# Max-Pressure Traffic Signal Timing: A Summary of Methodological and Experimental Results

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**Abstract:** Max-pressure control is a new approach to signal timing with mathematically proven network throughput properties. Over the past decade, max-pressure control has emerged from a novel mathematical idea in a simple store-and-forward queueing model to include practical aspects like a signal cycle, realistic traffic flow models, measurement errors, and intersection access for alternative modes. Furthermore, max-pressure control is adaptive to local queues and to changes in network demand. Max-pressure control is also decentralized and easy to compute. A broad range of numerical results in calibrated microsimulation models have mostly demonstrated superior performance from max-pressure control compared with existing signal timing methods, and some researchers have started to experiment with max-pressure control on actual roads. Given these benefits, the purpose of this review paper is to provide a summary of the mathematical approach, methodological improvements, and numerical results. This summary is intended for researchers interested in continuing methodological or numerical work and for practitioners exploring the potential use of this state-of-the-art signal timing method. **DOI: 10.1061/JTEPBS.TEENG-7578.** *© 2023 American Society of Civil Engineers.*



## Introduction

Intersections controlled by traffic signals are a major bottleneck for traffic in many urban road networks. Consequently, effective timing of traffic signals is highly important for improving traffic flow through cities. Waiting times for vehicles must be balanced with providing sufficient throughput because capacity is lost from phase changes. Even after decades of implementation in practice, new research is still being conducted on various approaches for improved signal timing. From the practical side, traffic engineers use a mix of fixed, adaptive, and coordinated signal timing plans for the different demand periods throughout the day. Traffic signal timing plans are also revised every few years based on changes to demand patterns.

This review article summarizes the progress on a novel approach to signal timing known as max-pressure or back-pressure control, which provides new methods for addressing many of the challenges of timing traffic signals. Max-pressure control is based on work by Tassiulas and Ephremides (1990) on network communications packet routing with mathematically proven throughput properties. However, vehicles moving through a traffic network behave much differently than the movement of packets through communications networks. Network communications involves computers sending packets of data through a network at nearly the speed of light. Transmission between computers is therefore almost instantaneous. Furthermore, packet size is tiny relative to computer memory, so the space required for storing packets is rarely a concern. The movement of packets can also violate first-in-first-out behavior because there are not any physical constraints to enforce it. Tassiulas and Ephremides (1990) were also concerned with the routing of packets, not only their transmission by computers. In contrast, vehicle travel times on roads are significant, and the space available for queueing is limited. First-in-first-out discipline is usually required, and drivers choose their routes independently. Therefore, the application of max-pressure control to traffic networks requires significant study.

In building from the work of Tassiulas and Ephremides (1990), most methodological studies of max-pressure control for traffic networks used one or more of the previous assumptions designed for network communications. Although these assumptions are not realistic, they simplified the already-complex mathematics to admit formal proofs of mathematical properties. Therefore, it is currently unknown whether the favorable mathematical properties hold in realistic traffic. However, many max-pressure papers have worked on incorporating more realistic assumptions into methodology and/or numerical results. For example, microsimulation studies have demonstrated numerical benefits in realistic simulation models, and Li and Jabari (2019) derived mathematical properties using the kinematic wave theory to model traffic flow. These results suggest significant potential for using max-pressure control in traffic networks.

Max-pressure concepts were applied to an isolated intersection by Wunderlich et al. (2007), but Wongpiromsarn et al. (2012) and Varaiya (2013) developed the first max-pressure controls for networks of signalized intersections. Although the use of long queues in adaptive signal timing existed previously (e.g., Arel et al. 2010), this paper specifically focuses on methods that aim to use the mathematical throughput properties possible with max-pressure control. Max-pressure control specifies a phase selection algorithm that selects phases in real time based on the most recent traffic conditions. This selection is significantly different from typical actuated or adaptive signal timings due to its mathematical properties, and the name *max-pressure* comes from the mathematical form of a key calculation used in the control policy. Max-pressure control has several key benefits that make it potentially attractive:

- Throughput/stability properties: Methodological work on max-pressure control usually defines it within a model of network traffic flow including stochasticity in vehicle demand, and various forms of max-pressure control have all been mathematically proven to serve all network demand whenever possible. These properties apply to a city road network and not just one intersection or corridor, and reduce the bottleneck limitations of

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Note. This manuscript was published online on January 30, 2023. Discussion period open until June 30, 2023; separate discussions must be submitted for individual papers. This paper is part of the ***Journal of Transportation Engineering, Part A: Systems***, © ASCE, ISSN 2473-2907.

signalized intersections on the movement of vehicles through networks. These are also referred to as *stability properties* because the mathematical proofs are based on an equivalent definition of network stability.

- Adaptive: Max-pressure control selects phases in real time based on actual traffic conditions, which makes it fairly responsive to those traffic conditions at a local level. At the network level, max-pressure control can serve all demand whenever possible, but does not depend on information about vehicle trips. Therefore, it can respond effectively to unusual demand patterns associated with evacuations, events, or other circumstances for which engineers have not developed timing plans.
- Decentralized control: Despite its network-level throughput properties, most forms of max-pressure control can be decentralized or distributed, meaning the phase selection at any individual intersection depends only on the traffic conditions at incoming or outgoing roads to that intersection. This makes the control easy to compute and reduces the input data needed at any individual intersection.

The purpose of this review article is to accelerate research on and implementation of max-pressure control in three ways. First, max-pressure methodologies are complex, involve some difficult mathematical proofs, and include mathematical subtleties that are not always explained well. We present a summary of the methodological approach to help readers understand the general concepts in papers on max-pressure control. Second, we review methodological developments to describe the current state of max-pressure control, build intuition about possible extensions, and discuss open problems. Finally, we summarize the simulation work that has been conducted on comparing max-pressure control to other signal timing methods. We hope these aggregate simulation results will encourage practitioners to seriously investigate whether max-pressure control might be effective in their jurisdictions.

The remainder of this paper is organized as follows. We first present an explanation of the methodological approach to max-pressure control, followed by a review of improvements made to the methodologies. We then summarize the simulation results, and briefly dicuss some applications of max-pressure control beyond traffic signals. Finally, we conclude and discuss important open problems with max-pressure control.

## Methodological Approach

Developing a max-pressure control with the desired throughput properties involves several complex mathematical components. Identifying these components in papers focused on methodological extensions may be difficult, and certain important subtleties may not be well explained. In this section, we present the general approach to help readers understand the methodological process. Afterward, we discuss methodological changes and improvements that other papers have made to this general form.

The approach starts by describing a typical network model of the signalized traffic for max-pressure control. This model must capture the time-dependent impacts of signal phase selection on the stochastic movement of vehicles while being simple enough to work with mathematically. Once the model is established, we define the max-pressure control. Afterward, we give a mathematical definition of stability that is equivalent to the desired throughput properties, and discuss methods of proving the stability properties.

### Network Model

Consider a network $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ with nodes $\mathcal{N}$ and directed links $\mathcal{A}$. The set of links is further divided into $\mathcal{A}_r$, the set of source links, $\mathcal{A}_s$, the set of sink links, and $\mathcal{A}_i$, the set of internal links. Source links represent entry points to the network for parked vehicles, such as parking garages and other similar locations. Sink links represent vehicles exiting to those same locations. Internal links connect two intersections. Each type of link has a different flow model. Source and internal links have similar flow models that are discussed subsequently, and vehicles that reach sink links are assumed to exit the network immediately.

Most papers on max-pressure control discretize time and use the store-and-forward queueing model developed by Wongpiromsarn et al. (2012) or Varaiya (2013). The store-and-forward queue assumes that each link has a free-flow travel time of one time step. Vehicles that enter a link are stored there, and can be forwarded on to downstream links at subsequent time steps depending on the signal activation and link capacity. The assumption on one time step of free-flow time is not limiting because a long road can be separated into multiple links connected in series. The nodes in between such links can be thought of as having an imaginary traffic signal that is always green. Consequently, the store-and-forward queueing model is equivalent to the point queue model of traffic flow.

The store-and-forward queueing model can either be based on link occupancies (Wongpiromsarn et al. 2012) or turning movement occupancies (Varaiya 2013). We describe the turning movement version and discuss the changes involved for the link occupancy version subsequently. For link occupancies, let $x_{ij}(t)$ be the number of vehicles on link $i$ that will next move to link $j$ at time $t$. Sometimes $x_{ij}(t)$ is called the queue on link $i$ waiting for $j$, but that should not be confused with a queue of stopped vehicles at a red light. In its count, $x_{ij}(t)$ includes vehicles that are moving, even at free-flow speed, but within the bounds of link $i$. In other words, $x_{ij}(t)$ is a measure of occupancy, or the number of vehicles on the link. Occupancy differs from the density, or the number of vehicles per mile. Let $y_{ij}(t)$ be the number of vehicles leaving link $i$ for $j$ at time $t$. Let $p_{ij}(t)$ be the turning ratio at time $t$, and it is usually assumed that $p_{ij}(t)$ is independent random variables that are identically distributed over time with known mean $\bar{p}_{ij}$. However, the distribution of $p_{ij}(t)$ can vary with $i$ and $j$. We want to define the evolution of occupancies from time $t$ to time $t+1$. For internal links, the state evolution is given by

$$x_{jk}(t+1) = x_{jk}(t) - y_{jk}(t) + \sum_{i\in\mathcal{A}} y_{ij}(t) p_{jk}(t) \qquad (1a)$$

where $\sum_{i\in\mathcal{A}} y_{ij}(t)$ = flow entering link $j$; and $y_{jk}(t)$ = flow moving from link $j$ to link $k$ at time $t$. For entry links, a similar conservation law applies, but entering flow is based on demand $d_i(t)$:

$$x_{jk}(t+1) = x_{jk}(t) - y_{jk}(t) + d_j(t) p_{jk}(t) \qquad (1b)$$

The entering demand $d_j(t)$ is usually assumed to be an independent random variable that is identically distributed over time with mean $\bar{d}_j$. Max-pressure papers usually do not make any assumptions on the distribution of $d_j(t)$. Furthermore, demand can have a different distribution (and different mean) for each $j$; it does not have to be evenly distributed over the network. Eqs. (1a) and (1b) is therefore stochastic due to the inclusion of $p_{jk}(t)$ and $d_j(t)$.

The outflows $y_{ij}(t)$ are defined in terms of signal activation $s_{ij}(t) \in \{0, 1\}$, turning movement capacity $Q_{ij}$, and occupancy $x_{ij}(t)$:

$$y_{ij}(t) = \min\{Q_{ij} s_{ij}(t), x_{ij}(t)\} \qquad (2)$$

If desired, it is possible to replace a deterministic $Q_{ij}$ with a capacity that varies randomly over time with known mean (Varaiya 2013).



J. Transp. Eng., Part A: Systems, 2023, 149(4): 03123001

The symbol $s_{ij}(t) \in \{0,1\}$ represents either a protected green light or a red light for movement from $i$ to $j$. In other words, the turning movement model is effective when individual turning movements have separate right-of-way, e.g. left- and right-turn bays. Let $\mathbf{s}(t)$ be the vector of all $s_{ij}(t)$ activations; $\mathbf{s}(t)$ could also be described as a block-diagonal matrix where the block-diagonal form comes from the fact that $s_{ij}(t) = 0$ if $i$ and $j$ are not upstream and downstream links to the same node, respectively (Varaiya 2013). For safety reasons we cannot have $s_{ij}(t) = 1$ (representing a green light) for all turning movements simultaneously. Let $\mathcal{S}$ be the set of feasible phase activations, i.e., those determined by a dual-ring controller. At any time step, $\mathbf{s}(t) \in \mathcal{S}$. Eq. (2) therefore suggests a constraint on the length of a time step; a time step should be at least as long as the minimum activation time of any phase.

### *Max-Pressure Control*

Varaiya (2013) gives the max-pressure control for the turning occupancy model as follows. Define the weight $w_{ij}(t)$ for movement $(i,j)$ at time $t$ as

$$w_{ij}(t) = x_{ij}(t) - \sum_{k \in \mathcal{A}} x_{jk}(t) \bar{p}_{jk} \tag{3}$$

Then the max-pressure control is to choose

$$\mathbf{s}^*(t) \in \underset{\mathbf{s}(t) \in \mathcal{S}}{\operatorname{argmax}} \left\{ \sum_{(i,j) \in \mathcal{A}^2} w_{ij}(t) Q_{ij} s_{ij}(t) \right\} \tag{4}$$

There is some intuition behind why this control might be a good idea. Eq. (3) increases with upstream occupancy $x_{ij}(t)$ and decreases with downstream occupancies $x_{jk}(t)$, so Eq. (4) is essentially trying to move vehicles from long queues to short queues. However, it is not obvious from intuition why $\bar{p}_{jk}$ appears in Eq. (3). This demonstrates how relying on an intuitive explanation can be misleading. Because the primary goal of max-pressure control is usually to establish mathematical stability properties, those stability properties necessitate that the max-pressure control takes on a certain form. In other words, when deriving a max-pressure control, it is usually more efficient to attempt a proof of the stability properties and identify the precise form of the max-pressure control that will make the proof hold.

### *Stability Definition*

The throughput properties depend on the following definition of stability:

*Definition 1:* The network is strongly stable if there exists a $\kappa < \infty$ such that

$$\lim_{T \to \infty} \sup \frac{1}{T} \sum_{t=1}^{T} \sum_{(i,j) \in \mathcal{A}^2} \mathbb{E}[x_{ij}(t)] \leq \kappa \tag{5}$$

where $T$ = time horizon. In other words, stability means that the average number of vehicles in the network remains bounded. This is useful because the number of vehicles in the network will on average increase by the inflow (demand) and decrease by the outflow each time step. For Definition 1 to hold, the average outflow must be greater than or equal to the average inflow, meaning vehicles exit the network at the same rate at which they enter. In other words, stability implies that all demand is served.

Stability is related to positive recurrence of Markov chains, but is a stronger condition. Although the throughput properties do not explicitly require a Markov chain, they are highly related to Markov chain theory. Indeed, the network model given here describes a Markov decision process (MDP) with state variables $x_{ij}(t)$, transition function given by Eqs. (1) and (2), and decision variables $s_{ij}(t)$. Solving this MDP computationally suffers from the curse of dimensionality. Instead, the approach is to define the max-pressure control policy, converting the MDP into a Markov chain, and prove that certain favorable properties hold. These properties are still valid for non-Markovian network models, although the method to establish them mathematically may change.

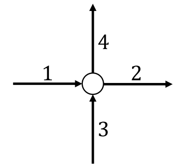


**Fig. 1.** Example network to demonstrate the relationship between demand, capacity, and stability.

The objective is to prove that max-pressure control achieves stability whenever possible, which we refer to as *maximum stability*. It is trivial to create an example that is impossible to stabilize. Consider the network in Fig. 1 in which movement (1,2) and movement (3,4) conflict and cannot simultaneously be given green lights. Suppose that capacities are $Q_{12} = Q_{34} = 1{,}800$ vehicles per hour (vph). Any demand that exceeds $\bar{d}_1 + \bar{d}_3 > 1{,}800$ vph is impossible to be served by any feasible signal control. We define $\mathcal{D}$ to be the stable region, or the set of demands for which there exists a signal timing $\pi$ that will stabilize the network. Maximum stability means a control will achieve stability for any demand $\bar{\mathbf{d}} \in \mathcal{D}$. Because max-pressure control does not make any assumptions about the distribution of the random variables, the magnitude of the variance, or deviations from the mean, does not affect the stability properties. Mathematically, stability properties hold even under large variances because Definition 1 is a property of the long-run average.

Every signal timing will achieve stability for some set of demand, but may not achieve maximum stability. Therefore, the goal of the stability properties is to prove that max-pressure control achieves maximum stability. However, maximum stability is not quite the same as achieving maximum throughput. Maximum stability is equivalent to maximum throughput for any demand $\bar{\mathbf{d}} \in \mathcal{D}$, but does not say anything about the throughput for demand $\bar{\mathbf{d}} \notin \mathcal{D}$.

### *Characterizing the Stable Region*

In the process of proving stability, papers often give equations that explicitly define $\mathcal{D}$. Typically, papers prove stability only for the interior of the stable region (excluding its boundary). Let $\mathcal{D}^0$ be the interior of $\mathcal{D}$ so there is an $\epsilon$ difference between any $\bar{\mathbf{d}} \in \mathcal{D}^0$ and the boundary of $\mathcal{D}$. This $\epsilon$ can be obtained by recalling the definition of $\mathcal{D}$ as the set of demands that can be stabilized by some control (Wongpiromsarn et al. 2012) or by an explicit analytical characterization. For instance, we can define the average demand for link $i$ as $\bar{f}_i$ via

$$\bar{f}_i = \bar{d}_i \quad \forall\ i \in \mathcal{A}_r \tag{6a}$$

$$\bar{f}_j = \sum_{i \in \mathcal{A}} \bar{f}_i \bar{p}_{ij} \quad \forall\ j \in \mathcal{A}_i \tag{6b}$$

Then we define $\hat{\mathcal{D}}$ as the set of demands such that there exists an $\bar{\mathbf{s}}$ in the convex hull of $\mathcal{S}$ satisfying

$$\bar{f}_i \bar{p}_{ij} \leq \bar{s}_{ij} Q_{ij} \quad \forall\ (i,j) \in \mathcal{A}^2 \tag{6c}$$

For demands in the interior of the stable region, Eq. (6c) should be a strict inequality.

Eq. (6) defines a set of demand rates $\hat{\mathcal{D}}$. It is not immediately obvious that $\hat{\mathcal{D}} = \mathcal{D}$, but that can be proven. We first prove that if $\bar{\mathbf{d}} \notin \hat{\mathcal{D}}$, then the network cannot satisfy Definition 1 of stability for any control policy. If we also prove that $\bar{\mathbf{d}} \in \hat{\mathcal{D}}^0$ (where $\hat{\mathcal{D}}^0$ is the interior of $\hat{\mathcal{D}}$) implies the network can be stabilized, then we establish that $\hat{\mathcal{D}} = \mathcal{D}$. Typically, the latter proof is conducted for the max-pressure control: we prove that the max-pressure control will stabilize the network if $\bar{\mathbf{d}} \in \hat{\mathcal{D}}^0$.

The $\bar{\mathbf{s}}$ used in Eq. (6c) can equivalently be defined as the average proportion of time that $(i,j)$ gets a green light in some signal activation sequence $\mathbf{s}(t)$ (Varaiya 2013)

$$\bar{\mathbf{s}} = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbf{s}(t) \tag{7}$$

Note that $s_{ij}(t) \in \{0,1\}$, representing the activation of a red light or a green light, whereas $\bar{s}_{ij} \in [0,1]$ is the proportion of time that $s_{ij}(t) = 1$. In other words, $\mathbf{s}(t) \in \mathcal{S}$ and $\bar{\mathbf{s}}$ is in the convex hull of $\mathcal{S}$. This demonstrates that the signal activation proportions used by max-pressure control to achieve maximum stability can also be replicated by a fixed-time signal control. However, finding such a fixed-time signal control would require solving Eq. (6) with correct knowledge of $\bar{\mathbf{d}}$, whereas max-pressure control adapts to any $\bar{\mathbf{d}} \in \mathcal{D}$ without explicit knowledge of $\bar{\mathbf{d}}$.

### Proof of Stability Properties

We will not give a full proof here, but we will provide an outline of the methods used to develop a proof. The general approach is based to define a Lyapunov function $\nu(t)$ satisfying the following condition: there exists a $K < \infty$ and $\epsilon > 0$ such that

$$\mathbb{E}[\nu(t+1) - \nu(t) | \mathbf{x}(t)] \leq K - \epsilon |\mathbf{x}(t)| \tag{8}$$

where $\mathbf{x}(t)$ = vector of $x_{ij}(t)$; and $|\mathbf{x}(t)|$ = L1 norm. Eq. (8) implies that Definition 1 of stability holds (Theorem 2 of Varaiya 2013). The problem then involves finding an appropriate $\nu(t)$ function. Let $\delta_{ij}(t) = x_{ij}(t+1) - x_{ij}(t)$. We can usually show that

$$\mathbb{E}[\delta_{jk}(t)|\mathbf{x}(t)] \leq K + \mathbb{E}\left[\sum_i y_{ij}(t) p_{jk}(t)\right] - \mathbb{E}[s_{jk}(t) Q_{jk}] \leq K - \epsilon \tag{9}$$

by converting Eq. (2) to $K - \mathbb{E}[s_{jk}(t) Q_{jk}]$ (see Varaiya 2013). Note that $|\mathbf{x}(t+1)| - |\mathbf{x}(t)|$ will not achieve the $-\epsilon|\mathbf{x}(t)|$ term in Eq. (8). Using

$$\nu(t) = \sum_{(i,j)\in\mathcal{A}^2} (x_{ij}(t))^2 \tag{10}$$

results in

$$\mathbb{E}[\nu(t+1) - \nu(t)|\mathbf{x}(t)] = 2\sum_{(i,j)} x_{ij}(t)\delta_{ij}(t) + \sum_{(i,j)} (\delta_{ij}(t))^2 \tag{11}$$

Then the goal is to show that $(\delta_{ij}(t))^2$ is bounded and that $\sum_{(i,j)} x_{ij}(t)\delta_{ij}(t) \leq K - \epsilon|\mathbf{x}(t)|$.

The remaining piece is to show that $\mathbb{E}[\sum_i y_{ij}(t) p_{jk}(t)] - \mathbb{E}[s_{jk}(t) Q_{jk}] \leq -\epsilon$. This holds by rewriting Eq. (6c), which is a strict inequality when $\bar{\mathbf{d}} \in \mathcal{D}^0$. Then proving stability usually requires getting to the following comparison:

$$\max_{\mathbf{s}(t)\in\mathcal{S}} \left\{ \sum_{(i,j)\in\mathcal{A}^2} w_{ij}(t)(\bar{f}_i \bar{p}_{ij} - s_{ij}(t) Q_{ij}) \right\}$$

$$\leq \sum_{(i,j)\in\mathcal{A}^2} w_{ij}(t)(\bar{f}_i \bar{p}_{ij} - \bar{s}_{ij} Q_{ij}) \tag{12}$$

$$\leq -\epsilon \sum_{(i,j)\in\mathcal{A}^2} w_{ij}(t) \tag{13}$$

where a phase $\mathbf{s}^*(t)$ satisfying

$$\mathbf{s}^*(t) \in \operatorname*{argmax}_{\mathbf{s}(t)\in\mathcal{S}} \left\{ \sum_{(i,j)\in\mathcal{A}^2} w_{ij}(t)(\bar{f}_i \bar{p}_{ij} - s_{ij}(t) Q_{ij}) \right\} \tag{14}$$

is compared with a control $\bar{\mathbf{s}}$ that satisfies Eq. (6c). Eq. (12) holds because $\bar{\mathbf{s}}$ is in the convex hull of $\mathcal{S}$, which is totally unimodular, and Eq. (13) holds from Eq. (6c), which is a strict inequality when $\mathbf{d} \in \mathcal{D}^0$. Because $\bar{f}_i$ and $\bar{p}_{ij}$ are constants for the purposes of phase selection, the choice of $\mathbf{s}^*(t)$ defined by Eq. (14) is equivalent to the max-pressure control of Eq. (4).

Some papers explicitly characterize $\hat{\mathcal{D}}$ (Varaiya 2013; Xiao et al. 2015a; Xu et al. 2022). Other papers do not explicitly write Eq. (6c) (Wongpiromsarn et al. 2012), or even leave out Eq. (6) entirely (Li and Jabari 2019). The reader is referred to the these papers for more details on different proof strategies.

### Implementation Properties

Eq. (4) could potentially find it optimal to give a green light fewer turning movements than is typical for traffic signal phases, or even no movements at all. This is because the weight function in Eq. (3) decreases with downstream vehicle occupancies, so long downstream queues could cause $w_{ij}(t)$ to be negative for all turning movements at an intersection. It is possible to restrict $\mathcal{S}$ to be the set of dual-ring signal controller phases and exclude the selection of an all-red phase. Alternatively, Noaeen et al. (2021) proposed a phase improvement step in which turning movements that do not conflict with the $\mathbf{s}^*(t)$ from Eq. (4) would also be given a green light. Chang et al. (2020) discussed the implementation of max-pressure control from the perspective of cyberphysical system design.

As specified by Eq. (4), the required information to implement max-pressure control is the turning movement occupancies $x_{ij}(t)$ for the current time step, capacities $Q_{ij}$, and average turning proportions $\bar{p}_{ij}$. Most forms of max-pressure control do not depend on either $\bar{\mathbf{d}}$ or $\mathbf{d}(t)$, so their existence is used only to establish throughput properties. The lack of dependency on demand could admit some potential robustness against changes to $\bar{\mathbf{d}}$. If $\bar{\mathbf{d}}$ changes to $\bar{\mathbf{d}}'$, as long as $\bar{\mathbf{d}}'$ is in the stable region $\mathcal{D}$, then max-pressure control should achieve long-run stability.

Most forms of max-pressure control also exhibit the decentralized or distributed property. The signal phase selection at node $n$ in Eq. (4) does not depend on any constraints that would affect the phase selection at another node $n' \neq n$. Therefore, these phase selections can be made independently while still achieving the maximum objective value. In other words, the signal controller at each intersection can independently select the correct max-pressure phase without any central system coordination. Furthermore, the phase selection at node $n$ is the activation of a green light for some turning movements $(i,j)$ centered around $n$. That selection is based on $w_{ij}(t)$, which is a function of upstream occupancies $x_{ij}(t)$ and downstream occupancies $x_{jk}(t)$. Consequently, the only information required for the phase selection at node $n$ is the occupancies of links immediately upstream and downstream of $n$.

## Previous Methodological Work

Many studies have made various methodological improvements or modifications to the original max-pressure controls of Wongpiromsarn et al. (2012) and Varaiya (2013). Using the basic max-pressure control framework described previously as a starting point, we discuss various modifications to the original structure.

### *Link Occupancy Model*

An alternative approach to the turning movement occupancy model $x_{ij}(t)$ is to define link occupancies $x_i(t)$, where $x_i(t)$ is the number of vehicles on link $i$ at time $t$ (Wongpiromsarn et al. 2012). The occupancy evolution Eqs. (1) and (2) are modified accordingly, although they still retain the familiar conservation of vehicles structure. Then maximum stability can be achieved with a weight function of

$$w_{ij} = x_i(t) - x_j(t) \tag{15}$$

This max-pressure control does not require knowledge of the turning proportions $\bar{p}_{ij}$ (Gregoire et al. 2014a), which can be advantageous when those turning proportions are unknown or expected to change. However, the link occupancy model $x_i(t)$ also assumes that all vehicles on link $i$ share the same right-of-way, which may not accurately represent left- or right-turn bays. Nevertheless, this link occupancy model has been used in some other extensions of max-pressure control (e.g., Le et al. 2015; Liu et al. 2018).

### *Realistic Traffic Flow*

The store-and-forward queueing model lacks both jam density and a realistic flow-density relationship, so any throughput properties proven in the store-and-forward model may not apply to actual traffic. Therefore, extending the throughput properties to a more realistic traffic flow model is important for achieving those benefits in practice. Furthermore, a more realistic flow model will likely necessitate a change to the specific form of the max-pressure control, resulting in a control that may perform better for actual traffic. The relevant methodological papers are summarized in Table 1. Many other papers conducted simulations using more realistic traffic models, but did not incorporate those traffic models into their methodologies, and will be discussed subsequently in the section on simulation results.

The first change was to add a jam density to the store-and-forward queueing model by preventing $x_{ij}(t)$ from exceeding a maximum occupancy $X_{ij}$. Some papers (e.g., Xiao et al. 2014) refer to the maximum occupancy as the queue capacity. To distinguish between road capacity (maximum vph), we refer to the maximum occupancy via the jam density (maximum vehicles per mile). Given a link of fixed length, the jam density can be converted into maximum occupancy. Constraining the maximum occupancy involves adding the constraint

$$\sum_{i \in \mathcal{A}} y_{ij}(t) p_{jk}(t) + x_{jk}(t) \leq X_{jk} \tag{16}$$

This constraint can be implemented as a restriction on forward movement in Eq. (2) (Gregoire et al. 2014b). Alternatively, Noaeen et al. (2021) use Eq. (16) as a constraint on $s_{ij}(t)$ so a green light activation for $(i, j)$ may result in $s_{ij}(t) < 1$ (i.e., part of the green light cannot be used). Either way, the addition of finite maximum occupancies creates the potential for loss of work conservation (Gregoire et al. 2014b). In other words, a signal activation pattern $\mathbf{s}(t)$ with corresponding $\bar{\mathbf{s}}$ satisfying Eq. (6c) may not provide sufficient capacity because vehicles cannot move forward during a green light due to the occupancy constraint.

The use of Definition 1 for stability is not immediately clear under these changes. Definition 1 requires that vehicle occupancies remain bounded on average, which would seem to be trivally true if queues have a finite occupancy. Typically, studies assume that queue lengths on entry links remain unbounded so that unserved demand will accumulate in the network and result in instability even though internal road links have a finite maximum occupancy.

Gregoire et al. (2014a) demonstrated how the loss of work conservation could prevent max-pressure control from achieving stability guarantees, and proposed a new pressure term that achieves work conservation. However, they did not prove any stability properties of their modified max-pressure control. Xiao et al. (2014) proposed a different pressure term and proved limited stability properties. Their policy achieves stability for a subset of $\mathcal{D}^0$, but requires a larger gap between any $\bar{\mathbf{d}}$ and the boundary of $\mathcal{D}$ for their stability proof to hold and therefore does not achieve maximum stability.

Adding a maximum occupancy essentially converts the store-and-forward queueing model from a point queue to a spatial queue model of traffic flow. However, mesoscopic traffic flow models have coalesced around extending the kinematic wave theory (Lighthill and Whitham 1955; Richards 1956), and several of those models are potentially well suited for max-pressure control. For instance, the cell transmission model (Daganzo 1994) has a similar state evolution as the store-and-forward queueing model with a maximum occupancy constraint. Li and Jabari (2019) identified a position-weighted back-pressure policy that was proven to achieve maximum stability for traffic flow following a very general kinematic wave model. Unlike previous work, their control is also defined in continuous time as opposed to discrete time steps. Their weight function involved integrals of position and density, unlike Eq. (3), which involves only the occupancies. This appears to be the first successful max-pressure control for a kinematic wave model of traffic flow, and could be a useful starting point for later work. Noaeen et al. (2021) developed a different pressure calculation based on the saturated green time and effective outflow under the assumption of a triangular flow-density relationship and with discretized time, and were able to prove maximum stability for their control. Yu et al. (2021) also developed a pressure-based phase selection for the link transmission model (Yperman et al. 2005), but did not study the stability properties.

**Table 1.** Methodological work on realistic traffic flow models

| Paper | Realistic aspects | Stability proof? |
|---|---|---|
| Gregoire et al. (2014b) | Maximum occupancy | No |
| Xiao et al. (2014) | Maximum occupancy | Yes |
| Li and Jabari (2019) | Kinematic wave theory | Yes |
| Noaeen et al. (2021) | Maximum occupancy | Yes |
| Yu et al. (2021) | Kinematic wave theory | No |

### *Cycle-Based Max-Pressure Control*

The selection of phases by max-pressure control is simply the phase defined by Eq. (4). Because this phase is based on link occupancies,

**Table 2.** Methodological work on cycle-based max-pressure control

| Paper | Fixed cycle length? | Pressure calculation | Stability proof? |
|---|---|---|---|
| Le et al. (2015) | Yes | Logit model | Yes |
| Pumir et al. (2015) | Yes | Link occupancy | Yes |
| Anderson et al. (2018) | Yes | Link occupancy | Yes |
| Levin et al. (2020) | No | Link occupancy | Yes |
| Xu et al. (2021) | No | Path-based occupancy | No |
| Ma et al. (2020) | No | Link occupancy | No |

which are stochastic, the phase selection is not guaranteed to follow any cycle structure. This can cause two potential problems for implementation in practice. First, drivers who are accustomed to observing traffic signal cycles may become confused when they observe their phase being skipped due to the alternating selection of other phases by Eq. (4). Second, any individual turning movement could have an arbitrarily long time before getting a green light, resulting in high waiting times for individual vehicles. Both of these situations could lead to repeated driver complaints or, in the worst case, drivers believing the signal controller to be erroneous and running a red light.

Consequently, several studies have taken various approaches to adding a signal cycle to max-pressure control. These studies are summarized in Table 2. In this context, a signal cycle is an exogenous and fixed ordering of a set of phases, and the signal controller must iterate through each phase in sequence. However, the duration of each phase and of the cycle could change in real time. Le et al. (2015) and Pumir et al. (2015) both proposed cycle-based max-pressure controls with a fixed cycle length but changing durations of each phase, and both proved that their controls achieved maximum stability. Pumir et al.'s (2015) work was closely based on Varaiya's (2013) control, using the turning movement occupancies of Eq. (1) and selecting phase durations based on Eq. (3). Le et al. (2015) used a link occupancy model, but the main difference is their use of a logit model to determine phase durations. Also, their time step represented one cycle instead of one phase selection, which seems to require that all intersections have the same cycle length. Anderson et al. (2018) extended the phase duration selection of Pumir et al. (2015) with each time step representing one signal cycle as in Le et al. (2015).

Levin et al. (2020) extended the model of Le et al. (2015), but instead of using a fixed cycle duration, they proposed a model predictive control (MPC) approach to updating the phase durations each time step. They assumed that each time step activated exactly one phase, and that each phase had to be activated at least once per cycle, although cycle durations could vary up to an exogenous maximum. This ensures that each turning movement gets green time at least once per cycle, but also reduces the size of the stable region because the range of possible values for $\bar{s}_{ij}$ is smaller than [0,1]. They proved the maximum stability property among all signal controls with a similar limitation, and also simplified the MPC to a single phase selection each time step. Xu et al. (2021) and Ma et al. (2020) also used MPC in their cycle-based max-pressure control, but did not prove maximum stability. Xu et al. (2021) used a path-based queueing model by adding a path index to each $x_{ij}(t)$ variable, which ignores first-in-first-out interactions between vehicles on different paths using the same link. Ma et al. (2020) applied the max-pressure approach to a central intersection coordinating phases with peripheral intersections.

### Lost Time

Most methodological studies have ignored lost time due to phase switching. Lost time includes both the all-red actuation in between phases and any time lost due to change intervals and startup delays. Lost time could be modeled by representing a green light with $s_{ij}(t) < 1$ to represent that part of the green light activation time is lost. However, this approach adds lost time when $s_{ij}(t+1) = s_{ij}(t)$, i.e., the signal activation does not change. Le et al. (2015) and Anderson et al. (2018) included lost time by assuming that some time per cycle is lost. Levin et al. (2020) modeled lost time by setting the green light signal value to $0 < s_{ij}(t) < 1$ when switching phases. Wang et al. (2022) suggested modifying the weight function $w_{ij}(t)$ into a more general form and adjusting it with reinforcement learning to reduce the impacts of lost time on performance.

### Handling Measurement Errors

Implementation in practice requires measurement of $x_{ij}(t)$, $Q_{ij}$, and $\bar{p}_{ij}$, all of which are subject to errors due to sensor malfunctions or data limitations. Therefore, several studies have investigated the stability properties of max-pressure control under measurement noise. Varaiya (2013) briefly discussed that if $Q_{ij}$ and $\bar{p}_{ij}$ had measurement noise but converged to the true values, then the max-pressure control would still achieve maximum stability. Xiao et al. (2015b, a) explicitly added measurement errors to their model and used an online estimation for the turning proportions. The proof of maximum stability is achieved by hiding the error terms in the constant $K$ in Eq. (8).

Alternatively, Cao et al. (2020) and Zhang et al. (2020) suggested using connected vehicle data to estimate queue lengths, and integrated a triangular flow-density relationship into their estimation. However, the stability properties of their control are not clear. Li et al. (2021a) also used estimated queue lengths from connected vehicles for their max-pressure control. Their method converted speed estimations from connected vehicles into density estimations, which were then used in the pressure calculation.

Yen et al. (2021) considered the impact of deliberate measurement errors introduced by a cyberattacker on max-pressure control, and used a 0–1 knapsack problem to choose the optimal attack strategy. Unfortunately, traffic signal systems are generally vulnerable to cyberattacks (Ghena et al. 2014) with potentially large implications for network performance (Perrine et al. 2019). However, the security of signal controllers is mostly outside the scope of this paper.

### Integration with Reinforcement Learning

Although the maximum stability property is beneficial, max-pressure control does not make any claims about achieving optimality in other performance characteristics such as vehicle delay. Furthermore, because multiple policies can achieve maximum stability, there are options to choose a policy out of the class of maximum-stable policies with favorable performance in other areas. For instance, Xiao et al. (2015a) added phase weights to prioritize certain phases. Other studies have combined reinforcement learning with max-pressure control to improve performance while retaining maximum stability. Wang et al. (2022) proved maximum stability for a generalized weight function $w_{ij}(t)$ that is monotonically increasing with respect to queue length $x_{ij}(t)$. They then applied reinforcement learning to determine $w_{ij}(t)$, with favorable simulation comparisons to other max-pressure methods. Wei et al. (2019) proposed a $Q$-learning approach that was combined with the max-pressure weight function. Other studies added reinforcement learning without explicitly retaining the stability properties. Boukerche et al. (2021) suggested using the pressure as the reward function for signal control, and included data transmission delays. Maipradit et al. (2019, 2021) used $Q$-learning to estimate route congestion for the traffic phase selection.

### Alternative Modes with Separate Right-Of-Way

Eqs. (1) and (2) define links as separate rights-of-way that interact only at intersections. This model is applicable to some degree for road links because there are still some right-of-way interactions between turning movements sharing the same approach. More generally, the same concept can be applied to other types of right-of-way that only interact with roads at intersections. For instance, Xu et al. (2022) modified max-pressure control to include bus rapid transit driving on separate bus lanes with transit signal priority. Bus lanes interact with private vehicle lanes only at intersections where conflicts between bus movement and private vehicle movement can occur. They proved that their max-pressure control achieved maximum stability among all other controls providing transit signal priority. Chen et al. (2020) used max-pressure control to decide how to activate a combination of pedestrian crosswalks and vehicle movement, with pedestrians traveling on separate pedestrian links that only interact with road links through crosswalks. Although they used autonomous intersection management (Dresner and Stone 2004) for their intersection vehicle control, the concept could also be applied to traffic signals.

## Simulation and Experimental Results

Although the throughput properties of max-pressure control are well established mathematically, at best it is proven to achieve the maximum throughput possible. The mathematical results establish that a properly designed fixed-time control could achieve the same throughput as max-pressure control. Therefore, alternative signal timing methods might perform better than max-pressure control in terms of other metrics, such as delay. Furthermore, the mathematical properties of max-pressure control are usually based on an unrealistic model of traffic flow, so it is unclear how well they will apply to actual traffic. The purpose of this section is to summarize the extensive simulation and experimental results that have been published so far. Many studies include some simulation results as part of their presentation. In this section, we focus on papers that attempted a comparison with realistic traffic conditions. Those include simulations that were calibrated based on real-world measurements and experiments on actual roads.

### Comparisons with Alternative Signal Timing Methods

Many papers compared max-pressure control with fixed or actuated signals. Some of the aforementioned studies conducted simulations in artificial grid networks. Although these studies are valuable for comparison purposes, other studies included simulations on calibrated models of actual roads in commercial microsimulation, and we believe these studies are more convincing for obtaining insights about the performance on actual roads. By calibration, we mean models that were based on observations of approach volumes, turning ratios, and current signal timings.

Because current microsimulators do not natively support max-pressure control, such studies had to create a modified signal control in the microsimulation software of choice. As a result, some of the studies mentioned here were entirely about simulation comparisons. Some other studies included simulations as part of a methodological contribution. Most studies used Wongpiromsarn et al.'s (2012) and Varaiya's (2013) acyclic max-pressure control and/or Le et al.'s (2015) cyclic max-pressure controls in their simulations. Overall, most studies report that max-pressure performs better than fixed-time or actuated-coordinated signal timing methods. Table 3 summarizes simulation results comparing max-pressure control with more standard signal timing methods using realistic simulations. Many of the methodological papers included simple simulations using store-and-forward queueing models on grid networks, but those are not included in Table 3 to highlight the comparisons using realistic simulation.

Simulation studies generally looked for changes to throughput, delay, travel times, vehicle occupancies, vehicle speeds, and/or number of stops. Reductions in delay are related to increases in

**Table 3.** Comparisons with alternative signal controls

| Paper | Max-pressure control | Simulation software | No. of intersections | Location | Comparison control | Improvements |
|---|---|---|---|---|---|---|
| Wongpiromsarn et al. (2012) | Acyclic | MITSIMLab | 15, network | Sweden | SCATS | Queue length |
| Xiao et al. (2015a) | Cyclic | Vissim | 5, corridor | Singapore | Fixed time | Delay, stops |
| Xiao et al. (2015c) | Cyclic | Vissim | 14, network | Singapore | Fixed time | Delay |
| Le et al. (2015) | Acyclic, cyclic | SUMO | 72, network | Melbourne, Australia | Proportional | Delay, queue length |
| Dakic et al. (2015) | Acyclic | Vissim | 5, corridor | Salt Lake City | Fixed time, actuated | None |
| Pumir et al. (2015) | Cyclic | Aimsun | 11, network | San Diego | Fixed time, actuated coordinated | Queue length, stops |
| Lioris et al. (2016) | Acyclic | PointQ | 16, corridor | Los Angeles | Fixed time | Decongestion |
| Le et al. (2017) | Acyclic | SUMO | 16, network | Melbourne, Australia | Fixed time | Travel time |
| Sun and Yin (2018) | Acyclic, cyclic | Vissim | 12, corridor | Gainesville, Florida | Actuated coordinated | Delay, stops |
| Anderson et al. (2018) | Cyclic | Aimsun | 11, network | San Diego | Actuated coordinated | None |
| Wang and Abbas (2019) | Acyclic | Vissim | 3, corridor | Blacksburg, Virginia | Optimized | Delay |
| Li and Jabari (2019) | Acyclic | SCOOT | 12, network | Abu Dhabi, United Arab Emirates | Fixed time | Queue lengths, delay |
| Sha and Chow (2019) | Acyclic, cyclic | SUMO | 12, network | Grid | Centralized | None |
| Ramadhan et al. (2020) | Acyclic, cyclic | Vissim | 6, network | Bandung, Indonesia | Fixed | Queue lengths |
| Bai and Bai (2021) | Acyclic | SUMO | 2, isolated 25, network | Changchun, China | Fixed time | Travel time, queue lengths |
| Li et al. (2021a) | Acyclic | Microsimulation | 12, network | Abu Dhabi, United Arab Emirates | Fixed time | Delay, queue lengths |
| Barman and Levin (2022) | Acyclic, cyclic | SUMO | 7, corridor | Minneapolis | Actuated coordinated | Delay |





J. Transp. Eng., Part A: Systems, 2023, 149(4): 03123001

throughput because vehicle demand around peak hours is usually much less than the network capacity. For instance, over a 4-h simulation comparing actuated and max-pressure controls, all vehicles might exit for both types of signal timing resulting in similar average throughput measurements (Barman and Levin 2022). However, if peak hour traffic can pass through the network with higher throughput from max-pressure control, that higher throughput may appear in the aggregate results as smaller average travel times or delays.

### Wongpiromsarn et al.'s (2012) Acyclic Max-Pressure Control

Several studies compared Wongpiromsarn et al.'s (2012) acyclic max-pressure control with alternative signal timing methods in calibrated microsimulation models. Wongpiromsarn et al. (2012) compared their own max-pressure control with sydney coordinated adaptive traffic system (SCATS) on a 14-intersection corridor in Sweden using MITSIMLab, and found that max-pressure control reduced queue lengths. Le et al. (2017) and Bai and Bai (2021) compared Wongpiromsarn et al.'s (2012) control to fixed-time traffic signals in SUMO and found reductions in delay. Le et al.'s (2017) results used SUMO to model a 16-intersection network from Melbourne, Australia. Bai and Bai (2021) modeled a 25-intersection network in Changchun, China. Unlike other studies, Dakic et al. (2015) found that optimized fixed-time or actuated signal timings performed better than max-pressure control on five intersections in Salt Lake City using Vissim.

### Varaiya's (2013) Acyclic Max-Pressure Control

Varaiya (2013) presented an alternate acyclic max-pressure control tracking vehicle occupancies per turning movement instead of per link like Wongpiromsarn et al. (2012). Varaiya's (2013) max-pressure control has been the subject of more simulation studies. However, we did not find any studies directly comparing Varaiya's (2013) control with Wongpiromsarn et al.'s (2012) control in simulation.

Wang and Abbas (2019) compared Varaiya's (2013) control with several signal optimizations including Vistro and a model predictive control method they proposed in a Vissim model of a corridor in Blacksburg, Virginia. Li and Jabari (2019) compared their control, Varaiya's (2013) and Gregoire et al.'s (2014b) controls, and fixed-time signals in a network from Abu Dhabi, United Arab Emirates, in SCOOT. Li et al. (2021a) simulated fixed-time signals and Varaiya's (2013) max-pressure control with and without estimating queue lengths from connected vehicles on calibrated models in Abu Dhabi, United Arab Emirates. The three aforementioned studies observed that max-pressure control reduced delays, with Li and Jabari (2019) and Li et al. (2021a) also observing reductions in vehicle occupancies. However, Lioris et al. (2016) found that pretimed signals achieved lower delays on a corridor in Los Angeles using the PointQ discrete event simulation. Max-pressure control was still observed to be more effective at handling periods of high congestion.

### Le et al.'s (2015) Cyclic Max-Pressure Control

After the development of Le et al.'s (2015) cyclic max-pressure control, some studies started using it in simulation due to the practical benefits of having a signal cycle. Xiao et al. (2015a, c) compared it with fixed-time signals using Webster's formula on networks in Singapore using Vissim. They observed reductions in delay and the average number of stops. Xiao et al. (2015c) further explored through simulation the integration of bus and pedestrian intersection behaviors, which adds additional constraints to max-pressure control. Xu et al. (2022) developed a modified max-pressure control specifically to integrate bus rapid transit.

Other studies compared Le et al.'s (2015) control with both Varaiya's (2013) control and alternative methods. Le et al. (2015) themselves compared their cyclic control, Varaiya's (2013) acyclic control, and proportional signal timings on a 72-intersection network based on Melbourne, Australia, in SUMO. Sun and Yin (2018) modeled a 12-intersection corridor from Gainsville, Florida, in Vissim to compare the existing actuated-coordinated control. Ramadhan et al. (2020) modeled a six-intersection network in Bandung, Indonesia, in Vissim based on observed data. Sun and Yin (2018) and Ramadhan et al. (2020) found that both types of max-pressure control performed better than the alternative signal timings. Furthermore, they observed that Varaiya's (2013) control had lower delays than Le et al.'s (2015) control, which is possibly because Varaiya (2013) is not constrained to follow a signal cycle and can respond at shorter time steps. Varaiya's (2013) control selects one phase per time step, whereas Le et al.'s (2015) time step represents an entire signal cycle. However, Le et al. (2015) found that their control achieved lower delays than Varaiya's (2013) control. Le et al. (2015) and Ramadhan et al. (2020) also found that max-pressure control reduced queue lengths. In particular, Ramadhan et al. (2020) observed that max-pressure control could prevent gridlock in a disturbed network caused by a partial road closure.

In contrast, Sha and Chow (2019) compared Varaiya's (2013) and Le et al.'s (2015) controls with the centralized (non-max-pressure) control of Diakaki et al. (2002). With fixed route choice, the centralized control had lower delays, but after route choices were updated it performed similarly to max-pressure control.

### Other Cyclic Max-Pressure Controls

Besides the cyclic control of Le et al. (2015), other studies have proposed alternative cyclic max-pressure controls and compared them in simulation. Pumir et al. (2015) and Anderson et al. (2018) modeled 11 intersections in San Diego in Aimsun and compared their cycle-based max-pressure controls with actuated-coordinated signals. Performance was similar between the max-pressure and actuated-coordinated controls, with Pumir et al. (2015) observing some reductions in queue lengths. Barman and Levin (2022) conducted detailed SUMO microsimulations of seven intersections across two corridors at different time periods, calibrated to match Hennepin County, Minnesota, data. They compared the current actuated-coordinated signals with Varaiya's (2013) acyclic and Levin et al.'s (2020) cyclic controls. Although performance varied, most intersections had some max-pressure parameters that would increase performance.

## *Comparisons between Different Types of Max-Pressure Controls*

Several studies compared several variants of max-pressure control against each other in realistic networks, and these are worth mentioning separately. Levin et al. (2020) compared Varaiya's (2013), Le et al.'s (2015), and their cyclic max-pressure control in a mesoscopic point queue model of Austin, Texas, and found that Varaiya's (2013) acyclic control had lower delays than both cyclic max-pressure controls. Also, Levin et al.'s (2020) cyclic control performed better than Le et al.'s (2015) cyclic control due to the variable cycle lengths. Robbennolt et al. (2022) studied Varaiya's (2013) and Levin et al.'s (2020) cyclic control, and a new semicyclic control in a SUMO model of Austin, Texas. They also found that Varaiya's (2013) control had lower network occupancies than Levin et al.'s (2020) control. Their new semicyclic control achieved similar performance to Varaiya's (2013) control while retaining some aspects of a signal cycle.

### Experiments on Actual Roads

Implementation on actual roads is made difficult by the fact that most current signal controller hardware does not support the implementation of novel adaptive algorithms such as max-pressure control. Nevertheless, two studies were still able to implement max-pressure control in reality by installing their own hardware. Unfortunately, both studies relied on waiting times or travel times instead of road occupancies, leaving open the question of how conventional occupancy-based max-pressure control performs on actual roads. Mercader et al. (2020) implemented a max-pressure control based on waiting times, which were obtained by a Bluetooth detector that registered nearby Bluetooth devices. Their max-pressure controller appeared to perform better than a fixed-time controller in a comparison to historic data, but their results were limited. Their estimation of waiting times was also incomplete because not all vehicles contained Bluetooth devices. Dixit et al. (2020) used crowdsourced travel time data from Google Maps to determine phase durations within a fixed cycle similar to Le et al. (2015). They conducted field experiments on seven intersections across Indonesia and India using an Arduino control board to replace the signal controller with multiple days of experiments. They reported a net decrease in intersection delay at all intersections, with most individual approaches receiving statistically significant benefits.

## Applications beyond Traffic Signals

Max-pressure control itself is adapted from back-pressure routing of communications networks (Tassiulas and Ephremides 1990), so it is natural to apply the concepts of max-pressure control to other transportation systems where achieving maximum throughput is challenging. We discuss three such applications that have been studied in the literature. There are probably more problems within transportation engineering that could benefit from max-pressure concepts.

### Combined Signal and Route Control

Since Tassiulas and Ephremides (1990) combined route choice and node service of communications networks, several studies integrated route choice and traffic signals. Because drivers ultimately control their vehicle movements, route choice can be modeled as an imposed control or instead as an advisory, and both assumptions have been studied. There are at least two approaches to including destinations. Gregoire et al. (2016) and Le et al. (2017) assumed that the turning movements $p_{ij}(t)$ were controllable within Wongpiromsarn et al.'s (2012) link occupancy model. Other studies added a destination index to the occupancies, i.e., defining $x_i^s(t)$ to be the number of vehicles on link $i$ at time $t$ destined for $s$ (Zaidi et al. 2016; Maipradit et al. 2021). The latter approach is useful for ensuring that vehicles are routed toward their actual destination, but potentially introduces first-in-first-out issues in the flow modeling on link $i$ because vehicle queues are separated by destination. Another common modeling choice is the use of a shadow or virtual network that tracks all real vehicles but also has a small $\epsilon$ probability of adding an additional fake vehicle when a real vehicle enters the network. This $\epsilon$ probability was said to help with stability, although many of the route control studies did not discuss the mathematical details of stability properties.

Most studies assumed that route controls are imposed. Zaidi et al. (2015, 2016) used a model with virtual link queues tracking a destination index. Liu et al. (2018) presented a similar model with a bias term that encouraged following the shortest path. Taale et al. (2015) used a route-based pressure model with a cyclic max-pressure control based on Le et al. (2015). Maipradit et al. (2019, 2021) used a shadow network with a destination index, but also added a bias term to their weight function that was determined by $Q$-learning. None of the aforementioned papers included a proof of the stability properties.

Gregoire et al. (2016) and Le et al. (2017) both modeled a partial compliance scenario in which a limited proportion of vehicles would comply with route controls. Both Gregoire et al. (2016) and Le et al. (2017) used link occupancy models without explicit tracking of destination indexes, and included a proof of the stability properties. The proof technique is generally similar to stability proofs for max-pressure control without route control, but includes additional complexity around the turning proportions.

Overall, simulation results generally showed that max-pressure with route control reduced travel times beyond max-pressure alone. However, Le et al. (2017) observed that a large proportion of vehicles following route control could increase travel times due to a large emphasis in their max-pressure on routing to avoid congestion.

### Automated Intersection Control

Autonomous intersection management (Dresner and Stone 2004) is very different from the phase-based structure of traffic signals because vehicles in conflicting turning movements use the intersection simultaneously, with the speed and timing of individual vehicles controlled by the intersection to prevent conflicts (Levin and Rey 2017). This precise individualized control is designed for automated vehicles, and all relevant max-pressure studies have assumed that only automated vehicles use autonomous intersection management. Nevertheless, a similar max-pressure approach can be applied to achieve maximum stability of autonomous vehicle network flows, except with $s_{ij}(t) \in [0,1]$ instead of in $\{0,1\}$ to model simultaneous activation of conflicting turning movements. A different model, such as the conflict region model (Levin et al. 2016), is then applied to describe how conflicts restrict intersection flows. Rey and Levin (2019) used max-pressure control to decide between activating a traffic signal phase for human-driven vehicles or autonomous intersection management for automated vehicles, which were on separate rights-of-way. They were the only study to include a mixture of human-driven and automated vehicles, and modeled the mixed behavior by separating human-driven and automated vehicles on different lanes and preventing simultaneous intersection use. Chen et al. (2020) integrated automated intersection management and pedestrian crosswalk activation, and used max-pressure control to decide when to activate crosswalks. Levin et al. (2019) controlled a combination of autonomous intersection management and dynamic lane reversals (Hausknecht et al. 2011). Both Chen et al. (2020) and Levin et al. (2019) assumed that all vehicles were automated. All of these studies presented proofs of maximum stability.

### Ridesharing Dispatch

Although unrelated to intersection control, the dispatch of ridesharing or (autonomous) mobility-on-demand vehicles benefits from throughput guarantees in terms of serving passenger requests. Kang and Levin (2021), Li et al. (2021b), and Levin (2022) applied max-pressure concepts to this setting with proofs of maximum stability. Kang and Levin (2021) tracked the number of passengers waiting for service, and used a model predictive control approach for the max-pressure control. Li et al. (2021b) and Levin (2022) instead tracked the waiting time of the longest-waiting customer at each location, and developed a maximum-stable control based on required service times. Although max-pressure control does not translate directly into ridesharing due to differences in the network evolution dynamics, these papers demonstrate that the maximum stability concept may be useful in other contexts.

## Conclusions and Future Work

The previous discussion has several main takeaways. First, we summarized the methodological approach to max-pressure control. Although other methodological papers have varying organizations, we suggest that readers look for the individual components presented here. We also hope that the discussion will explain some methodological concepts that may not have been made clear in other papers. Next, we discussed several methodological improvements to max-pressure control. Researchers have found ways to add more realistic traffic flow models and practical constraints such as cycle structure and lost time while retaining the maximum stability properties. These results suggest that further methodological extensions are possible without giving up the nice mathematical properties.

Of course, the main determinant for use in practice is its performance. Many studies included microsimulations on networks based on using data from observing actual roads. Most of these simulations observed benefits in delay, queue lengths, travel times, vehicle speeds, and/or number of stops compared with alternative signal timing methods. A few studies suggested that optimized fixed-time or centrally controlled signal timings would perform better than max-pressure control. Based on the theoretical guarantees, it is likely that optimized signals could achieve better delays than max-pressure control on some networks, but that optimization must be adaptive to variations in demand both due to stochasticity and time-of-day changes. Max-pressure control may be able to achieve good performance with less extensive engineering effort.

Finally, we discuss some open problems with max-pressure control that could improve its performance. We hope that future researchers will address these issues in their work.

Network impacts have mostly been measured through average delays, travel times, queue lengths, or number of stops. However, alternative metrics exist, and an analysis of max-pressure control using these could yield different insights. For instance, Salomons and Hegyi (2016) derived macroscopic fundamental diagrams from a comparison of Wongpiromsarn et al.'s (2012) control with actuated signals on 16 intersections in Vissim. They found that actuated signals accumulated flow faster in uncongested situations, but max-pressure control was more effective in reducing queues when the network was congested. However, more realistic data on max-pressure performance would be beneficial, both in highly congested scenarios and in low-demand periods with sparse occupancy where max-pressure phase activation may be based on vehicles that are on a link but far from the intersection.

Most methodological and simulation studies of max-pressure control are based on the store-and-forward queueing model, which lacks some aspects of realistic traffic behavior. Although some studies have started developing max-pressure control based on more realistic traffic flow models, they have yet to be used by many of the other methodological and numerical studies. In particular, Li and Jabari's (2019) max-pressure control established throughput properties for kinematic wave models, which are standard tools for traffic flow modeling. Nevertheless, stability properties of max-pressure control have not been established for the most realistic car-following models. Therefore, it is unclear whether the stability properties apply to actual traffic. Simulation studies designed to probe the boundaries of the stable region might answer this question, but prior microsimulation studies have focused on comparing max-pressure with other signal timing methods.

Dixit et al. (2020) and Mercader et al. (2020) both conducted experiments on actual roads, but both used modified versions of max-pressure control that may have different performance. Future experimental work, especially involving max-pressure controls based on methodologies with mathematical throughput properties, would be beneficial for achieving a better understanding of actual road performance.

Many corridors of traffic signals have their signal timings coordinated to reduce the number of stops for corridor travel. Some degree of coordination implicitly exists in max-pressure control. As a platoon of vehicles moves along a corridor, the presence of the platoon vehicles will affect pressure calculations and corresponding phase selections. However, platoons may be interrupted and possibly separated when side queues are longer than the remaining platoon due to the pressure-based phase selection. Only the link occupancy from the vehicles comprising the platoon is used in pressure calculations, not the existence of a platoon itself. No explicit coordination exists yet, and it is not known how to modify max-pressure control to include signal coordination while retaining its throughput properties.

Most existing signal controllers are not equipped to implement the novel algorithm of max-pressure control. Consequently, implementing max-pressure control could involve expensive replacement of signal control hardware. The corresponding network design problem has yet to be addressed: given a limited budget for max-pressure signal controllers, where are the optimal locations to install them in a network of signalized intersections?

The mathematical throughput properties of max-pressure control usually assume that demand and turning proportions are identically distributed over time. Obviously, that is not true for most city networks; peak periods exhibit different trip demands and route choices than other times. Although some simulation results have explored the performance of max-pressure control at varying times of day, it is not known how the mathematical throughput properties apply to these daily variations in demand.

Traffic signal timing in general is often optimized for observed link demands and turning ratios, but signal timing itself can affect the route choices that determine link demands (Smith 1979). Max-pressure control is no exception; some versions (e.g., Varaiya 2013) use turning ratios explicitly, and the mathematical proofs of throughput properties usually assume fixed route choices. Very little work has been done on how max-pressure control affects the route choices of independent vehicles. Smith et al. (2019) showed that max-pressure control's throughput properties do not apply under route choice changes, and developed a modified pressure-based policy that maximizes throughput under route choices changes. However, it is not even clear whether this is a common issue in practice.

In summary, the relatively recent emergence of max-pressure control has left many questions unanswered. However, the many performance results on different calibrated networks even from initial versions of max-pressure control suggest that further research could be highly beneficial for traffic performance.

## Data Availability Statement

No data, models, or code were generated or used during the study.

## Acknowledgments

The authors gratefully acknowledge the support of the National Science Foundation, Award No. 1935514.

## Notation

*The following symbols are used in this paper:*

$\mathcal{A}$ = set of links;
$\mathcal{A}_i \subset \mathcal{A}$ = set of internal links;
$\mathcal{A}_r \subset \mathcal{A}$ = set of source links;
$\mathcal{D}$ = stable region of demand with $\mathcal{D}^0$ being the interior of $\mathcal{D}$;
$d_i(t)$ = number of vehicles entering source link $i$ during time step $t$, with mean $\bar{d}_i$ and vector of means $\bar{\mathbf{d}}$;
$\bar{f}_i$ = average volume on link $i$;
$\mathcal{G}$ = network;
$K$ = positive constant;
$\mathcal{N}$ = set of nodes;
$p_{ij}(t)$ = proportion of vehicles entering link $i$ during time step $t$ that will next move to link $j$, with mean $\bar{p}_{ij}$;
$Q_{ij}$ = capacity for movement from link $i$ to link $j$;
$\mathcal{S}$ = set of feasible phase activations;
$\bar{s}_{ij}$ = average proportion of time that movement $(i,j)$ gets a green light;
$s_{ij}(t)$ = signal activation for turning movement $(i,j)$ during time step $t$, with vector $\mathbf{s}(t)$;
$T$ = time horizon;
$w_{ij}(t)$ = weight (pressure) for movement $(i,j)$ at time $t$;
$X_{ij}$ = maximum occupancy possible for the queue on link $i$ waiting to turn to link $j$;
$x_{ij}(t)$ = number of vehicles on link $i$ waiting to turn to link $j$ at time $t$;
$y_{ij}(t)$ = number of vehicles moving from link $i$ to link $j$ during time step $t$;
$\delta_{ij}(t) = x_{ij}(t+1) - x_{ij}(t)$;
$\epsilon$ = small positive constant; and
$\nu(t)$ = Lyapunov function.

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