included in the present sets of data. It is also envisioned to add more quantities to the data tables that might be relevant for astrophysical applications and beyond, in particular transport quantities.

Part III.

For Users

6. Models for the equation of state

There is a large number of model equations of state. They fall into different classes depending on the theoretical approach. The main difficulty in generating an equation of state is the description of the strongly interacting constituents, hadrons and/or quarks. See reference [2] for a in-depth review. The EoS tables on the CompOSE website are not classified according to the model but to their structure of tabulation and possible application.

Details and characteristic parameters of each particular EoS can be found in a data sheet that accompanies the data tables on the CompOSE web site

<https://compose.obspm.fr>.

For electrons and muons, a uniform Fermi gas model is employed in many cases. This assumption simplifies the calculation significantly in case of spatially inhomogeneous charge distributions. Details on the treatment of electrons and muons are given for each EoS separately on the data sheet.

At finite temperatures, photons contribute to the thermodynamic properties of the system. Their treatment is discussed in section 3.9. For each EoS in the database, a remark is given whether photons are considered or not.

The definition of commonly used nuclear matter parameters is given in section 6.1. They characterize the essential properties of nuclear matter and can serve in a first comparison of the various models. Be, however, careful since these parameters are only valid in the vicinity of saturation for symmetric, i.e. same number of protons and neutrons, matter. Specific values of these parameters are given in the data sheet for the individual models and in a summary table on the CompOSE website. Collections of nuclear matter parameters are available for widely used Skyrme Hartree-Fock models [22] and for nonrelativistic Hartree-Fock models with Gogny-type interactions [23]. Results for relativistic mean-field models can be found in [24]. See the review [2] for recent experimental constraints.

In the end of this section, further remarks of caution are given.

6.1. Nuclear matter properties

Realistic models of nuclear, i.e. purely hadronic, matter without leptons show some general features that are related to the occurrence of the saturation phenomenon. Due to the symmetries of the strong interaction, uniform nuclear matter at zero temperature reaches a state with the largest binding energy per nucleon at a finite saturation density n_b^{sat} with equal concentrations of neutrons and protons¹. For $T = 0$ MeV, the energy

¹This is exactly true only if the mass difference between neutrons and protons is neglected.

per nucleon $\mathcal{E}$ can be considered as a function of the baryon number density n_b and the asymmetry

$$\alpha = \frac{n_n - n_p}{n_n + n_p} = 1 - 2Y_q \quad [\text{dimensionless}] \quad (6.1)$$

that vanishes for symmetric nuclear matter, i.e. $\alpha = 0$ and $Y_q = 0.5$. It can be expanded in a power series around the saturation point as

$$\begin{aligned} \mathcal{E}(n_b, \alpha) = & -B_{\text{sat}} + \frac{1}{2}Kx^2 + \frac{1}{6}Qx^3 + \dots \\ & + \alpha^2 \left(J + Lx + \frac{1}{2}K_{\text{sym}}x^2 + \dots \right) + \dots \quad [\text{MeV}] \end{aligned} \quad (6.2)$$

with the relative deviation of the actual density from the saturation density

$$x = \frac{n_b - n_b^{\text{sat}}}{3n_b^{\text{sat}}} \quad [\text{dimensionless}] \quad (6.3)$$

and the asymmetry α . The coefficients $B_{\text{sat}}, K, Q = -K', J, L, K_{\text{sym}}$ [MeV], $\dots$ and n_b^{sat} [fm⁻³] characterize the behaviour of the EoS. They can, of course, only give an indication of the general features of the EoS, since these quantities are relevant essentially in the vicinity of the saturation density at zero temperature and close to symmetric nuclear matter (equal number of protons and neutrons). Extrapolations based on this polynomial expansion might be dangerous. The numerical factor in equation (6.3) is of historical origin since the Fermi momentum k_F [fm⁻¹] instead of the density $n_b = 2k_F^3/(3\pi^2)$ of symmetric nuclear matter at zero temperature was originally used as the expansion parameter. Note that contributions linear in α and linear in x for $\alpha = 0$ are absent in the expansion due to the condition of minimum energy per baryon.

In order to study the dependence on the asymmetry α , it is also convenient to introduce the symmetry energy

$$E_{\text{sym}}(n_b) = \frac{1}{2} \frac{\partial^2 \mathcal{E}(n_b, \alpha)}{\partial \alpha^2} \Big|_{\alpha=0} \quad [\text{MeV}] \quad (6.4)$$

that is a function of the baryon number density n_b only. In many cases the quadratic approximation in α in equation (6.2) is sufficient. Then the symmetry energy can also be calculated from the finite difference approximation

$$E_{\text{sym}}(n_b) = \frac{1}{2} [\mathcal{E}(n_b, -1) - 2\mathcal{E}(n_b, 0) + \mathcal{E}(n_b, 1)] \quad [\text{MeV}] \quad (6.5)$$

comparing symmetric matter with pure neutron and pure proton matter.

In the following, the coefficients appearing in equation (6.2) are discussed in more detail.

- The **saturation density** n_b^{sat} [fm⁻³] of symmetric nuclear matter is defined by the condition that the pressure vanishes, i.e.

$$p = n_b^2 \frac{d\mathcal{E}(n_b, 0)}{dn_b} \Big|_{n_b=n_b^{\text{sat}}} = 0 \quad [\text{MeV fm}^{-3}] \quad (6.6)$$

and the energy per baryon becomes minimal.

- The **binding energy at saturation** B_{sat} [MeV] of symmetric nuclear matter can be obtained from Bethe-Weizsäcker mass formulas for nuclei ${}^A Z$ by an extrapolation of $A = 2Z$ to infinity.
- The **incompressibility of bulk nuclear matter** K [MeV] quantifies the curvature of the binding energy per baryon with respect to the density variation at saturation since it is defined by

$$K = 9n_b^2 \frac{\partial^2 \mathcal{E}(n_b, 0)}{\partial n_b^2} \bigg|_{n_b=n_b^{\text{sat}}} = 9n_b \frac{\partial(p/n_b)}{\partial n_b} \bigg|_{n_b=n_b^{\text{sat}}} \quad [\text{MeV}]. \quad (6.7)$$

It is related to the isothermal compressibility κ_T (3.44) at zero temperature, zero asymmetry and saturation density by

$$K = \frac{9}{\kappa_T n_b^{\text{sat}}} \quad [\text{MeV}]. \quad (6.8)$$

Thus K would be better called compression modulus. It can be derived, e.g. from studies of isoscalar giant monopole excitations of nuclei. The extraction from data on isoscalar giant monopole resonances depends on the density dependence of the nuclear symmetry energy, a quantity under intensive debate in recent years. See also a recent comprehensive study of giant monopole resonances [25].

- The **skewness coefficient of bulk nuclear matter** $Q = -K'$ [MeV] is defined by the third derivative of the energy per baryon as

$$Q = 27n_b^3 \frac{\partial^3 \mathcal{E}(n_b, 0)}{\partial n_b^3} \bigg|_{n_b=n_b^{\text{sat}}} = 27n_b^2 \frac{\partial^2(p/n_b)}{\partial n_b^2} \bigg|_{n_b=n_b^{\text{sat}}} - 6K \quad [\text{MeV}]. \quad (6.9)$$

The value of Q in combination with K determines the surface properties of nuclei, e.g. the ratio of the surface tension to the surface thickness. Thus, in order to fix Q in a particular model, it is important to include quantities like radii and the surface thickness in the procedure to fit the model parameters.

- The **symmetry energy at saturation** J [MeV] is just given by

$$J = E_{\text{sym}}(n_b^{\text{sat}}) \quad [\text{MeV}]. \quad (6.10)$$

This coefficient mainly determines the isospin dependence of the binding energy of nuclei. It is important in predicting masses of exotic nuclei far away from the valley of stability in the chart of nuclei.

- The **symmetry energy slope coefficient** L [MeV] that is obtained from

$$L = 3n_b \frac{dE_{\text{sym}}(n_b)}{dn_b} \bigg|_{n_b=n_b^{\text{sat}}} \quad [\text{MeV}]. \quad (6.11)$$

This coefficient is related to the density dependence of the neutron matter EoS near the saturation density. It is strongly correlated with the neutron skin thickness of heavy nuclei like ^{208}Pb . Unfortunately, experimentally determined values of the neutron skin thickness are not very precise so far and a large range of values for L is found by comparing different EoS models.

- The **symmetry incompressibility** K_{sym} [MeV] quantifies the curvature of the symmetry energy with respect to the density variation at saturation. It is defined by

$$K_{\text{sym}} = 9n_b^2 \left. \frac{d^2 E_{\text{sym}}}{dn_b^2} \right|_{n_b=n_b^{\text{sat}}} \quad [\text{MeV}]. \quad (6.12)$$

For each EoS in the CompOSE data base, the actual values of the nuclear matter coefficients are given if available.

6.2. Remarks of caution

We provide equations of state tables in the CompOSE data base that cover large ranges in temperature, baryon number density and charge fraction of strongly interacting particles. However, we cannot guarantee that the physical description of matter under these conditions is close to reality. The user has to judge whether a particular EoS is suitable for her or his application.

The user also has to keep in mind that the thermodynamic and compositional properties of nuclear matter without electrons (and muons) and dense stellar matter with electrons (and muons) are rather different because in the former case the Coulomb interaction is artificially neglected. This can have drastic consequences for the occurrence and type of phase transitions. Thus it is not recommended to generate a dense matter EoS from a nuclear matter EoS simply by adding the contribution from electrons (and muons).

In many cases it is desirable to extend a particular EoS to densities and temperatures below or above the available or recommended parameter ranges. In these regimes, an equation of state depending only on temperature, charge fraction of strongly interacting particles and baryonic density to describe matter might not be sufficient, e.g. at very low densities and temperatures where the time scales of thermodynamic equilibration reaction rates become large and full reaction networks have to be considered. However, for many purposes a detailed description of matter in these regime is not necessary, such that we decided to furnish tables for these conditions, too.

Our knowledge about the QCD phase diagram suggests that there could be a transition from a hadronic phase to a quark-gluon plasma within the range of densities and temperatures reachable in core-collapse supernovae, hence within the range of our tables. Of course, there are lots of uncertainties about this phase transition, so that its occurrence cannot be affirmed, but the possibility has to be kept in mind when employing a purely hadronic EoS up to densities well above nuclear matter saturation density and temperatures as high as several tens of MeV. Within CompOSE there will be tables

available including this phase transition. Even without thinking about a QCD phase transition, other forms of (exotic) matter shall appear at high densities and temperatures. Already for cold matter EoS used for neutron star modelisation for a long time, hyperons, pions and kaons have been considered. At temperatures above about 20 MeV, this point becomes even more crucial. Thus using a purely nuclear EoS in this regime can be a first approximation, technically very appealing, but again it has to be kept in mind that probably the considered physics is too poorly described.

7. Online services and data handling

7.1. Access to CompOSE

The EoS data and routines provided by CompOSE can be downloaded from the web site

`https://compose.obspm.fr` .

The tables and codes are offered free of charge but come without warranty. Please acknowledge and give proper reference to CompOSE if you use our service for your applications, presentations and publications (see appendix E). If you encounter any difficulties, it is possible to contact the CompOSE team by sending an email to

`develop.compose@obspm.fr` .

To subscribe to the CompOSE newsletter, distributed via the email list “compose.info”, where changes and updates will be announced, send an e-mail with subject “Subscribe” to `develop.compose@obspm.fr`.

7.2. EoS data sheets

Each EoS of the CompOSE data base is accompanied with a data sheet available for download from the web site. The data sheet provides essential information that allows the user to decide whether the EoS is suitable for her/his application. It contains information on the origin and creation of the EoS table, a short abstract of the physical model, references, the parameter ranges and considered particle species, a summary of the available data files and fundamental quantities that characterize the EoS, such as nuclear matter and neutron star properties, if available.

7.3. Options for using EoS data

There are different options for downloading and/or generating EoS data and tables. It is possible to obtain EoS tables for different models and for different purposes. The original EoS tables are given as plain ASCII files that allow every user to read the data without the need for further codes. The general idea is that a user chooses a particular model from those available on the CompOSE web site. Then, there are three different ways of accessing EoS data:

1. The user downloads data tables of an available EoS in the original form and uses her or his own routines to handle the data. For every model EoS there are three

tables that contain the details of the discretization of the parameters: `eos.t`, `eos.nb` and `eos.yq`. The original EoS data are stored as functions of temperature T , baryon number density n_b and charge fraction of strongly interacting particles Y_q . The last quantity is identical to the electron fraction Y_e if electrons are the only charged leptons considered in the EoS model. The actual data on the thermodynamic, compositional and microscopic properties are stored in the files `eos.thermo`, `eos.compo` and `eos.micro`, respectively, as far as available for that particular model. See section 4.2 for the format of all the tables. If available for an EoS, the file `eos.mr` contains information of the properties of compact stars and can be read directly without the need to use the CompOSE software.

2. The user downloads the original data tables for a particular EoS as described above and, in addition, several files that contain routines for reading, testing, interpolating and transforming the data. These codes serve four major purposes:
 - to interpolate the original EoS data tables in order to obtain the quantities at parameter values different from the original tabulation,
 - to calculate additional quantities that are not given in the original data files,
 - to select those quantities that are relevant for a particular application and to store them in separate data files in a format more convenient for the user,
 - to provide EoS data tables in the advanced HDF5 format (see <http://www.hdfgroup.org/HDF5/>) that is widely used in the astrophysics community.

Details on how to work with the subroutines are described in the section 7.5.

3. Using the tools provided on the webpage to manipulate and visualise EoS data. In contrast to downloading data and the CompOSE software, the access to running the web tools is password restricted. Please contact `develop.compose(at)obspm.fr` if you wish an account to be created for you.

7.4. Different families of EoS data

There is now a large number of different EoS models for which data is available on the web page for free download. In order to more easily find the data for a particular application, the different models have been ordered into different categories:

1. **Cold Neutron Star EoS:** this category contains models designed to describe cold (zero temperature) matter in weak (β) equilibrium. They all contain the contribution of electrons (and for some of them muons, too) and are directly applicable to construct cold neutron star models, e.g. by using the EOS_COMPOSE class with the LORENE library¹.

¹<https://lorene.obspm.fr>

2. **Neutron Star Crust EoS:** category for EoS that describe the non-homogeneous matter of the outer crust and or inner crust, e.g. the Baym-Pethick-Sutherland (BPS) EoS. The set of crust EoS is divided into **outer** crust and **inner** crust and may be used, together with a core EoS, to construct a complete EoS. Some EoS account for the appearance of pasta phases in the transition region from the inner crust to the homogeneous core. In order for radii of low mass NS not to be affected by the crust EoS, the lowest considered density in the table should be sufficiently small, i.e., it has to reach small densities of the order of 10^{-9} fm^{-3} , where iron group nuclei appear in the outer crust. This is not the case for all models. If the model provides at least an inner crust, any outer crust model can be matched, the exact choice of the outer crust EoS does not much affect global compact star properties such as radius or mass. On the contrary, if the model only contains a core EoS, then some attention should be paid to the choice of the inner crust EoS for matching and the matching conditions, since in particular radii and tidal deformabilities can depend on that choice. The choice is optimal if the symmetry energy properties are similar to the ones of the core EoS. The crust thickness is sensitive to the crust-core transition density which in turn is sensitive to the medium modifications of the cluster properties and the treatment of the pasta phases. A small routine that allows for a thermodynamically consistent matching of the crust EoS to the core EoS will be made available in the near future on the `CompOSE` web page, e.g. it checks that $p_{\text{crust}} = p_{\text{core}}$ together with $\mu_{\text{crust}} = \mu_{\text{core}}$ at the transition.
3. **Cold Matter EoS:** this category contains models at zero temperature for different charge fractions. It is divided into different entrances according to the underlying physical approach.
4. **Neutron Matter EoS:** this category contains models for neutron matter, i.e. for $Y_q = 0$ (the net abundances of charged particles and antiparticles is zero) at both $T = 0$ and finite temperature.
5. **General Purpose EoS:** this category contains models which cover a large range of different temperatures, charge fractions and baryon number densities, as required in particular for simulations of core-collapse supernovae and binary neutron star mergers. For convenience, most models are provided in two versions, one with the contribution of electrons, positrons and photons included and one containing only the baryonic part.

In addition to this very general distinction between EoS models, based mainly on distinguishing the different possible applications of the EoS data, there are sub-families. These sub-families first describe the particular assumptions for the particle content of matter:

1. **Nucleonic models,** EoS models that assume matter to be composed of nucleons, charged leptons and potentially nuclear clusters,

2. **Hybrid (quark-hadron) models**, category for EoS models with a transition from hadronic matter at low densities to quark matter at high densities,
3. **Models with hyperons**, category for EoS models allowing for hyperons (strange baryons) to be populated,
4. **Models with hyperons and Δ -resonances**, category for EoS models allowing for hyperons (strange baryons) and Δ -resonances to be populated,
5. **Models with condensates**, category for EoS models allowing for kaon or pion condensates to set in,
6. **Quark models**, category for EoS models containing quark matter.

Other sub-families might be added in the future. Then, there are sub-families distinguishing a particular framework to determine the EoS (see the review [2]). This concerns first the description of homogeneous matter, i.e. the interaction of nucleons, hyperons etc, and the corresponding many-body treatment:

1. **Non-relativistic density functional models**, EoS, where the interaction of baryons is based on non-relativistic density functional theory, in particular this concerns Skyrme like interactions, e.g., SLy4, or Gogny-type interactions. In these models, the total energy density of homogeneous baryonic matter is given in terms of the sum of a kinetic and a potential energy density functional in the space defined by the different particle species number densities: proton and neutron densities for nucleonic matter and proton, neutron and hyperons for hyperonic matter.
2. **Relativistic density functional models**, EoS based on covariant density functional theory, e.g. the STOS EoS (H Shen et al). These models are generally applied in the mean-field approximation. Baryons, nucleons and, when included, hyperons and or Delta-resonances, are described by fermionic fields that interact with purely phenomenological mesonic fields introduced in order to describe the nuclear interaction. The quantum numbers of the interaction channel give the name to the mesons. The most frequently used are the scalar and vector-isoscalar mesons named, respectively, σ and ω -mesons, and the vector-isovector ρ -meson. Some models also include the scalar-isovector δ -meson, and when hyperons are included also the scalar and vector-isoscalar σ^* and ϕ -mesons with hidden strangeness ($\bar{s}s$ mesons) may be included. These models are divided into two categories: models with constant couplings and non-linear mesonic terms (TM1 and SFHo are two examples) and models with density dependent couplings and only quadratic mesonic terms (such as DD2 and DDME2). If the actually used values of the couplings differ from those of the original publications, these new values are provided in the data sheet.
3. **Microscopic calculations**, EoS based on ab initio calculations of nuclear matter, e.g. the BHF technique or variational calculations or Monte Carlo methods.

Within these approaches different many-body methods are used to include correlations in the many-body system starting from a few-body interaction that has been fitted to observables in nucleon-nucleon scattering in vacuum and properties of bound few-nucleon systems,

4. **Holographic models**, EoS based on a holographic approach to determine homogeneous matter.

In a second step, different treatments of inhomogeneous matter are distinguished:

1. **NSE** models, using a nuclear statistical equilibrium approach to describe inhomogeneous matter at nonzero temperature. These category concerns mainly the general purpose EoS models, e.g. the models by Hempel & Schaffner-Bielich. Cold neutron star EoS obtained from the lowest temperature entry of a general purpose without a dedicated description of the crust are ranged in this category, too.
2. **SNA** models, using the single nucleus approximation to describe inhomogeneous matter at nonzero temperature. These concern the general purpose EoS models, e.g. the famous EoS by Lattimer & Swesty or Shen et al.
3. **Unified EoS** For “Cold neutron star EoS” this keyword indicates models where the same nuclear interaction has been used to describe the neutron star crust and core, all other models have a crust model matched to a core EoS and enter the category
4. **Non-unified models (crust-core matched)**, see the description of the particular EoS table for details.
5. **Thomas-Fermi calculations** concern the NS crust models and indicate the method to determine the crust properties.

Let us mention that a very convenient way to find a particular table is the bibliography search, where a function exists to search for tables related to publications of one particular author (see section 7.6).

7.5. Handling and transforming EoS data

The handling of EoS data is considerably simplified by using the software provided on the CompOSE web page. In the most recent version of the `compose` code, the generation of the necessary parameter files to define output quantities and format is guided via terminal input. There are only few steps required in order to generate customized data tables from the original sets of EoS tables.

All quantities describing the thermodynamic, compositional and microscopic properties of the matter are found for arbitrary values of the table parameters with an interpolation scheme that is described in detail in appendix A.1. It is possible to create tables with different mesh settings and ranges. Of course, the ranges should be chosen only within the ranges of the basis table for each model. If they are outside these ranges, error messages will indicate the problems.

7.5.1. Downloading and compiling

There are four FORTRAN90 files that are needed in every case to create the executable `compose` program. The file `composemodules.f90` contains all the necessary modules and the file `compose.f90` all required (sub-)routines and functions in order to read, interpolate and write the EoS data in ASCII format. The file `out_to_json.f90` is only relevant for the program version used on the CompOSE webpage. All necessary files are most simply obtained by downloading the file `code.zip`.

Using the present version of the program `compose`, it is not necessary to provide input files beyond those that were downloaded from the CompOSE website. The input files that define the selected variables and tabulation scheme are created automatically. Their structure is specified in appendix B.

If output of the EoS data in HDF5-format is needed, the files `hdf5compose.f90` and `hdf5writecompose.c` are required in addition to `compose.f90` and `composemodules.f90`.

All the files can be downloaded from the corresponding CompOSE web page and they have to be compiled with an appropriate FORTRAN90 or C compiler. The program was written using the GNU compilers `gfortran` and `gcc` and the use of these compilers is encouraged. A sample `Makefile` is available on the CompOSE web page. It contains in line 26 a switch to select the compilation without (`HDF = 0`, default setting) and with (`HDF = 1`) the option for the HDF5 output. For using the HDF5 routines, the HDF5 library has to be installed. Compiling the program files with the provided `Makefile` generates an executable with the name `compose`.

7.5.2. Direct use of `compose`

In this subsection, the details are described how customized EoS tables from the EoS models on the CompOSE web site can be obtained by running the program `compose`.

The operation proceeds in the following steps:

1. Selection of EoS

In a first step the user selects the EoS that is appropriate for her/his application from the list given on the CompOSE web pages. Often, there are different types of tables available for a single EoS model as described in subsection 4.2. The tabulated data can depend on one, two or three of the independent parameters T , n_b and Y_q . There is an option to replace the temperature T by the entropy per baryon $\mathcal{S}$ as parameter when running the standard version of `compose`. There are at least four different files needed for the successful application of the program `compose` as specified in subsection 4.2.8: three files containing information on the parameter grid and at least one file with the thermodynamic data.

The downloaded parameter and EoS files are always identified with the same name and a file name extension that specifies the type of stored quantities. The program `compose` uses these generic names for the input files. Thus the downloaded files are called “`eos.t`”, “`eos.nb`”, “`eos.yq`”, “`eos.thermo`”, “`eos.compo`” (if available), “`eos.micro`” (if available), and “`eos.init`” (if available).

For some EoS, additional files are available: `eos.mr`, the properties of a cold β -equilibrated spherically symmetric compact star, i.e. mass and radius of a spherically symmetric star, together with optionally tidal deformability and corresponding central density, obtained with the particular EoS model; `eos.thermo.ns` and `eos.nb.ns`, baryon number densities and thermodynamic quantities for cold β -equilibrated matter for direct use within LORENE [18] via the `eoscompose` class to compute, e.g., initial data for a binary neutron star. The latter files exist only for general purpose EoS tables and have been extracted from the lowest temperature entry of the corresponding table, i.e., in general for a nonzero (but very small) temperature. `eos.thermo.ns` contains as additional quantity the electron fraction Y_e obtained in β -equilibrium and the enthalpy density.

Note that for some of the general purpose EoS tables, the information in the file `eos.thermo.ns` is not sufficient for building compact stars up to the maximum mass. The reason is that for densities exceeding a certain value, Y_q corresponding to β -equilibrated matter becomes lower than the lowest value considered in the table, typically $Y_q = 0.01$ and thus at high densities no β -equilibrium solution is found. In this case, for some of these models, corresponding cold Neutron Star Matter tables are provided containing the complementary information.

If the user does not run the program `compose` in the standard version (see 2) he or she has to supply two files (in ASCII format) in addition to the downloaded files. They specify the parameter values and the quantities that will be stored in the customized EoS output table. See appendix B for details of their structure. The input files `eos.parameters` and `eos.quantities` are created automatically when the standard version of the program `compose` is used. Thus it is recommended to run the standard version, in particular for first-time users of CompOSE.

2. Running compose

The user has to run the `compose` code three times:

- a) to select the output quantities
- b) to define the tabulation parameters
- c) to generate the file `eos.table` with the customized EoS table.

In each case, the program explicitly asks questions and gives information for possible answers. The files `eos.quantities` and `eos.parameters` are created automatically. See the “quick guide for users” for examples.

In the case that there is no file `eos.init` provided on the webpage for a particular EoS, it is created automatically. During the execution of the `compose` program, the relevant input data files are read: first the files with the parameter grids `eos.t`, `eos.nb` and `eos.yq`, second the EoS data files `eos.thermo`, `eos.compo` (if available), and `eos.micro` (if available). The read data are analyzed, checked for consistency and the information is stored in the file `eos.init`.

Running the `compose` code for task 1, a file `eos.quantities` is created. When task 2 is selected, a file `eos.parameters` is generated. The structure of the files

`eos.init`, `eos.quantities`, and `eos.parameters` is described in detail in appendix B.

Each row of the output file `eos.table` contains the three parameter values of the user-chosen grid and then the selected quantities. Note that for each quadruple index I_i four quantities ($Y_{I_i}^{\text{av}}$, $A_{I_i}^{\text{av}}$, $Z_{I_i}^{\text{av}}$, and $N_{I_i}^{\text{av}}$) are given.

7.5.3. HDF5 table

The organization of the data in case of the HDF5 table is different as compared to the ASCII table. All data are stored in a single data file that is denominated `eoscompose.h5`. The names of the data sets are given by the identifiers in tables 7.7 and 7.8. Note, too, that the values of the different quantities at all grid points are stored together in one data set. The data sets can in addition contain a group of quantities, e.g., the N_{thermo} thermodynamic quantities, see tables 7.7 and 7.8 for details.

7.5.4. Additional output

During the execution of task three of the program `compose`, a file `eos.report` is generated that contains information on the EoS derived from the tabulated data. The information is used for the creation of the data sheet that accompanies each EoS on the CompOSE web pages. If possible, a second file `eos.beta` is produced with the EoS of β -equilibrated matter for the lowest available temperature for all grid points in baryonic density that are available for the specific EoS and where a solution is found. Each row consists of four quantities, the density n_b , the charge fraction Y_q , the (free) energy density f (including rest mass contributions) and the pressure p . These data are used to derive characteristic neutron star parameters for the data sheet. Finally, a third file `eos.info.json` is created that is used for the interactive web version of the program.

7.5.5. Error messages

During the execution of the subroutines, errors can occur due to several reasons, e.g. parameters out of range etc. In this case, the execution is stopped and a corresponding error message is generated.

7.6. Web tools

The web tools are constructed as an interface to the COMPOSE software, i.e. all features of the code described in the previous section are available. The obtained parameter files `eos.parameters`, `eos.quantities` as well as the output table `eos.table` can be downloaded. In addition, it is possible to visualise the results, choosing the axis from the different computed quantities on a linear or a logarithmic scale.

Table 7.1.: Thermodynamic quantities in the data tables with their units.

index J	quantity/ expression	unit	description
1	p	MeV fm ⁻³	total pressure
2	$\mathcal{S} = s/n_b$	dimensionless	total entropy per baryon
3	$\mu_b - m_n$	MeV	baryon chemical potential with respect to neutron mass
4	μ_q	MeV	charge chemical potential
5	μ_l	MeV	lepton chemical potential
6	$\mathcal{F}/m_n - 1$	dimensionless	scaled free energy per baryon
7	$\varepsilon = \mathcal{E}/m_n - 1$	dimensionless	scaled internal energy per baryon
8	$\mathcal{H}/m_n - 1$	dimensionless	scaled enthalpy per baryon
9	$\mathcal{G}/m_n - 1$	dimensionless	scaled free enthalpy per baryon
10	$\left. \frac{\partial p}{\partial n_b} \right _{\mathcal{E}}$	MeV	partial derivative of pressure with respect to baryon number density
11	$\left. \frac{\partial p}{\partial \mathcal{E}} \right _{n_b}$	fm ⁻³	partial derivative of pressure with respect to internal energy per baryon
12	c_s^2	dimensionless	speed of sound squared
13	c_V	dimensionless	specific heat capacity at constant volume
14	c_p	dimensionless	specific heat capacity at constant pressure
15	$\Gamma = c_p/c_V$	dimensionless	adiabatic index
16	α_p	MeV ⁻¹	expansion coefficient at constant pressure
17	β_V	fm ⁻³	tension coefficient at constant volume
18	κ_T	fm ³ /MeV	isothermal compressibility
19	κ_S	fm ³ /MeV	adiabatic compressibility

Table 7.2.: Additional thermodynamic quantities in the data tables with their units.
(Quantities 25 - 28 are not yet available with the `CompOSE` code.)

index J	quantity/ expression	unit	description
20	$\mathcal{F}$	MeV	free energy per baryon
21	$\mathcal{E}$	MeV	energy* per baryon
22	$\mathcal{H}$	MeV	enthalpy per baryon
23	$\mathcal{G}$	MeV	free enthalpy per baryon
24	$e = \mathcal{E}n_b$	MeV fm ⁻³	energy density
25	ζ	MeV fm ⁻²	bulk viscosity
26	η	MeV fm ⁻²	shear viscosity
27	κ	fm ⁻²	thermal conductivity
28	σ	fm ⁻¹	electrical conductivity

* This corresponds to the "internal energy" in the thermodynamic language including the rest mass contribution.

Table 7.3.: Values and derivatives of the free energy per baryon which are stored in the data tables with their units.

index J	quantity/ expression	unit	description
1	$\mathcal{F}$	[MeV]	free energy per baryon
2	$\frac{\partial \mathcal{F}}{\partial T}$	dimensionless	first temperature derivative of free energy per baryon
3	$\frac{\partial^2 \mathcal{F}}{\partial T^2}$	[MeV $^{-1}$]	second temperature derivative of free energy per baryon
4	$\frac{\partial^2 \mathcal{F}}{\partial T \partial n_b}$	[fm 3]	mixed second derivative of free energy per baryon
5	$\frac{\partial^2 \mathcal{F}}{\partial T \partial Y_q}$	dimensionless	mixed second derivative of free energy per baryon
6	$\frac{\partial \mathcal{F}}{\partial n_b}$	[MeV fm 3]	first baryon density derivative of free energy per baryon
7	$\frac{\partial^2 \mathcal{F}}{\partial n_b^2}$	[MeV fm 6]	second baryon density derivative of free energy per baryon
8	$\frac{\partial^2 \mathcal{F}}{\partial n_b \partial Y_q}$	[MeV fm 3]	mixed second derivative of free energy per baryon
9	$\frac{\partial \mathcal{F}}{\partial Y_q}$	[MeV]	first hadronic charge fraction derivative of free energy per baryon
10	$\frac{\partial^2 \mathcal{F}}{\partial Y_q^2}$	[MeV]	second hadronic charge fraction derivative of free energy per baryon

Table 7.4.: Quantities containing information on the composition which are stored in the data tables with their units. The particle index I_i has been defined in tables 3.2, 3.3, 3.4 and 3.5. The symbols % denotes the group of nuclei.

index J	quantity/ expression	unit	description
	I_{phase}	dimensionless	phase index
	Y_{I_i}	dimensionless	fraction of particle I_i
1	$Y_{\%}^{\text{av}}$	dimensionless	combined fraction of group % of nuclei
2	$A_{\%}^{\text{av}}$	dimensionless	average mass number of group % of nuclei
3	$Z_{\%}^{\text{av}}$	dimensionless	average charge number of group % of nuclei
4	$N_{\%}^{\text{av}}$	dimensionless	average neutron number of group % of nuclei

Table 7.5.: Microscopic quantities which are stored in the data tables with their unit.

index J	quantity/ expression	unit	description
40	$m_{I_i}^L/m_{I_i}$	dimensionless	effective Landau mass with respect to the particle mass
41	$m_{I_i}^D/m_{I_i}$	dimensionless	effective Dirac mass with respect to the particle mass
50	U_{I_i}	MeV	nonrelativistic single-particle potential
51	V_{I_i}	MeV	relativistic vector self-energy
52	S_{I_i}	MeV	relativistic scalar self-energy
60	Δ_{I_i}	MeV	gap

Table 7.6.: Error quantities which are stored in the data tables with their unit.

index J	quantity/ expression	unit	description
1	$\Delta\mathcal{F}$	MeV	absolute error estimate of free energy per baryon
2	$\Delta\mathcal{F}/\mathcal{F}$	[dimensionless]	relative error estimate of free energy per baryon
3	$\Delta\mathcal{E}$	MeV	absolute error estimate of internal energy per baryon
4	$\Delta\mathcal{E}/\mathcal{E}$	[dimensionless]	relative error estimate of internal energy per baryon
5	$\Delta\left(\frac{p}{n_b}\right)$	MeV	absolute error estimate of pressure-to-density ratio
6	$\Delta\left(\frac{p}{n_b}\right) / \left(\frac{p}{n_b}\right)$	[dimensionless]	relative error estimate of pressure-to-density ratio
7	$\Delta\mathcal{S}$	[dimensionless]	absolute error estimate of entropy per baryon
8	$\Delta\mathcal{S}/\mathcal{S}$	[dimensionless]	relative error estimate of entropy per baryon

Table 7.7.: Thermodynamic quantities which are stored in the HDF5 data file together with their units and the name of the corresponding data set. If there are two lines in the second column, the first one always corresponds to $I_{\text{tab}} = 0$ and the second one to $I_{\text{tab}} \neq 0$.

name of data set	quantity/ expression	unit	description
t	T	MeV	temperature
pointst	$\frac{N_{\text{data}}}{N_T}$	dimensionless	number of points in T
nb	n_b	fm^{-3}	baryon number density
pointsnb	$\frac{N_{\text{data}}}{N_{n_b}}$	dimensionless	number of points in n_b
yq	Y_q	dimensionless	charge fraction of strongly interacting particles
pointsyq	$\frac{N_{\text{data}}}{N_{Y_q}}$	dimensionless	number of points in Y_q
thermo	$Q(N_{\text{data}}, 1, 1, N_{\text{thermo}})$ $Q(N_{n_b}, N_T, N_{Y_q}, N_{\text{thermo}})$	varying	array of thermodynamical quantities, see tables 7.1, 7.2
pointstthermo	N_{thermo}	dimensionless	number of thermodynamic quantities, see tables 7.1, 7.2.
index_thermo	$J(N_{\text{thermo}})$	dimensionless	index identifying the thermodynamic quantities, see tables 7.1, 7.2.
thermo_add	$q(N_{\text{data}}, 1, 1, N_{\text{add}})$ $q(N_{n_b}, N_T, N_{Y_q}, N_{\text{add}})$	varying	array of additional quantities $q_{\%}$ in file <code>eos.thermo</code> .
pointsadd	N_{add}	dimensionless	number of additional thermodynamic quantities in file <code>eos.thermo</code> .
index_thermo_add	$I_{\text{add}}(N_{\text{add}})$	dimensionless	index identifying the additional quantities $q_{\%}$ from file <code>eos.thermo</code> , see eq. (4.18)

Table 7.8.: Compositional and microscopic quantities which are stored in the HDF5 data file together with their units and the name of the corresponding data set. If there are two lines in the second column, the first one always corresponds to $I_{\text{tab}} = 0$ and the second one to $I_{\text{tab}} \neq 0$.

name of data set	quantity/ expression	unit	description
yi	$Y_I(N_{\text{data}}, 1, 1, N_{\text{p}})$ $Y_I(N_{n_b}, N_T, N_{Y_q}, N_{\text{p}})$	dimensionless	array of particle fractions
pointspairs	N_{p}	dimensionless	number of pairs for compositional data
index_yi	$I_i(N_{\text{p}})$	dimensionless	index identifying the particle i , see tables 3.2, 3.3, 3.4
yav	$Y^{\text{av}}(N_{\text{data}}, 1, 1, N_{\text{q}})$ $Y^{\text{av}}(N_{n_b}, N_T, N_{Y_q}, N_{\text{q}})$	dimensionless	array of combined fractions of groups of nuclei
aav	$A^{\text{av}}(N_{\text{data}}, 1, 1, N_{\text{q}})$ $A^{\text{av}}(N_{n_b}, N_T, N_{Y_q}, N_{\text{q}})$	dimensionless	average mass number of groups of nuclei
zav	$Z^{\text{av}}(N_{\text{data}}, 1, 1, N_{\text{q}})$ $Z^{\text{av}}(N_{n_b}, N_T, N_{Y_q}, N_{\text{q}})$	dimensionless	average charge number of groups of nuclei
nav	$N^{\text{av}}(N_{\text{data}}, 1, 1, N_{\text{q}})$ $N^{\text{av}}(N_{n_b}, N_T, N_{Y_q}, N_{\text{q}})$	dimensionless	average neutron number of groups of nuclei
pointsav	N_{q}	dimensionless	number of quadruples for compositional data
index_av	$I_{\text{av}}(N_{\text{q}})$	dimensionless	index identifying the group % of nuclei, see tables 3.2, 3.3, 3.4.
micro	$q_{\text{mic}}(N_{\text{data}}, 1, 1, N_{\text{mic}})$ $q_{\text{mic}}(N_{n_b}, N_T, N_{Y_q}, N_{\text{mic}})$	varying	array of microscopic quantities
pointsmicro	N_{mic}	dimensionless	number of microscopic quantities
index_micro	$K(N_{\text{mic}})$	dimensionless	index identifying the microscopic quantities, see subsection 4.2.4
error	$q_{\text{err}}(N_{\text{data}}, 1, 1, N_{\text{err}})$ $q_{\text{err}}(N_{n_b}, N_T, N_{Y_q}, N_{\text{err}})$	varying	array of error quantities
pointserr	N_{err}	dimensionless	number of microscopic quantities
index_err	$J_{\text{err}}(N_{\text{err}})$	dimensionless	index identifying the error quantities, see table 7.6.

Part IV.

Appendix

A. Technical details

The basic EoS tables only give a selected set of thermodynamic quantities at the grid points that are identified with the index triple (i_T, i_{n_b}, i_{Y_q}) . From the stored quantities Q_i , $i = 1, \dots, 6$, see section 4.2.2, further relevant thermodynamic quantities can be derived by a smooth interpolation for all possible values of the parameters T , n_b and Y_q within the tabulated ranges. For these quantities, thermodynamic consistency should be respected as far as possible. When the thermodynamic quantities are interpolated separately, however, this condition is usually not exactly fulfilled. On the other hand, a separate interpolation often leads to smoother dependencies of the thermodynamic quantities on the parameters avoiding unphysical oscillations. In addition, different ways of determining a single quantity gives the opportunity to estimate the error in the interpolation. Thus, in the present code `compose.f90` the strategy of a direct interpolation of individual quantities is followed.

A.1. Interpolation

The interpolation scheme for a quantity in the thermodynamic, compositional and microscopic EoS tables generally proceeds in several steps. In the following, the procedure will be explained for a three-dimensional general purpose EoS table. The interpolation scheme follows the method proposed in Ref. [26] using polynomials of sufficiently high order.

In the sequel, the generic symbol Q will be used for any tabulated quantity. Its values are given at the grid points that are specified by a triple of indices $(i_T, i_{n_b}, i_{Y_q}) = (i, j, k)$, see subsection 4.1.1, corresponding to temperature, baryon number density and charge fraction of strongly interacting particles. Thus all values $Q(T(i), n_b(j), Y_q(k))$ are known at the grid points. In order to calculate the quantity Q at given values of T , n_b , and Y_q , first the values of the indices $i = i_T$, $j = i_{n_b}$ and $k = i_{Y_q}$ are determined such that

$$T(i) \leq T < T(i+1), \quad (\text{A.1})$$

$$n_b(j) \leq n_b < n_b(j+1), \quad (\text{A.2})$$

$$Y_q(k) \leq Y_q < Y_q(k+1). \quad (\text{A.3})$$

Then the interpolation parameters

$$\xi = \frac{T - T(i)}{T(i+1) - T(i)} \quad (\text{A.4})$$

$$\eta = \frac{n_b - n_b(j)}{n_b(j+1) - n_b(j)} \quad (\text{A.5})$$

$$\zeta = \frac{Y_q - Y_q(k)}{Y_q(k+1) - Y_q(k)} \quad (\text{A.6})$$

are introduced with

$$0 \leq \xi < 1 \quad 0 \leq \eta < 1 \quad 0 \leq \zeta < 1. \quad (\text{A.7})$$

For the interpolation of a quantity Q at given (T, n_b, Y_q) we need the tabulated values at least at the eight corners of the cube with grid points (i, j, k) , $(i+1, j, k)$, $(i, j+1, k)$, $(i, j, k+1)$, $(i+1, j+1, k)$, $(i+1, j, k+1)$, $(i, j+1, k+1)$, $(i+1, j+1, k+1)$.

The interpolation proceeds in two steps: first, an interpolation in the variable Y_q such that the three-dimensional grid is mapped to a two-dimensional grid with four corners of each square; second, a two-dimensional interpolation in the variables T and n_b .

For the interpolation in Y_q in the three-dimensional cube as defined above, four separate one-dimensional interpolations along the lines that connect the grid points (i, j, k) and $(i, j, k+1)$, $(i+1, j, k)$ and $(i+1, j, k+1)$, $(i, j+1, k)$ and $(i, j+1, k+1)$, $(i+1, j+1, k)$ and $(i+1, j+1, k+1)$ have to be performed. After this first step, the interpolation proceeds in the parameters T and n_b by a two-dimensional scheme as discussed in subsection A.1.2.

The interpolation in the variables T , n_b and Y_q can be of different order in general, defined by the value I of the variables `ipl_t`, `ipl_n` and `ipl_y`, see section 7.5. In the following, the case of highest order ($I = 3$) is considered first.

A.1.1. Interpolation in one dimension

The order $I = 3$ of the interpolation requires that the function values, its first and second derivatives are continuous at the two corner points of each line, i.e. six values have to be reproduced by the interpolation polynomial. Hence, a polynomial of at least fifth degree has to be used. The six independent coefficients q_n of a single polynomial

$$Q(T(i), n_b(j), Y_q) = \sum_{n=0}^5 q_n \zeta^n \quad (\text{A.8})$$

can be determined from the function values and derivatives at the corner points directly. One finds

$$q_0 = Q_{ijk}^{(0)} \quad q_1 = Q_{ijk}^{(1)} \quad q_2 = \frac{1}{2} Q_{ijk}^{(2)} \quad (\text{A.9})$$

with

$$Q_{ijk}^{(n)} = [Y_q(k+1) - Y_q(k)]^n \left. \frac{\partial^n Q}{\partial Y_q^n} \right|_{T(i), n_b(j), Y_q(k)} \quad (\text{A.10})$$

for $n = 0, 1, 2$, and

$$Q_{ijk}^{(0)} = \left. \frac{\partial^0 Q}{\partial Y_q^0} \right|_{T(i), n_b(j), Y_q(k)} = Q(T(i), n_b(j), Y_q(k)) \quad (\text{A.11})$$

in particular. The remaining three coefficients are given by

$$q_3 = 10A - 4B + \frac{1}{2}C \quad (\text{A.12})$$

$$q_4 = -15A + 7B - C \quad (\text{A.13})$$

$$q_5 = 6A - 3B + \frac{1}{2}C \quad (\text{A.14})$$

with

$$A = Q_{ijk+1}^{(0)} - Q_{ijk}^{(0)} - Q_{ijk}^{(1)} - \frac{1}{2}Q_{ijk}^{(2)} \quad (\text{A.15})$$

$$B = Q_{ijk+1}^{(1)} - Q_{ijk}^{(1)} - Q_{ijk}^{(2)} \quad (\text{A.16})$$

$$C = Q_{ijk+1}^{(2)} - Q_{ijk}^{(2)}. \quad (\text{A.17})$$

Alternatively, the approach of Ref. [26] can be followed. The value of the quantity Q at given ζ is found with the help of the quintic basis functions

$$\psi_0^{(0)}(z) = 1 - 10z^3 + 15z^4 - 6z^5 \quad (\text{A.18})$$

$$\psi_1^{(0)}(z) = z - 6z^3 + 8z^4 - 3z^5 \quad (\text{A.19})$$

$$\psi_2^{(0)}(z) = \frac{1}{2}(z^2 - 3z^3 + 3z^4 - z^5) \quad (\text{A.20})$$

that have the properties

$$\psi_m^{(n)}(0) = \delta_{nm} \quad \psi_m^{(n)}(1) = 0 \quad (\text{A.21})$$

for

$$\psi_m^{(n)}(z) = \frac{d^n \psi_m^{(0)}}{dz^n} \quad (\text{A.22})$$

with $n, m \in \{0, 1, 2\}$. Then one has

$$Q(T(i), n_b(j), Y_q) = \sum_{n=0}^2 \left[Q_{ijk}^{(n)} \psi_n^{(0)}(\zeta) + (-1)^n Q_{ijk+1}^{(n)} \psi_n^{(0)}(1 - \zeta) \right] \quad (\text{A.23})$$

with six interpolation coefficients that are given by directly by the function values and derivatives at the corner points defined by the indices. Derivatives are easily found as

$$\begin{aligned} & \frac{\partial^s}{\partial Y_q^s} Q(T(i), n_b(j), Y_q) \\ &= \frac{1}{[Y_q(k+1) - Y_q(k)]^s} \sum_{n=0}^2 \left[Q_{ijk}^{(n)} \psi_n^{(s)}(\zeta) + (-1)^{n+s} Q_{ijk+1}^{(n)} \psi_n^{(s)}(1 - \zeta) \right]. \end{aligned} \quad (\text{A.24})$$

The numerical determination of the derivatives depends on the order of interpolation. For $I = 3$, centered five-point finite difference formulas are used to calculate the first and second derivative of a function. Close to the boundaries of the EoS, centered difference formulas cannot be applied. In this case, off-center formulas are used. In the case $I = 2$, three-point finite difference formulas are employed. A continuity of the second derivatives at the corner points is not required. Only the function and the first derivative are demanded to be continuous, corresponding to four independent quantities. Hence, a polynomial of third degree is sufficient and $q_4 = q_5 = 0$ in Eq. (A.8). Then, the coefficients of the second derivatives have to be defined as

$$Q_{ijk}^{(2)} = 6 \left[Q_{ijk+1}^{(0)} - Q_{ijk}^{(0)} \right] - 2Q_{ijk+1}^{(1)} - 4Q_{ijk}^{(1)} \quad (\text{A.25})$$

$$Q_{ijk+1}^{(2)} = -6 \left[Q_{ijk+1}^{(0)} - Q_{ijk}^{(0)} \right] + 4Q_{ijk+1}^{(1)} + 2Q_{ijk}^{(1)}. \quad (\text{A.26})$$

For $I = 1$, the function has to be continuous at the grid points but there is no condition on the derivatives. In this case, one sets

$$Q_{ijk}^{(1)} = Q_{ijk+1}^{(1)} = Q_{ijk+1}^{(0)} - Q_{ijk}^{(0)} \quad Q_{ijk}^{(2)} = Q_{ijk+1}^{(2)} = 0 \quad (\text{A.27})$$

for given function values $Q_{ijk}^{(0)}$ and $Q_{ijk+1}^{(0)}$. Then the polynomial (A.8) reduces to a linear function with $q_2 = q_3 = q_4 = q_5 = 0$.

A.1.2. Interpolation in two dimensions

The interpolation in two dimensions with conditions on the continuity of the function and its derivatives is more complicated than in the one-dimensional case. For an interpolation order $I = 3$ in both variables ξ and η , there are four function values, eight first derivatives and the twelve second derivatives, hence 24 values in total, that can be used to determine the coefficients of a polynomial in ξ and η . From this consideration, it would seem to be sufficient to use a polynomial

$$Q(T, n_b, Y_q) = \sum_{m=0}^4 \sum_{n=0}^4 q_{mn} \xi^m \eta^n \quad (\text{A.28})$$

that includes powers up to four in both variables, resulting in $5 \times 5 = 25$ coefficients q_{nm} in total, i.e. one more than required. However, such a form does not guarantee that the function and the derivatives are continuous not only at the grid points but also along the boundaries of the interpolation square. In fact, a continuity of the function Q , the first derivatives $\partial Q / \partial T$ and $\partial Q / \partial n_b$, the second derivatives $\partial^2 Q / \partial T^2$, $\partial^2 Q / \partial T \partial n_b$ and $\partial^2 Q / \partial n_b^2$, the third derivatives $\partial^3 Q / \partial T^2 \partial n_b$ and $\partial^3 Q / \partial T \partial n_b^2$ and the fourth derivative $\partial^4 Q / \partial T^2 \partial n_b^2$ at the corners has to be demanded, determining $4 \times 9 = 36 = 6 \times 6$ coefficients of a polynomial

$$Q(T, n_b, Y_q) = \sum_{m=0}^5 \sum_{n=0}^5 q_{mn} \xi^m \eta^n \quad (\text{A.29})$$

with degree six in each variable.

Instead of determining the coefficients directly as in the case of a one dimensional interpolation, it is more advantageous to use the biquintic interpolation scheme as in Ref. [26] with

$$\begin{aligned}
Q(T, n_b, Y_q) & \quad (A.30) \\
= & \sum_{m=0}^2 \sum_{n=0}^2 \left[Q_{ij}^{(mn)} \psi_m^{(0)}(\xi) \psi_n^{(0)}(\eta) + (-1)^m Q_{i+1j}^{(mn)} \psi_m^{(0)}(1-\xi) \psi_n^{(0)}(\eta) \right. \\
& \left. + (-1)^n Q_{ij+1}^{(mn)} \psi_m^{(0)}(\xi) \psi_n^{(0)}(1-\eta) + (-1)^{m+n} Q_{i+1j+1}^{(mn)} \psi_m^{(0)}(1-\xi) \psi_n^{(0)}(1-\eta) \right]
\end{aligned}$$

and $4 \times 9 = 36$ coefficients

$$Q_{ij}^{(mn)} = [T(i+1) - T(i)]^m [n_b(j+1) - n_b(j)]^n \frac{\partial^{m+n} Q}{\partial T^m \partial n_b^n} \Big|_{T(i), n_b(j), Y_q} \quad (A.31)$$

$$Q_{i+1j}^{(mn)} = [T(i+1) - T(i)]^m [n_b(j+1) - n_b(j)]^n \frac{\partial^{m+n} Q}{\partial T^m \partial n_b^n} \Big|_{T(i+1), n_b(j), Y_q} \quad (A.32)$$

$$Q_{ij+1}^{(mn)} = [T(i+1) - T(i)]^m [n_b(j+1) - n_b(j)]^n \frac{\partial^{m+n} Q}{\partial T^m \partial n_b^n} \Big|_{T(i), n_b(j+1), Y_q} \quad (A.33)$$

$$Q_{i+1j+1}^{(mn)} = [T(i+1) - T(i)]^m [n_b(j+1) - n_b(j)]^n \frac{\partial^{m+n} Q}{\partial T^m \partial n_b^n} \Big|_{T(i+1), n_b(j+1), Y_q} \quad (A.34)$$

where $m, n \in \{0, 1, 2\}$. Of course, it is possible to express the coefficients q_{mn} of the polynomial (A.29) through the coefficients $Q_{ij}^{(mn)}$, as done in the code `compose.f90`. The derivatives are obtained in the same way as in the one-dimensional case for the different interpolations orders I . Derivatives of the function Q with respect to Y and n_b can be found with the help of the relation

$$\begin{aligned}
& \frac{\partial^{s+t}}{\partial T^s \partial n_b^t} Q(T, n_b, Y_q) \quad (A.35) \\
= & \frac{1}{[T(i+1) - T(i)]^s} \frac{1}{[n_b(j+1) - n_b(j)]^t} \\
& \times \sum_{m=0}^2 \sum_{n=0}^2 \left[Q_{ij}^{(mn)} \psi_m^{(s)}(\xi) \psi_n^{(t)}(\eta) + (-1)^{m+s} Q_{i+1j}^{(mn)} \psi_m^{(s)}(1-\xi) \psi_n^{(t)}(\eta) \right. \\
& \left. + (-1)^{n+t} Q_{ij+1}^{(mn)} \psi_m^{(s)}(\xi) \psi_n^{(t)}(1-\eta) + (-1)^{m+n+s+t} Q_{i+1j+1}^{(mn)} \psi_m^{(s)}(1-\xi) \psi_n^{(t)}(1-\eta) \right].
\end{aligned}$$

Above considerations apply to the interpolation in all three parameters for a general purpose EoS table. For neutron matter, i.e. $Y_q = 0$ the interpolation in the charge fraction of strongly interacting particles is trivial since $\zeta = 0$ and the tabulated values of the quantities can be used directly in the second interpolation step in T and n_b . A similar procedure applies to the case of an EoS for symmetric nuclear matter of β -equilibrium. In the case of a zero-temperature table, the second interpolation step reduces from two to one dimension.

A.2. Matter in beta equilibrium

In the case of matter in β equilibrium, there is an additional condition on the chemical potentials that reduces the number of independent parameters by one. Requiring that the weak interaction reactions

$$p + e^- \leftrightarrow n + \nu_e \quad p + \bar{\nu}_e \leftrightarrow n + e^+ \quad (\text{A.36})$$

and, if muons are considered in the model,

$$p + \mu^- \leftrightarrow n + \nu_\mu \quad p + \bar{\nu}_\mu \leftrightarrow n + \mu^+ \quad (\text{A.37})$$

are in weak equilibrium, the relations

$$\mu_p + \mu_e = \mu_n + \mu_{\nu_e} \quad (\text{A.38})$$

and

$$\mu_p + \mu_\mu = \mu_n + \mu_{\nu_\mu} , \quad (\text{A.39})$$

respectively, should hold. The chemical potentials of the neutrinos are considered to be zero, i.e. $\mu_{\nu_e} = \mu_{\nu_\mu} = 0$. Because $\mu_p = \mu_n + \mu_q$, $\mu_e = \mu_{le} - \mu_q$ and $\mu_\mu = \mu_{l\mu} - \mu_q$, the constraint can be formulated as

$$\mu_l = 0 \quad (\text{A.40})$$

with the effective lepton chemical potential $\mu_l = \mu_{le} = \mu_{l\mu}$. For given temperature T and baryon number density n_b there is a unique charge fraction of strongly interacting particles Y_q that is found by determining the zero of the function

$$f(Y_q) = \mu_l(Y_q) . \quad (\text{A.41})$$

B. Structure of input files

During the execution of the program `compose` for tasks 1 and 2, three files (in ASCII format) are generated that are used for task 3 as input files. The structure of these files is described below. Experienced users of `compose` can modify the files `eos.parameters` and `eos.quantities` according to their needs when producing a new EoS table with modified parameters by running only task 3. The file `eos.init` has to be left untouched.

B.1. `eos.init`

There are 11 rows in the file `eos.init`. The first three numbers in the first row indicate whether the EoS dependence on the temperature, the baryon density, and the hadronic charge fraction (index = 1) or not (index = 0). The following two integers are presently set to zero and will be used only for future extensions of the `compose` code. The next three rows give the minimum and maximum values for the variables T , n_b , and Y_q . In the fifth row the number zero appears twice in the present version as dummy floating-point variable. Row six contained four integers. The first indicates whether leptons are included in the EoS (1: yes, 0: no). The second number gives the number of additional quantities in the file `eos.thermo` and the third number is the total number of thermodynamic quantities in each row of the file `eos.thermo`. The last integer indicates whether a calculation of an EoS table for the condition of β equilibrium is possible (1) or not (0). Row seven gives the number of pairs and quadruples in the file `eos.compo` and the following two lines contain the corresponding indices according to tables 3.2, 3.3, 3.4 and 3.5 for pairs (first line) and according to the data sheet for quadruples (second line). In row ten the number of microscopic quantities is given and their indices are listed in the last row of the file `eos.init`.

B.2. `eos.quantities`

This file contains 18 rows. They determine which and how quantities will be stored in the final EoS table with the name `eos.table`. Rows with odd number (one, three, ...) are comment lines beginning with the character `#`. The second and fourth lines define the thermodynamic quantities that will be stored in the output file `eos.table`. The first entry N_{thermo} in row two specifies the number of thermodynamic quantities that are selected from tables 7.1 and 7.2 followed by the number N_{add} of quantities that are selected from the stored quantities of the EoS file `eos.thermo` in addition to the seven standard quantities as described in subsection 4.2.2. The last number N_{deriv} in row two is the number of quantities from table 7.3 that contains the free energy per baryon and

various derivatives of it. The fourth line of the file `eos.quantities` lists the N_{thermo} indices (if $N_{\text{thermo}} > 0$) from the second column of tables 7.1 and 7.2. that define the available thermodynamic quantities, the N_{add} indices (if $N_{\text{add}} > 0$) of the additionally stored quantities and the N_{deriv} indices (if $N_{\text{deriv}} > 0$) from the first column of table 7.3. E.g., the first four lines

```
# number of regular, additional and derivative quantities
3 1 1
# indices of regular, additional and derivative quantities
6 1 2 1 1
```

of an example file `eos.quantities` denote that the quantities $\mathcal{F}/m_n - 1$, p , $\mathcal{S}$, q_1 and $\mathcal{F}$ will appear in the output file `eos.table` or `eoscompose.h5`. Regular thermodynamic quantities are those that are obtained by a direct interpolation of the tabulated quantities $Q_1, \dots, Q_7$ stored in the file `eos.thermo` or those that are obtained from them by applying the thermodynamic relations of section 3.6. The indices of the regular thermodynamic quantities can have values of 1, 2, $\dots$, 23 corresponding to the definition in tables 7.1 and 7.2. Additional thermodynamic quantities are those that are denoted $q_1, \dots, q_{N_{\text{add}}}$ in the file `eos.thermo`. Their number N_{add} and meaning depends on the specific EoS table. Their values are found by a direct interpolation. The values for $\mathcal{F}$ and its derivatives are found from a direct interpolation and subsequent derivatives of the interpolating function. If N_{thermo} , N_{add} and N_{deriv} are zero the fourth line of the file `eos.quantities` is empty.

The sixth row of the file `eos.quantities` contains the numbers N_p and N_q of pairs and quadruples that are selected from the compositional quantities that are stored in the file `eos.compo` as described in subsection 4.2.3. In the eighth line the N_p particle indices $I_1, \dots, I_{N_p}$ as defined in tables 3.2, 3.3, 3.4 and 3.5 are followed by the N_q indices that identify a particular group of nuclei, see subsection 4.2.3. E.g., the four lines

```
# number of pairs and quadruples for composition data
3 1
# indices of pairs and quadruples for composition data
10 11 0 1
```

denote that the fractions Y_n , Y_p , Y_e are stored in the file `eos.table` followed by Y_1^{av} , A_1^{av} , Z_1^{av} , and N_1^{av} of the first set of nuclei. If both N_p and N_q are zero the eighth line of the file `eos.quantities` is empty.

The tenth line of the file `eos.quantities` contains the number N_{mic} of microscopic quantities in the file `eos.micro` to be stored in the output file `eos.table`. Then line 12 is a list of N_{mic} composite indices K_i as defined in subsection 4.2.4. E.g., the four lines

```
# number of microscopic quantities
2
# indices of microscopic quantities
10050 11050
```

denote that the nonrelativistic single-particle potentials U_n and U_p are stored in the file `eos.table`. If N_{mic} is zero the twelfth line of the file `eos.quantities` is empty.

In addition to the quantities considered above, error estimates for the interpolation of the thermodynamic quantities $\mathcal{F}$, $\mathcal{E}$, p/n_b and $\mathcal{S}$ are available. The free energy per baryon $\mathcal{F}$ can be obtained by direct interpolation or by using the homogeneity condition (4.5) with the interpolated values for the pressure and chemical potentials. An estimate for the absolute error in $\mathcal{F}$ is then given by

$$\Delta\mathcal{F} = \mathcal{F} + \frac{p}{n_b} - (\mu_b + Y_q\mu_q) \quad (\text{B.1})$$

in case of an EoS without leptons and

$$\Delta\mathcal{F} = \mathcal{F} + \frac{p}{n_b} - (\mu_b + Y_l\mu_l) \quad (\text{B.2})$$

in case of an EoS with leptons and charge neutrality. The relative error is then given by $\Delta\mathcal{F}/\mathcal{F} - 1$. The consideration above also apply to the internal energy per baryon $\mathcal{E}$. An estimate for the absolute error in p/n_b is obtained by comparing the directly interpolated pressure p with that derived from the first derivative of the free energy per baryon, i.e.

$$\Delta\frac{p}{n_b} = \frac{p}{n_b} - n_b \left. \frac{\partial\mathcal{F}}{\partial n_b} \right|_{T, Y_q}. \quad (\text{B.3})$$

Similarly, an error estimate for the entropy per baryon is calculated from

$$\Delta\mathcal{S} = \mathcal{S} - \left. \frac{\partial\mathcal{F}}{\partial T} \right|_{n_b, Y_q}. \quad (\text{B.4})$$

The sixteenth line of the file `eos.quantities` defines the number N_{err} of error quantities to be stored in the output file. Line 18 is a list of the N_{err} indices J as defined in table 7.6. E.g., the four lines

```
# number of error quantities
2
# indices of error quantities
1 2
```

denote that the estimates $\Delta\mathcal{F}$ and $\Delta\mathcal{F}/\mathcal{F}$ are stored in the file `eos.table`. If N_{err} is zero the eighteenth line of the file `eos.quantities` is empty.

The last row of the file `eos.quantities` defines the format of the output file by a single integer I . There are two methods that are used for storing the EoS data. The first ($I = 1$) is the simple ASCII format using the file name `eos.table` that easily allows to read the data without the need for further codes. The second method ($I \neq 1$) uses the more advanced HDF5 format (see <http://www.hdfgroup.org/HDF5/>) that is widely used in the astrophysics community. In this case, the output file carries the name `eoscompose.h5` and each data set is designated with a particular identifier \$ of the stored

quantity Q . Tables 7.7 and 7.8 give a list of all tabulated quantities and corresponding identifiers \$.

Note that $\varepsilon = \mathcal{E}/m_n - 1$ in table 7.1 is *not* the specific internal energy, since the total mass density is not given by $m_n n_b$. We do not store $\mathcal{E}/m_n$ because it is in many regimes largely dominated by the rest mass contribution, such that it is numerically difficult to keep trace of small variations in the internal energy in that case. The same argument applies to the other thermodynamic quantities $\mathcal{F}$, $\mathcal{G}$ and $\mathcal{H}$. The present choice of storing ε is also motivated by the fact that the total mass density is not a conserved quantity throughout the hydrodynamic evolution, but only the baryon number density, n_b .

The individual chemical potentials for all the particles (baryons, mesons, nuclei, quarks, ...) can be calculated from the three ones given in the tables. Note that we use the relativistic definition of the chemical potentials.

Particle fractions Y_i are stored in the data sets with identifier $y\#$ where $\#$ stands for the particle index I_i as defined in tables 3.2, 3.3, 3.4 and 3.5. E.g., the data identifier `y20` and `y400` contain the particle fractions of Δ^- and K^- particles, respectively. Similarly, `yav1` stands for the fraction of the first group of nuclei.

Combining the examples above, the file `eos.quantities` assumes the form

```
# number of regular, additional and derivative quantities
3 1 1
# indices of regular, additional and derivative quantities
6 1 2 1 1
# number of pairs and quadruples for composition data
3 1
# indices of pairs and quadruples for composition data
10 11 0 1
# number of microscopic quantities
2
# indices of microscopic quantities
10050 11050
# number of error quantities
2
# indices of error quantities
1 2
```

and, consequently, each line of the output file `eos.table` will contain the following entries:

$$\begin{array}{cccccccccccc} T & n_b & Y_q & \mathcal{F}/m_n - 1 & p & S & q_1 & \mathcal{F} & Y_n & Y_p & Y_e \\ & & & Y_1^{\text{av}} & A_1^{\text{av}} & Z_1^{\text{av}} & N_1^{\text{av}} & U_n & U_p & \Delta\mathcal{F} & \Delta\mathcal{F}/\mathcal{F} \end{array}$$

i.e., in total 19 quantities.

B.3. eos.parameters

There are at least ten lines in the file `eos.parameters`. Rows one, three, five and seven of the file are comment lines indicated by the first character `#`. The second row of this file contains three integer numbers I_T , I_{n_b} and I_{Y_q} that specify the interpolation scheme for the temperature, baryon number density and charge fraction of strongly interacting particles, respectively. Presently, there is the choice between first order ($I_x = 1$, $x = T, n_b, Y_q$), second order ($I_x = 2$) and third order ($I_x = 3$) interpolation available with continuity of all quantities $Q(x)$, all quantities $Q(x)$ and their first derivative $\partial Q/\partial x$ or all quantities $Q(x)$, their first and second derivatives $\partial Q/\partial x$ and $\partial^2 Q/\partial x^2$, respectively. See appendix A.1 for details on the interpolation scheme.

The fourth row of the file `eos.parameters` contains two integers I_β and I_{entr} . The first determines whether the EoS in the output file `eos.table` will be generated for matter in β equilibrium ($I_\beta = 1$) or not ($I_\beta \neq 1$). Of course, this option is only effective for EoS tables that include electrons (and muons) and depend on the parameter Y_q , e.g. three-dimensional general purpose EoS tables and two-dimensional zero-temperature EoS tables. The second integer determines whether the EoS in the output file `eos.table` will be generated for given entropy per baryon $\mathcal{S}$ ($I_{\text{entr}} = 1$) or not ($I_{\text{entr}} \neq 1$). If yes, the input parameters that usually define the temperature values will be interpreted as defining the entropy per baryon. This option can only be used for EoS that depend on the temperature. However it is not guaranteed that there are solutions, found by interpolation, for the given parameter values.

In the sixth row of the file, the tabulation scheme for the parameters is defined by the integer I_{tab} . For $I_{\text{tab}} = 0$ the parameter values of T (or $\mathcal{S}$), n_b and Y_q are listed explicitly in the file `eos.parameters` as follows: The eighth row contains the number of data points N_{data} to be generated and each line of the next N_{data} rows contains the three parameter values in the form

$$T \quad n_b \quad Y_q \tag{B.5}$$

or

$$\mathcal{S} \quad n_b \quad Y_q \tag{B.6}$$

that define the grid points. The EoS data that will be stored in the final EoS table will appear in the same sequence of the parameters as given in the file `eos.parameters`. For $I_\beta = 1$, Y_q can be set to any value. For two- and one-dimensional EoS tables, the parameters that are not used can be set to arbitrary values, e.g. for a zero-temperature EoS the first entry in each row can have any finite value. An example file `eos.parameters` for the tabulation for a zero-temperature EoS the first entry in each row can have any finite value. An example file `eos.parameters` for the tabulation scheme with $I_{\text{tab}} = 0$ is given by

```
# interpolation rules for T, n_b and Y_q
3 3 3
# calculation of beta equilibrium (1: yes, else: no) and for given
                                entropy (1: yes, else: no)
```

```

0 0
# tabulation scheme of parameter values (see manual)
0
# parameter values depending on tabulation scheme
5
0.1 0.1 0.5
0.1 0.2 0.5
0.1 0.3 0.5
0.1 0.4 0.5
0.1 0.5 0.5

```

for generating a table using third order interpolations in T , n_b and Y_q . Five data points are calculated for constant $T = 0.1$ MeV and $Y_q = 0.5$ at densities of $n_b = 0.1, 0.2, 0.3, 0.4$, and 0.5 fm^{-3} .

For $I_{\text{tab}} \neq 0$ the parameter values are generated in a similar way as suggested in subsection 4.1.1. In this case, only four additional rows follow the comment line below the row with the specification of I_{tab} . Each column of these rows (in the order temperature, density, charge fraction) contains the four quantities

$$\begin{array}{c}
p_{\min} \\
p_{\max} \\
N_p \\
I_p
\end{array} \tag{B.7}$$

where p stands for the parameters T , n_b or Y_q , respectively. The minimum and maximum values of the parameter p in the EoS table are denoted by $p_{\min}$ and $p_{\max}$, respectively. $N_p > 0$ is the number of data points in the mesh of the parameter p and I_p defines the discretization scheme. If $I_p = 0$, the individual points are given by a linear interpolation

$$p_i = p_{\min} + (p_{\max} - p_{\min}) \frac{i - 1}{N_p - 1} \tag{B.8}$$

for $i = 1, \dots, N_p$ with $N_p > 1$ and

$$p_1 = p_{\min} \tag{B.9}$$

for $N_p = 1$. For $I_p \neq 0$, a logarithmic scaling is used with

$$p_i = p_{\min} \left(\frac{p_{\max}}{p_{\min}} \right)^{\frac{i-1}{N_p-1}} \tag{B.10}$$

for $i = 1, \dots, N_p$ with $N_p > 1$ and

$$p_1 = p_{\min} \tag{B.11}$$

for $N_p = 1$. For this tabulation scheme an example file `eos.parameters` has the form

```

# interpolation rules for T, n_b and Y_q
3 3 3
# calculation of beta equilibrium (1: yes, else: no) and for given
                                entropy (1: yes, else: no)
0 0
# tabulation scheme of parameter values (see manual)
1
# parameter values depending on tabulation scheme
5.0 0.01 0.3
5.0 1.00 0.3
1 201 1
0 1 0

```

Here, $T = 5$ MeV and $Y_q = 0.3$ are kept constant and the densities n_b cover the range from 0.01 fm^{-3} to 1.0 fm^{-3} in 200 intervals with a logarithmic scaling.

In case the option $I_\beta = 1$ is set in the third row of the file `eos.parameters`, the specification of the parameter grid in Y_q is irrelevant.

C. Structure of the `compose.f90` program

(Remark: This appendix is not yet updated from version 2.00 of the manual.)

Instead of using the code `compose.f90` as given, the user can employ the subroutines directly in her/his program. The file `compose.f90` contains one main program, 31 subroutines and one function. The structure of the program with the dependencies of the subroutines and functions is depicted in tables C.1, C.2 and C.3. The file `composemodules.f90` contains two modules.

The user needs to call the following three subroutines

```
init_eos_table(iwr)
define_eos_table(iwr)
get_eos(t,n,y,b,ipl,i_beta,i_entr)
```

from the file `compose.f90` in order to generate EoS data. The subroutine

```
init_terminal(iwr)
```

is called only from the main program for running the standard terminal version. It is usually not relevant for users that want to use subroutines within their own programs. The first two mentioned subroutines depend on a single integer parameter `iwr`. For `iwr = 1` a progress report during the execution of the subroutine will be written to the terminal. Otherwise, this action will be suppressed. The subroutine `get_eos` has three double precision parameters (`t`, `n`, `y`) and three integer parameters (`ipl`, `i_beta`, `i_entr`) as arguments, where the first is a vector of dimension three. Be sure to include all (sub)routines, functions and the two modules `eos_tables` and `compose_internal` in your code.

The subroutine `init_eos_table` has to be called only once in order to initialize all relevant tables and quantities. The files `eos.t`, `eos.nb`, `eos.yq` and `eos.thermo` have to exist in order that `init_eos_table` is executed properly. The files `eos.compo` and `eos.micro` are optional.

The subroutine `define_eos_table` defines the quantities that are interpolated and that will be stored in the EoS table. It reads the file `eos.quantities`. The subroutine needs to be called only once. Instead of calling `define_eos_table` in the user's program, it is possible that the relevant parameters are defined directly in the user's code. Thus it is possible to change the selection of quantities that are interpolated during the execution of the user's program. In the following, the variable names to be specified by the user are given according to the structure of the input file `eos.quantities`:

```
# number of regular, additional and derivative quantities
n_qty  n_add  n_df
```

```

# indices of regular, additional and derivative quantities
idx_qty(1)...idx_qty(n_qty) idx_add(1)...idx_add(n_add) idx_df(1)
...idx_df(n_df)
# number of pairs and quadruples for composition data
n_p   n_q
# indices of pairs and quadruples for composition data
idx_p(1) ... idx_p(n_p)   idx_q(1) ... idx_q(n_q)
# number of microscopic quantities
n_m
# indices of microscopic quantities
idx_m(1) ... idx_m(n_m)
# number of error quantities
n_err
# indices of error quantities
idx_err(1) ... idx_err(n_err)
# format of output file
iout

```

Note that all variables are `integers`. The appearing vectors with fixed size are defined in the module `compose_internal`:

```

idx_qty(dim_qtyt)  with  dim_qtyt = 23
idx_err(dim_qtye)  with  dim_qtye = 8 .

```

where the dimensions are defined in the module `eos_tables`.

Finally, the subroutine `get_eos` can be called as often as needed with the appropriate arguments. The `double precision` variables `t`, `n` and `y` define the temperature T [MeV], baryon number density n_b [fm^{-3}] and the charge fraction of strongly interacting particles Y_q [dimensionless], respectively, where the EoS is evaluated. The `integer` indices `ipl_t`, `ipl_n` and `ipl_y` define the interpolation rule in T , n_b and Y_q , respectively. For `ipl_t` = 1 all selected quantities are interpolated linearly such that the interpolated values agree with the tabulated values at the grid points of the EoS table. For `ipl_t` = 2 the interpolated values of a quantity and its first derivative with respect to temperature agree with those at the grid points. For `ipl_t` = 3 a continuity of also the second derivatives at the grid points are demanded. Similarly, the interpolation rules for n_b and Y_q are determined. If the index is outside the range [1,3] it is set to 3. See appendix A.1 for details of the interpolation scheme.

The results of the interpolation are stored in seven `double precision` vectors/arrays with dimensions defined in the module `eos_tables` or defined according to the EoS input files:

```

eos_thermo(dim_qtyt)  with  dim_qtyt = 23
eos_thermo_add(dim_qty)
eos_compo_p(dim_qtyp)
eos_compo_q(dim_qtyq,3)

```

```

eos_micro(dim_qtym)
eos_err(dim_qtye)  with  dim_qtye = 8
eos_df(dim_df)    with  dim_df = 10 .

```

The vector index in `eos_thermo` corresponds to the index J given in tables 7.1 and 7.2. The vector index in `eos_thermo_add` is just the index $i = 1, \dots, N_{\text{add}}$ of the additional quantities stored in each row of the file `eos.thermo`. The (first) index of the vectors/arrays `eos_compo_p`, `eos_compo_q` and `eos_compo_m` corresponds to the index of the vectors `idx_p`, `idx_q` and `idx_m` that are defined in the file `eos.quantities` or by the user before the subroutine `get_eos` is called. The second index of the array `eos_compo_q` is correlated with the index J as given in table 7.4. The vector index in `eos_err` corresponds to the index J given in table 7.6. The vector index in `eos_df` corresponds to the index J given in table 7.3.

Table C.1.: Modules, (sub)routines and functions in the file `compose.f90` with their dependencies (continued on next page).

	uses MODULE	uses SUBROUTINE	uses FUNCTION
PROGRAM			
compose		run_terminal init_eos_table define_eos_table get_eos_table	
SUBROUTINE			
init_eos_table		read_eos_tables_tnyb read_eos_table_thermo read_eos_table_compo read_eos_table_micro get_diff_rules init_ipl_rule get_eos_report	
define_eos_table	compose_internal	write_errors	
get_eos_table	compose_internal m_out_to_json	get_eos write_errors out_to_json_write	
get_eos	compose_internal	get_eos_beta get_eos_sub write_errors	
get_eos_beta	compose_internal	get_eos_sub	
get_eos_sub	compose_internal	get_eos_grid_para eos_interpol	
get_eos_grid_para	compose_internal		
eos_interpol	compose_internal	eos_interpol.d1 eos_interpol.d2 eos_interpol.d3	
eos_interpol.d1	textttcompose_internal	get_interpol_x write_errors	get_ipl_rule
eos_interpol.d2	compose_internal	get_idx_arg2 get_diff_rules2 get_derivatives get_coefficients get_interpol_xy write_errors	get_ipl_rule

Table C.2.: Modules, (sub)routines and functions in the file `compose.f90` with their dependencies (continued from previous page).

	uses MODULE	uses SUBROUTINE	uses FUNCTION
SUBROUTINE			
<code>eos_interpol_d3</code>	<code>compose_internal</code>	<code>get_interpol_x</code> <code>get_idx_arg2</code> <code>get_diff_rules2</code> <code>get_derivatives</code> <code>get_coefficients</code> <code>get_interpol_xy</code> <code>write_errors</code>	<code>get_ipl_rule</code>
<code>get_idx_arg2</code>	<code>compose_internal</code>		
<code>get_diff_rules</code>	<code>compose_internal</code>		
<code>get_diff_rules2</code>	<code>compose_internal</code>		<code>get_ipl_rule</code>
<code>get_interpol_x</code>	<code>compose_internal</code>		
<code>get_interpol_xy</code>	<code>compose_internal</code>		
<code>get_derivatives</code>	<code>compose_internal</code>		
<code>get_coefficients</code>	<code>compose_internal</code>		
<code>run_terminal</code>	<code>compose_internal</code>	<code>init_eos_table_term</code> <code>init_quant</code> <code>init_para</code> <code>get_eos_table_term</code>	
<code>init_eos_table_term</code>	<code>compose_internal</code>	<code>read_eos_tables_tnyb</code> <code>read_eos_table_thermo</code> <code>read_eos_table_compo</code> <code>read_eos_table_micro</code>	
<code>get_eos_table_term</code>	<code>compose_internal</code>	<code>read_eos_tables_tnyb</code> <code>read_eos_table_thermo</code> <code>read_eos_table_compo</code> <code>read_eos_table_micro</code> <code>get_diff_rules</code> <code>get_ipl_rule</code> <code>get_eos_report</code> <code>define_eos_table</code> <code>get_eos_table</code>	
<code>init_quant</code>	<code>compose_internal</code>		
<code>init_para</code>	<code>compose_internal</code>		
<code>init_ipl_rule</code>	<code>compose_internal</code>		

Table C.3.: Modules, (sub)routines and functions in the file `compose.f90` with their dependencies (continued from previous page).

	uses MODULE	uses SUBROUTINE	uses FUNCTION
FUNCTION			
<code>get_eos_report</code>	<code>compose_internal</code>	<code>get_eos_nmp</code> <code>get_eos</code> <code>write_errors</code>	
<code>get_eos_nmp</code>	<code>compose_internal</code>	<code>get_eos_sub</code>	
<code>write_errors</code>	<code>compose_internal</code>		
<code>read_eos_tables_tnyb</code>	<code>compose_internal</code>	<code>write_errors</code>	
<code>read_eos_table_thermo</code>	<code>compose_internal</code>	<code>write_errors</code>	
<code>read_eos_table_compo</code>	<code>compose_internal</code>	<code>write_errors</code>	
<code>read_eos_table_micro</code>	<code>compose_internal</code>	<code>write_errors</code>	
<code>get_ipl_rule</code>	<code>compose_internal</code>		

D. Organization of the CompOSE team

CompOSE is developed in close collaboration with the communities the service is meant for. These are mainly physicists who develop equations of state or simulate astrophysical phenomena numerically on the computer. Direct interaction with the users is an essential part in the concept of the project.

The following persons are engaged in the preparation of the web site, the manual and the tools, numerical codes and tables.

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E. Citing CompOSE

If you make use of the tables provided in CompOSE, you will be guided on the CompOSE web pages to the scientific publications where the particular EoS models have been described in detail. Please cite them when using the tables for scientific purposes together with a reference to the CompOSE website (<https://compose.obspm.fr>) and/or the original CompOSE publications:

```
@article{Oertel:2016bki,  
  author = "Oertel, M. and Hempel, M. and Klaehn, T. and Typel, S.",  
  title = "{Equations of state for supernovae and compact stars}",  
  eprint = "1610.03361",  
  archivePrefix = "arXiv",  
  primaryClass = "astro-ph.HE",  
  doi = "10.1103/RevModPhys.89.015007",  
  journal = "Rev. Mod. Phys.",  
  volume = "89",  
  number = "1",  
  pages = "015007",  
  year = "2017"  
}  
  
@article{Typel:2013rza,  
  author = "Typel, S. and Oertel, M. and Klaehn, T.",  
  title = "{CompOSE CompStar online supernova equations of state  
    harmonising the concert of nuclear physics and astrophysics  
    compose.obspm.fr}",  
  eprint = "1307.5715",  
  archivePrefix = "arXiv",  
  primaryClass = "astro-ph.SR",  
  doi = "10.1134/S1063779615040061",  
  journal = "Phys. Part. Nucl.",  
  volume = "46",  
  number = "4",  
  pages = "633--664",  
  year = "2015"  
}
```

F. Acknowledgments

CompOSE would not be possible without the financial and organisational support from a large number of institutions and individual contributors. We gratefully acknowledge support by CompStar, NewCompStar, and PHAROS, Research Networking Programs of the European Science Foundation (ESF) and the birthplace of the CompOSE project, by a grant from the Polish Ministry for Science and Higher Education (MNiSW) supporting the “CompStar”-activity, by the Instytut Fizyki Teoretycznej of the Uniwersytet Wrocławski, the National Science Centre Poland (Narodowe Centrum Nauki, NCN) within the “Maestro”-programme under contract No. DEC-2011/02/A/ST2/00306, by the “hadronphysics3” network within the seventh framework program of the European Union, by the GSI Helmholtzzentrum für Schwerionenforschung GmbH, by the Helmholtz International Center for FAIR within the framework of the LOEWE program launched by the state of Hesse via the Technical University Darmstadt, by the Helmholtz Association (HGF) through the Nuclear Astrophysics Virtual Institute (VH-VI-417), by the ExtreMe Matter Institute EMMI in the framework of the Helmholtz Alliance ‘Cosmic Matter in the Laboratory’, by the DFG cluster of excellence “Origin and Structure of the Universe”, by the DFG through grant SFB 1245, by the SN2NS project ANR-10-BLAN-0503 and by the MPNS COST action MP1304 “Exploring fundamental physics with compact stars (NewCompStar)” within the EU framework programme. We thank Jean-Yves Giot for creating the first version of the web site.

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