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Curbing cruising-as-substitution-for-parking in automated mobility

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ABSTRACT

This study considers dynamic parking management in the era of driving automation that users of privately-owned automated vehicles (AVs) come to a downtown area to conduct various activities and choose between three parking options: outside parking, on-street parking, and cruising-as-substitution-for-parking. In the latter, AVs cruise in downtown until summoned by their users after concluding their activities. Given the distribution of users' activity time in downtown, we propose a system of ordinary differential equations to model AVs' parking choice and capture the impact of cruising-as-substitution-for-parking on traffic congestion. With the proposed model, we further investigate dynamic time-based tolling and parking provision to optimize the system performance. Results of our numerical experiments demonstrate the validity of our model and the potential of dynamic tolling and parking provision on managing downtown parking of AVs.

1. Introduction

Parking is not only a daily struggle for many drivers but also is a challenging problem for transportation policymakers. According to Boltze and Puzicha (1995), travelers might bear up to a 40% increase in their original travel time due to the search for a parking space. Shoup (2006) showed that searching for parking contributes between 8% to 74% of traffic in downtown and might be one of the major causes for congestion. Therefore, parking management strategies and their impacts on traffic congestion have received significant attention in the literature (see, e.g., Axhausen et al., 1994; Anderson and De Palma, 2004; Arnott and Inci, 2006; Van Ommeren et al., 2012; Qian and Rajagopal, 2014; Geroliminis, 2015; Weinberger et al., 2020). The majority of these prior studies focus on addressing the parking problem in the current state of the transportation system. The advent of automated vehicles (AVs) brings new opportunities and challenges to solving the parking problem in the future transportation system.

One of these challenges is the congestion created by cruising-as-substitution-for-parking. Once arriving at their destinations, users of AVs can instruct their AVs to drive back home, find a place to park, or cruise until summoned. The latter, i.e., cruising-as-substitution-for-parking, is particularly relevant in locations such as a downtown area where finding a parking space is costly and AV users' activity time may be short. Millard-Ball (2019) considered the impact of such a cruising behavior on downtown San Francisco and suggested that AVs may collaborate and choose the most congested cruising path to make a gridlock to decrease their parking cost. Bahrami (2019) used agent-based simulation to examine AVs' parking choices in downtown Toronto and showed that cruising-as-substitution-for-parking could be the most economical option for many users and then introduced a time-based toll to discourage it. More recently, Bahrami et al. (2021) developed a static equilibrium model to describe the parking choice of privately-owned AVs with different activity time duration in a downtown area and showed that cruising AVs can cause a gridlock even if they do not collude with each other.

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This study advances the static analysis by Bahrami et al. (2021) to be in a dynamic setting, attempting to capture the interaction between the evolution of downtown traffic congestion and AVs' parking choices and then explore strategies to manage congestion created by cruising-as-substitution-for-parking. Specifically, given the distribution of users' activity time in downtown, we propose a system of ordinary differential equations to model AVs' parking choice and downtown congestion. Then, we optimize dynamic time-based tolling and on-street parking provision to improve the system performance. The contributions of this paper are twofold: (i) to our best knowledge, this paper is the first to develop a dynamic modeling framework to describe and control the interaction between cruising-as-substitution-for-parking and the formation and dissipation of downtown traffic congestion; and (ii) this work enriches the literature of macroscopic modeling of downtown traffic congestion by contributing a new class of problems where heterogeneity is due to the varying time spent in the downtown rather than the varying length of trips (e.g., Vickrey, 2020; Jin, 2020). This new class of problems uniquely arises in automated mobility and has not been investigated.

The remainder of this paper is organized as follows. First, in Section 2, we review the relevant literature. Section 3 describes our mathematical framework, followed by an investigation on time-based dynamic tolling strategies and parking provision in Sections 4 and 5. Finally, the last section concludes this paper.

2. Relevant literature

Given the abundant literature on parking and congestion modeling, below we review those highly relevant to our work, which are classified into three categories: modeling of cruising-for-parking and traffic congestion, dynamic modeling of AV parking, and macroscopic modeling of downtown traffic congestion.

2.1. Cruising-for-parking and traffic congestion

Cruising-for-parking, rather than cruising-as-substitution-for-parking of interest to this paper, has been the focus of prior studies that investigate the impact of parking on traffic congestion. There are two critical issues in this quest: estimating the time or distance of cruising before finding a parking space and capturing how cruising traffic impacts traffic congestion. For a more detailed discussion or comprehensive review, see, e.g., Arnott and Williams (2017) for the former and Inci (2015) for the latter.

A series of studies has analyzed the interaction between cruising-for-parking and congestion using structural models (e.g., Arnott and Inci, 2006, 2010; Arnott and Rowse, 2009). These studies decompose vehicles into several pools based on their parking states, such as parked, cruising-for-parking, and moving, then model the transitions between them. These studies typically analyze the steady states of the proposed dynamic system and subsequently derive optimal parking policies.

Another stream of studies aims to develop dynamic models to capture the interaction between traffic congestion and cruising-for-parking evolution and then investigate real-time operational strategies to manage parking and traffic congestion. These studies typically exploit a macroscopic fundamental diagram (MFD) approach (e.g., Godfrey, 1969; Geroliminis and Daganzo, 2008; Mahmassani et al., 2013) to model traffic congestion. As one of the pioneering studies, Geroliminis (2015) characterized the impact of cruising-for-parking on downtown traffic using an accumulation-based MFD approach, while Leclercq et al. (2017) later proposed a trip-based counterpart. An extension to Geroliminis (2015) was made by Zheng and Geroliminis (2016) that developed a network-wide feedback control scheme as a parking pricing policy in a bi-modal network. More recently, Gu et al. (2020) proposed a macroscopic dynamic framework that considers on-street and off-street parking operations.

Substantial efforts have also been made to integrate parking into the morning commute problem (Vickrey, 1969) to investigate how parking availability and relevant parking management schemes such as reservation would impact rush-hour congestion (e.g., Zhang et al., 2011; Qian et al., 2011, 2012; Fosgerau and De Palma, 2013; Yang et al., 2013). However, most of these studies do not model the impacts of cruising-for-parking. In contrast, Liu and Geroliminis (2016) considered cruising-for-parking in a bottleneck model characterized by MFD and developed a network-level dynamic pricing scheme to reduce schedule delay costs and cruising time in the morning commute problem.

2.2. Dynamic modeling of AV parking

A few recent efforts have been made to capture the interaction of parking choice and traffic congestion in the era of AVs (e.g., Liu, 2018; Zhang et al., 2019; Su and Wang, 2020). The impacts of AVs considered by these studies are as follows: (i) commuters do not waste time on cruising for parking and walking from park space to workplace, and (ii) parking location may not necessarily be near the workplace due to self-driving, e.g., the vehicle can drive back home (Zakharenko, 2016). However, these studies consider that cruising-as-substitution-for-parking is not economical for commuters with long activity times and thus ignore its impact.

More specifically, to capture the impacts of walking distance elimination on the morning commute problem, Liu (2018) adopted Vickery's bottleneck model on a linear corridor to analyze traffic dynamics and commuters' departure time choice. The research is further extended by Zhang et al. (2019) by examining an integrated morning–evening commute problem. More recently, with the consideration of off-street parking, home parking, and shared parking, Su and Wang (2020) analyzed the attractiveness of each parking option for users with different distances from downtown.

2.3. Macroscopic modeling of downtown traffic congestion

Vickery's bottleneck model (Vickrey, 1969) has been utilized by many to model rush-hour traffic congestion. However, this model considers congestion as a queue behind a bottleneck with fixed capacity and thus fails to capture the so-called "hypercongestion" condition. In the economic literature, the theoretical method that can consider hypercongestion is called a "bathtub" model. Vickrey (2020) initiated the bathtub model for the evolution of downtown traffic condition based on three premises: (i) consideration of the road network as a single bathtub; (ii) existence of speed-density relationship at the network level, and (iii) the time-independent negative exponential distribution for trip lengths. The first two premises are practically acceptable as per the abundant literature of MFD. However, the last assumption may yield unreasonable results for other types of time-dependent trip length distributions. Jin (2020) thus relaxed this assumption by developing a generalized bathtub model that works for any distribution of trip lengths. Prior efforts have also been made to address trip length heterogeneity in the MFD-based morning commute problem (e.g., Lamotte and Geroliminis, 2018).

In this paper, to capture the interaction between cruising-as-substitution-for-parking and downtown congestion, we consider trip time rather than trip length heterogeneity. This new setting uniquely arises in automated mobility and has not been previously investigated.

3. Modeling AV parking choices in downtown

3.1. Problem setting

We consider a downtown area where a continuum of users of privately-owned AVs come to engage in activities whose duration varies from user to user but follows a known distribution. The arrival rates of these AV users to downtown are considered given in this paper and can vary with time. Upon arrival, users choose one of the following three parking options for their AVs: (i) cruise inside downtown as a substitution for parking (we hereinafter call these AVs as "cruisers" and often attach "c" as subscript to associated variables like parking choice probability); (ii) search for an on-street parking spot immediately after dropping off their users and park after finding the vacant spot ("searchers" or "s"); or (iii) exit downtown streets and then use an outside parking option ("parkers" or "p"). Because this paper focuses on downtown congestion, we use outside parking to encapsulate other parking possibilities such as parking in an outskirt parking lot, sending AVs home and using downtown off-street parking. For the latter, AVs are considered not to cruise for it and thus do not contribute to downtown congestion.

Anticipating the conclusion of their activities, users are assumed to summon their AVs in advance and be picked up right after completing their activities. This implies that AVs are able to determine appropriate cruising routes to return to their users on time. To facilitate our modeling development, we further assume that AVs experience no delay during ingress or egress of the downtown area. This implies that the time a cruiser or searcher spends in downtown will be equal to the activity time of its user; while parkers must revisit downtown to pick up their users prior to their departure, they do not spend any time in downtown. It is worth noting that these assumptions are made to isolate the contribution of cruising to traffic congestion. They are not necessarily restrictive and can be relaxed if needed.

Fig. 1 presents a downtown representation that makes sense of our simplification. In this figure, the locations where users of AVs conduct activities are uniformly distributed along the perimeter of the downtown area shown by small blue dots, and AVs can access these points without interacting with the downtown traffic flow, which consists of three groups: (i) cruisers who cruise in downtown and return to activity points upon being summoned; (ii) searchers who cruise until they find an available parking spot; and (iii) background traffic that traverses downtown with a given trip length.

3.2. Notation

As described above, the activity time of an AV user, α, varies across the user population, and we denote its probability density function by $f(.)$, whose support is between a minimum and a maximum activity time $\check{\alpha}$ and $\hat{\alpha}$, respectively. Given an inflow rate of AVs to downtown at time t, denoted by $\lambda(t)$, and the choice probability $\eta_m(t,\alpha)$, $m \in \{c,s,p\}$, we calculate the inflow rate of cruisers, $\lambda_c(t)$, and searchers, $\lambda_s(t)$, to estimate AVs' accumulation in downtown. The probability of choosing a parking option may depend on users' estimation of cost of cruising, $\mu_c(t,\alpha)$, searching, $\mu_s(t,\alpha)$, and outside parking $\mu_p(t)$ when they enter the downtown. It is worth mentioning that on-street parking may involve cruising during an estimated search time $S(t)$ and park for the remaining activity time $\alpha - S(t)$ with a unit on-street parking cost of $\mu_o(t)$. As a result, the estimated cost for this option depends on the cost of cruising-for-parking and on-street parking.

To model downtown congestion, we further consider that the average speed inside downtown $v(t)$ is given by an MFD denoted by $M(.)$ based on the accumulation, denoted by $n(t)$, which is contributed by cruisers with the accumulation labeled by $n_c(t)$, searchers, $n_s(t)$, who cruise until they find an available parking spot, and background traffic, $n_b(t)$, who traverses downtown with a trip length of L_b. For the later, its inflow rate $\lambda_b(t)$ is specified by an elastic demand function denoted by $D(.)$. Throughout this paper, we study a horizon $[0,T]$, where T is the end of the horizon.

For readers' convenience, Table 1 presents the nomenclature list for the proposed model.

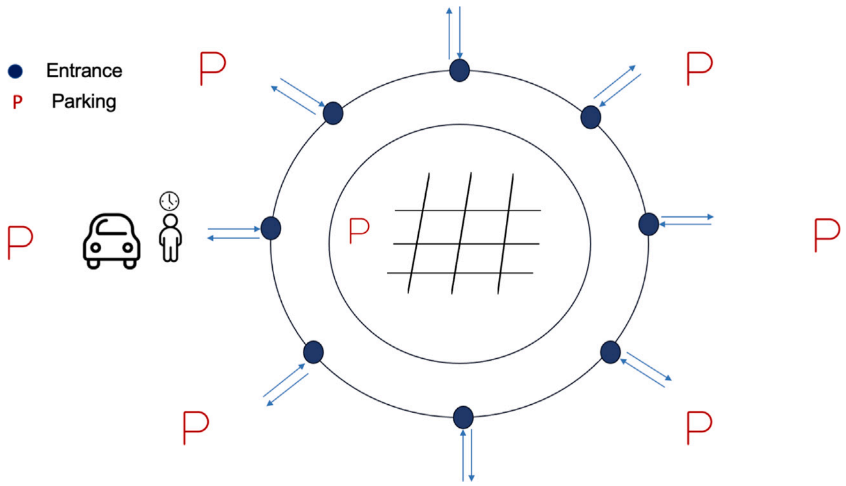


Fig. 1. Downtown representation.

Table 1
Nomenclature.

Parameters	Definition
$\lambda(t)$	AV users' inflow rate to downtown at the time t
$f(\alpha)$	Probability density function for activity time
μ_c^l	Unit cruising cost per unit distance of driving
$\mu_o(t)$	Unit on-street parking cost at the time t
$\mu_p(t)$	Unit outside parking cost at the time t
N_o	Number of on-street parking spots
L_b	Average travel distance for background traffic
T	Duration of study period
L	Downtown lane length
Variables	**Definition**
$n_c(t)$	Number of cruisers in downtown at the time t
$n_s(t)$	Number of vehicles cruising for parking in downtown at the time t
$n_o(t)$	Number of vehicles parked on-street in downtown at the time t
$n_b(t)$	Number of background traffic vehicles in downtown at the time t
$\lambda_c(t)$	Cruisers' inflow rate at the time t
$\lambda_s(t)$	Searchers' inflow rate at the time t
$\lambda_b(t)$	Background traffic demand at the time t
α	Activity time that varies between the minimum $\check{\alpha}$ and the maximum $\hat{\alpha}$
$\eta_m(t,\alpha)$	Probability of choosing option m at the time t with activity time α
$\mu_c(t,\alpha)$	Cruising cost at the time t with activity time α
$\mu_s(t,\alpha)$	Searching cost at the time t with activity time α
$S(t)$	Search time for on-street parking
$v(t)$	Traffic speed in downtown at the time t
$M(n(t))$	Macroscopic speed-density relationship
$\tau(t)$	Toll rate at the time t
$q(t)$	System throughput at the time t

3.3. Modeling parking choice

It is assumed that users are myopic when evaluating parking options and thus base their choices on perceived costs that are true instantaneous costs plus zero-mean perception errors. We further assume that once a parking decision is reached upon arrival, AVs do not revisit it during activity times of their users. We note that a similar treatment has been adopted in the literature of dynamic traffic assignment (e.g., Boyce et al., 2001). It is also worth noting that another behavioral consideration (e.g., users being more strategic and choosing based on actual rather than instantaneous travel costs) can be accommodated in the proposed modeling framework. However, they may lead to a model that is more computationally challenging.

With the above consideration, the proportion of users with activity time α choosing a parking option upon their arrival at time t can be estimated as follows:

$$\eta_m(t,\alpha) = P_m(\mu_c(t,\alpha), \mu_s(t,\alpha), \mu_p(t,\alpha)) \quad m \in \{c, s, p\} \tag{1}$$

where the functional form $P_m(.)$ can depend on the distribution of the perception errors. For example, it can be a multinomial logit model.

In the estimation of the instantaneous cost for a cruiser whose activity time is α and arrives at time t, we have the following:

$$\mu_c(t,\alpha) = \mu_c^l v(t)\alpha + \tau(t)\alpha$$

where μ_c^l represents the cost of cruising per unit distance, and $\tau(t)$ is the toll rate levied on downtown at the arrival instance if a time-based dynamic tolling scheme is implemented (see its details in Section 4). As such, the first term represents the driving cost of a cruiser, and the second term is the amount of toll perceived by the cruiser for the entire activity time of its user.

Similarly, we have the instantaneous cost of searchers:

$$\mu_s(t,\alpha) = (\mu_c^l v(t) + \tau(t)) \min(\alpha, S(t)) + \mu_o(t) \max(\alpha - S(t), 0)$$

where the first term represents the driving cost for cruising-for-parking while the second term is the instantaneous parking cost perceived by an on-street parker. The parking search time $S(t)$ generally depends on the number of AVs searching for parking, parking availability, and downtown congestion. For example, it can be estimated by following Zheng and Geroliminis (2016): considering a homogeneous distribution of parking spots on the road of length l and parking occupancy rate $\frac{n_o}{N_o}$ where n_o represents the number of vehicles parked on street and N_o is the total on-street parking, the average search time for a vacant parking spot can be approximated as follows:

$$S(t) = \frac{l}{(1 - \frac{n_o(t)}{N_o}) \times v(t)}$$

Finally, without loss of generality, we consider the cost of outside parking $\mu_p(t,\alpha)$ to be $\mu_p\alpha$, where μ_p denotes the cost of outside parking for unit time, which is constant and given.

3.4. Modeling congestion

As previously stated, we apply an MFD approach to model downtown congestion. More specifically, we have:

$$v(t) = M(n(t))$$

where, $n(t) = n_c(t) + n_s(t) + n_b(t)$.

Below we discuss how these accumulations can be estimated. We first compute the inflow rate of cruisers. Noticing that Eq. (1) conditions on the activity time of an AV user $\alpha \in [\breve{\alpha}, \hat{\alpha}]$, integrating over the support of α yields the proportion of drivers who choose to be cruisers upon their arrival. The inflow rate of cruisers at time t is thus given as follows:

$$\lambda_c(t) = \lambda(t) \int_{\breve{\alpha}}^{\hat{\alpha}} f(\alpha)\eta_c(t,\alpha)d\alpha$$

For the outflow rate of cruisers at time t, we condition on the activity time of their users, $\alpha \in [\breve{\alpha}, \hat{\alpha}]$. Note that users departing at time t with activity time α enter downtown at time $t-\alpha$ with a rate of $\lambda(t-\alpha)\eta_c(t-\alpha,\alpha)$. Similarly, integrating over the support of α leads to the outflow rate of cruisers at time t, λ_c^{out}, as follows:

$$\lambda_c^{out}(t) = \int_{\min\{t,\breve{\alpha}\}}^{\min\{t,\hat{\alpha}\}} \lambda(t-\alpha) f(\alpha)\eta_c(t-\alpha,\alpha)d\alpha$$

where we limit the range of integral to $[\min\{t,\breve{\alpha}\}, \min\{t,\hat{\alpha}\}]$, since $t-\alpha$ cannot be negative by definition. With the inflow and outflow rate of cruisers, it is straightforward to derive the following to describe the rate of change of the cruiser accumulation in the downtown area:

$$\frac{dn_c(t)}{dt} = \lambda_c(t) - \lambda_c^{out}(t)$$

Similar to cruisers, we compute the inflow rate for searchers as follows:

$$\lambda_s(t) = \lambda(t) \int_{\breve{\alpha}}^{\hat{\alpha}} f(\alpha)\eta_s(t,\alpha)d\alpha$$

For searchers, during the activity time of their owners, they may experience two phases: cruising-for-parking and parked on-street. The former contributes to the downtown traffic accumulation. To calculate the accumulation of vehicles in cruising-for-parking, i.e., searchers in their cruising-for-parking phase, we first introduce the following function that defines the entrance time κ for those who have found a parking spot at time t:

$$\phi(t) = \kappa$$

Denoting the search time for those who enter downtown at the time κ as $S(\kappa)$, we have $\kappa + S(\kappa) = t = \phi^{-1}(\kappa)$. In other words, $\kappa = \phi(\kappa + S(\kappa))$.

To make our model more tractable, we assume the function $\phi(.)$ to be strictly increasing, which implies first-come-first-park. At time t, searchers with activity times in the range $[\max(t-\phi(t), \check{\alpha}), \hat{\alpha}]$ who entered downtown at time $\phi(t)$ will find a parking spot. A searcher with activity time $\alpha \in [\check{\alpha}, \min(\hat{\alpha}, \max(t-\phi(t), \check{\alpha}))]$ who entered downtown at time $t-\alpha$ would leave the system before finding a spot. So, we calculate the outflow rate from the cruising-for-parking phase as follows:

$$\lambda_s^{out}(t) = \int_{\max(t-\phi(t),\check{\alpha})}^{\hat{\alpha}} \lambda(\phi(t)) f(\alpha)\eta_s(\phi(t),\alpha)d\alpha + \int_{\check{\alpha}}^{\min(\hat{\alpha},\max(t-\phi(t),\check{\alpha}))} \lambda(t-\alpha)\eta_s(t-\alpha,\alpha)d\alpha$$

where the first term reflects the outflow rate due to those who find a parking spot at time t while the second term is attributed to those who leave at the end of their activities before finding a parking spot. Using the same procedure as for cruisers, the accumulation of vehicles in cruising-for-parking is calculated as:

$$\frac{dn_s(t)}{dt} = \lambda_s(t) - \lambda_s^{out}(t)$$

The first term in $\lambda_s^{out}(t)$ is essentially the inflow rate to the on-street parking system. Thus, the evolution of on-street parking occupancy can be described as follows:

$$\frac{dn_o(t)}{dt} = \int_{\max(t-\phi(t),\check{\alpha})}^{\hat{\alpha}} \lambda(\phi(t)) f(\alpha)\eta_s(\phi(t),\alpha)d\alpha - \int_{\max(t-\phi(t),\check{\alpha})}^{\min(t,\hat{\alpha})} \lambda(t-\alpha) f(\alpha)\eta_s(t-\alpha,\alpha)d\alpha$$

where the second term on the right-hand side represents the outflow from the parking system, denoted by $\lambda_o^{out}(t)$, which is due to the completion of user activities.

For the background traffic, it is assumed that its inflow rate is a function of instantaneous travel time and toll rate if a time-based tolling scheme is implemented. With an average trip length L_b of the background traffic, the demand function can be specified as $D(n(t), \tau(t))$. Also, we employ Little's formula to approximate the outflow rate of the background traffic $\lambda_b^{out}(t)$ by $\frac{v(t)}{L_b} n_b(t)$ (Small and Chu, 2003; Daganzo, 2007). The rate of change for the background traffic accumulation is thus:

$$\frac{dn_b(t)}{dt} = D(n(t), \tau(t)) - \frac{v(t)}{L_b} n_b(t)$$

As a system performance measure, the downtown throughput, denoted by $q(t)$ can thus be estimated as:

$$q(t) = \lambda_c^{out}(t) + \lambda_b^{out}(t) + \lambda_o^{out}(t) + \int_{\check{\alpha}}^{\min(\hat{\alpha},\max(t-\phi(t),\check{\alpha}))} \lambda(t-\alpha)\eta_s(t-\alpha,\alpha)d\alpha$$

In the above, the third and fourth components represent the overall outflow rate of the searcher option.

3.5. Mathematical model

For readers' convenience, below we present the full model that consists of a set of Eqs. (2a)–(2s):

$$\mu_c(t,\alpha) = (\mu_c^l v(t) + \tau(t))\alpha \tag{2a}$$

$$s(t) = \frac{d}{(1 - \frac{n_o(t)}{N_o}) \times v(t)} \tag{2b}$$

$$\phi(t + S(t)) = t \tag{2c}$$

$$\mu_s(t,\alpha) = (\mu_c^l v(t) + \tau(t)) \min(\alpha, S(t)) + \mu_o(t) \max(\alpha - S(t), 0) \tag{2d}$$

$$\eta_m(t,\alpha) = P_m(u_c(t,\alpha), u_s(t,\alpha), u_p(t,\alpha)) \quad m \in \{c, s, p\} \tag{2e}$$

$$\lambda_c(t) = \lambda(t) \int_{\check{\alpha}}^{\hat{\alpha}} f(\alpha)\eta_c(t,\alpha)d\alpha \tag{2f}$$

$$\lambda_c^{out}(t) = \int_{\min\{t,\check{\alpha}\}}^{\min\{t,\hat{\alpha}\}} \lambda(t-\alpha) f(\alpha)\eta_c(t-\alpha,\alpha)d\alpha \tag{2g}$$

$$\frac{dn_c(t)}{dt} = \lambda_c(t) - \lambda_c^{out}(t) \tag{2h}$$

$$\lambda_s(t) = \lambda(t) \int_{\check{\alpha}}^{\hat{\alpha}} f(\alpha)\eta_s(t,\alpha)d\alpha \tag{2i}$$

$$\lambda_s^{out}(t) = \int_{\max(t-\phi(t),\check{\alpha})}^{\hat{\alpha}} \lambda(\phi(t)) f(\alpha)\eta_s(\phi(t),\alpha)d\alpha + \int_{\check{\alpha}}^{\min(\hat{\alpha},\max(t-\phi(t),\check{\alpha}))} \lambda(t-\alpha)\eta_s(t-\alpha,\alpha)d\alpha \tag{2j}$$

$$\frac{dn_s(t)}{dt} = \lambda_s(t) - \lambda_s^{out}(t) \tag{2k}$$

$$\lambda_o^{out}(t) = \int_{\max(t-\phi(t),\check{\alpha})}^{\min(t,\hat{\alpha})} \lambda(t-\alpha) f(\alpha)\eta_s(t-\alpha,\alpha)d\alpha \tag{2l}$$

$$\frac{dn_o(t)}{dt} = \int_{\max(t-\phi(t),\check{\alpha})}^{\hat{\alpha}} \lambda(\phi(t)) f(\alpha) \eta_s(\phi(t), \alpha) d\alpha - \lambda_o^{out}(t) \tag{2m}$$

$$\lambda_b^{out}(t) = \frac{v(t)}{L_b} n_b(t) \tag{2n}$$

$$\frac{dn_b(t)}{dt} = D(n(t), \tau(t)) - \lambda_b^{out} \tag{2o}$$

$$n(t) = n_c(t) + n_s(t) + n_b(t) \tag{2p}$$

$$q(t) = \lambda_c^{out}(t) + \lambda_b^{out}(t) + \lambda_o^{out}(t) + \int_{\check{\alpha}}^{\min(\hat{\alpha},\max(t-\phi(t),\check{\alpha}))} \lambda(t-\alpha)\eta_s(t-\alpha,\alpha)d\alpha \tag{2q}$$

$$v(t) = M(n(t)) \tag{2r}$$

$$\lambda_c(t), \lambda_s(t), \lambda_b(t), n_c(t), n_s(t), n_o(t), n_b(t) \geq 0 \tag{2s}$$

where Eq. (2a) illustrates the instantaneous cost for cruisers entering at the time t; (2b) and (2c) estimate the search time for a searcher entering at the time t; (2d) represents the instantaneous cost for the on-street parking option at time t; (2e) outlines the parking choice probabilities at the time t; (2f) and (2i) represent the inflow rate of cruisers and searchers at the time t; (2h), (2k), (2m), and (2o) describe the evolution of the accumulations; (2p) reflects that the accumulation comes from cruisers, searchers in cruising-for-parking, and background traffic; (2q) shows the system throughput using the input from (2g), (2j), (2l), and (2n). (2r) represents the speed in the downtown area as a function of network accumulation and lastly, (2s) illustrates the non-negativity conditions.

Given the initial accumulation $n(0)$ and inflow $\lambda(t)$ at each time t, the above model can be solved to describe the evolution of the dynamic system. More specifically, to solve this system of first-order ordinary differential equations (ODEs), we discretize the study period into a set of time intervals $i \in \{1, 2, \dots, I\}$ of equal length δt. Subsequently, we convert this system of ODEs into a system of nonlinear equations via finite difference, presented in Appendix, which is readily solvable.

It is worth pointing out that the model does not explicitly include physical capacity constraints on downtown accumulation and parking occupancy. The former should be less than the jam accumulation while the latter less than 1. To explicitly include these two constraints, "boundary" queues to the downtown area and to the parking system would need to be considered (see, e.g., Guo and Ban, 2020, for a summary of treatments to address the boundary queue problem). Since this consideration does not provide additional insight pertaining to the focus of this paper, we do not consider these boundary queues. During our implementation, this first constraint of downtown capacity is respected by choosing a reasonable inflow profile of $\lambda(t)$, while the violation of the second constraint of parking capacity does not generally arise due to the fact that the probability of choosing on-street parking $\eta_s(t,\alpha)$ converges to 0 when the parking occupancy tends to 1.

3.6. Numerical example

This section presents a numerical example to demonstrate the proposed model and reveal how cruising-as-substitution-for-parking contribute to downtown traffic congestion. Following the relevant literature (Arnott, 2013; Bahrami et al., 2021), we model the relationship between speed and accumulation using a Greenshields speed-density function (Greenshields et al., 1935).

$$v(t) = v_f[1 - \frac{n(t)}{k_j A_u}] \tag{3}$$

where, v_f is the downtown free-flow speed, and $k_j A_u$ implies the downtown jam accumulation denoted by n_j. Note that, with the Greenshields function, the maximum throughput n^* is obtained when $n^* = \frac{n_j}{2}$.

In order to estimate the inflow rate for the background traffic, we adopt a linear elastic demand function as presented in Eq. (4).

$$D(n(t), \tau(t)) = d_0 - \zeta(\frac{L_b}{v(t)})(VOT + \tau(t)) \tag{4}$$

where d_0 denotes the potential background traffic, ζ denotes the elastic demand parameter, and VOT denotes the value of time.

We further consider AV users' activity time is uniformly distributed between 0 and 3 h, i.e., $U[0, 3]$. We further use a multinomial logit model with a dispersion parameter θ being 3 for AV users' parking choice.

Lastly, we consider an eight-hour period for the analysis and discretize it into intervals of six minutes. The corresponding inflow rate profile is shown in Fig. 2.

Table 2 lists the default values for the parameters in the numerical example section.

The default value for the unit cruising cost is estimated according to the AAA publication for users' driving costs in 2021 (AAA, 2021); the cost of parking is estimated using the data from the City of Ann Arbor (Ann Arbor Downtown Development Authority, 2021) and the value of time (VOT) is from Zhong et al. (2020). The computational tests are conducted with an Apple MacBook Pro computer with 8 GB RAM and the 64-bit version of the IOS operating system. The run time for all computational studies is less than one minute.

Results of our analysis in Fig. 3 demonstrate the evolution of traffic accumulation and speed in the downtown area, starting from an empty network as the initial condition. From Fig. 3(a), we observe that initially users choose on-street parking and thus the on-street parking occupancy increases to 90% during the second and third hour. However, the growth in parking occupancy elevates

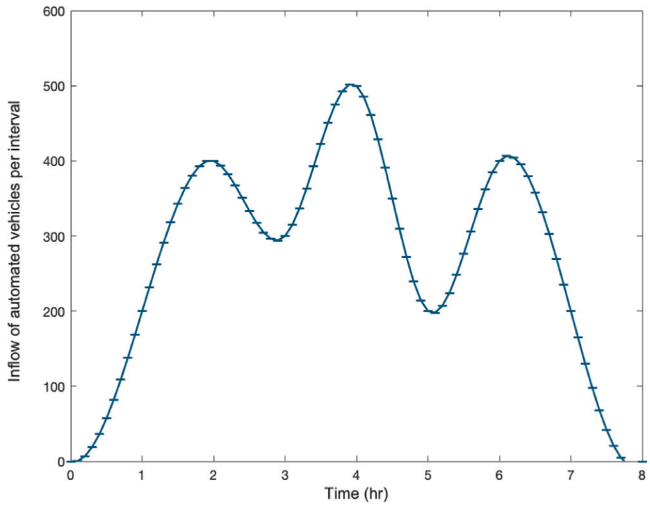


Fig. 2. Time-variant inflow rate distribution.

Table 2
Default values for the parameters in numerical experiments.

Notation	Definition	Default value
μ_c^l	Unit cruising cost [$/mi]	0.06
μ_p	Unit outside parking cost [$/h]	1.5
μ_o	Unit on-street parking cost [$/h]	1.3
θ	Dispersion parameter	3
v_f	Free-flow speed [mi/h]	30
k_j	jam density [veh/mi lane]	300
d_0	Potential background traffic [veh/h]	600
ζ	Elastic demand parameter	30
L_b	Average travel distance of the background traffic [mi]	5
N_o	Number of on-street parking spots	250
VOT	Value of time for the background traffic [$/h]	10
L	Total downtown lane miles [mi]	250

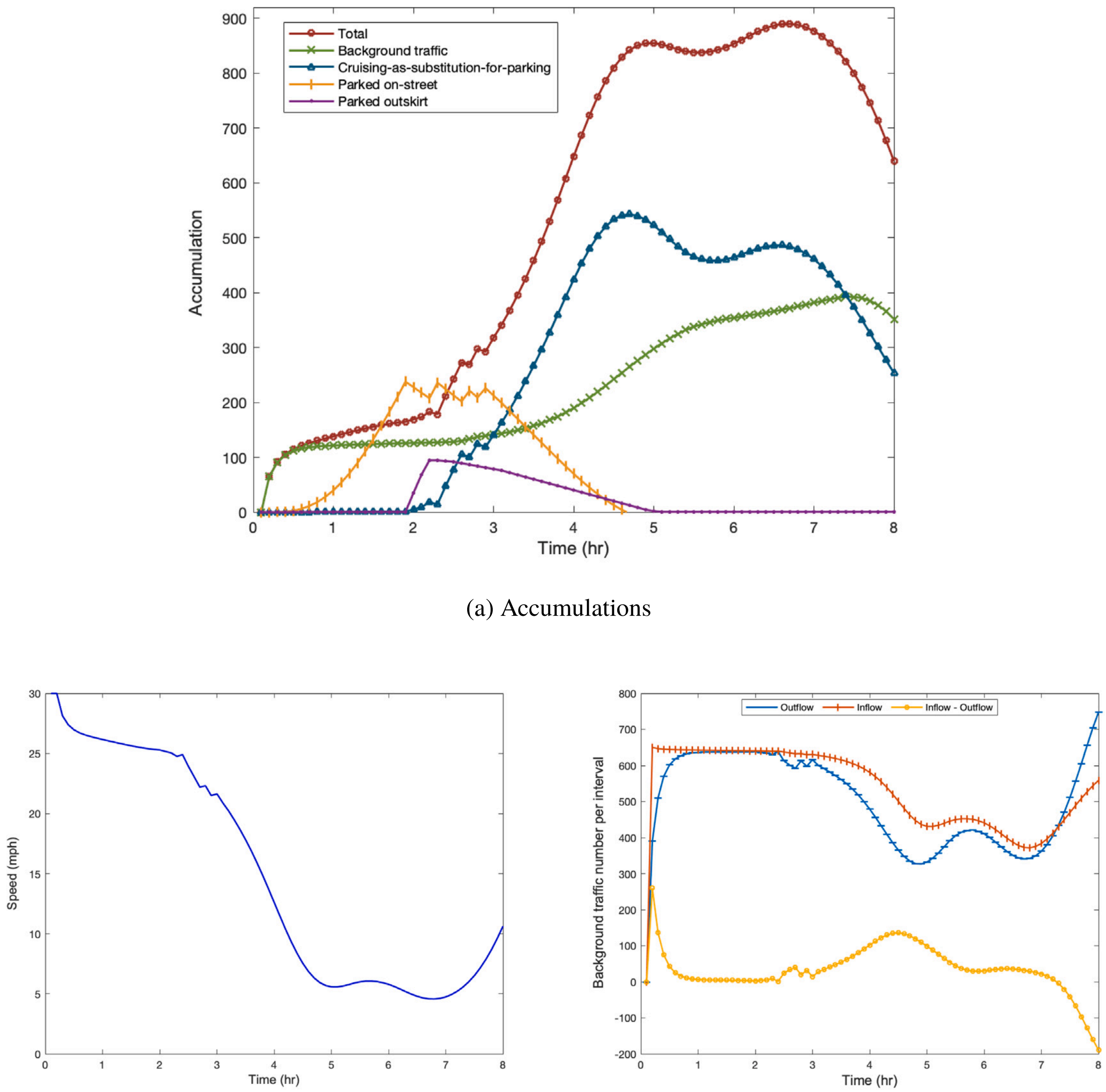
users' parking search time, thereby increasing traffic congestion. Another contributor to the congestion growth in downtown is the background traffic. Initially, with an empty network, we observe a surge in background traffic inflow, which gradually decreases with the increase in congestion (see Eq. (4)). At the same time, the outflow rate also decreases due to a prolonged time to traverse a given trip length in downtown. This yields a net positive background flow rate, shown in Fig. 3(c), which increases the background traffic accumulation and thus contributes to the congestion growth.

The increasing congestion results in a reduction in downtown speed, as shown in Fig. 3(b), which makes cruising-as-substitution more attractive, thereby increasing the number of cruisers and further worsening the traffic condition. Specifically, cruising-for-parking and the background traffic in the first two hours reduce downtown speed to 25 mph, which triggers some users' intention to cruise as a substitution for parking. The surge in the number of cruisers encourages more users who would choose either on-street or outside parking to become a cruiser, and this phenomenon drastically reduces downtown speed to be as low as 4 mph. The situation eventually improves thanks to the reduction in the inflow rate, as shown in Fig. 2. Notice that both on-street parking and outside parking become underutilized during the peak hour because of the lower cost of cruising-as-substitution-for-parking.

This numerical example highlights that cruising-as-substitution-for-parking can create an extremely congested regime. Without this parking option, cruising is self-restraint, as longer cruising time would discourage users from choosing on-street parking and thus preventing the congestion from getting even worse. However, with this option, cruising becomes self-reinforcing, i.e., worse congestion reduces the cost of cruising and further encourages more AVs to adopt cruising-as-substitution-for-parking. System managers need to find appropriate congestion mitigation strategies to break this vicious circle.

3.7. Sensitivity analysis

In this section, we conduct sensitivity analysis to further reveal the adverse impact of cruising-as-substitution-for-parking on downtown congestion accompanied by background traffic as another contributing factor. Our analysis considers four scenarios: Scenario I and II do not include cruising-as-substitution-for-parking with potential background traffic being 600 and 900 veh/h respectively; in comparison, Scenario III and IV have the same potential background traffic but consider cruising-as-substitution-for-parking.



(a) Accumulations

(b) Downtown speed

(c) Background traffic

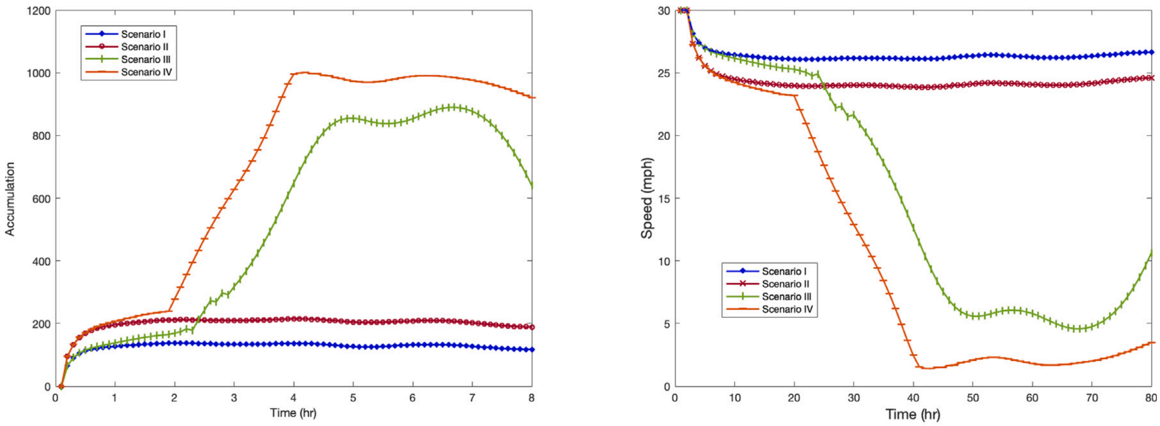
Fig. 3. Evolution of downtown traffic.

Results in Fig. 4(b) demonstrate that cruising-as-substitution-for-parking is substantially more destructive than a 50% increase in the potential background traffic in our simulation. As previously pointed out, cruising-as-substitution-for-parking is self-reinforcing. Worse congestion reduces the cost of cruising, thereby encouraging more AVs to adopt this option. This is evident in Fig. 4(a) that it leads to an increase in accumulation over time. In contrast, background traffic is self-restraint. More potential background traffic will cause worse congestion, which discourages vehicles to traverse through downtown and thus prevents congestion from becoming even worse.

This analysis further reveals the significance of the congestion problem that cruising-as-substitution-for-parking may create. The following sections discuss dynamic time-based tolling and on-street parking provision as two strategies for parking management and congestion mitigation.

4. Dynamic time-based tolling strategy

To mitigate congestion incurred by cruising-as-substitution-for-parking, Bahrami et al. (2021) suggested that time-based tolling, which charges AVs based on the time (rather than distance) they traverse in downtown, is particularly effective in discouraging AVs from driving the system toward gridlock for their personal cost saving. This section optimizes this tolling strategy in a dynamic setting with two parking options, cruising-as-substitution-for-parking and outside parking. Other parking options can be



(a) Accumulations (b) Downtown speed

Fig. 4. Performance sensitivity to cruising and background traffic volume.

easily incorporated in our optimization framework, but we choose not to model them in this section to highlight the impact of cruising-as-substitution-for-parking on downtown traffic congestion.

4.1. Model predictive control for dynamic tolling

Using a binomial logit model for the choice of two parking options, the system of ordinary differential equations in Eq. (2) can be simplified to be the following:

$$\mu_c(t,\alpha) = (\mu_c^l v(t) + \tau(t))\alpha \tag{5a}$$

$$\eta_c(t,\alpha) = \frac{\exp(-\theta\mu_c(t,\alpha))}{\exp(-\theta\mu_c(t,\alpha)) + \exp(-\theta\mu_p\alpha)} \tag{5b}$$

$$\lambda_c(t) = \lambda(t)\int_{\check{\alpha}}^{\hat{\alpha}} f(\alpha)\eta_c(t,\alpha)d\alpha \tag{5c}$$

$$\frac{dn_c(t)}{dt} = \lambda_c(t) - \int_{\min\{t,\check{\alpha}\}}^{\min\{t,\hat{\alpha}\}} \lambda(t-\alpha)f(\alpha)\eta_c(t-\alpha,\alpha)d\alpha \tag{5d}$$

$$\frac{dn_b(t)}{dt} = D(n(t),\tau(t)) - \frac{v(t)}{L_b}n_b(t) \tag{5e}$$

$$n(t) = n_c(t) + n_b(t) \tag{5f}$$

$$v(t) = M(n(t)) \tag{5g}$$

$$q(t) = \frac{v(t)}{L_b}n_b(t) + \int_{\min\{t,\check{\alpha}\}}^{\min\{t,\hat{\alpha}\}} \lambda(t-\alpha)f(\alpha)\eta_c(t-\alpha,\alpha)d\alpha \tag{5h}$$

$$\lambda_c(t), \lambda_b(t), n_c(t), n_b(t) \geq 0 \tag{5i}$$

Our goal here is to find an optimal dynamic time-based tolling scheme that maximizes a system performance measure. As an example, below, we devise a model predictive control (MPC) problem with an objective function of maximizing the total system throughput while controlling the toll fluctuation during the study period:

$$\max_{\tau} \int_0^T q(t)dt - w\int_0^T \left|\frac{\partial\tau(t)}{\partial t}\right| dt \tag{6a}$$

$$s.t.\text{(5a)–(5i)} \tag{6b}$$

$$\tau(t) \geq 0, t \in [0,T] \tag{6c}$$

$$n(0) = n_0 \tag{6d}$$

where w is a weighting factor and n_0 represents the initial traffic accumulation in the downtown. The first component in the objective function represents the accumulative system throughput, while the second shows the absolute value for toll fluctuations during the study period.

With a discretized ODE via finite difference, the above MPC problem can be converted into a nonlinear program (see the Appendix for the nonlinear program), which can be readily solved to prescribe the optimal tolling scheme. However, this program is only appropriate for offline implementation, as it requires the knowledge of AV inflow rates from $[0, T]$. For online implementation, a rolling horizon scheme can be adopted, where the arrival rates of AVs over the rolling horizon need to be predicted. This prediction task is critical as the control efficiency largely depends on the prediction accuracy. Nonetheless, this task is out of the scope of this paper.

Below we also present two policy benchmarks for the comparison purpose: (i) simple feedback control and (ii) a myopic solution to the above MPC without taking the toll fluctuation into account.

4.2. Feedback control scheme

Feedback-based control has been widely applied in various traffic management applications, such as ramp metering known as ALINEA (Papageorgiou et al., 1991), pricing of high-occupancy toll lanes (Yin and Lou, 2009; Jin et al., 2020) and cordon pricing (Zheng et al., 2012).

In feedback control, the toll rate at $(i+1)$th time interval, τ_{i+1}, depends on the toll rate and the downtown accumulation at the end of ith time interval, n_i. The toll increases when the downtown accumulation is beyond its optimal value and decreases when it is less. As such, this reactive method attempts to maintain the downtown accumulation near the optimal accumulation n^* to maximize the system throughput. Eq. (7) represents this relationship.

$$\tau_{i+1} = \tau_i + \beta(n_i - n^*) \tag{7}$$

where $n_i - n^*$ denotes the deviation of ith time interval from the optimal accumulation level, and β is the control disturbance regulator parameter. This parameter can be determined via manual fine-tuning. A large value for β corresponds to a more sensitive tolling control where the toll rate can highly fluctuate. See, e.g., Wang et al. (2020), for additional discussions on this controller.

The feedback control is simple yet effective, but it does not guarantee the maximization of the objective function of the MPC.

4.3. Myopic MPC pricing scheme

Notice that for the MPC problem (6), if we do not consider the toll fluctuation, the model can be decomposed with respect to each time step; as long as we maximize throughput at each step, we can solve the model. Maximizing throughput at each step turns out to be straightforward. Conceptually, given the current system state, we can estimate the available "capacity" at the next time interval, i.e., the number of vehicles allowed to enter the downtown to reach the optimal level of accumulation of n^*. If no capacity exists, the toll must force new arrivals to be a parker; if the available capacity is sufficient to accommodate all the new arrivals, the toll would be zero to allow all new arrivals to be a cruiser. Otherwise, the toll rate can be determined so that the available "capacity" is fully utilized to maintain the system's maximum throughput. In this case, the toll rate is the solution to the following equation (to avoid introducing additional notation, we present a continuous version, although its discrete counterpart is solved):

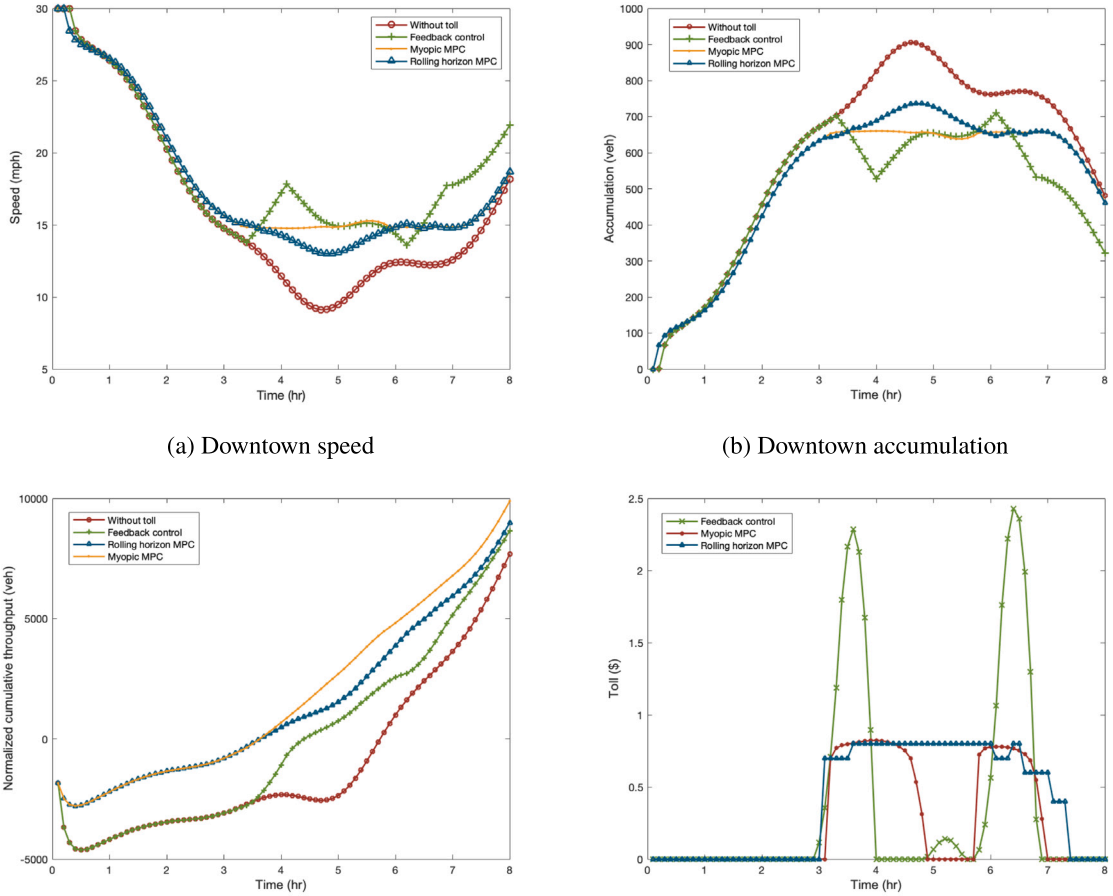
$$\begin{aligned} \lambda(t)\int_{\check{\alpha}}^{\hat{\alpha}} f(\alpha)\eta_c(t,\alpha)d\alpha - \int_{\min\{t,\check{\alpha}\}}^{\min\{t,\hat{\alpha}\}} \lambda(t-\alpha) f(\alpha)\eta_c(t-\alpha,\alpha)d\alpha \\ + D(n(t),\tau(t)) - \frac{v(t)}{L_b} n_b(t) = n^* - n(t) \end{aligned} \tag{8}$$

4.4. Numerical example for dynamic time-based tolling

In this section, we discuss the impacts of each tolling strategy on downtown traffic characteristics in the same numerical setting in Section 3.6. As shown in Fig. 5(a), without imposing toll, downtown speed drastically decreases to 9 mph as a result of the increased accumulation observable in Fig. 5(b). However, time-based tolling improves the overall system performance by increasing the cost of cruising and discouraging users from entering downtown. Fig. 5(c) compares the cumulative throughput before and after tolling. To make this comparison more observable, we define the normalized cumulative throughput for the system as follows (Cassidy, 1998):

$$Q(t) = \int_0^t (q(t) - c)dt,$$

where $Q(t)$ denotes the normalized cumulative throughput at time t, and c is a constant, set to be 9 after fine-tuning. As expected, all three tolling policies are effective in increasing the total system throughput, with the MPC method being the most effective. Compared to the rolling horizon implementation, the myopic MPC can improve the system throughput (also see hours 4–6 in Figs. 5(a) and 5(b)) since it solely focuses on the throughput maximization while ignoring the toll fluctuation (See Fig. 5(d)). In fact, the myopic MPC can be viewed as a special case of the rolling horizon scheme with the weighing factor $w = 0$. Here, we consider $w = 5$ as the weighing factor of the rolling horizon MPC after fine-tuning. In comparison to the feedback control with the fine-tuned feedback control parameter of $\beta = 15$, the two MPC strategies show less fluctuation and higher system throughput. One reason is that feedback control is reactive to the changes in the accumulation and therefore lags in toll adjustment whenever the accumulation changes drastically in a short time. Also, these two control methods pursue fundamentally different objectives. A feedback controller aims to guarantee stability around a predetermined set point, while MPC has the objective of maximizing the system performance. Note that the sudden surge in the toll rate using feedback controller, e.g., in the third hour in Fig. 5(d), affects users' parking choice in the following time steps by discouraging them from entering downtown and cruising. However, higher



(a) Downtown speed (b) Downtown accumulation

(c) Normalized cumulative throughput (d) Toll values

Fig. 5. Downtown characteristics after imposing different tolling strategies.

values of toll might result in the under-utilization of downtown, which is undesirable for throughput maximization (see the third hour in Fig. 5(b)).

From the system manager's point of view, the feedback control scheme is less beneficial since it might lead to the under-utilization of downtown and the resulting system performance is less predictable compared to the two MPC strategies (see the discussion on the cumulative throughput). In comparison between the rolling horizon and myopic MPC, the former considers users' preference of a fair less-fluctuating toll, while the latter maintains the system performance at its maximum at all times. As such, depending on their objective, managers might exploit either of the MPC tolling schemes to discourage users from entering downtown and cruise as a substitution for parking.

4.5. Sensitivity analysis

To examine how parking rate impacts the tolling schemes and better reflect cities with higher parking rate like Manhattan and downtown Chicago, we increase the parking rate μ_p from 1.5\$/h to 3\$/h.

Results in Fig. 6 demonstrate that, as expected, an increase in parking price yields higher toll rates compared to Fig. 5(d). Also, we observe one peak in the feedback-control toll, which reduces the demand for the background traffic and cruising during the 3rd – 5th h. As such, there is no need to impose toll in the following hours. Other observations from this sensitivity analysis are consistent.

5. Parking provision strategy

The potential of on-street parking provision on reducing cruising-for-parking has been well studied in the literature (e.g., Xu et al., 2017; Ye et al., 2020; Najmi et al., 2021). Similarly, on-street parking may attract cruisers to park and thus decrease cruising-as-substitution-for-parking. However, providing extra on-street parking reduces road capacity and possibly yields more congestion.

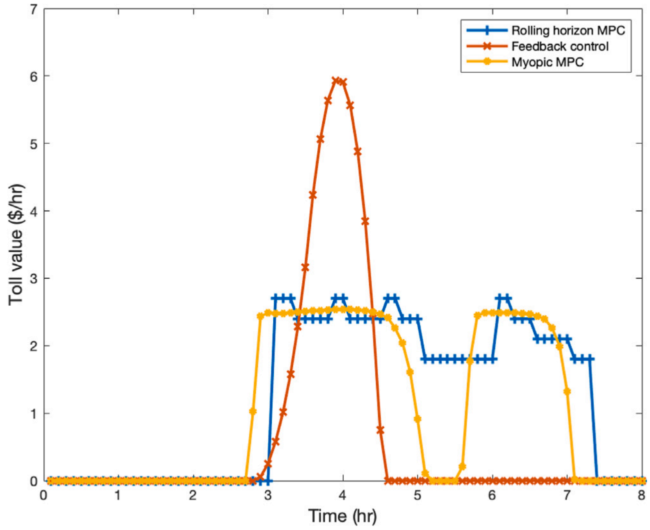


Fig. 6. Sensitivity of toll values to the parking rate.

The research question is therefore how to find appropriate balance to determine the optimal provision of on-street parking. Below we present a modeling framework to capture the trade-off to improve the overall system performance.

In modeling the effect of on-street parking provision on reducing the utilizable road area A_u, we follow Xu et al. (2017) by using a modified Greenshields speed-density function as follows:

$$v(t) = v_f[\frac{n_j - n(t)}{k_j(A_u - N_o A_o)}]$$

where A_o represents the area dedicated to one on-street parking spot.

We are now ready to present a modeling framework to optimize on-street parking provision together with its pricing. As an example, we consider an objective function of maximizing the total system throughput.

$$\max_{N_o,\mu_o} \int_0^T q(t)dt \tag{9a}$$

$$s.t.(2a)–(2q), (2s) \tag{9b}$$

$$v(t) = v_f[\frac{n_j - n(t)}{k_j(A_u - N_o A_o)}] \tag{9c}$$

$$\tau(t) \geq 0, t \in [0, T] \tag{9d}$$

$$n(0) = n_0 \tag{9e}$$

Similarly, via finite difference, the above MPC problem can be converted into a nonlinear program easy to solve in an offline fashion.

5.1. Numerical example for on-street parking provision

To highlight the trade-off in on-street parking provision, we consider parking pricing to be given (without optimizing it) in this example. The scheme is given as follows:

$$\mu_o(t) = \min(\mu_p(t) + \epsilon, \mu_c(t) - \epsilon) + \gamma e^{N_o - n_o(t) - n_s(t)}$$

where $\min(\mu_p(t) + \epsilon, \mu_c(t) - \epsilon)$ ensures outside parkers are not attracted to on-street parking while cruisers are encouraged to choose on-street parking. The second term,i.e. $\gamma e^{N_o - n_o(t) - n_s(t)}$, is a barrier function, which assures the feasibility of the model (see Section 3.5 for the feasibility discussion).

Fig. 7 demonstrates how the cumulative system throughput varies with the number of provided on-street parking slots. The aforementioned trade-off makes the throughput initially increase thanks to the reduction of cruising and then decrease because of the reduction of roadway capacity.

Numerically, Fig. 7 suggests that the provision of 500 parking spots maximizes the downtown cumulative throughput. We provide other traffic features during the study period in Fig. 8 under this parking provision.

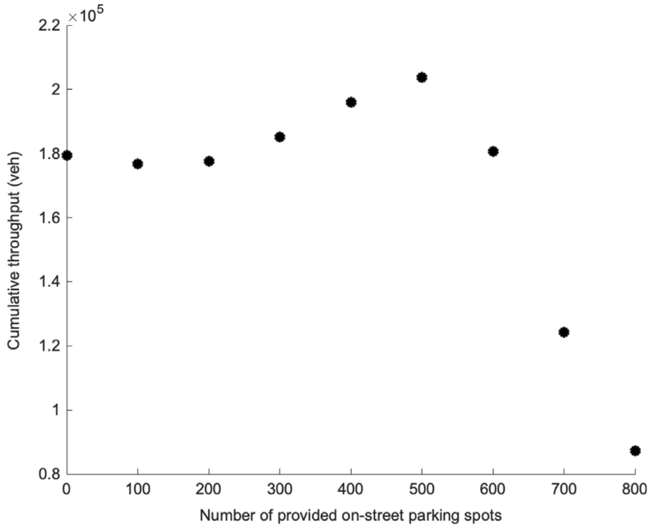
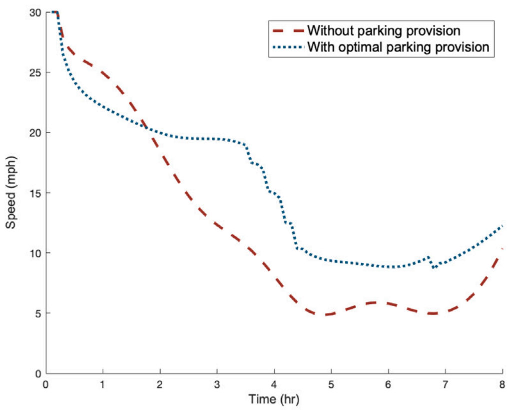
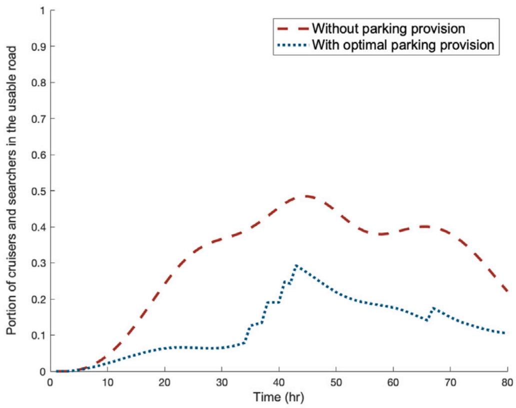


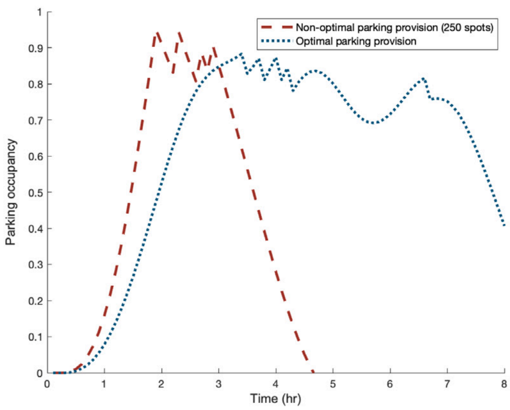
Fig. 7. Cumulative throughput with varying on-street parking provision.



(a) Downtown speed



(b) The portion of utilizable capacity used by cruisers and searchers



(c) On-street parking occupancy

Fig. 8. Downtown characteristics with the optimal parking provision.

Fig. 8(a) shows the improvement in the traffic speed during the study period resulting from providing an optimal number of parking spots under the given parking pricing scheme. Fig. 8(b) demonstrates the percentage of cruising vehicles (including cruising-for-parking and cruising-as-substitution-for-parking) in the utilizable area of downtown before and after the optimal on-street parking provision. Specifically, the number of cruising vehicles in the former are compared to n_j, while it is compared to $n_j - n_o$ in the latter. In comparison, the optimal parking provision decreases cruising in downtown since more vehicles choose to park on-street instead. For further comparison, we provide Fig. 8(c) and observe that parking occupancy remains high for a longer duration in the optimal scheme, especially between the third to the sixth hour with the maximum accumulation (minimum speed), compared to the non-optimal parking provision discussed in Section 3.6.

From a managerial point of view, on-street parking provision can improve the system performance up to a certain level constrained by the utilizable road area. It is also critical for managers to choose a proper time-based on-street parking pricing scheme to ensure those who would choose outside parking do not find downtown parking attractive and those who would choose to cruise switch to on-street parking. Note that this strategy might be incapable of breaking the vicious cycle of cruising-as-substitution-for-parking revealed in Section 3.6. Once cruising-as-substitution-for-parking becomes attractive, more AVs will be encouraged to choose this option and make it less expensive. So, if we reduce the price for on-street parking as an attempt to attract more users to this parking option, the associated cruising-for-parking reduces the cost of cruising-as-substitution-for-parking by decreasing the traffic speed in downtown and further encourage cruising.

6. Conclusion and discussion

This study proposed a dynamic framework for parking in a downtown area in the era of AVs. Given the distribution of users' activity time in downtown, we developed a system of ordinary differential equations to model AVs' choice between outside parking, on-street parking, and cruising-as-substitution-for-parking. After solving the model, we observed that in the absence of proper congestion management strategies, cruising-as-substitution-for-parking heavily contributes to creating a severely congested regime and even gridlock in downtown. This condition worsens as the system grows more congested since the cost of cruising decreases at lower speeds, attracting more users to choose this option. Also, an increase in the number of cruisers impacts other options' users. For example, increasing the number of cruisers elevates the search time for finding vacant on-street parking. Additionally, the significant speed decrease resulted from the increased number of cruisers traps the background traffic in downtown for a long time since they must drive a certain distance with a lower system speed.

We investigated dynamic time-based tolling and parking provision to manage the cruising-as-substitution-of-parking and break its vicious cycle. Our objective in the dynamic time-based tolling strategy was to maximize the system throughput while taking toll rate fluctuations into account. We modeled the optimization problem as an MPC and regulated the toll values in a rolling horizon fashion. As a result, downtown congestion resulted by cruising significantly improves. Also, outside parking becomes a more appealing option. For on-street parking provision, it can enhance the system performance by attracting cruisers to park and by decreasing the search time for on-street parkers. However, its provision may decrease the utilizable road capacity. We thus developed a model to capture this trade-off to determine an optimal provision as well as its pricing. Results of our analysis demonstrate the effectiveness of this management scheme for a time-based on-street parking pricing strategy. However, it reveals that on-street parking provision might be incapable of breaking the vicious cycle of cruising-as-substitution-for-parking and preventing it from creating severe congestion.

The analysis in this paper suffers a few limitations. In particular, we did not consider the ingress/egress time for AVs. Unlike the activity time of users, the ingress/egress time depends on the system congestion level. To consider a non-zero ingress or egress distance, it is necessary to integrate the trip-length-based approach in the bathtub model with our time-based framework. This integration not only leads to a significantly complex mathematical model but also to some extent mitigate the self-reinforcing nature of cruising-as-substitution-parking. Although this parking option continues to benefit from a low speed, the pronged ingress/egress time would discourage AVs from creating extreme congestion like gridlock. This study also did not consider the boundary queue that may cause model infeasibility. Ideas from the MFD literature can be borrowed to address this boundary queue limitation (e.g., Ni and Cassidy, 2020).

As a first step towards understanding and managing the impacts of AVs' parking on downtown congestion, this paper poses some interesting problems for future research. One is to extend this study to consider variable inflow rates of AVs to capture how control methods trigger mode and departure time shift for AVs, and then examine how these shifts impact downtown congestion. Another topic is to analyze a dynamic parking provision strategy via dynamic allocation of roadway spaces between traveling and parking lanes (Xu et al., 2017). The trade-off between the increase in parking provision and the decrease in road capacity is interesting and complicated to analyze in the setting of this paper. Lastly, it is also interesting to investigate comprehensive curb utilization strategies for managing AV parking in the era of shared mobility, as this paper focuses on addressing the parking needs from privately-owned AVs (Marsden et al., 2020).

CRediT authorship contribution statement

Tara Radvand: Methodology, Validation, Formal analysis, Investigation, Writing – original draft, Visualization, Review & editing. **Sina Bahrami:** Validation, Review & editing. **Yafeng Yin:** Conceptualization, Methodology, Review & editing. **Ken Laberteaux:** Conceptualization, Funding acquisition.

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Appendix

Discretized counterpart for equation set (2)

This section presents the system of equations to solve the discretized counterpart for the ODE presented in the equation sets (2). As a common pre-process to obtain a well-defined discretized version of function $\phi(.)$, we extrapolate the function as $\phi(t) = \max_{t' \in \mathbb{Z}} \left\{ t' | t' + s(t') \leq t \right\}$. Consequently, we obtain the following first-order discretized approximation of the system:

$$s(i) = \frac{d}{(1 - \frac{n_o(i-1)}{N_o}) \times v(i-1)} \tag{10a}$$

$$\mu_c(i,\alpha) = (\mu_c^l v(i-1) + \tau(i-1))\alpha \tag{10b}$$

$$\mu_o(i,\alpha) = (\mu_c^l v(i-1) + \tau(i-1)) \min(\alpha, S(i-1)) + \mu_o(i-1) \max(\alpha - S(i-1), 0) \tag{10c}$$

$$\eta_m(i,\alpha) = P_m(n_o(i-1) + n_s(i-1), n(i-1), \alpha) \quad m \in \{c, s, p\} \tag{10d}$$

$$\lambda_c(i) = \lambda(i-1) \sum_{\check{\alpha}}^{\hat{\alpha}} f(\alpha)\eta_c(i-1,\alpha) \tag{10e}$$

$$\lambda_s(i) = \lambda(i-1) \sum_{\check{\alpha}}^{\hat{\alpha}} f(\alpha)\eta_s(i-1,\alpha) \tag{10f}$$

$$n_c(i) = n_c(i-1) + \delta_t(\lambda_c(i-1) - \sum_{\min\{i-1,\check{\alpha}\}}^{\min\{i-1,\hat{\alpha}\}} \lambda(i-1-\alpha) f(\alpha) \eta_c(i-1-\alpha,\alpha)) \tag{10g}$$

$$\phi(i + s(i)) = i \tag{10h}$$

$$\begin{aligned} n_s(i) = {} & n_s(i-1) + \delta_t(\lambda_s(i-1) - \sum_{\max(i-1-\phi(i),\check{\alpha})}^{\hat{\alpha}} \lambda(\phi(i-1)) f(\alpha) \eta_s(\phi(i-1),\alpha) \\ & - \sum_{\check{\alpha}}^{\min(\hat{\alpha},\max(i-1-\phi(i),\check{\alpha}))} \lambda(i-1-\alpha)\eta_s(i-1-\alpha,\alpha)) \end{aligned} \tag{10i}$$

$$\begin{aligned} n_o(i) = {} & n_o(i-1) + \delta_t(\sum_{\max(i-1-\phi(i),\check{\alpha})}^{\hat{\alpha}} \lambda(\phi(i)) f(\alpha) \eta_s(\phi(i),\alpha) \\ & - \sum_{\max(i-1-\phi(i),\check{\alpha})}^{\min(i-1,\hat{\alpha})} \lambda(i-1-\alpha) f(\alpha) \eta_s(i-1-\alpha,\alpha)) \end{aligned} \tag{10j}$$

$$n_b(i) = n_b(i-1) + \delta_t(D(n(i-1), \tau(i-1)) - \frac{v(i-1)}{L_b} n_b(i-1)) \tag{10k}$$

$$n(i) = n_c(i) + n_s(i) + n_b(i) \tag{10l}$$

$$v(i) = M(n(i)) \tag{10m}$$

Discretized counterpart for the MPC

To transform the MPC problem (6) into a nonlinear program, we apply our discretization approach to the ODE (2). The nonlinear program is then formulated as follows:

$$\max_{\tau} \sum_i q(i)\delta t - w \sum_0^T \left| \tau_{i+1} - \tau_i \right| dt \tag{11a}$$

$$s.t. \mu_c(i,\alpha) = (\mu_c^l v(i-1) + \tau(i-1))\alpha \tag{11b}$$

$$\eta_c(i,\alpha) = \frac{\exp(-\theta\mu_c(i,\alpha))}{\exp(-\theta\mu_c(i,\alpha)) + \exp(-\theta\mu_p\alpha)} \tag{11c}$$

$$\lambda_c(i) = \lambda(i-1) \sum_{\check{\alpha}}^{\hat{\alpha}} f(\alpha)\eta_c(i-1,\alpha) \tag{11d}$$

$$n_c(i) = n_c(i-1) + \delta_t\left(\lambda_c(i-1) - \sum_{\min\{i-1,\check{\alpha}\}}^{\min\{i-1,\hat{\alpha}\}} \lambda(i-1-\alpha) f(\alpha) \eta_c(i-1-\alpha,\alpha)\right) \tag{11e}$$

$$n_b(i) = n_b(i-1) + \delta_t\left(D(n(i-1),\tau(i-1)) - \frac{v(i-1)}{L_b} n_b(i-1)\right) \tag{11f}$$

$$n(i) = n_c(i) + n_b(i) \tag{11g}$$

$$v(i) = M(n(i)) \tag{11h}$$

$$\tau(t) \geq 0, t \in [0, T] \tag{11i}$$

$$n(0) = n_0 \tag{11j}$$

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