

# Dynamic Modeling and Analysis of Multi-Product Flexible Production Line

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## Abstract

Mass customization has been a major challenge in the manufacturing sector. For this purpose, Flexible Manufacturing Systems (FMS) are usually designed to accommodate variations and manufacture different types of products in batches, but the products follow a single rigid path using a conveyor and in case of a failure on a single machine, the whole production line is affected. However, a multi-product dynamic production system can operate under an elongated failure on a single machine. A novel dynamic modeling method for a multi-product production line is developed to investigate dynamic properties of the system under random disruptions such as machine failures and market demand changes. Permanent production loss (PPL) and demand dissatisfaction for multi-production lines are formally defined as real-time performance metrics. A real-time analysis method is developed to evaluate the PPL and demand dissatisfaction. Numerical case studies are presented to validate the fidelity of the real-time multi-product production line model and the effectiveness of the real-time analysis method.

**Keywords:** Flexible manufacturing system, multi-products, dynamic modelling, demand dissatisfaction, permanent production loss

## 1. Introduction

Traditional manufacturing systems comprise a series of machines that performs a set of operations to produce the end product. Each part goes through all the machines in a sequence, resulting in a final product (Browne et al. 1984). Such lines are also called serial production lines. These lines are usually designed for the mass production of a single product where the parts usually move through a conveyor, but the growing and customized customer requirements need a flexible line that can accommodate the regular changes quickly (Yadav and Jayswal 2018).

Conventionally, manufacturing systems comprise sequential, ordered and task-oriented workstations. Production flows are defined and immutable. Thus, there is no insight into the production lines. Since the introduction of FMS and the emergence of industry 4.0, production lines are now comprised of flexible, value-adding units that easily deal with demand variations in terms of variety and volume (Oztemel and Gursev 2020, Singh et al. 2022).

FMS was introduced with its definition as ‘an integrated system that is capable of processing medium-sized batches of different product types’ (Browne et al. 1984). This definition is very generic, and the specific meaning of flexibility is unknown, as the term ‘flexible’ can be used to address higher product varieties, large production volumes, lower costs, or minimum setup times. However, it does clarify that an FMS can adopt changes (Bhatta, Huang, and Chang 2022, Waseem et al. 2021). With the increasing and varying customer demands, the FMS must be quickly ramped up. In addition, these systems need to be updated with the advent of new/improved technologies. However, higher flexibility may lead to higher complexity of FMS, making modeling such systems more challenging. Increasing production throughput for demand satisfaction has become a significant issue for manufacturing industries such as automotive, battery, and composite, etc.

Unlike a typical serial production line, which is usually designed for a single product type, and uses a conveyor for the parts’ handling, a multi-product production line is a complex system, using a mobile robot for the products’ handling among the buffers and machines. The performance is not only determined by the machines or buffers’ status but also depends on the availability of the mobile robot. Further, unlike serial lines, even an elongated downtime on the slowest machine may not result in a permanent production loss, as the multi-product line processes multiple products, following a unique sequence, and a failure on a single machine may only result in demand dissatisfaction for the corresponding product type passing through the disrupted machine.

This paper is devoted to the modeling and analysis of multi-product production system. The major contribution of this paper is twofold: 1) A dynamic mathematical system model is built for multi-product production lines with mobile robots constituting material handling, and a recursive algorithm is developed to quickly evaluate system real-time state. 2) Based on the developed model, an analytical method is proposed to efficiently identify real-time permanent production loss (PPL) attributed to each machine due to random disruption events and to evaluate demand dissatisfaction for each product type. This dynamic modeling and analysis framework for multi-product manufacturing systems provide theoretical tools for understanding system dynamic properties and facilitate real-time production control.

The rest of the paper is organized as follows: Section 2 provides an extensive literature review. Section 3 describes the system. In Section 4, a dynamic system model for a multi-product production system is established. Section 5 evaluates the system's dynamic performance. Numerical experiments are presented in Section 6. Conclusion and future work are addressed in Section 7.

## 2. Literature Review

A multi-product production system is a dynamic system, that is subject to unpredictable changes such as demand variations and machines' random disruptions. In literature, the multi-product systems are studied in the context of batch manufacturing, where each batch follows the same production path irrespective of the product type and the line is set up for each product type. For example (Jarrahi and Abdul-Kader 2015) evaluated the performance of a multi-product system using the queuing model with each product type as a batch. To enable flexibility, (Bavelos et al. 2021) developed a framework integrating the shop floor and robot workers. Robot perception functions and sensor data from the shop floor were used to analyze the integration level. Lei et al.

(Ren et al. 2021) established a mathematical model for lithium batteries to estimate their remaining useful life. Although the model was tested successfully, due to some assumptions, its applications cannot be applied to other product types. A predictive model was developed by (Long, Li, and Chen 2022) to compare the productivity of three different aircraft. This model works efficiently on the defined number of product types, but it does not guarantee to efficiently satisfy the product customization based on the users' demands. This challenge was addressed by (Mueller-Zhang, Antonino, and Kuhn 2021), whose main objective was to optimize the process plans and make sure that each customized demand is satisfied even if it is only single product demand. However, their model was based on batch manufacturing, where the setup time is critical. To satisfy varying market demands, (Huang, Chang, and Arinez 2020) used a distributed production scheduling for a multi-product system. However, it does not address the line performance under system failures. Park et al. (Park and Li 2019) studied a case of a motorcycle manufacturing plant, where a multi-product machining line is used. The production process is transformed into a two-stage Bernoulli model and Markov chain analysis is used for the throughput analysis. Although these studies consider a multi-product system, they follow a rigid single production path using a conveyor for all product types.

In general, dynamic systems have received significant attention in the literature (Qu et al. 2019). There are two major methods followed to address these systems: analytical and simulation methods. Analytical methods are more of a mathematical representation of a real system, which can be categorized into exact solutions and approximate solutions. Whereas simulation methods are based on experimentation and are mostly used to analyze complex systems, it is time-consuming, and the simulation-based results are difficult to interpret. For the analytical approaches, exact analytical results are available only for two-machine and one-buffer systems (Li

et al. 2009). These methods are approximated for complex systems as (Colledani and Gershwin 2013) developed a decomposition method for the performance evaluation of continuous flow lines with machines characterized by general Markovian fluid models and finite buffers. An aggregation method was proposed by (Li and Meerkov 2007) to analyze Markovian serial production lines. Following the work of (Li 2005), (Li and Meerkov 2007) developed a new method to evaluate the performance of complex systems with rework, parallel, scrap, and assembly operations. Similarly, (Zhang et al. 2013) further extended the aggregation method of (Li and Meerkov 2007) for the transient analysis of serial production lines. However, most of these analytical approaches focus on a single product type and address the system's steady-state behaviour to optimize long-term production schedule plans.

In modern manufacturing systems, the extensive use of sensors has made it possible to collect real-time information, which is used for data-driven modelling (Rossit and Tohmé 2022). These data-driven models use the plant floor information, e.g., buffer levels, and machine downtimes, for the performance evaluation of the system. A dynamic model was developed by (Ma et al. 2022) for scheduling a smart shop floor. The model analyzes production performance by using shop floor knowledge. Similarly, (Zheng et al. 2020) discussed a model for assembly lines to focus on process time minimization. The model was tested on data generated from Monte Carlo simulation. It also assumed machine random disruptions and uncertain cycle times to update the scheduling accordingly. Oscar et al. (Serradilla et al. 2022) facilitated the implementation of a predictive maintenance system by using domain knowledge with a dynamic model. Although these models optimize the system efficiency in general, they cannot be used to address a multiproduct system, as they consider the system's overall production rather than the product variations. Besides, the response variables also vary from model to model, which makes it difficult to implement a single

model in each environment. While considering the applications of big data, (Rossit, Tohmé, and Frutos 2019) developed a model for scheduling smart manufacturing systems. The model uses sensors' data to update the schedules accordingly. Another data-driven method was developed by (Zou, Chang, Arinez, and Xiao 2017) to find the impacts of machine downtime on the system output. Even though these methods serve to be very useful for the scheduling optimization or performance evaluation of typical manufacturing systems, to our knowledge, the literature lacks an analytical method to evaluate the dynamic performance of a multi-product system, where a mobile robot handles the products, and each product type follows a unique sequence rather than a single fixed production path.

Market demand satisfaction/dissatisfaction plays a critical role in the performance analysis of multi-product dynamic production systems. Market demand is addressed by (Kück and Freitag 2021). Considering the customized market requirements and variable demands, (Kuo et al. 2021) interlinked the supply chain members with the material resource management, which enabled both the manufacturers and distributors to observe the production plan and market demand. Because of the demand variations, the manufacturers try to forecast the market demand and produce in advance, but usually, the forecasting methods ignore the marketing factors. Therefore, (Kumar, Shankar, and Aljohani 2020) developed a data-driven framework based on forecasting with a focus on mix-marketing variables. Similarly, (Shao et al. 2018) developed a dynamic optimization model to analyze the performance of a gear manufacturing line. Although the model worked properly, it was designed for a specific product only, which may not be suitable for other product types. These models try to improve demand satisfaction, but they do not address its impacts on shop floor activities. Besides, mostly these models focus on single product types rather than different product types.

### 3. System Description

The system considers a multi-product flexible production line that consists of  $M$  machines and  $M - 1$  intermediate buffers as shown in Fig. 1. Machines are represented by rectangles and denoted as  $S_i, i = 1, 2, \dots, M$ . Buffers are represented by the circles and denoted as  $B_i, i = 2, 3, \dots, M$ . The source provides raw products to the system, while the sink receives finished products. The system uses a mobile robot for material handling among machines and buffers. As shown in Fig. 1, the different flowline styles between stations and buffers represent the material flows of different product types. For clarity, only three types of products are shown with each flowline representing individual product types  $l, l = 1, 2, 3$ .

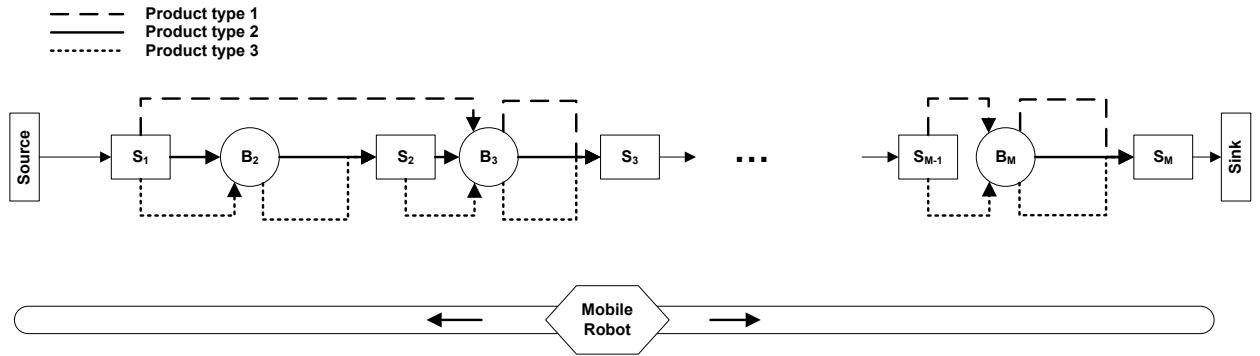


Figure 1: General multi-product flexible production line with a mobile robot

In this system, the following notations are used.

- (1)  $S_i, i = 1, 2, \dots, M$  represents the  $i^{th}$  machine
- (2)  $B_i, i = 2, 3, \dots, M$  represents the  $i^{th}$  buffer. With the abuse of notation, it is also denoted as the capacity of the  $i^{th}$  buffer

$$(3) \mathcal{P} = \begin{bmatrix} P_{11} & P_{12} & \dots & P_{1M} \\ P_{21} & P_{22} & \dots & P_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ P_{K1} & P_{K2} & \dots & P_{KM} \end{bmatrix}, \text{ where } P_{li}, l = 1, 2, \dots, K \text{ and } i = 1, 2, \dots, M \text{ denotes}$$

whether product type  $l$  is processed at the machine  $S_i$ .  $P_{li}$  has only two values: 1 or 0.

A value of 1 denotes the presence of operation by machine  $S_i$  on product type  $l$  and 0 denotes the absence of operation by the machine  $S_i$  on product type  $l$ .

- (4)  $p_{li}(t)$  denotes the quantity of product type  $l$  in buffer  $B_i$  at time  $t$
- (5)  $T_i, i = 1, 2, \dots, M$  represents the processing time of the  $i^{th}$  machine
- (6)  $T_{travel}$  is the travel time of a mobile robot between two adjacent machines, and  $T_{travel}^{ij} = |j - i| \times T_{travel}, i, j = 1, 2, \dots, M, and i \neq j$  represents the travelling time of mobile robot from machine  $S_i$  to machine  $S_j$ . Machine  $S_i$  denotes the current position of the robot while machine  $S_j$  is the final position of the robot
- (7)  $\vec{b}_i(t) = [p_{1i}(t) \dots p_{li}(t) \dots p_{Ki}(t)]'$  is the  $i^{th}$  buffer level at time  $t$ , representing the quantity of each product type  $l, l = 1, 2, \dots, K$ .
- (8)  $\vec{e}_i = (j, t_i, d_i)$ , denotes the  $i^{th}$  disruption event that machine  $S_j$  is down at time  $t_i$  for a duration of  $d_i$ .
- (9)  $\mathbf{E} = [\vec{e}_1, \vec{e}_2, \dots, \vec{e}_n]$ , represents a sequence of disruption events
- (10)  $MTBF_i$  and  $MTTR_i$  represent the mean time between failure and mean time to repair of machine  $S_i$ , respectively.
- (11)  $S_{M^*}$  denotes the slowest machine of the production line, while  $T_{M^*} + T_{travel}^{iM^*}$  represents the cycle time of the slowest machine.
- (12) A machine  $S_i$  is critical if it processes all product types of the system. For example, if a machine  $S_i$  is critical then  $[P_{1i} \ P_{2i} \ \dots \ P_{Ki}]' = [1 \ 1 \ \dots \ 1]'$ . The  $i^{th}$  critical machine is denoted as  $S_{ic}$ , whereas  $T_{ic} + T_{travel}^{j,ic}$  represents the cycle time of the  $i^{th}$  critical machine.

(13)  $S_{sc}$  represents the slowest critical machine of the production line, while  $T_{sc} + T_{travel}^{j,sc}$  represents the cycle time of the slowest critical machine.

(14)  $D_l(t)$  is the market demand of product type  $l$  at time  $t$

(15)  $N_l(t)$  represents the net loss of product type  $l$  at time  $t$

(16)  $Q_l(t)$  represents a production loss threshold for product type  $l$  during time  $t$

The following assumptions are adopted in this system:

(1) The term ‘part’ and ‘product’ in this study are used interchangeably

(2) The system is capable of processing  $l$  types of products. Each product type follows a unique sequence of operations, for example,  $S_1 \rightarrow S_2 \rightarrow S_4$  and  $S_1 \rightarrow S_3 \rightarrow S_4$  are two different product types

(3) Each part must pass through the first and last machine. In other words, machines  $S_1$  and  $S_M$  are always critical. The sequence of operations required on each part must be in the forward direction.

(4) Each machine must process at least one type of product in the system

(5) The buffers have a limited capacity, denoted as  $B_i, i = 2, 3, \dots, M$

(6) Immediate upstream buffer has only those product types which are to be processed by the consecutive next machine

(7) Following the unique sequence of each product type, parts produced by machine  $S_i$  will be delivered to the upstream buffer of its next-level operating station by the mobile robot

(8) Each machine  $S_i$  operates at a rated speed of  $\frac{1}{T_i + T_{travel}^{ji}}$

(9) Machines follow the exponential reliability models. The failure rate for machine  $S_i$  is  $\lambda_i$ , and the repair rate for machine  $S_i$  is  $\mu_i$

- (10) The part loading and unloading time is assumed to be included in the travelling time of the mobile robot
- (11) First machine is never starved, and the last machine is never blocked
- (12) A machine is said to be blocked if it is operational and its downstream buffer is full. Whereas a machine is said to be starved if it is operational and its upstream buffer is empty

#### 4. Dynamic System Model

The major difference of this kind of multi-product dynamic system from that of a traditional line/serial production system is that it does not follow a single mainstream flow but has multiple interlaced tributaries. Besides, it uses a mobile robot for the material handling among machines and buffers, which is another difference from that of a typical serial production line, using conveyors for the material handling. The material flow in this system is not only constrained by the interaction between machines and buffers but also dependent on the mobile robot's movement. This system can be modelled as a dynamic system, where each buffer's level is the system state, and external disturbances come from random disruption events such as machine failures. In addition, machines may wait for mobile robots to load/unload parts, which is dependent on the scheduling policy for mobile robots. This study only considers the system modelling and analysis and leaves the control problem of mobile robots to be future work. Therefore, the control input is treated as a given determined policy. Based on the prior discussion, our dynamic system can be presented by the following state-space equation

$$\dot{\mathbf{b}}(t) = \mathbf{F}(\mathbf{b}(t), \mathbf{U}(t), \mathbf{W}(t)) \quad (1)$$

$$\mathbf{Y}(t) = \mathbf{H}(\mathbf{b}(t)) \quad (2)$$

In the context of this multi-product system, the parameters are defined as

$\mathbf{b}(t) = [\vec{b}_2(t), \vec{b}_3(t), \dots, \vec{b}_M(t)]'$  represents the buffer levels at time  $t$ . It is a vector of all product types and  $\vec{b}_i(t) = [p_{1i}(t) \dots p_{li}(t) \dots p_{Ki}(t)]'$

$\mathbf{W}(t) = [w_1(t), w_2(t), \dots, w_M(t)]'$  represents the disturbances at time  $t$ , where  $w_j(t)$  describes whether  $S_j$  suffers from a disruption at time  $t$ . If  $\exists \vec{e}_k \in \mathbf{E} . s.t. \vec{e}_k = (i, t_k, d_k)$  and  $t \in [t_k, t_k + d_k]$ , then,  $w_i(t) = 1$ , otherwise,  $w_i(t) = 0$ . The authors define  $\theta_i(t)$  as the status of machine  $S_i$  at time  $t$  i.e.,  $\theta_i(t) = 1 - w_i(t)$ . A machine  $S_i$  is up at time  $t$  when  $w_i(t) = 0$ , and down when  $w_i(t) = 1$ ;

$\mathbf{Y}(t) = [\vec{Y}_1(t), \vec{Y}_2(t), \dots, \vec{Y}_M(t)]'$  denotes the system output at time  $t$ , where  $\vec{Y}_i(t) = [p_{1i}(t) \dots p_{li}(t) \dots p_{Ki}(t)]', i = 1, \dots, M$ , and  $l = 1, 2, \dots, K$ , denotes the production count of machine  $S_i$  for all product types up to time  $t$ ;

$\mathbf{F}(\cdot) = [f_1(\cdot), f_2(\cdot), \dots, f_M(\cdot)]'$ , where  $f_j(\cdot)$  denotes the dynamic function for machine  $S_j$ ;  
 $\mathbf{H}(\cdot) = [H_1(\cdot), H_2(\cdot), \dots, H_M(\cdot)]'$ , where  $H_j(\cdot)$  denotes the observation function for machine  $S_j$ ;

$\mathbf{U}(t)$  is the control input, which is the mobile robot's availability to load/unload a product based on a certain control and scheduling policy. Since the control problem is not considered in this paper and is treated as a given, which in this case is 1, the state space equations can be simplified as:

$$\dot{\mathbf{b}}(t) = \mathbf{F}(\mathbf{b}(t), \mathbf{W}(t)) \quad (3)$$

$$\mathbf{Y}(t) = \mathbf{H}(\mathbf{b}(t)) \quad (4)$$

#### 4.1. Dynamic System Derivation

Using the conservation of flow, the accumulated production count within a time  $[0, t]$  between any two machines  $S_i$  and  $S_j$ ,  $\forall i, j \in 1, 2, \dots, M$ , for product type  $l$ , where  $l = 1, 2, \dots, K$  satisfy the below equation:

$$\vec{Y}_i(t) - \vec{Y}_j(t) = \vec{\tau}_{ij}^l(t) = [\tau_{ij}^1(t) \dots \tau_{ij}^l(t) \dots \tau_{ij}^K(t)]' =$$

$$\begin{cases} \sum_{n=i+1}^j [p_{1n}(t) \dots p_{ln}(t) \dots p_{Kn}(t)]' - \sum_{n=i+1}^j [p_{1n}(0) \dots p_{ln}(0) \dots p_{Kn}(0)]', & i < j \\ 0 & i = j \\ \sum_{n=j+1}^i [p_{1n}(0) \dots p_{ln}(0) \dots p_{Kn}(0)]' - \sum_{n=j+1}^i [p_{1n}(t) \dots p_{ln}(t) \dots p_{Kn}(t)]', & i > j \end{cases} \quad (5)$$

Denote  $\tau_{ij}(t) = \tau_{ij}^1(t) + \dots + \tau_{ij}^l(t) + \dots + \tau_{ij}^K(t)$  to be the total production count including all product types produced between  $S_i$  and  $S_j$  up to time  $t$ , note that  $\tau_{ij}(t)$  has an upper bound and follows the condition that all buffers between machine  $S_i$  and  $S_j$  for all product types are empty (for  $i > j$ ) or full (for  $i < j$ ). This boundary is denoted as  $\beta_{ij}$  and can be derived as

$$\beta_{ij} = \begin{cases} \sum_{n=i+1}^j B_n - \sum_{n=i+1}^j [p_{1n}(0) + \dots + p_{ln}(0) + \dots + p_{Kn}(0)], & i < j \\ 0 & i = j \\ \sum_{n=j+1}^i [p_{1n}(0) + \dots + p_{ln}(0) + \dots + p_{Kn}(0)], & i > j \end{cases} \quad (6)$$

where  $B_n$  is buffer capacity of buffer  $n$ .

Therefore,  $\tau_{ij}(t) \leq \beta_{ij}$  always holds. If  $\tau_{ij}(t) < \beta_{ij}$ , machine  $S_i$  is not starved or blocked by  $S_j$ , and it operates at its own rated speed. If  $\tau_{ij}(t) = \beta_{ij}$ , the processing speed of machine  $S_i$  will be constrained by machine  $S_j$ .

Let  $\xi(t) = [\xi_{ij}(t)]_{M \times M}$  be a matrix used to indicate the interactions among machines  $S_i$  and  $S_j$  at time  $t$  as:

$$\xi_{ij}(t) = \begin{cases} 1, & \text{if } \tau_{ij}(t) = \beta_{ij}, i \neq j \\ \infty, & \text{otherwise} \end{cases} \quad (7)$$

where  $\xi_{ij}(t)$  indicates the starvation or blockage of machine  $S_i$  by  $S_j$ . If machine  $S_i$  is constrained by machine  $S_j$ , then machine  $S_i$  must operate at the operating speed of machine  $S_j$

$$v_i(t) = \min \left\{ \frac{\xi_{ij}(t) \theta_j(t)}{T_j + T_{travel}^{ij}}, \frac{\theta_i(t)}{T_i + T_{travel}^{ji}} \right\} \quad (8)$$

where  $v_i(t)$  is the operating speed of machine  $S_i$  at time  $t$ .  $v_i(t)$  can be extended to consider the interactions of machine  $S_i$  with all other machines of the production system.

$$v_i(t) = \min \left\{ \frac{\frac{\xi_{i1}(t)\theta_1(t)}{T_1+T_{travel}^{j1}},}{\frac{\xi_{i2}(t)\theta_2(t)}{T_2+T_{travel}^{j2}}, \dots, \frac{\frac{\theta_i(t)}{T_i+T_{travel}^{ji}},}{\frac{\xi_{iM}(t)\theta_M(t)}{T_M+T_{travel}^{jM}}, \dots} \right\} \quad (9)$$

The change rate of  $\vec{b}_i(t)$  is the speed difference between  $S_i$  and  $S_{i-1}$ .

$$\begin{aligned} \dot{\vec{b}}_i(t) &= \left\{ [v_i(t)][P_{1i} \dots P_{li} \dots P_{Ki}]' - [v_{i-1}(t)][P_{1(i-1)} \dots P_{l(i-1)} \dots P_{K(i-1)}]'\right\} \odot [P_{1i} \dots P_{li} \dots P_{Ki}]' \\ &= F_i(\mathbf{b}(t), \mathbf{W}(t)) \end{aligned} \quad (10)$$

where  $\odot$  represents the Hadamard product, and  $P_{li}, i = 1, 2, \dots, M, l = 1, 2, \dots, K$  denotes whether machine  $S_i$  processes product type  $l$ .

The system output is the production count of the last machine  $S_M$  at time  $t$  and it can be written as follows:

$$\mathbf{Y}(t) = \vec{Y}_M(t) = [p_{1M}(t) \dots p_{lM}(t) \dots p_{KM}(t)] = H_i(\mathbf{b}(t)) \quad (11)$$

Hence, the dynamic function  $\mathbf{F}(*)$  and observation function  $\mathbf{H}(*)$  are derived for our production line. This model can be used to obtain the system states and other important variables at any time if the sequence of operations required for each product type,  $\vec{P}_{li}$  and machine random failures  $\mathbf{W}(t)$  are known.

## 5. System Dynamic Performance Evaluation

A real production line constantly faces random disruption events, which can significantly affect the system's efficiency and throughput. For a typical production line with a single product type, a

random disruption event might stop the processing at one machine and thus block the upstream machines or starve the downstream machines. However, due to the presence of finite buffers and machines' variable cycle times, the permanent production loss (PPL) at the end of the line after a certain time is not guaranteed (Ou et al. 2017). In our previous study, an ideal clean case is defined, which represents a virtual scenario when there are no random disruptions and there is always a robot available to load/unload a machine. Then the PPL is defined as the difference between the output of the ideal clean case and the real output of a production line (Ou et al. 2017). This PPL is fundamentally different from the steady-state metric of throughput where long-term throughput is the same for each machine.

In this paper, multi-product production lines are considered. The dynamic performance of such lines is not only affected by random disruptions but also depends on the customer demands for various products. Therefore, two metrics are defined to evaluate real-time performance: PPL and demand dissatisfaction. For a critical machine  $S_{ic}$ , since all product types need to go through this machine, the authors want to evaluate PPL caused by random disruptions on this machine. For other machines such as  $S_i$ , a disruption on machine  $S_i$  will only affect the production of product type  $l$  passing through machine  $S_i$ . Since there is no objective reference for product type  $l$ , the demand dissatisfaction will be used as the evaluation metric.

### **5.1. Permanent Production Loss Evaluation**

PPL evaluation depends on the system performance under failures. The system status is measured by the Opportunity window that directly relates to the resilience to the disruption events (Zou, Chang, Arinez, and Xiao 2017). Zou et al. also defined the opportunity window and PPL for a single-product serial production line. Using the similar concept, PPL evaluation methods will be

developed for the multi-product system. To make this paper self-sufficient, a summary of the basic concepts is provided without detailed proof.

Opportunity window of a machine  $S_j$ , denoted as  $OW_j(T_d)$ , is the longest possible downtime on machine  $S_j$  at time  $T_d$  such that the end-of-line machine  $S_M$  does not result in PPL. It can be defined as:

$$OW_j(T_d) = \sup\{d \geq 0: s. t. \exists T^*(d),$$

$$\int_0^T s_M(t)dt = \int_0^T \tilde{s}_M(t; \vec{e})dt, \forall T \geq T^*(d)\} \quad (12)$$

where  $\int_0^T \tilde{s}_M(t; \vec{e})dt$  and  $\int_0^T s_M(t)dt$  represents the production count of the last machine  $S_M$  with and without disruption event  $\vec{e} = (M, T_d, d)$  respectively.

Our previous study (Zou, Chang, Arinez, and Xiao 2017) defined the opportunity window of a machine  $S_j$  along a serial production line with finite buffers as the time it takes for the buffers between machines  $S_j$  and the slowest machine  $S_{M^*}$  to become empty ( $j < M^*$ ) or full ( $j > M^*$ ) when  $S_j$  is forced down.

Besides, the authors have proved that there will be a permanent production loss only if the duration of disruption event exceeds the opportunity window. Such a permanent production loss due to an elongated disruption is also permanent to other machines of the line (Zou, Chang, Arinez, Xiao, et al. 2017). In case of a disruption event  $\vec{e}_i = (j, t_i, d_i)$ , which is greater than its corresponding opportunity window  $OW_j(t)$ , then for any machine  $S_j$ , there exists  $T^* \geq t + d$ , which is also dependent on the relative position of machine  $S_j$  with respect to the slowest machine  $S_{M^*}$ , such that:

$$\int_0^T S_j(t')dt' - \int_0^T S_j(t', \vec{e})dt' = \frac{(d - OW_j(t))}{T_{M^*}}, \quad \forall T \geq T^* \quad (13)$$

PPL is a key measure to analyze the production system's performance. Any stoppage of the slowest machine  $S_{M^*}$  contributes to the PPL (Zou, Chang, Arinez, Xiao, et al. 2017). To extend this concept to a multi-product production line, a disruption on the slowest machine  $S_{M^*}$  may not necessarily result in PPL. Therefore, the permanent production loss for this system can be defined as follows.

*Remark 1:* For a production line with a mobile robot subject to random failures, if the slowest critical machine  $S_{sc}$  stops for a duration of  $D_{sc}(T)$  during a time  $[0, T]$ , the permanent production loss can be defined as

$$PPL(T) = \frac{D_{sc}(T)}{T_{sc} + T_{travel}^{j,sc}} \quad (14)$$

It is proved by (Li et al. 2014) that any downtime on the slowest machine  $S_{M^*}$  adds to permanent production loss, which is given as

$$PPL(T) = \frac{D}{T_{M^*}}$$

where  $D$  represents the length of the duration for which the slowest machine is down and  $T_{M^*}$  represents its cycle time.

For a multi-product system, a stoppage on only the slowest critical machine  $S_{sc}$  will result in permanent production loss, as it processes all product types of the system. Also, a part can only be finished if it is loaded, processed, and unloaded, so the cycle time of the slowest critical machine  $S_{sc}$  equals  $T_{sc} + T_{travel}^{j,sc}$ , where the loading and unloading times are included in the travel time of the mobile robot. Therefore, permanent production loss of a multi-product dynamic system is

$$PPL(T) = \frac{D_{sc}(T)}{T_{sc} + T_{travel}^{j,sc}}$$

### **5.2. Permanent Production Loss Attribution**

For a deterministic scenario, where the disruptions  $\mathbf{W}(t)$  are known, the total PPL can be attributed to an individual machine and can identify the impact of each machine's downtime on the system.

For the system with multiple products, the state  $\mathbf{b}(t)$ , which is the buffer level of each buffer  $\vec{b}_i$  at time  $t$ , can be measured by using Eq. (9)-(11). For a multi-product system subject to known disruption events  $\vec{e}_i = (j, t_i, d_i)$  within time  $[0, T]$ , we have  $Y_j(t) = Y_j(t_i), \forall t \in [t_i, t_i + d_i]$ , since machine  $S_j$  stops from  $t_i$ . It is clear from equation (14) that every unit of PPL can be related to the downtime on the slowest critical machine  $S_{sc}$ . The PPL attribution is discussed in three cases.

### 5.2.1. Case 1

Only one disruption event occurs. A disruption event  $\vec{e}_i = (j, t_i, d_i)$  does not overlap with other disruption events within  $[t_a, t_b] \subseteq [t_i, t_i + d_i]$ , the production loss due to  $\vec{e}_i$  during this interval is

$$PPL_{\vec{e}_i}[t_a, t_b] = \max\{0, ((t_b - t_a)/(T_{sc} + T_{travel}^{j,sc})) - (Y_j(t_a) - Y_{sc}(t_a) + \beta_{j,sc})\} \quad (15)$$

where  $PPL_{\vec{e}_i}[t_a, t_b]$  represents the production loss due to  $\vec{e}_i$  within  $[t_a, t_b]$  under deterministic scenario.

Eq. (15) shows that the PPL will start to accumulate after the buffers between machine  $S_j$  and the critical slowest machine  $S_{sc}$  become full ( $j > sc$ ) or empty ( $j < sc$ ).

### 5.2.2. Case 2

A disruption event  $\vec{e}_1 = (j_1, t_1, d_1)$  overlaps with another disruption event  $\vec{e}_2 = (j_2, t_2, d_2)$  within  $[t_a, t_b]$ , where  $t_a \geq \max(t_1, t_2)$  and  $t_b \leq \min(t_1 + d_1, t_2 + d_2)$ . In such a case, the permanent production loss in  $[t_a, t_b]$  can be attributed to either event based on any reasonable principles. The permanent production loss is attributed to the disruption event whose corresponding machine has a smaller opportunity window. In case of equal opportunity windows, the permanent production

loss is equally attributed to the involved disruption events. Using the similar concept used by (Zou, Chang, Arinez, Xiao, et al. 2017), the production losses attributed to the overlapping disruption events during the time  $[t_a, t_b]$  can be calculated based on equation (15) as follows.

$$PPL_{\vec{e}_1}[t_a, t_b] = \begin{cases} PPL_{\vec{e}_1}[t_a, t_b], & \text{if } Y_{j1}(t_a) + \beta_{j1,sc} < Y_{j2}(t_a) + \beta_{j2,sc} \\ \frac{PPL_{\vec{e}_1}[t_a, t_b]}{2}, & \text{if } Y_{j1}(t_a) + \beta_{j1,sc} = Y_{j2}(t_a) + \beta_{j2,sc} \\ 0, & \text{if } Y_{j1}(t_a) + \beta_{j1,sc} > Y_{j2}(t_a) + \beta_{j2,sc} \end{cases} \quad (16)$$

$$PPL_{\vec{e}_2}[t_a, t_b] = \begin{cases} PPL_{\vec{e}_2}[t_a, t_b], & \text{if } Y_{j2}(t_a) + \beta_{j2,sc} < Y_{j1}(t_a) + \beta_{j1,sc} \\ \frac{PPL_{\vec{e}_2}[t_a, t_b]}{2}, & \text{if } Y_{j2}(t_a) + \beta_{j2,sc} = Y_{j1}(t_a) + \beta_{j1,sc} \\ 0, & \text{if } Y_{j2}(t_a) + \beta_{j2,sc} > Y_{j1}(t_a) + \beta_{j1,sc} \end{cases} \quad (17)$$

where  $Y_j(t_a) - Y_{sc}(t_a) + \beta_{j,sc}$  is the opportunity window of machine  $S_j$ . OW, as discussed in section 5.1, is the time it takes for the buffers between machine  $S_j$  and  $S_{sc}$  to become empty ( $j < sc$ ) or full ( $j > sc$ ). Therefore, in case of disruption events on two machines  $S_{j1}$  and  $S_{j2}$ , the OW decides the attribution of production loss.

### 5.2.3. Case 3

Multiple disruption events  $\vec{E} = (\vec{e}_1, \vec{e}_2 \dots, \vec{e}_n)$  occurs during the time period  $[t_a, t_b]$ . In this case, the production loss needs to be further attributed to each machine  $S_i$  and disruption event  $\vec{e}_i$ . Based on the discussion in case 2 above, the production loss can be attributed to the disruption event  $\vec{e}_i = (j_i, t_i, d_i)$ ,  $1 \leq i \leq n$  during the time  $[t_a, t_b]$  as follows

$$PPL_{\vec{e}_i}[t_a, t_b] = PPL[t_a, t_b]/\rho \quad (18)$$

where  $\rho$  denotes the number of all the corresponding events that are equally contributing to the whole production loss of the line during a time  $[t_a, t_b]$ .

The production loss can be further attributed to individual machines. For a sequence of disruption events  $\vec{e}_{j,1}, \dots, \vec{e}_{j,n}$  on machine  $S_j$  during a time  $[0, T]$ , the PPL caused by machine  $S_j$  within  $[0, T]$  can be measured as:

$$PPL_j[0, T] = \sum_{q=1}^n PPL_{\vec{e}_{j,q}} \quad (19)$$

### 5.3. Demand Dissatisfaction

Demand  $D_l(t)$  is defined as the customer's order for a product type  $l$  at a time  $t$ . Demand is a major factor in analyzing the performance of a production line and its satisfaction/dissatisfaction decides the line's capability. A demand  $D_l(t)$  is satisfied if the production line produces the required quantity during a time  $[0, t]$ , otherwise, it is dissatisfied. In a real-time production environment, it is very challenging to satisfy market demand because the production line faces random disruption events, and the demand is not consistent. There is significant literature available on the optimization of demand satisfaction and production lines' throughput (Turki, Sauvey, and Rezg 2018, Rachih, Mhada, and Chiheb 2022). However, the scope of this paper excludes the optimization part and focuses on its analysis only.

For a multi-product production line, the system faces permanent production loss only if the disruption is on a critical machine  $S_{lc}$ . If a machine  $S_i$  other than the critical one faces an elongated failure exceeding the opportunity window, the system may dissatisfy the demand for product type  $l$ , which is passing through the disrupted machine  $S_i$ .

Demand dissatisfaction is considered as an equivalent permanent production loss in this paper and for each product type  $l$ . The authors assume the demand  $D_l(t)$  to be equal to the production quantity of each product type  $l$  under an ideal scenario at time  $t$ , i.e.,  $D_l(t) = Y_M^l(t)$ . Due to the

stochastic nature of the system, the actual production is always less than the ideal production. This difference in the production quantities under different scenarios is called net loss, represented as  $N_l(t)$ , then

$$N_l(t) = Y_M^l(t) - Y_M^l(t, \vec{e}) \quad (20)$$

where  $Y_M^l(t)$  represents the end-of-line machine  $S_M$  production count for product type  $l$  under ideal conditions and  $Y_M^l(t, \vec{e})$  represents the real production of end-of-line machine  $S_M$  for product type  $l$  under disruption events  $\vec{e}$ .

In addition,  $Q_l(t)$  is defined as a production loss threshold for product type  $l$  during time  $t$ .  $Q_l$  is a qualitative parameter and is defined based on the demand history of the respective product type  $l$ . Therefore, the demand satisfaction/dissatisfaction can be defined as:

$$\begin{aligned} N_l(t) \leq Q_l(t) & \quad \text{Demand is satisfied} \\ N_l(t) > Q_l(t) & \quad \text{Demand is dissatisfied} \end{aligned}$$

A demand  $D_l(t)$  for a product type  $l$  during the time  $[0, t]$  is said to be satisfied if the net loss of product type  $l$ , i.e.,  $N_l(t)$  does not exceed the production loss threshold  $Q_l(t)$  during the time  $[0, t]$ , otherwise, there is a demand dissatisfaction for the product type  $l$ .

## 6. Case Study

In this section, a case study is presented to numerically validate the model. First, the fidelity of the proposed multi-product dynamic mathematical model is validated for which the proposed model is compared with a discrete event simulation model. Second, the authors verify the proposed methods for the permanent production loss evaluation and attribution and demand dissatisfaction.

To demonstrate the aforementioned goals, extensive research is carried out under deterministic scenarios. For the validation reliability, 100 different production lines are used for the evaluation of the model fidelity, which consists of practical industries like automotive assembly lines, battery, and semi-conductor lines etc.

For demonstration purpose, the authors use a segment of a mobile robot-assisted multi-product production line consisting of 5 machines, 4 intermediate buffers and one mobile robot. A schematic diagram of the line is given in below Fig. 2. The production line processes 3 types of products, as shown in Fig. 2. The parameters used in this study are given in Tables 1, 2, and 3.

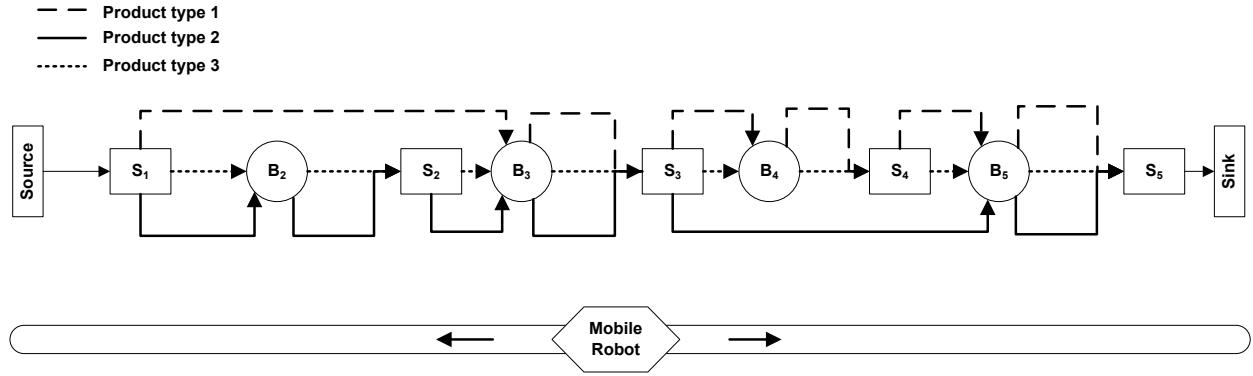


Figure 2: 5-Machines, 4-buffers line with a mobile robot for 3 types of products

Table 1: Machines' operation on each product type,  $P_{li}$

		Machine	1	2	3	4	5
Product							
1			1	0	1	1	1
2			1	1	1	0	1
3			1	1	1	1	1

Table 2: Initial buffers and buffer capacity

Initial Buffers	Buffer	Buffer	Buffer	Buffer
	1	2	3	4
	0	0	0	0

<b>Buffer's Capacity</b>				
<b>Product type 1</b>	15	25	35	25
<b>Product type 2</b>	15	25	35	25
<b>Product type 3</b>	15	25	40	15

*Table 3: Machines' cycle time, MTBF and MTTR*

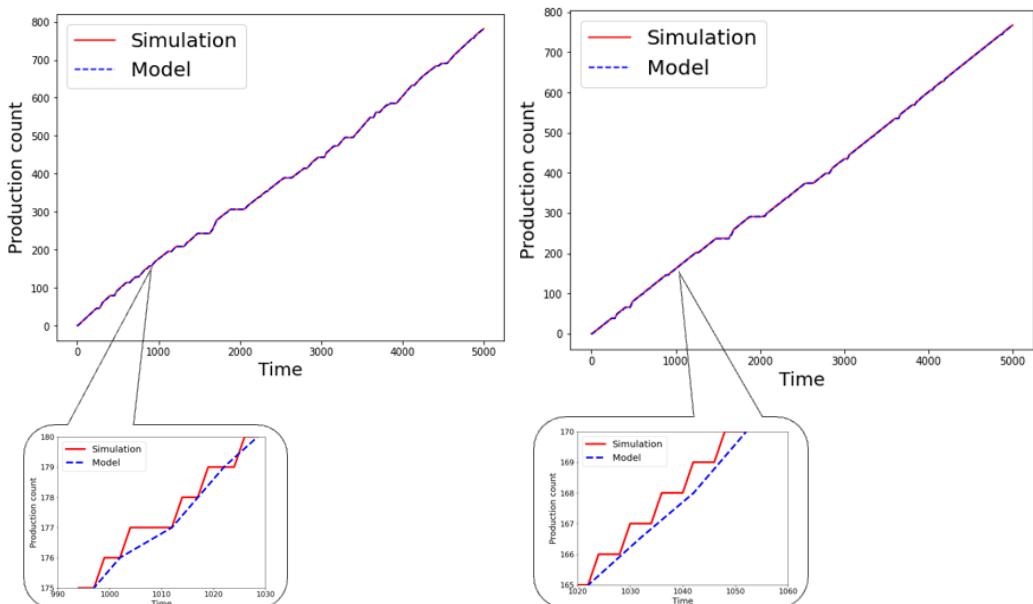
<b>Machine</b>	<b>M1</b>	<b>M2</b>	<b>M3</b>	<b>M4</b>	<b>M5</b>
<b>Cycle Time, <math>T_i + T_{travel}^{ji}</math></b>	2	4	6	3	5
<b>MTBF</b>	300	400	250	350	300
<b>MTTR</b>	40	50	30	40	35

### **6.1. Fidelity of the Proposed Model**

For validation purpose, the production count from the proposed dynamic math model and simulation built-in Simul8 are used. The time duration is 5000 minutes. The production count for each product type by the mathematical model and simulation model is compared. From the numerical study for the 100 lines, the average error between the mathematical and simulation model is observed to be 1.7%. Since about 5% error is considered acceptable (Kumar and Varaiya 2015, Li and Meerkov 2008), the model is claimed to be accurate.

As an illustration, the results for the production line in Fig. 2 from both the proposed model and simulation are studied. The data is divided into three groups: input data, randomly generated data, and model output data. Input data comprises machines' cycle time, given in Tables 3 and 4, operations' sequence for individual product type, given in Table 1, initial buffers, and buffers' capacity, given in Table 2, machines' MTTR and MTBF, given in Table 3, and production loss

threshold for each product type, given in Table 6. Randomly generated data includes the machines' random disruption events. Using exponential distribution, these disruptions are generated and fed into the model for the system's analysis. Based on the input and random data, the model results in the system state at each time step. This data includes the machines' production and buffer levels at a certain time. Table 5 presents the production data for the machine  $S_5$  with and without disruptions. Figure 3 and 4 represents the detailed production data and its comparison based on the proposed model. The production data is generated based on the developed mathematical model. The dynamic mathematical model assumes a continuous flow model, while the simulation is based on a discrete model. Therefore, the discrete flow and continuous flow properties can be observed from the result. However, the systems' variation follows the same pattern for both models. From the results, both the continuous flow model and discrete flow model highly resemble each other. For the demonstration purpose, the production counts of the fifth machine  $S_5$  is shown in Fig.3. It can be observed that the result of the simulation model (red solid line) and dynamic model (blue dashed line) overlaps with each other. It validates the fidelity of the model and thus it can be used for the evaluation of production performance for a multi-product production line.



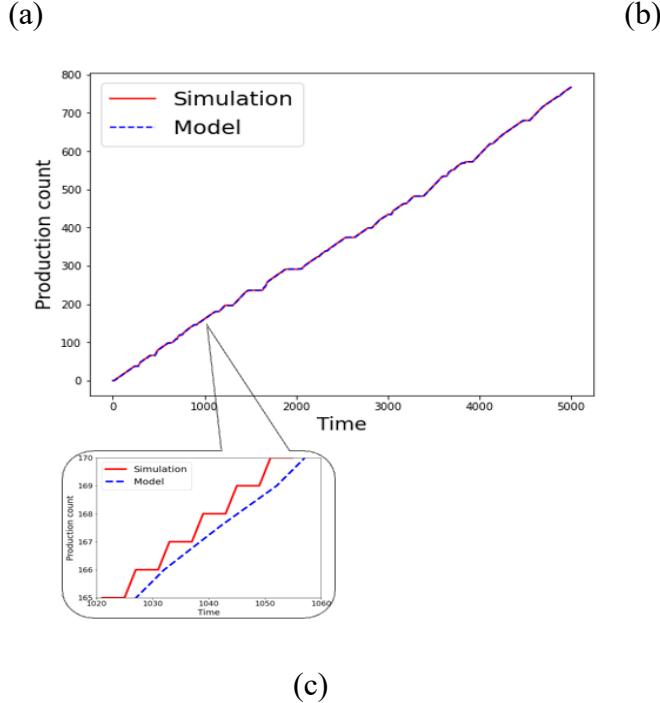


Figure 3: (a) Model validation for product type 1, (b) product type 2, (c) product type 3,

## 6.2. Validation of Performance Evaluation

### 6.2.1. PPL evaluation

To demonstrate the PPL evaluation, let the critical slowest machine be  $S_3$  that processes all product types  $l$  with the highest cycle time in Table 3,  $T_3 + T_{travel}^{j3} = 6$ . First, using the suggested method for production loss identification and attribution as Eqs. (18) and (19), the PPL attribution to each event and machine is obtained. For example, a production loss of 36 units for product type 2 and 3 is attributed to the fifth machine  $S_5$ .

Next, the production improvement is evaluated in a controlled simulation, where downtime is eliminated on each machine one at a time and then its impact on the overall production output is calculated. Both the PPL attribution and production improvement are represented in Fig. 4. It also depicts individual product type  $l$  loss attribution. The PPL attribution and overall production improvement bars in Fig. 4 represent the average attribution and improvement for all product types.

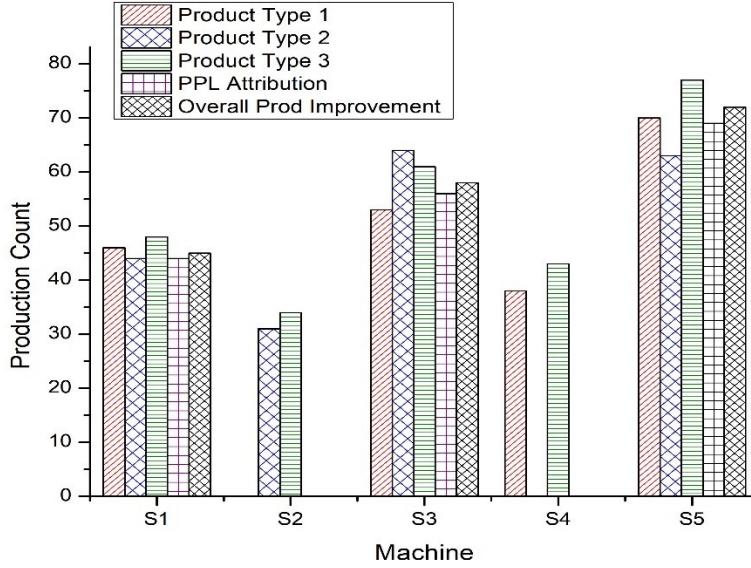


Figure 4: Overall production improvement and PPL distribution

The permanent production loss is only attributed to the critical machines  $S_1$ ,  $S_3$ , and  $S_5$ . A disruption on these machines affects the production of all product types and thus results in a permanent production loss if the downtime exceeds the corresponding opportunity window. Whereas disruption on the non-critical machines  $S_2$  and  $S_4$  can only affect the production of product types 2 and 3, or product types 1 and 3 respectively. If the downtime on these non-critical machines exceeds the opportunity window, there may be a demand dissatisfaction for the corresponding product types.

Results in Fig.4 show that the PPL is in close agreement with the overall productivity improvement in the corresponding controlled simulation experiments for critical machines  $S_1$ ,  $S_3$ , and  $S_5$ . This means that the PPL attribution can accurately identify and rank each machine's responsibility for system-level production loss.

#### 6.2.2. Demand Dissatisfaction

For demand dissatisfaction analysis, the same production line is studied but with different cycle times,  $T_i + T_{travel}^{ji}$ . New cycle times are given in Table 4. This scenario addresses the demand dissatisfaction for each product type. From Tables 1 and 4, the slowest machine  $S_2$  is not the critical machine and thus, under elongated disruptions, there will only be a demand dissatisfaction.

The model is run for the same period of 5000 minutes. Using the same controlled experimentation, the production count of the fifth machine  $S_5$  with and without downtime for each product type can be observed from Table 5.

*Table 4: Machine cycle time for scenario 2*

Machine	M1	M2	M3	M4	M5
Cycle time, $T_i + T_{travel}^{ji}$	5	6	3	4	2

*Table 5: Production count of  $S_5$  with and without disruptions for each product type*

Production Count	With disruption	Without disruption, $D_l$	Net Loss, $N_l$
<b>Product 1</b>	797	832	35
<b>Product 2</b>	773	841	68
<b>Product 3</b>	765	837	72

Product types 2 and 3 are more affected by downtime as compared to type 1. The main reason is that product types 2 and 3 are passing through the slowest machine  $S_2$ . In case of disruption on  $S_2$ , product types 2 and 3 may be affected and result in demand dissatisfaction if there is an elongated downtime. However, there is no PPL, as the other machines are still processing product

type 1. For the fifth machine  $S_5$ , it can be observed that there is a significant difference between production counts for each product type. In addition, the production loss threshold for each product type at time  $t$ ,  $Q_l(t)$  is given in Table 6, which is defined based on the ideal capability of the production line and demand history for each product type during a time  $[0, t]$ .

*Table 6: Production loss threshold for each product type,  $l$*

Product, $l$	1	2	3
Threshold, $Q_l$	40	40	40

A production loss for product type  $l$  under the threshold  $Q_l$  can happen due to any random disruptions on the production line. Only product type 1 satisfies the demand, while product type 2 and 3 exceeds  $Q_l$  and thus dissatisfies the market demand.

## 7. Conclusion and Future Work

In this paper, a multi-product production line, handled by a mobile robot is modelled. Using the concept of a serial production line, a dynamic model is extended to include multiple products. A product follows a unique sequence of operations along the production line. The paper addresses the performance of the whole production line in case of a disruption event on an individual machine. As compared to a serial production line, a single-machine disruption for an elongated time may not result in a permanent production loss. However, there might be a demand dissatisfaction for the product type passing through the disrupted machine. A mobile robot is used for the products' handling among machines and buffers, based on a given determined policy. The control problem of mobile robot will be addressed in future work.

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No potential conflict of interest was reported by the author(s).

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