

THEORY

All Heavy Tetraquarks: The Dynamical Diquark Model and Other Approaches

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Abstract—The 2020 announcement by LHCb of a narrow structure $X(6900)$ in the di- J/ψ spectrum—a potential $c\bar{c}c\bar{c}$ state—has opened a new era in hadronic spectroscopy. In this talk, we briefly survey theory works preceding this event, examine key features of the observed spectrum, and then discuss how subsequent theory studies (including with the author’s own dynamical diquark model) have interpreted these features. We conclude with proposals for experiments capable of distinguishing competing interpretations.

Keywords: LHCb, tetraquarks, pentaquark, dynamical diquark model

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1. INTRODUCTION

This talk nominally discusses all-heavy tetraquarks in general, but in fact it focuses almost exclusively upon developments related to the interesting structures reported by LHCb in the di- J/ψ ($c\bar{c}c\bar{c}$) channel [1], particularly a putative resonance called $X(6900)$. Early indications for the observation of possible di- $\Upsilon(1S)$ ($b\bar{b}b\bar{b}$) states above 18 GeV [2, 3] were not supported by subsequent searches [4, 5]. Furthermore, no structures have yet been reported in other all-heavy channels, such as $b\bar{b}c\bar{c}$, $b\bar{c}b\bar{c}$, etc. Nevertheless, quark-flavor universality suggests that any approach successful for $c\bar{c}c\bar{c}$ should be applicable to all of these sectors.

All-heavy tetraquark production at hadron colliders is believed to be proceed primarily through gluon-gluon fusion, with both single- and double-parton scattering being important for nonresonant $Q\bar{Q}Q'\bar{Q}'$ production [6]. Heavy-ion collisions have also been identified as important potential production sources for such states [7, 8].

$X(6900)$ joins a large family (now over 50!) of heavy-quark tetraquark (and pentaquark) candidates discovered in the past two decades [9], including hidden-charm, hidden-bottom, and open-charm candidates. Of these, the Particle Data Group [10] identifies 15 as fully established. Moreover, a naive count through the various flavor sectors, upon employing any widely discussed substructure (hadronic

molecules, diquark compounds, etc.), suggests that over 100 more such exotic states await discovery.

2. FEATURES OF THE LHCb DATA

The LHCb data contains not only the $X(6900)$, but also several other features of interest. Figure 1 adapts a portion of Fig. 3a of [1], focusing upon the region of interest that starts at the di- J/ψ threshold $\simeq 6200$ MeV and extends slightly beyond the $c\bar{c}c\bar{c}$ open-flavor threshold at $2m_{\Xi_{cc}} \simeq 7240$ MeV. One immediately notes not only the prominent peak near 6900 MeV, but also a broad excess above background around 6400–6500 MeV, a sharp dip near 6750 MeV, and a rapid transition from a dip to an excess just above 7200 MeV.

$X(6900)$ is the only obvious peak, and although it lies about 700 MeV above $2m_{J/\psi}$, it is likely not wider ($\lesssim 200$ MeV) than the ρ , and potentially is much narrower. In fact, a $c\bar{c}c\bar{c}$ state behaving as a “traditional” di-hadron molecule would bind through exchanging conventional charmonium states; since the expected molecular binding energies are expected to be $\leq O(10)$ MeV, the exchanged charmonium that binds $X(6900)$ would lie far off mass shell. Typical $c\bar{c}$ mean charge radii from potential models are about 0.35 fm for $1S$ states, and about twice that for $1P$ and $2S$ states. One concludes that J/ψ exchange, in particular, would be very short-ranged, implying a molecular state in which all quarks occupy the same volume. Alternately, one may consider non-traditional di-hadron molecules in which the J/ψ

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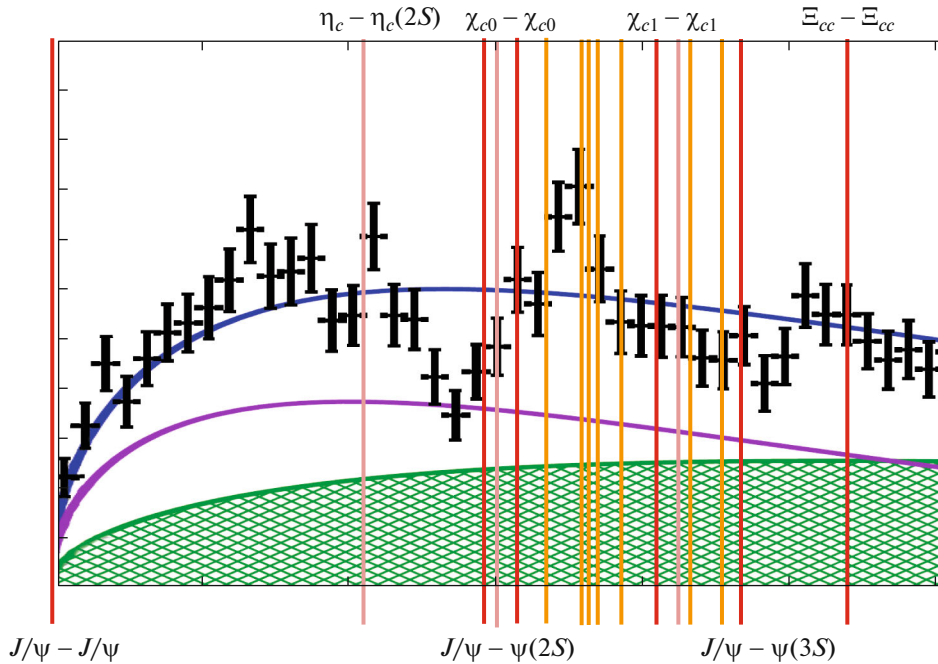


Fig. 1. Di- J/ψ production data, adapted from [1]. Ticks on the horizontal axis mark every 200 MeV in the range 6200–7400 MeV. Vertical lines denote thresholds for di-hadron states containing $c\bar{c}c\bar{c}$, red and orange indicating pairs of hadrons with the same $c\bar{c}$ spin structure as J/ψ . The curves indicate fits to components of the nonresonant contribution carried out in [1].

pair are bound through Pomeron (multi-gluon) exchanges [11] or through the exchange of soft gluons that hadronize into light-meson (π , K) exchange quanta [12].

Of particular note are the allowed J^{PC} quantum numbers for an identical J/ψ (1^{--}) pair. In the S wave one can have 0^{++} or 2^{++} , while the allowed P waves are 0^{-+} , 1^{-+} , and 2^{-+} . These J^{PC} values refer both to di- J/ψ resonances and the background. Alternately, if one considers the di- J/ψ system only in terms of identical cc and $\bar{c}\bar{c}$ fermion pairs, then in their respective S waves, only (color- $\bar{\mathbf{3}}$, spin-1) and (color- $\mathbf{6}$, spin-0) combinations are allowed.

3. THEORETICAL STUDIES

3.1. Prior to the LHCb Observation

Remarkably, the first discussion of a di- J/ψ bound resonance [13] followed the 1974 discovery of charmonium [14, 15] by only one year. Afterwards, however, only 1 further paper (by the same author as [13]) followed in the 1970s, 5 in the 1980s, 1 in the 1990s, and 3 in the 2000s. The reason for this dearth of effort appears to stem from the realization that di- J/ψ states were likely to lie above the di- J/ψ threshold. As one of the early works stated, “All the states are unbounded and consequently rather uninteresting.”

2010 saw first physics from the LHC, and it was very quickly appreciated that the collider would produce a great deal of $gg \rightarrow J/\psi J/\psi$, which would be straightforward for LHCb to reconstruct via $J/\psi \rightarrow \mu^+ \mu^-$. One theory collaboration analyzed this effect and predicted (using a diquark model) the spectrum of $c\bar{c}c\bar{c}$ states that could be produced [16, 17]. While one might have expected a subsequent flurry of work using other theoretical approaches, no further papers studying the $c\bar{c}c\bar{c}$ system appeared until 2016, with [18] more papers (perhaps anticipating the trove of LHC Run 2 data) appearing from 2016 to the middle of 2020.

3.2. Subsequent to the LHCb Observation

The LHCb observations were announced in a talk on June 16, 2020 [18], and the collaboration’s preprint was posted on June 30, 2020 [1]. In that 2-week period alone, 8 more theory papers appeared. Since then (as of the time of writing), 58 additional papers discussing either the production or spectroscopy of these states have been produced. The length limit imposed upon this document precludes even the mere listing of these references, let alone discussing all of them in any detail. Instead, we examine the features in the LHCb data, with an eye to their interpretation as proposed in a select few papers.

A wide variety of theoretical approaches have been employed to understand the data, including: string

junction models; quark models with chromomagnetic interactions; quark potential models; chiral quark models; diquark models; effective theories with light-meson exchanges; threshold effects with coupled charmonium channels; threshold effects plus compact tetraquarks; QCD sum rules; lattice simulations; Regge phenomenology including Pomeron exchange; holography; spin-chain (Bethe Ansatz) algebraic methods; and the Bethe-Salpeter approach. $X(6900)$ has even been proposed to be a Higgs-like boson [19].

Are there any solid conclusions or points of consensus uniting these analyses? First, not many authors dispute that $X(6900)$ seems to be a genuine resonance, even when including the effects due to the presence of multiple thresholds (see Fig. 1) that might explain other $c\bar{c}c\bar{c}$ structure, as considered in, e.g., [20–22]. However, other authors (e.g., [23]) suggest that even $X(6900)$ itself might be generated by the $\chi_{c0}-\chi_{c1}$ threshold.

Second, virtually all models (back to the very first papers [13]) predict ground-state ($1S$) $c\bar{c}c\bar{c}$ resonances to lie much lower than $X(6900)$, typically from 6000–6400 MeV. So then, is $X(6900)$ a $P = -$, $1P$ state (e.g., [24]), or a $P = +$, $2S$ state (e.g., [25, 26])?

The broad structure around 6400–6500 MeV lies at the upper limit of where models predict $1S$ ground states to occur (e.g., [27, 28]). In LHCb’s Model I [1], for example, the structure is treated as a superposition of (at least) two resonances, irrespective of quantum numbers. Indeed, what is meant by “ $1S$ ”, which suggests a 2-body description? Since “traditional” molecules are problematic for $c\bar{c}c\bar{c}$, and no good thresholds lie in the 6400–6500 MeV range (Fig. 1), then the diquark $(cc)\bar{\mathbf{3}}(\bar{c}\bar{c})\mathbf{3}$ structure seems most natural for identifying possible quantum numbers. But not everyone agrees! Noting that $\mathbf{6}-\bar{\mathbf{6}}$ attraction is stronger than $\bar{\mathbf{3}}-\mathbf{3}$ (despite quark repulsion in a $\mathbf{6}$ diquark), [29] finds that the ground states mix both configurations, while the $\bar{\mathbf{3}}-\mathbf{3}$ configuration dominates excited states.

The dip around 6750 MeV suggests destructive interference with $X(6900)$. LHCb’s Model II [1] posits interference between the broad 6400–6500 MeV structure and a second resonance. A $\chi_{c0}-\chi_{c0}$ threshold effect provides an alternate explanation [26]. Furthermore, if $X(6900)$ is $2S$ ($P = +$), then $1P$ states ($P = -$) are expected near 6750 MeV [25], although $P = \pm$ configurations do not interfere with each other; again, measuring parities is crucial.

The structure near 7200 MeV may be associated with open-flavor decays of $c\bar{c}c\bar{c}$, which first appear at the $\Xi_{cc}-\Xi_{cc}[(ccu)(\bar{c}\bar{c}\bar{u})]$ threshold 7242.4(1.0) MeV. Likely, no narrow $c\bar{c}c\bar{c}$ structures occur above this

point. This is the point in diquark models where the color flux tube breaks [25], or in holographic models where new string junctions become possible [24].

How many states are expected in a diquark model? If both $\bar{\mathbf{3}}$ and $\mathbf{6}$ diquarks are allowed, one finds a lot [30]: 17 with $C = +$ and $J \leq 2$ are predicted to lie below the $\Xi_{cc}-\Xi_{cc}$ threshold. If one adopts a minimal ansatz of allowing only $\bar{\mathbf{3}}$ diquarks, then only about half as many states occur. Further taking quark spin couplings to be large only within diquarks (a defining property of the dynamical diquark model [25, 31]), then all S -wave multiplets consist of 3 degenerate states: 0^{++} , 1^{+-} , and 2^{++} , and all P -wave multiplets consist of 7 states (3 with $C = +$) that follow an equal-spacing mass rule if tensor couplings are neglected.

4. SOME PARTING PROPOSALS

Most obviously, we desperately need J^P information in order to disentangle the di- J/ψ spectrum. An excellent suggestion (e.g., [32, 33]) is to look at the $J/\psi-\psi(2S)$ spectrum, even though its threshold is 700 MeV higher, and $\psi(2S)$ production is lower than that of J/ψ . Note in this regard that BESIII sees distinct Y exotics decaying to J/ψ or to $\psi(2S)$.

The gg production of di- J/ψ is $C = +$; is there much ggg ($C = -$) production? If so, one could find the 1^{+-} resonance via $J/\psi-\eta_c$, although η_c is harder to reconstruct (but note that $\text{B.R.}(\eta_c \rightarrow p\bar{p}) = 1.45 \times 10^{-3}$). Alternately, $J/\psi-\chi_{cJ}$ also has $C = -$, but less phase space. And don’t forget about $c\bar{c}b\bar{b}$ and $b\bar{b}b\bar{b}$ production! $c\bar{c}b\bar{b}$ (via $J/\psi-\Upsilon$) should produce many more resonances, by evading the identical-fermion constraint. Such experiments provide important tests of quark-flavor universality.

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CONFLICT OF INTEREST

The author declares that he has no conflicts of interest.

REFERENCES

1. R. Aaij et al. (LHCb Collab.), “Observation of structure in the J/ψ -pair mass spectrum,” *Sci. Bull.* **65**, 1983 (2020); arXiv: 2006.16957 [hep-ex].
2. S. Durgut (CMS Collab.), <http://meetings.aps.org/Meeting/APR18/Session/U09.6>.

3. L. Bland et al. (ANDY Collab.), “Observation of Feynman scaling violations and evidence for a new resonance at RHIC,” arXiv: 1909.03124 [nucl-ex] (2019).
4. R. Aaij et al. (LHCb Collab.), “Search for beautiful tetraquarks in the $\Upsilon(1S)\mu^+\mu^-$ invariant-mass spectrum,” J. High Energy Phys. **2018**, 086 (2018); arXiv: 1806.09707 [hep-ex].
5. A. Sirunyan et al. (CMS Collab.), “Measurement of the $\Upsilon(1S)$ pair production cross section and search for resonances decaying to $\Upsilon(1S)\mu^+\mu^-$ in proton-proton collisions at $\sqrt{s} = 13$ TeV,” Phys. Lett. B **808**, 135578 (2020); arXiv: 2002.06393 [hep-ex].
6. R. Maciula, W. Schäfer, and A. Szczurek, “On the mechanism of $T_{4c}(6900)$ tetraquark production” Phys. Lett. B **812**, 136010 (2021); arXiv: 2009.02100 [hep-ph].
7. J. Zhao, S. Shi, and P. Zhuang, “Fully-heavy tetraquarks in a strongly interacting medium,” Phys. Rev. D **102**, 114001 (2020); arXiv: 2009.10319 [hep-ph].
8. A. Esposito, C. Manzarì, A. Pilloni, and A. Polosa, “Hunting for tetraquarks in ultra-peripheral heavy ion collisions,” Phys. Rev. D **104**, 114029 (2021); arXiv: 2109.10359 [hep-ph].
9. R. Lebed, R. Mitchell, and E. Swanson, “Heavy-quark QCD exotica,” Prog. Part. Nucl. Phys. **93**, 143 (2017); arXiv: 1610.04528 [hep-ph].
10. P. Zyla et al. (Particle Data Group), “Review of particle physics,” Prog. Theor. Exp. Phys. **2020**, 083C01 (2020).
11. C. Gong, M.-C. Du, Q. Zhao, X.-H. Zhong, and B. Zhou, “Nature of $X(6900)$ and its production mechanism at LHCb,” Phys. Lett. B **824**, 136794 (2022); arXiv: 2011.11374 [hep-ph].
12. X.-K. Dong, V. Baru, F.-K. Guo, C. Hanhart, A. Nefediev, and B.-S. Zou, “Is the existence of a $J/\psi J/\psi$ bound state plausible?,” Sci. Bull. **66**, 1577 (2021); arXiv: 2107.03946 [hep-ph].
13. Y. Iwasaki, “A possible model for new resonances-exotics and hidden charm,” Prog. Theor. Phys. **54**, 492 (1975).
14. J. Aubert et al. (E598 Collab.), “Experimental observation of a heavy particle J ,” Phys. Rev. Lett. **33**, 1404 (1974).
15. J. Augustin et al. (SLAC-SP-017), “Discovery of a narrow resonance in e^+e^- annihilation,” Phys. Rev. Lett. **33**, 1406 (1974).
16. A. Berezhnoy, A. Likhoded, A. Luchinsky, and A. Novoselov, “Double J/ψ -meson production at LHC and $4c$ -tetraquark state,” Phys. Rev. D **84**, 094023 (2011); arXiv: 1101.5881 [hep-ph].
17. A. Berezhnoy, A. Luchinsky, and A. Novoselov, “Tetraquarks composed of 4 heavy quarks,” Phys. Rev. D **86**, 034004 (2012); arXiv: 1111.1867 [hep-ph].
18. L. An (LHCb Collab.), “Latest results on exotic hadrons at LHCb,” (2020).
<https://indico.cern.ch/event/900972/>
19. J.-W. Zhu, X.-D. Guo, R.-Y. Zhang, W.-G. Ma, and X.-Q. Li, “A possible interpretation for $X(6900)$ observed in four-muon final state by LHCb: A light Higgs-like boson?,” arXiv: 2011.07799 [hep-ph] (2020).
20. X.-K. Dong, V. Baru, F.-K. Guo, C. Hanhart, and A. Nefediev, “Coupled-channel interpretation of the LHCb double- J/ψ spectrum and hints of a new state near the $J/\psi J/\psi$ threshold,” Phys. Rev. Lett. **126**, 132001 (2021).
21. X.-K. Dong, V. Baru, F.-K. Guo, C. Hanhart, and A. Nefediev, Phys. Rev. Lett. **127**, 119901(E) (2021); arXiv: 2009.07795 [hep-ph].
22. Z.-H. Guo and J. Oller, “Insights into the inner structures of the fully charmed tetraquark state $X(6900)$,” Phys. Rev. D **103**, 034024 (2021); arXiv: 2011.00978 [hep-ph].
23. J.-Z. Wang, D.-Y. Chen, X. Liu, and T. Matsuki, “Producing fully charm structures in the J/ψ -pair invariant mass spectrum,” Phys. Rev. D **103**, 071503 (2021); arXiv: 2008.07430 [hep-ph].
24. J. Sonnenschein and D. Weissman, “Deciphering the recently discovered tetraquark candidates around 6.9 GeV,” Eur. Phys. J. C **81**, 25 (2021); arXiv: 2008.01095 [hep-ph].
25. J. Giron and R. Lebed, “Simple spectrum of $c\bar{c}c\bar{c}$ states in the dynamical diquark model,” Phys. Rev. D **102**, 074003 (2020); arXiv: 2008.01631 [hep-ph].
26. M. Karliner and J. Rosner, “Interpretation of structure in the $\text{Di-}J/\psi$ spectrum,” Phys. Rev. D **102**, 114039 (2020); arXiv: 2009.04429 [hep-ph].
27. B.-C. Yang, L. Tang, and C.-F. Qiao, “Scalar fully-heavy tetraquark states $QQ'\bar{Q}\bar{Q}'$ in QCD sum rules,” Eur. Phys. J. C **81**, 324 (2021); arXiv: 2012.04463 [hep-ph].
28. Z. Zhao, K. Xu, A. Kaewsnod, X. Liu, A. Limphirat, and Y. Yan, “Study of charmonium like and fully-charm tetraquark spectroscopy,” Phys. Rev. D **103**, 116027 (2021); arXiv: 2012.15554 [hep-ph].
29. C. Deng, H. Chen, and J. Ping, “Towards the understanding of fully-heavy tetraquark states from various models,” Phys. Rev. D **103**, 014001 (2021); arXiv: 2003.05154 [hep-ph].
30. M. Bedolla, J. Ferretti, C. Roberts, and E. Santopinto, “Spectrum of fully-heavy tetraquarks from a diquark+antidiquark perspective,” Eur. Phys. J. C **80**, 1004 (2020); arXiv: 1911.00960 [hep-ph].
31. R. Lebed, “Spectroscopy of exotic hadrons formed from dynamical diquarks,” Phys. Rev. D **96**, 116003 (2017); arXiv: 1709.06097 [hep-ph].
32. J.-M. Richard, “About the $J/\psi J/\psi$ peak of LHCb: Fully-charmed tetraquark?,” Sci. Bull. **65**, 1954 (2020); arXiv: 2008.01962 [hep-ph].
33. Q.-F. Cao, H. Chen, H.-R. Qi, and H.-Q. Zheng, “Some remarks on $X(6900)$,” Chin. Phys. C **45**, 103102 (2021); arXiv: 2011.04347 [hep-ph].