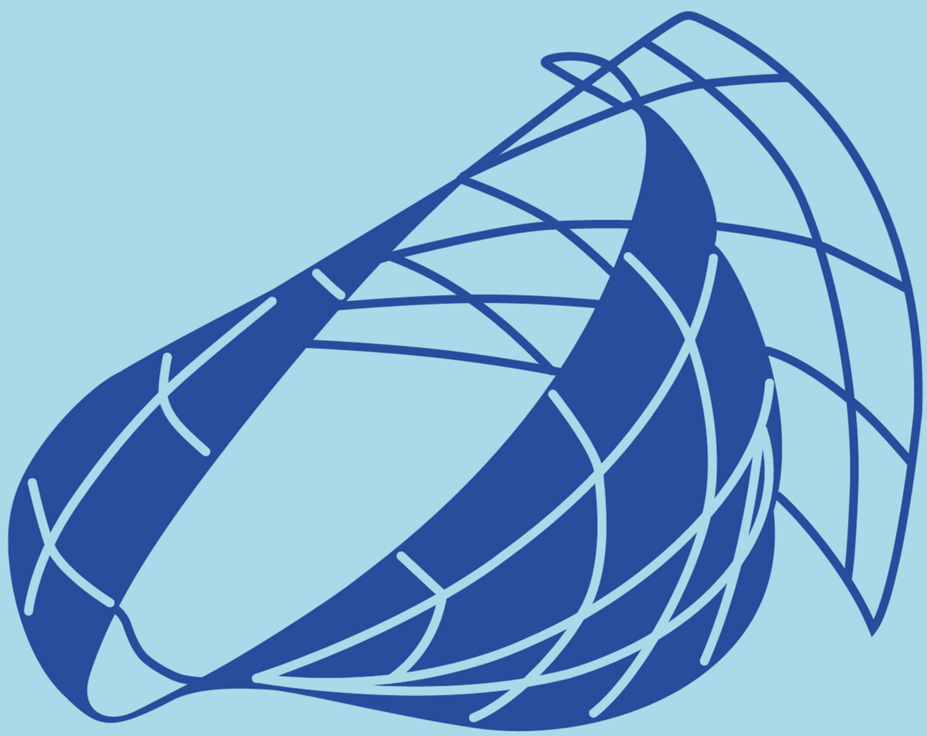
PART A

PROCEEDINGS OF THE SIXTEENTH

MARCEL GROSSMANN MEETING ON GENERAL RELATIVITY



Editors

Remo Ruffini
Gregory Vereshchagin

World Scientific

PART A

THE SIXTEENTH
MARCEL GROSSMANN MEETING

On Recent Developments in Theoretical and Experimental General Relativity, Astrophysics and Relativistic Field Theories

Proceedings of the MG16 Meeting
on General Relativity
Online, 5–10 July 2021

Editors

Remo Ruffini
University of Rome "La Sapienza", Rome, Italy
International Center for Relativistic Astrophysics Network (ICRANet), Pescara, Italy

Gregory Vereshchagin
International Center for Relativistic Astrophysics Network (ICRANet), Pescara, Italy

Series Editor

Remo Ruffini
University of Rome "La Sapienza", Rome, Italy
International Center for Relativistic Astrophysics Network (ICRANet), Pescara, Italy

World Scientific

NEW JERSEY • LONDON • SINGAPORE • BEIJING • SHANGHAI • HONG KONG • TAIPEI • CHENNAI

Published by

World Scientific Publishing Co. Pte. Ltd.
5 Toh Tuck Link, Singapore 596224
USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601
UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

Library of Congress Cataloging-in-Publication Data
Names: Marcel Grossmann Meeting on General Relativity (16th : 2021 : Online) |
Ruffini, Remo, editor. | Vereshchagin, Gregory, editor.
Title: The sixteenth Marcel Grossmann meeting on recent developments in theoretical and experimental general relativity, astrophysics, and relativistic field theories : proceedings of the MG16 meeting on general relativity online, 5–10 July 2021 / editors: Remo Ruffini, University of Rome "La Sapienza", Rome, Italy, International Center for Relativistic Astrophysics Network (ICRANet), Pescara, Italy, Gregory Vereshchagin, International Center for Relativistic Astrophysics Network (ICRANet), Pescara, Italy.
Other titles: Proceedings of the MG16 meeting on general relativity online, 5–10 July 2021
Description: New Jersey : World Scientific, [2023] | Includes bibliographical references.
Identifiers: LCCN 2022047088 | ISBN 9789811269769 (set ; hardcover) | ISBN 9789811266584 (v. 1) | ISBN 9789811266591 (v. 2) | ISBN 9789811266607 (v. 3) | ISBN 9789811266614 (v. 4) | ISBN 9789811269776 (set ; ebook)
Subjects: LCSH: General relativity (Physics)--Congresses. | Gravitation--Congresses. | Quantum gravity--Congresses. | Cosmology--Congresses. | Astrophysics--Congresses.
Classification: LCC QC173.6 .M37 2021 | DDC 523.01--dc23/eng/20220929
LC record available at https://lccn.loc.gov/2022047088

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library.

For any available supplementary material, please visit
https://www.worldscientific.com/worldscibooks/10.1142/13149#t=suppl

Typeset by Stallion Press
Email: enquiries@stallionpress.com

Printed in Singapore

Study the effects of anisotropy on the highly magnetized white dwarfs

Debabrata Deb
Department of Physics, Indian Institute of Science, Bangalore 560012, India
E-mail: debabratadeb@iisc.ac.in

Banibrata Mukhopadhyay
Department of Physics, Indian Institute of Science, Bangalore 560012, India
E-mail: bm@iisc.ac.in

Fridolin Weber
Department of Physics, San Diego State University, San Diego, CA 92182, USA
Center for Astrophysics and Space Sciences, University of California at San Diego, La Jolla, CA 92093, USA
E-mail: fweber@sdsu.edu

The equilibrium configuration of white dwarfs composed of anisotropic fluid distribution in the presence of a strong magnetic field is investigated in this work. By considering a functional form of the anisotropic stress and magnetic field profile, some physical properties of magnetized white dwarfs, such as mass, radius, density, radial and tangential pressures, are derived; their dependency on the anisotropy and central magnetic field is also explored. We show that the orientations of the magnetic field along the radial direction or orthogonal to the radial direction influence the stellar structure and physical properties of white dwarfs significantly. Importantly, we show that ignoring anisotropy governed by the fluid due to its high density in the presence of a strong magnetic field would destabilize the star. Through this work, we can explain the highly massive progenitor for peculiar over-luminous type Ia supernovae, and low massive progenitor for under-luminous type Ia supernovae, which poses a question of considering 1.4 solar mass white dwarf to be related to the standard candle.

Keywords: Magnetic fields; white dwarfs; super-Chandrasekhar white dwarf; massive stars

1. Introduction

In a recent article, we have shown that the presence of strong magnetic field, orientations of magnetic field and the combined anisotropy due to both the matter and magnetic field exhibit notable influence on the microscopic and thermodynamic properties of compact stars, viz., neutron stars and strange quark stars. We have also very briefly discussed in their study (hereinafter Paper I) that same effects can be observed for magnetized and anisotropic white dwarfs (WDs) which further may leads to possible violation of Chandrasekhar critical mass limit. Importantly Paper I shows that to achieve hydrostatic stable configuration for the highly magnetized compact stars, it is necessary to consider local pressure anisotropy due to matter besides the anisotropy due to magnetic field. Following Paper I, in the present work we assume two type of magnetic field orientations, such as *radial orientation (RO)* when the magnetic field is directed toward the radial direction and *transverse orientation (TO)* when field is oriented perpendicular to the radial direction (say along θ or ϕ directions). In this work, we investigate in detail the effects of magnetic field strength, their orientations and anisotropy on the highly magnetized WDs and their thermodynamic parameters.

WDs achieve stable stellar configuration via equilibrium of the forces when the inward gravitational pull is counterbalanced by the outward repulsive electron degeneracy pressure. During the accretion of mass from the companion stars when WDs surpass their critical mass limit, immediately a violent explosion triggers, which completely disrupt WDs and releases huge amount of energy of the order $\sim 10^{51}$ erg, known as type Ia supernova (SNIa). The critical mass limit was introduced by Chandrasekhar in his famous works[2,3] where he predicted that the critical mass limit for non-magnetized and non-rotating carbon-oxygen (C-O) WD is 1.4 $M_\odot$ which is known as "Chandrasekhar mass limit" (CML) for WDs.

The standard and specific set of relations among colour, intrinsic luminosity and light-curve width[4,5] make SNIa a perfect candidate for "standard candle" which helps astronomers to measure accurately the distance of the host galaxy. Behind the measurement of cosmological parameters, the disproportional relation between distance and luminosity of SNIa is used as a key technique. In fact, SNIa as a "standard candle" had pivotal role to confirm the accelerated expansion of universe.[6,7] However, the idea of "standard candle" faces a major setback with the recent series of observation of several peculiar over-luminous type Ia supernovae (SNeIa) such as *SN 2003cv*,[8] *SN2007fg*,[9] *SN2009if*[10] and *SN 2013dc*,[11] which are best explained through WDs as a progenitor having mass beyond CML. Howell and collaborators[8] predicted that to explain *SN 2003cv* it requires to power by the radioactive decay of $\sim 1.3M_\odot$ of ^{56}Ni which further predicts $\sim 2.1M_\odot$ WD as a progenitor, known as *super-Chandrasekhar progenitor WD* (SCPWD). Later researchers have predicted further massive SCPWD against the overluminous SN 2009if of mass as high as $\sim 2.8M_\odot$.

To explain these unexpected overluminous SNeIa, researchers suggested two possible conjectures such as (i) explosion of rapidly rotating WDs[8,12,13] and (ii) merger of two massive WDs.[14,15] Chen and Li[22] showed that the differentially rotating and accreting WD can not gain mass more than $1.7\,M_\odot$. On the other hand, further investigations showed that through the double degenerate scenario, it is not possible to explain such a highly massive 2.8 $M_\odot$ SCPWD.[16–21] The results from numerical simulation of massive WD merger suggest that due to off-center carbon burning accreting WD collapses into neutron star.[23,24] Hence, it is evident through the above discussion that the double degenerate scenario conjectures never leads to a SNIa which can explain the massive SCPWDs.

Introducing magnetic model, Mukhopadhyay and collaborators[25,26] are able to explain SCPWDs where they considered Landau levels due to high magnetic field in a plane perpendicular to the magnetic field axis and they[27] are also able to suitably explain $\sim 2.6\ M_\odot$ highly massive SCPWD. In another study, Das and Mukhopadhyay[28] showed that the average magnetic field and related magnetic pressure are significantly low compared to the matter pressure when inside magnetized WDs (hereinafter B-WDs) the strong magnetic field is tangled in a length scale larger than quantum length scale, which however leads to the Landau quantization to matter. The mass-radius relation for B-WDs deviates significantly beyond CML when mass accretion continues as the effective pressure counterbalances the inward gravitational pull.[25,29] Das and Mukhopadhyay[30] further predicated that it requires a time scale of 2×10^7 years to turn into a super-Chandrasekhar B-WD (SCBWD) for a $0.2\ M_\odot$ accreting WD having surface magnetic field (B_s) 10^9 G. In fact for the last several years, Mukhopadhyay and his group have explored various aspects of SCBWDs through different sophisticated models by considering variation of magnetic field and geometry, Landau quantization, a pure general relativistic effect, differential rotation, deviation from the spherical symmetric stellar configuration and thermal luminosity, etc.[31–38] They also explored the techniques of the possible direct detection of SCBWDs via continuous gravitational wave astronomy by the various upcoming detectors, such as BBO, DECIGO and LISA. The idea of SCBWDs has been further studied and supported by different independent researcher works.[41–47]

Bowers and Liang[48] raised a strong argument against the oversimplified assumption of the isotropic perfect fluid as the constituent matter distribution of compact stars and proposed a modified Tolman-Oppenheimer-Volkoff (TOV) equation,[49,50] which includes another force raised due to pressure anisotropy, besides the hydrodynamic and gravitational force. They also explored notable effect of local anisotropy due to matter on the different physical properties of the stellar objects, such as density, surface redshift, total mass and radius. Letelier[51] and Bayin[52] pointed out that the possible reason behind this local anisotropy is the presence and mixture of two (or more) fluid within stellar interior. On the other hand, the breaking of spatial rotational symmetry $\mathcal{O}(3)$ due to spontaneous creation of magnetic field instigates pressure anisotropy in the stellar interior.[53–55] Interestingly, during the cooling process of WDs at low temperature, they undergoes a first-order phase transition,[56–58] which is another possible reason for the presence of anisotropy inside the dense plasma. Stevenson[59] introduced a phase diagram through which they predicted that carbon and oxygen remains immiscible when WDs begin to freeze. Later, Garcia and collaborators[60] further explored and confirmed this idea. Nag and Chakrabarty[61] also supported the idea of first-order phase transition during the cooling of WDs through the Bose condensation. In fact, they proposed that "*crystalline normal crustal matter*" is present at the massive WDs core, which is also Bose condensed and similar as the ultra-dense structure of the neutron stars. So, based on the above discussion we are convinced to consider local anisotropy of matter inside WDs.

In this context, it is worth mentioning that though there are several articles[32,33,39,40,62–71] which study non-spherical magnetized compact stars in general relativity based on the perturbative approach to solve the the Einstein-Maxwell field equations or the numerical techniques (such as Lorene and XNS codes) by considering poloidal or toroidal or mixed field geometries, but they did not include the effects of anisotropy. Although, a recent work[72] has explored the effect of anisotropy on WDs in scalar-tensor (ST) theories of gravity but it did not consider magnetic field. The idea of magnetic field orientations viz. RO and TO was originally coined by Chu et al.[73] In the present study, we explore the combined effects of magnetic field strength, its orientations and anisotropy on the WDs to have a more generalized perspective of the WD stellar interior.

This paper is organized as follows: We discuss the basic mathematical formalism in Section 2. Our results and combined effects of the magnetic field, its orientations and anisotropy on the physical properties of WDs are discussed in Section 3. Finally, we summarise our work and present some concluding remarks in Section 4.

2. Basic mathematical formulation

Mukhopadhyay and group[32, 33, 39] have shown that the magnetized compact stars which are toroidally dominated approximately retains their spherical symmetric stellar structure. Hence in this work we describe B-WDs by the spherically symmetric spacetime metric given by

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

where we assume that the metric potentials ν and λ are functions of radial coordinate r. We choose $c = G = 1$, where G is Newton's gravitational constant and c is the speed of light.

Now neglecting the effect of the electric field by assuming that inside the B-WDs there are no macroscopic charges, the energy-momentum tensor, which describes anisotropic and magnetized matter distribution, reads

$$\begin{aligned} T_m^{\mu\nu} &= (\rho + p_t)u^\mu u^\nu - p_t g^{\mu\nu} + (p_r - p_t)\, v^\mu v^\nu \\ &\quad + \mathcal{M}B\left(g^{\mu\nu} - u^\mu u^\nu + \frac{B^\mu B^\nu}{B^2}\right) + \frac{B^2}{4\pi}\left(u^\mu u^\nu - \frac{1}{2}g^{\mu\nu}\right) - \frac{B^\mu B^\nu}{4\pi}, \end{aligned} \tag{2}$$

where $u^\mu = \delta^\mu_0 e^{-\nu(r)/2}$ is a timelike 4-velocity unit vector of fluid and $v^\mu = \delta^\mu_1 e^{-\lambda(r)/2}$ is a spacelike unit vector acting along the radial direction. It is noteworthy that these orthogonal vectors satisfy $u^\mu u_\mu = -v^\mu v_\mu = 1$ and $u^\mu v_\mu = 0$. Here, ρ, p_r and p_t are the density, radial pressure and tangential pressure of the matter inside B-WDs. $g_{\mu\nu}$ denotes metric tensor and $\mathcal{M}$ represents magnetization per unit volume. B is the magnetic field strength given by $B^\mu B_\mu = -B^2$. On the other hand, we do not consider the associated magnetization effect since magnetic pressure is one order higher in magnitude compared to the magnetization which has already been predicted by a few articles.[53, 74] So in this work the presented numerical results do not include effect of magnetization.

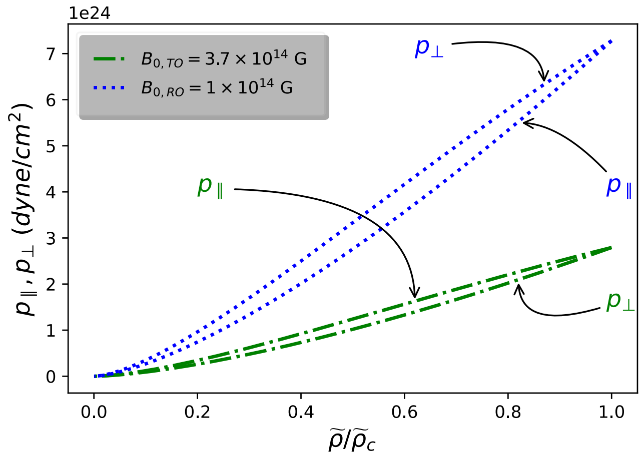


Fig. 1. Variation of parallel pressure ($p_\parallel$) and transverse pressure ($p_\perp$) with the system density ($\widetilde{\rho}$) normalized by the central system density ($\widetilde{\rho}_c$) for a 1.3 $M_\odot$ magnetized WD candidate.

Now using Eqs. (1) and (2) the conservation of energy-momentum tensor, i.e., $\nabla_\mu T^{\mu\nu} = 0$, leads to the essential stellar structure equations which describe static and anisotropic B-WDs as follows:

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi\widetilde{\rho}r^2, \tag{3}$$

$$\begin{cases} \dfrac{\mathrm{d}p_\parallel}{\mathrm{d}r} = -\left(\widetilde{\rho} + p_\parallel\right)\dfrac{4\pi r^3 p_\parallel + m}{r\,(r-2m)} + \dfrac{2}{r}\Delta, & \text{for RO,} \\ \dfrac{\mathrm{d}p_\perp}{\mathrm{d}r} = -\left(\widetilde{\rho} + p_\perp\right)\dfrac{4\pi r^3 p_\perp + m}{r\,(r-2m)} + \dfrac{2}{r}\Delta, & \text{for TO.} \end{cases} \tag{4}$$

where $\widetilde{\rho}$ is the system density defined as $\widetilde{\rho} = \rho + \frac{B^2}{8\pi}$, $p_\parallel$ and $p_\perp$ are the system pressures, which take the form based on the magnetic field orientations as follows:

$$p_\parallel = \begin{cases} p_r - \dfrac{B^2}{8\pi}, & \text{for RO,} \\ p_t - \dfrac{B^2}{8\pi}, & \text{for TO.} \end{cases} \tag{5}$$

$$\text{and} \tag{6}$$

$$p_\perp = \begin{cases} p_t + \dfrac{B^2}{8\pi}, & \text{for RO,} \\ p_r + \dfrac{B^2}{8\pi}, & \text{for TO.} \end{cases} \tag{7}$$

In Eq. (4), Δ denotes effective anisotropy of the stars and based on the choice of magnetic field orientations it takes the forms as $p_t - p_r + \frac{B^2}{4\pi}$ and $p_t - p_r - \frac{B^2}{8\pi}$ due to RO and TO, respectively. One may get the standard form of TOV equations[48,75] for non-magnetized and anisotropic starts for $B = 0$.

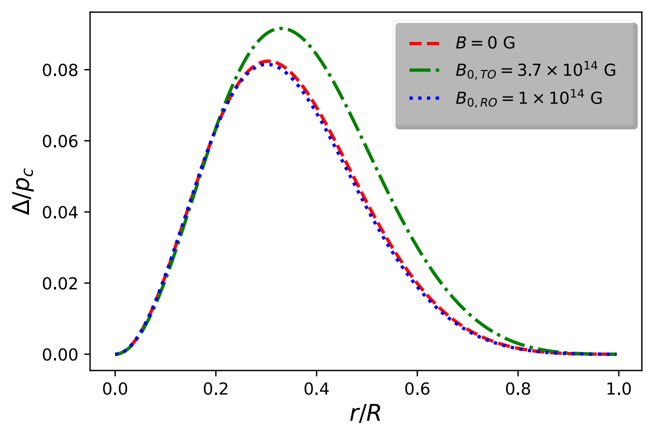


Fig. 2. Variation of anisotropy (Δ) normalized by matter central pressure (p_c) with the radial coordinate r/R for a 1.3 $M_\odot$ magnetized WD candidate.

Bowers and Liang proposed a generalized functional form for the anisotropic stress which has been widely used in different studies. However, since for the present work we require a functional form for anisotropy which includes both the effects of magnetic field strength and magnetic field orientation, we have suitably modified the Bowers-Liang anisotropic form which reads

$$\Delta = \begin{cases} \kappa \dfrac{\left(\widetilde{\rho} + p_{\parallel}\right)\left(\widetilde{\rho} + 3\,p_{\parallel}\right)}{(r - 2m)} r^3, & \text{for RO,} \\ \kappa \dfrac{\left(\widetilde{\rho} + p_{\perp}\right)\left(\widetilde{\rho} + 3\,p_{\perp}\right)}{(r - 2m)} r^3, & \text{for TO,} \end{cases} \tag{8}$$

where κ is a constant parameter that controls the amount of anisotropy within the star and we use κ maintaining its permissible range, given by $\left[-\frac{2}{3}, \frac{2}{3}\right]$.[76] Now, one may notice that if matter anisotropy is not considered, the anisotropic stress become $\sim\mid B^2 \mid$, which has a finite, non-zero value at the stellar center. Since at the center, the hydrodynamic and gravitational forces are zero, non-zero anisotropic force at the center should trigger instability due to nonequilibrium of the forces. We show this issue can easily be overcome considering combined anisotropy due to both the matter and magnetic field.

Further, to describe matter distribution of WDs we consider equation of state (EoS) proposed by Chandrasekhar which reads

$$p_r = \frac{\pi m_e^4 c^5}{3h^3}\left[x\left(2x^2 - 3\right)\sqrt{x^2+1} + 3\sinh^{-1} x\right],$$
$$\rho = \frac{8\pi \mu_e m_H \left(m_e c\right)^3}{3h^3} x^3, \tag{9}$$

where m_H and m_e denote mass of hydrogen atom and electron, respectively, $\mu_e = 2$ is the mean molecular weight., p_F is the Fermi momentum and $x = p_F/m_e c$. Note that for clarity here we introduce the speed of light c and Planck constant h in their appropriate places. For the present case of C-O WDs we have $\mu_e = 2$. We consider the exterior spacetime is defined by the well-known Schwarzschild exterior spacetime, given by

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \frac{1}{\left(1 - \frac{2M}{r}\right)} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \qquad (10)$$

where M represents total mass of the star.

Both the fossil field of the progenitor star[31,33,77] and dynamo effect[78] ensure stronger magnetic field at the center (B_c), which decreases gradually to reach its minimum value at the surface (B_s). Hence, in this work we consider a density dependent functional form for the magnetic field,[31,36,79,80] which can suitably mimic the spatial dependence of magnetic field strength, given by

$$B(\rho) = B_s + B_0 \left[1 - \exp\left\{-\eta \left(\frac{\rho}{\rho_0}\right)^\gamma\right\}\right], \qquad (11)$$

where the η and γ are the dimensionless constants which controls how fast magnetic field strength decays from the center to the surface. In this work we chose $\rho_0 = 10^9$ g/cm^3, $\gamma = 0.9$ and $\eta = 0.2$.[36] We also consider $B_s = 10^9$ G which is consistent with the Sloan Digital Sky Survey.[81]

3. Result and discussion

In this study, we explore that besides magnetic field strength, the magnetic field orientations (such as RO and TO) and combined anisotropy due to matter and field significantly influence the mass-radius curve and physical parameters of B-WDs. Note that to present some basic properties of thermodynamics and anisotropy we consider a $1.3\,M_\odot$ B-WD candidate having surface magnetic field 10^9 G in Figs. 1 and 2. Further, our choices of B_0 for RO and TO are different as they exhibit an asymmetric effect on the stellar mass. In the figures, we put RO and TO in the subscript of B_0 to specify the respective magnetic field orientations. It is also noteworthy that our choice of B_0 is also motivated by the highly massive SCPWDs of peculiar over-luminous SNeIa.

In Fig. 1 we feature combined effects of magnetic field strength and its orientations on the system parallel pressure $p_\parallel$ and transverse pressure $p_\perp$. We find that for TO, as B_0 increases, the slope of the system pressures decreases gradually, whereas, for RO with the increasing B_0, the system pressures become gradually stiffened. This phenomenon is because, as B_0 increases in the TO case, the stellar radius increases, which reduces gravitational force and consequently decreases the hydrodynamic force. However, for increasing B_0 in the RO case, the stellar radius of B-WDs decreases, consequently increasing outward hydrodynamic force to support the increased gravitation force. Importantly, note that at the center the effective anisotropy is zero since $p_\parallel = p_\perp$ at $r = 0$, which reflects consistency of TOV equations. In Fig. 2 we show the profile for system anisotropy Δ which also shows zero anisotropy at the center.

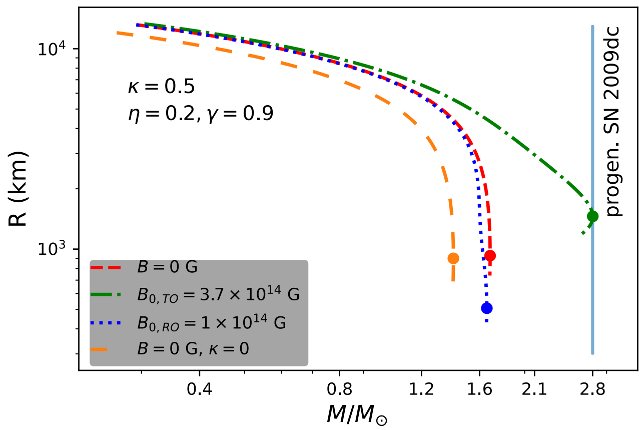


Fig. 3. Variation of the total mass in the units of solar mass ($M/M_\odot$) with the total radius (R) of the stars for varying B_0 and $\kappa = 0.5$. Solid circles represent maximum possible mass for the stars.

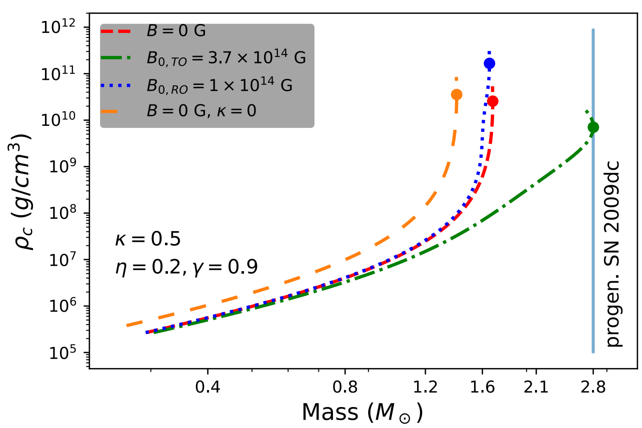


Fig. 4. Variation of $M/M_\odot$ as a function of the matter central density (ρ_c). Solid circles represent maximum possible mass for the stars.

Further, one may note that the maximum magnitude of anisotropy increases with increasing B_0 for the TO case, whereas it decreases for the RO case. We find that for $B_0 = 3.7 \times 10^{14}$ G the maximum magnitude of Δ is $\sim 92\%$ lower compared to the central matter pressure p_c which is insufficient to drive the spherical symmetric stellar structure of B-WDs to non-spherical symmetry.

The total mass and radius (M-R) curve for B-WDs due to different B_0 for both RO and TO is shown in Fig. 3. We find the maximum mass ($M_{\rm max}$) and the corresponding radius ($R_{\rm Mmax}$) in TO due to $B_0 = 3.7 \times 10^{14}$ G are 2.8 $M_\odot$ and 1457.24 km, respectively, whereas in RO due to $B_{0,\rm TO} = 10^{14}$ G they are 1.66 $M_\odot$ and 507.10 km, respectively. In TO due to $B_{0,\rm TO} = 3.7 \times 10^{14}$ G, $M_{\rm max}$ and $R_{\rm Mmax}$ increase compared to their non-magnetized anisotropic case of $\sim$66% and $\sim$57%, respectively, whereas in RO due to $B_{0,\rm RO} = 10^{14}$ G they decrease to $\sim$2% and $\sim$45%, respectively. Only considering local anisotropy due to matter and not

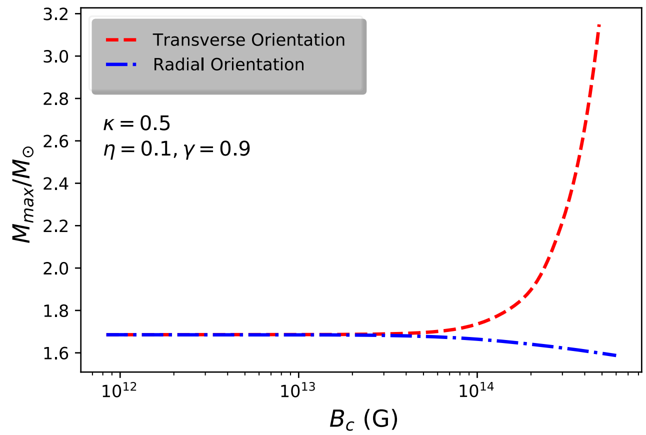


Fig. 5. Variation of $M/M_{\odot}$ with the central magnetic field (B_c).

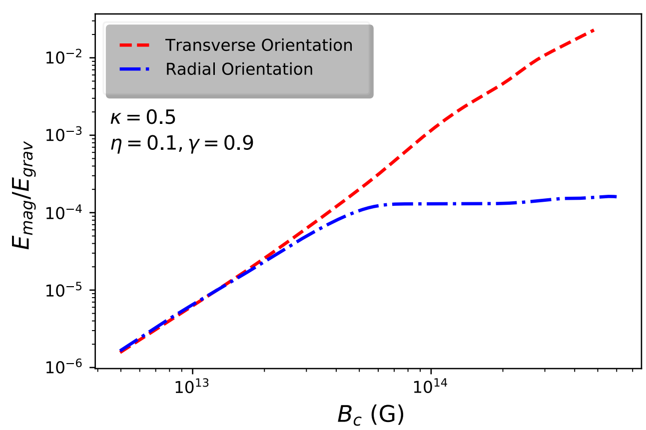


Fig. 6. Variation of the ratio of the magnetic energy ($E_{\rm mag}$) to the gravitational energy ($E_{\rm grav}$) with the central magnetic field (B_c).

considering magnetic field through our model one may have 1.81 $M_{\odot}$ WD having radius 959.63 km which are ∼29% and ∼7%, respectively, higher compared to their values for WD CML. Harrison et al.[82] predicted that for the stability of a spherically symmetric stellar model the M-R relation should be consistent with $\mathrm{d}M/\mathrm{d}\rho_{\rm c} > 0$ up to $M_{\rm max}$. It has been shown in Fig. 4, which confirms stable B-WDs upto maximum limiting mass.

In Fig. 5 we explore the effects of magnetic field orientations on the mass of B-WDs. The effects of magnetic field strength on $M_{\rm max}$ for the TO and RO cases are asymmetric which motivates us to chose different B_0 for different magnetic field orientations. One may find that for a specific set of constant parameter such as $\eta = 0.1$, $\gamma = 0.9$ and $\kappa = 0.5$ the asymmetry of maximum mass for the TO and RO cases due to $B_c = 4 \times 10^{14}$ G is ∼65% which confirms significant effect of

Table 1. Physical parameters of WDs with $\kappa = 0.5$, $\eta = 0.2$ and $\gamma = 0.9$ for different B_0.

Orientation of magnetic field	B_0 (Gauss)	Maximum mass ($M_\odot$)	Corresponding predicted radius (km)	Central magnetic field B_c (Gauss)	Central density $\tilde{\rho}_c$ (gm/cm^3)	Central pressure $\tilde{p}_c$ ($dyne/cm^2$)	$\frac{E_{\mathrm{mag}}}{E_{\mathrm{grav}}}$
Transverse Orientation	3.7×10^{14}	2.80	1457.24	2.538×10^{14}	7.040×10^{9}	2.809×10^{28}	1.405×10^{-2}
No magnetic Field	-	1.69	926.91	-	2.560×10^{10}	9.865×10^{28}	-
Radial Orientation	10^{14}	1.66	507.10	10^{14}	1.665×10^{11}	1.152×10^{30}	7.227×10^{-5}

magnetic field orientations on B-WDs. Fig. 5 features that for TO (RO) the increase (decrease) of M_{max} is rapid (slow) with the increase of central magnetic field B_c.

Chandrasekhar and Fermi[83] predicted that for magnetized spherical symmetric stable compact stars it is not sufficient that they are consistent with the inequality $\Gamma > 4/3$ as the strong magnetic field may lead to stellar instability. They further predicted that for the dynamical stability of the magnetized compact stars the necessary condition is $E_{\mathrm{mag}}/|E_{\mathrm{grav}}| << 1$, where E_{mag} and $|E_{\mathrm{grav}}|$ denote the magnetic and gravitational potential energies, respectively. In Fig. 6 we present that for both TO and RO due to different B_0, $|E_{\mathrm{grav}}|$ overpowers E_{mag} considerably to achieve dynamical stability of B-WDs.

For the better understanding of the readers, we present the obtained results for the different cases in Table 1. We present the variation of M_{max}, R, B_c, system central density ($\tilde{\rho}_c$), system central pressure ($\tilde{p_c}$) and $E_{\mathrm{mag}}/\mid E_{\mathrm{grav}} \mid$ with different B_0 for both TO and RO and non-magnetized cases in Table 1. Note that the presented results in Table. 1 are based on the chosen set of constants such as $\eta = 0.2$, $\gamma = 0.9$ and $\kappa = 0.5$. Clearly, for TO with the increase of B_0, B-WDs become massive, larger in size and less dense compact object, whereas for RO as B_0 increases B-WDs become gradually less massive, smaller in size and highly dense compact object compared to their non-magnetized case.

4. Conclusion

In the present work we explore the combined effects due to magnetic field strength, orientations of magnetic field and effective anisotropy raised due to matter and field on B-WDs. Although anisotropic WDs were studied by Chowdhury and Sarkar,[72] their study was based on the non-magnetised case. On the other hand, Chu et al.[73] explored the effect of magnetic field and their orientation on total mass of compact stars. However, they ignored important effect of anisotropy and did not show effects of magnetic field orientations on the different physical parameters of compact stars. In this work for the first time we have studied the combined effects of magnetic field strength, their orientations and anisotropy on WDs in the strong magnetic field. In conclusion, the two major results of our study on B-WDs are as follows:

(i) We show that to achieve magneto-hydrostatic stability at the stellar center of B-WDs, the effective anisotropy due to matter and magnetic field should be considered.

(ii) With the choice of appropriate set of constant parameters such as κ and B_0 our model can appropriately explain massive progenitors of peculiar over luminous SNeIa as high as 2.8 $M_\odot$, which further raise question regarding the idea of 1.4 $M_\odot$ WDs as the "standard candle".

Acknowledgments

Research of DD is funded by the C.V. Raman Postdoctoral Fellowship (Reg. No. R(IA)CVR-PDF/2020/222) from the Department of Physics, Indian Institute of Science. BM acknowledges partial support by a project of the Department of Science and Technology (DST), India, with Grant No. DSTO/PPH/BMP/1946 (EMR/2017/001226). All computations were performed in open source softwares, and the authors are sincerely thankful to the open source community.

References

1. D. Deb, B. Mukhopadhyay and F. Weber, *accepted for publication in ApJ, arXiv:2108.12436* (2021); Paper I.
2. S. Chandrasekhar, *Astrophys. J.* **74**, 81 (1931).
3. S. Chandrasekhar, *Mon. Not. R. Astron. Soc.* **95**, 207 (1935).
4. M. M. Phillips, *Astrophys. J. Lett.* **413**, L105 (1993).
5. G. Goldhaber, D. E. Groom, A. Kim, G. Aldering, P. Astier, A. Conley, S. E. Deustua, et al. *Astrophys. J.* **558**, 359 (2001).
6. A. G. Riess, A. V. Filippenko, P. Challis, A. Clocchiatti, A. Diercks, P. M. Garnavich, R. L. Gilliland, et al., *Astron. J.* **116**, 1009 (1998).
7. S. Perlmutter, G. Aldering, G. Goldhaber, R. A. Knop, P. Nugent, P. G. Castro, S. Deustua, et al., *Astrophys. J.* **517**, 565 (1999).
8. D. A. Howell, M. Sullivan, P. E. Nugent, R. S. Ellis, A. J. Conley, D. Le Borgne, R. G. Carlberg, et al., *Nature* **443**, 308 (2006).
9. R. A. Scalzo, G. Aldering, P. Antilogus, C. Aragon, S. Bailey, C. Baltay, S. Bongard, et al., *Astrophys. J.* **713**, 1073 (2010).
10. Taubenberger S., Benetti S., Childress M., Pakmor R., Hachinger S., Mazzali P. A., Stanishev V., et al., *Mon. Not. R. Astron. Soc.* **412**, 2735 (2011).
11. Y. Cao, J. Johansson, P. E. Nugent, A. Goobar, J. Nordin, S. R. Kulkarni, S. B. Cenko, et al., Astrophys. J. **823**, 147 (2016)
12. K. Boshkayev, L. Izzo, J. A. Rueda Hernandez and R. Ruffini, *Astron. Astrophys.* **555**, A151 (2013).
13. D. Branch, *Nature* **443**, 283 (2006).
14. I. Iben and A. V. Tutukov, *Astrophys. J., Suppl.* **54**, 335 (1984).
15. M. Hicken, P. M. Garnavich, J. L. Prieto, S. Blondin, D. L. DePoy, R. P. Kirshner and J. Parrent, *Astrophys. J. Lett.* **669**, L17 (2007).
16. K. Nomoto, *Astrophys. J.* **277**, 791 (1984).
17. H. Saio and K. Nomoto, *Astron. Astrophys.* **150**, L21 (1985).
18. K. Nomoto, *Astrophys. J.* **322**, 206 (1987).
19. K. Nomoto and Y. Kondo, *Astrophys. J. Lett.* **367**, L19 (1991).
20. F. X. Timmes, S. E. Woosley and R. E. Taam, *Astrophys. J.* **420**, 348 (1994).
21. H. Saio and K. Nomoto, *Astrophys. J.* **500**, 388 (1998).
22. W.-C. Chen and X.-D. Li, *Astrophys. J.* **702**, 686 (2009).

23. H. Saio and K. Nomoto, *Astrophys. J.* **615**, 444 (2004).
24. R. G. Martin, C. A. Tout and P. Lesaffre, *Mon. Not. R. Astron. Soc.* **373**, 263 (2006).
25. U. Das and B. Mukhopadhyay, *Int. J. Mod. Phys. D* **21**, 1242001 (2012a).
26. A. Kundu and B. Mukhopadhyay, *Mod. Phys. Lett. A* **27**, 1250084 (2012).
27. U. Das and B. Mukhopadhyay, *Phys. Rev. Lett.* **110**, 071102 (2013a).
28. U. Das and B. Mukhopadhyay, *Mod. Phys. Lett. A* **29**, 1450035 (2014a).
29. U. Das and B. Mukhopadhyay, *Phys. Rev. D* **86**, 042001 (2012).
30. U. Das and B. Mukhopadhyay, *A. R. Rao, Astrophys. J. Lett.* **767**, L14 (2013).
31. U. Das and B. Mukhopadhyay, *J. Cosmol. Astropart. Phys.* **2014**, 050 (2014).
32. U. Das and B. Mukhopadhyay, *J. Cosmol. Astropart. Phys.* **2015**, 016 (2015a).
33. S. Subramanian and B. Mukhopadhyay, *Mon. Not. R. Astron. Soc.* **454**, 752 (2015).
34. B. Mukhopadhyay and A. R. Rao, *J. Cosmol. Astropart. Phys.* **2016**, 007 (2016).
35. B. Mukhopadhyay, A. R. Rao and T. S. Bhatia, *Mon. Not. R. Astron. Soc.* **472**, 3564 (2017).
36. M. Bhattacharya and B. Mukhopadhyay, S. Mukerjee, *Mon. Not. R. Astron. Soc.* **477**, 2705 (2018).
37. A. Gupta, B. Mukhopadhyay and C. A. Tout, *Mon. Not. R. Astron. Soc.* **496**, 894 (2020).
38. B. Mukhopadhyay, A. Sarkar and C. A. Tout, *Mon. Not. R. Astron. Soc.* **500**, 763 (2021).
39. S. Kalita and B. Mukhopadhyay, *Mon. Not. R. Astron. Soc.* **490**, 2692 (2019).
40. S. Kalita and B. Mukhopadhyay, *T. Mondal, T. Bulik, Astrophys. J.* **896**, 69 (2020).
41. P. Federbush, T. Luo and J. Smoller, *Arch. Ration. Mech. Anal.* **215**, 611 (2015).
42. B. Franzon and S. Schramm, *Phys. Rev. D* **92**, 083006 (2015).
43. B. Franzon and S. Schramm, *Mon. Not. R. Astron. Soc.* **467**, 4484 (2017).
44. M. Moussa, *Ann. Phys.* **420**, 168263 (2020).
45. H. Shah and K. Sebastian, *Astrophys. J.* **843**, 131 (2017).
46. H. Sotani and T. Tatsumi, *Mon. Not. R. Astron. Soc.* **467**, 1249 (2017).
47. S. K. Roy, S. Mukhopadhyay, J. Lahiri and D. N. Basu, *Phys. Rev. D* **100**, 063008 (2019).
48. R. L. Bowers and E. P. T. Liang, *Astrophys. J.* **188**, 657 (1974).
49. R. C. Tolman, *Phys. Rev.* **55**, 364 (1939).
50. J. R. Oppenheimer and G. M. Volkoff, *Phys. Rev.* **55**, 374 (1939).
51. P. S. Letelier, *Phys. Rev. D* **22**, 807 (1980).
52. S. S. Bayin, *Phys. Rev. D* **26**, 1262 (1982).
53. E. J. Ferrer, V. de La Incera, J. P. Keith, I. Portillo and P. L. Springsteen, *Phys. Rev. C* **82**, 065802 (2010).
54. A. A. Isayev and J. Yang, *Phys. Rev. C* **84**, 065802 (2011).
55. A. A. Isayev and J. Yang, *Phys. Lett. B* **707**, 163 (2012).
56. D. A. Kirzhnits, *Zh. Eksp. Teor. Fiz.* **38**, 503 (1960); *Sov. Phys.—JETP* **11**, 365 (1960).
57. A. A. Abrikosov, *Zh. Eksp. Teor. Fiz.* **39**, 1798 (1960).
58. E. E. Salpeter, *Astrophys. J.* **134**, 669 (1961).
59. D. J. Stevenson, *J. Phys. Colloques* **41**, C2_61 (1980).
60. E. Garcia-Berro, M. Hernanz, R. Mochkovitch and J. Isern, *Astron. Astrophys.* **193**, 141 (1988).
61. N. Nag and S. Chakrabarty, arXiv:astro-ph/0008477 (2000).
62. M. Bocquet, S. Bonazzola, E. Gourgoulhon and J. Novak, *Astron. Astrophys.* **301**, 757 (1995).
63. K. Konno, T. Obata and Y. Kojima, *Astron. Astrophys.* **352**, 211 (1999).

64. C. Y. Cardall, M. Prakash and J. M. Lattimer, *Astrophys. J.* **554**, 322 (2001).
65. K. Ioka and M. Sasaki, *Astrophys. J.* **600**, 296 (2004).
66. A. Oron, *Phys. Rev. D* **66**, 023006 (2002).
67. K. Kiuchi and K. Kotake, *Mon. Not. R. Astron. Soc.* **385**, 1327 (2008).
68. K. Kiuchi, K. Kotake and S. Yoshida, *Astrophys. J.* **698**, 541 (2009).
69. N. Yasutake, K. Kiuchi and K. Kotake, *Mon. Not. R. Astron. Soc.* **401**, 2101 (2010).
70. J. Frieben and L. Rezzolla, *Mon. Not. R. Astron. Soc.* **427**, 3406 (2012).
71. S. Yoshida, K. Kiuchi and M. Shibata, *Phys. Rev. D* **86**, 044012 (2012).
72. S. Chowdhury and T. Sarkar, *Astrophys. J.* **884**, 95 (2019).
73. P.-C. Chu, L.-W. Chen and X. Wang, *Phys. Rev. D* **90**, 063013 (2014).
74. M. Sinha, B. Mukhopadhyay and A. Sedrakian, *Nucl. Phys. A* **898**, 43 (2013).
75. L. Herrera and W. Barreto, *Phys. Rev. D* **88**, 084022 (2013).
76. H. O. Silva, C. F. B. Macedo, E. Berti and L. C. B. Crispino, *Class. Quantum Gravity* **32**, 145008 (2015).
77. K. Fujisawa, S. Yoshida and Y. Eriguchi, *Mon. Not. R. Astron. Soc.* **422**, 434 (2012).
78. A. T. Potter, C. A. Tout, *Mon. Not. R. Astron. Soc.* **402**, 1072 (2010).
79. D. Bandyopadhyay, S. Chakrabarty and Pal S., *Phys. Rev. Lett.* 79 (1997) 2176.
80. D. Bandyopadhyay, S. Pal and S. Chakrabarty, *J. Phys. G* **24**, 1647 (1998).
81. G. D. Schmidt, H. C. Harris, J. Liebert, D. J. Eisenstein, S. F. Anderson, J. Brinkmann, P. B. Hall, et al., *Astrophys. J.* **595**, 1101 (2003).
82. B. K. Harrison, K. S. Thorne, M. Wakano and J. A. Wheeler, *Gravitation Theory and Gravitational Collapse, Chicago: University of Chicago Press*, (1965).
83. S. Chandrasekhar and E. Fermi, *Astrophys. J.* **118**, 116 (1953).