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On neutrino-mediated potentials in a neutrino background

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Abstract: The exchange of a pair of neutrinos with Standard Model weak interactions generates a long-range force between fermions. The associated potential is extremely feeble, $\propto G_F^2/r^5$ for massless neutrinos, which renders it far from observable even in the most sensitive experiments testing fifth forces. The presence of a neutrino background has been argued to induce a correction to the neutrino propagator that enhances the potential by orders of magnitude. In this brief note, we point out that such modified propagators are invalid if the background neutrino wavepackets have a finite width. By reevaluating the 2-ν exchange potential in the presence of a neutrino background including finite width effects, we find that the background-induced enhancement is reduced by several orders of magnitude. Unfortunately, this pushes the resulting 2-ν exchange potential away from present and near-future sensitivity of tests of new long-range forces.

Keywords: Neutrino Interactions, Non-Standard Neutrino Properties

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Contents

1 Introduction: short review of neutrino-mediated forces

Soon after the existence of neutrinos was proposed, it was pointed out that the exchange of neutrino pairs would induce a long-range force between nucleons [1–3]. Most of these early works estimated the potential to behave as $1/r$, with r the distance between two test particles, and even suggested that it could be responsible for gravitation [2, 4–6]. However, as early as 1936, Iwanenko and Sokolow [3] obtained that the potential due to the exchange of two massless fermions behaves as $1/r^5$. This did not prevent Feynman to consider a $1/r^3$ dependence in refs. [7, 8], reinforcing the idea that neutrinos could be mediators of a long-range interaction sourced by mass.

The confusion in the literature concerning the r-dependence of the potential was finally resolved by Feinberg and Sucher [9], who carried out the first calculation of the potential using the effective four-fermion interaction of weak charged currents of the Standard Model (SM) via dispersion techniques (the calculation was later complemented to include the neutral current contribution [10]). They found that, for the exchange of massless neutrino-antineutrino pairs, the interaction potential between two electrons separated by a distance r is given by

$$V(r) = -\frac{G_F^2}{4\pi^3 r^5}, \tag{1.1}$$

where G_F is the Fermi constant, thus confirming the $1/r^5$ behaviour. Hsu and Sikivie [11] obtained the same result using Feynman diagrammatic methods.

The effect of the neutrino mass on the potential was first introduced in ref. [12] for a single massive Dirac neutrino species, and in ref. [13] for the Majorana case; obtaining a potential with a finite range $\mathcal{O}(m_\nu^{-1})$. The effects from several neutrino masses and the associated flavor mixing was recently addressed in refs. [14–17]. In ref. [18] the general form of the neutrino forces generated by SM and beyond the Standard Model (BSM) interactions in the framework of effective field theories was derived. Finally, ref. [19] has included interactions beyond four-fermion contact interactions.

To sum up, the existence of the 2-ν mediated long-range force is a well-grounded prediction of the SM, but this force is extremely weak. The G_F^2 suppression together with the r^{-5} dependence implies that it is already much weaker than gravity at distances $\mathcal{O}(\text{nm})$. This renders the effect orders of magnitude below present and near-future sensitivity of experiments testing the gravitational inverse-square law [20–22] and the weak equivalence principle [23, 24]. Notice, however, that refs. [25, 26] pointed out that the very singular form of the potential may open up the posibility of improved sensitivity over $\sim$fm distances with atomic and nuclear spectroscopy.

However, if the interaction takes place in the presence of a background of neutrinos, the 2-ν mediated potential could be significantly modified. This was first discussed by Horowitz and Pantaleone [27] and later by Ferrer et al. [28, 29], who evaluated the 2-ν mediated potential in the presence of the cosmic neutrino background (CνB). For this case they found a modest enhancement. Most recently, the effect of neutrino backgrounds has been restudied and extended in ref. [30]. This work claims that the intense neutrino fluxes from the Sun, supernovae (SN), or nuclear reactors, may substantially enhance the potential (by up to 20 orders of magnitude), in particular in the direction of the incoming background neutrino flux. Such an enhancement could render the 2-ν potential close to the sensitivity of current and near future fifth force experiments, which motivated the revision of the effect performed in the rest of this note.

2 Neutrino-mediated forces in a neutrino background: general remarks

The key technical ingredient to evaluate neutrino background effects in refs. [27–30] is the use of the background-modified propagator obtained in finite temperature field theory (TQFT) (for reviews of TQFT, see for example refs. [31, 32]),

$$S_F(p) = (\not{p} + m)\left[\frac{i}{p^2 - m^2 + i\epsilon} - 2\,\pi\,\delta(p^2 - m^2)\left(\theta(p^0) f_\nu(\vec{p}) + \theta(-p^0) f_{\overline{\nu}}(\vec{p})\right)\right], \qquad (2.1)$$

where $f_\nu(\vec{p})$ and $f_{\overline{\nu}}(\vec{p})$ are the momentum distributions of neutrinos and antineutrinos, respectively, normalized so that $\int \frac{d^3\vec{p}}{(2\pi)^3} f_{\nu(\overline{\nu})}(\vec{p})$ gives their number density. In what follows we show that such a modified propagator does not *always* correctly quantify the effect of the neutrino background in the evaluation of the 2-ν-exchange potential. This is true, in particular, when the finite width of the wavepackets of the neutrinos of the medium are considered.

Our first observation is that the derivation of the background-modified fermion propagator in TQFT (eq. (2.1)) relies on the assumption that the fermions in the background are in thermal equilibrium, while neither the present CνB, nor the neutrino fluxes from the Sun, SN, or nuclear reactors are in thermal equilibrium close to Earth. Interestingly, one can still use QFT techniques to show that eq. (2.1) may be used for the neutrino propagator in the presence of *a class of* backgrounds, see [30]. This derivation, however, implicitly assumes that the neutrino background is well-described by an incoherent superposition of plane waves. Neither the present CνB, nor the neutrino fluxes from the Sun, SN, or nuclear reactors can be well-described as plane waves at *all distance scales*. Even more, on general grounds, the Pauli exclusion principle forbids a superposition of fermionic wavepackets with equal momenta over scales smaller than the size of the wavepacket. This

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results on constraints on the number density and energy distribution of a background of fermions at those distance scales.

We proceed now to evaluating the 2-ν-exchange in the presence of realistic neutrino backgrounds, accounting for the finite width of the wavepackets describing the background state.

3 Neutrino-mediated forces in a neutrino background: formalism

The static interaction potential plays a central role in non-relativistic scattering in quantum mechanics as well as in describing classical forces. To obtain it given a relativistic QFT, recall that for an interaction between two fermions f_1 and f_2 located at positions $\vec{r}_1$ and $\vec{r}_2$, the aforementioned potential reads, [33]

$$V(\vec{r} \equiv \vec{r}_1 - \vec{r}_2) = -\int d^3\vec{q}\, e^{i\vec{q}\cdot\vec{r}}\, T^{f_1 f_2}_{\rm NR}(\vec{q}) \,, \tag{3.1}$$

where $T^{ff'}_{\rm NR}(\vec{q})$ can be obtained from the fully relativistic scattering amplitude starting with the $\mathcal{S}$ matrix for the transition $|I\rangle \to |F\rangle$ computed in QFT in momentum space

$$\mathcal{S}_{IF} = \delta_{IF} + i(2\pi)^4 \delta^4(p_F - p_I) T^{f_1 f_2}_{IF} \,, \tag{3.2}$$

as

$$T^{f_1 f_2}_{\rm NR}(\vec{q}) \equiv \frac{T^{f_1 f_2}_{IF}}{4\, m_f\, m_{f'}} \,, \tag{3.3}$$

with the four-momentum transfer between the fermions given by $q \equiv (0, \vec{q}) \equiv p_1' - p_1 \equiv p_2 - p_2'$, and the helicities of f_1 and f_2 not changing in the process.

In the SM the relevant weak interaction Lagrangian leading to the 2-ν mediated potential can be written in the effective four-fermion interaction approximation as

$$\mathcal{L}_{\mathcal{I}} = -\frac{G_F}{2\sqrt{2}} \bar{\psi}_{\nu_i} \gamma^\mu (1-\gamma_5) \psi_{\nu_j} \bar{\psi}_f \gamma_\mu (g^f_{V_{ij}} - g^f_{A_{ij}} \gamma_5) \psi_f \,, \tag{3.4}$$

where ν_i are the neutrino fields with masses m_{ν_i} and f indicates the fermions in the external legs. The effective couplings are

$$g^e_{V_{ij}} = 2U_{ei}U_{ej} - (1 - 4s_w^2)\delta_{ij} \,, \tag{3.5}$$

$$g^p_{V_{ij}} = (1 - 4s_w^2)\delta_{ij} \,, \tag{3.6}$$

$$g^n_{V_{ij}} = -\delta_{ij} \,, \tag{3.7}$$

$$g^e_{A_{ij}} = g^n_{A_{ij}} = -g^p_{A_{ij}} = -\delta_{ij} \,. \tag{3.8}$$

In what follows, for simplicity we neglect leptonic mixing and assume Dirac neutrinos. Mixing effects and Majorana neutrinos do not affect our conclusions and can be easily accounted for.

The leading contribution to the 2-ν exchange potential generated in vacuum can be obtained from the amplitude represented by the Feynman diagram in figure 1. In the

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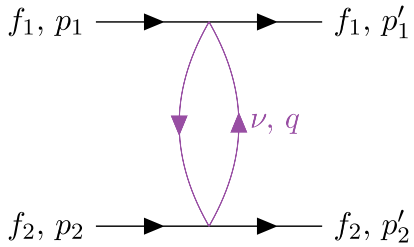

Figure 1. Feynman diagram contributing to the relevant scattering amplitude in vacuum.

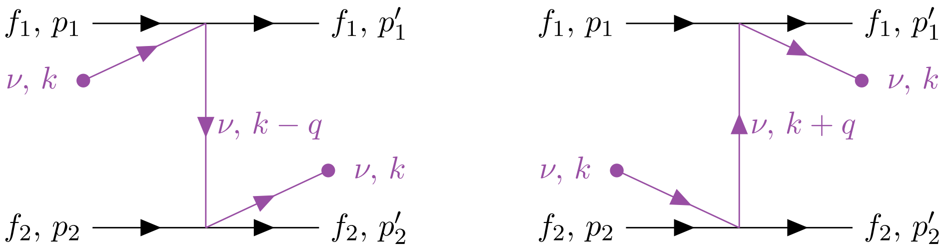

Figure 2. Feynman diagrams contributing to the relevant scattering amplitudes in the presence of a neutrino background.

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above notation, it corresponds to initial and final states $|I\rangle \equiv |p_1, s_1\rangle_{f_1} |p_2, s_2\rangle_{f_2}$ and $|F\rangle \equiv |p'{}_1, s_1\rangle_{f_1} |p'{}_2, s_2\rangle_{f_2}$, where p_i are the initial fermion 4-momenta (with $p_i^0 = \sqrt{|\vec{p}_i|^2 + m_i^2}$) and s_i their helicities. For massless neutrinos this leads to the well-known result

$$V(r) = -\frac{G_F^2\, g_V^{f_1}\, g_V^{f_2}}{16\,\pi^3}\frac{1}{r^5}\,. \tag{3.9}$$

In the presence of a neutrino background new diagrams contribute to the amplitude at the same order in perturbation theory. More concretely, there is a process capturing a neutrino from the background with momentum $\vec{k}$, exchanging a virtual neutrino, and returning another neutrino with momentum $\vec{k}$ as represented in figure 2 (below, we justify that both background neutrinos have the same momentum). The contributions from an antineutrino background can be trivially obtained by inverting the neutrino lines.[1]

Being of the same order in perturbation theory, these contributions should be taken into account for computing the potential in any environment where neutrinos are present. As mentioned above, the neutrino backgrounds that we shall consider are far from being in thermal equilibrium, and may even entail momentum distributions far from thermal. This is why the use of TQFT techniques is not justified *a priori*, and we will study the influence of the background explicitly.

[1]Notice that a diagram where both a neutrino and an antineutrino are absorbed or emitted would modify the fermion energy and hence it does not contribute to the static potential. This is also true for other scattering processes beyond those we consider in figure 2, and we ignore them.

The basic assumption in what follows is that the background can be well described by an incoherent superposition of neutrino wavepackets of the form[2]

$$|\psi(\vec{x},\vec{k}',s)\rangle = \int \frac{d^3\vec{k}}{(2\pi)^3\sqrt{2E_k}}\omega(\vec{k}',\vec{k})e^{-i\vec{k}\cdot\vec{x}}\,|\vec{k},s\rangle_\nu\ , \tag{3.10}$$

where $\omega(\vec{k}',\vec{k})$ are neutrino wavepackets centered at momentum of $\vec{k}'$ and with helicity s. The exponential factor centers the packet at position $\vec{x}$. In what follows, for concreteness in our quantifications we use Gaussian wavepackets,

$$|\omega(\vec{k}',\vec{k})|^2 = \frac{1}{\pi^{3/2}\sigma^3}e^{-\frac{|\vec{k}'-\vec{k}|^2}{\sigma^2}}\ , \tag{3.11}$$

with σ the width in momentum space (unlike the position-space width, that increases with time for free particles, this stays constant when the particles evolve freely).

To account for the stochastic properties of the medium, we parametrize it in terms of a density matrix. Assuming that the background is homogeneous (this can be realized by large enough densities over the probed distances or by averaging the force over a long enough time),

$$\begin{aligned}\rho^{bkg} &= \sum_s \int d^3\vec{k}' f_\nu(\vec{k}') \int d^3\vec{x}\,|\psi(\vec{x},\vec{k}',s)\rangle\,\langle\psi(\vec{x},\vec{k}',s)| \\ &= \sum_s \int d^3\vec{k}' f_\nu(\vec{k}') \int \frac{d^3\vec{k}}{(2\pi)^3\sqrt{2E_k}}|\omega(\vec{k}',\vec{k})|^2\,|\vec{k},s\rangle_\nu\,\langle\vec{k},s|_\nu\end{aligned} \tag{3.12}$$

with $f_\nu(\vec{k}')$ the momentum distribution of the neutrino ensemble which we assume to be spin-independent. For polarized ultrarelativistic backgrounds, such as solar or reactor neutrinos, only one of the helicities contributes, and hence this density matrix is equally applicable. As we see, when computing averages both background neutrinos will have the same momentum.

Before proceeding to the calculation of the potential, it is worth pointing out that the Pauli exclusion principle imposes relevant constraints on the possible form of a state composed of identical fermions. For a thermal neutrino background for which $f_\nu(\vec{k}')$ is a Dirac-Fermi distribution, eq. (3.12) is fully consistent for any width of the wave packet. On the contrary, for a non-thermal neutrino background, characterized by a general momentum distribution *with arbitrary normalization*, the previous description of the background as an incoherent sum of *single-particle states* is only physical for interparticle distances larger that the spatial extent of the wavepackets. For interparticle distances shorter than the extent of the wavepacket, the Pauli exclusion principle renders the assumed momentum distribution inconsistent. That said, in the physical scenarios we consider the background densities (ie the normalization of the f_ν) are low enough, and the interparticle distances are always larger than wavepacket extents. So in what follows we are quantifying exclusively the effects associated with the inclusion of the wavepacket, under the assumption that the

[2]One can add a random phase at each momentum state, but it does not play any role for the final observable and we ignore it.

normalization of the momentum distribution is consistent with the Pauli exclusion principle over the distances studied.

In this approach, we first compute the amplitudes in figure 2 and we later trace out the result with the state that describes the background, eq. (3.12). This corresponds to the following initial and final states in the evaluation of the amplitude

$$|I\rangle = |p_1, s_1\rangle_{f_1} |p_2, s_2\rangle_{f_2} |k, s\rangle_\nu \,, \tag{3.13}$$

$$|F\rangle = |p_1', s_1\rangle_{f_1} |p_2', s_2\rangle_{f_2} |k, s\rangle_\nu \,. \tag{3.14}$$

After some rearrangements, the corresponding $\mathcal{S}$ matrix element can be written as,

$$\begin{aligned}\mathcal{S}_{IF} = &-\frac{G_F^2}{8}\int d^4x_1\, d^4x_2\; {}_{f_1}\langle p_1', s_1|\, \bar\psi_f(x_1)\gamma_\mu(g_V^f - g_A^f\gamma_5)\psi_f(x_1)\, |p_1, s_1\rangle_{f_1} \times \\ &{}_{f_2}\langle p_2', s_2|\, \bar\psi_f(x_2)\gamma_\nu(g_V^f - g_A^f\gamma_5)\psi_f(x_2)\, |p_2, s_2\rangle_{f_2} \times \\ &\Big[{}_\nu\langle k, s|\, \bar\psi_\nu(x_1)\gamma^\mu(1-\gamma_5)\, iS_F(x_1 - x_2)\,\gamma^\nu(1-\gamma_5)\psi_\nu(x_2)\, |k, s\rangle_\nu + \\ &{}_\nu\langle k, s|\, \bar\psi_\nu(x_2)\gamma^\nu(1-\gamma_5)\, iS_F(x_1 - x_2)\,\gamma^\mu(1-\gamma_5)\psi_\nu(x_1)\, |k, s\rangle_\nu\Big]\,,\end{aligned} \tag{3.15}$$

where $S_F(x_1 - x_2)$ is the vacuum Feynman neutrino propagator in position space.

Fourier-expanding the fermion fields in terms of creation and annihilation operators with well-determined momenta and using the Fourier-space neutrino propagator, one can easily compute the expectation values. After factorizing out the global energy-momentum conservation Dirac delta, $i\,(2\pi)^4\delta^4(p_F - p_I) = i\,(2\pi)^4\delta^4(p_1 + p_2 - p_1' - p_2')$, and taking the non-relativistic limit we get,

$$\begin{aligned}\sum_s T_{NR}^{f_1 f_2\,\nu} = -G_F^2 g_V^{f_1} g_V^{f_2}\Bigg(&\int d^4x \int \frac{d^4q'}{(2\pi)^4}\,\frac{[2k^0q^0 - \vec{k}\cdot\vec{q}']}{q'^2 - m^2}\, e^{-ix(q-q'+k)} \\ &+ \int d^4x \int \frac{d^4q'}{(2\pi)^4}\,\frac{[2k^0q^0 - \vec{k}\cdot\vec{q}']}{q'^2 - m^2}\, e^{-ix(q+q'-k)}\Bigg)\,,\end{aligned} \tag{3.16}$$

where the superindex ν indicates that the background neutrino state is still not traced out. Using eq. (3.1) and computing the trace with the density matrix in eq. (3.12), we arrive at the following form for the potential between the two fermions,[3,4]

$$\begin{aligned}V_\sigma^{\rm bkg}(r) = G_F^2 g_V^{f_1} g_V^{f_2} &\int d^3\vec{q}\, e^{i\vec{q}\cdot\vec{r}} \int \frac{d^3\vec{k}'}{(2\pi)^3}\, f_\nu(\vec{k}') \int \frac{d^3\vec{k}}{(2\pi)^3 2E_k}\, |\omega(\vec{k}', \vec{k})|^2 \\ &\times \left[\frac{2|\vec{k}|^2 + m^2 + \vec{k}\cdot\vec{q}}{|\vec{k}|^2 + 2\vec{k}\cdot\vec{q}} + \frac{2|\vec{k}|^2 + m^2 - \vec{k}\cdot\vec{q}}{|\vec{k}|^2 - 2\vec{k}\cdot\vec{q}}\right].\end{aligned} \tag{3.17}$$

[3]Notice that the results only depend on the squared modulus of the momentum-space wavepacket$|\omega(k, k')|^2$. They are hence unaffected by position-space broadening induced by the past time evolution.

[4]We notice in passing that this wavepacket-size effect on the *background-mediated* potential, does not occur for *background-sourced* potentials, like the MSW potential for example, because in such cases the potential probes the background particle fields in a single point (while the background-mediated potential probes the background particle fields at two points, see (3.15)). Consequently if one evaluates the relevant matrix element using a density matrix (3.12) the result is independent of $\vec{k}$, the integral over $d^3\vec{k}$ is immediate, and any dependence on ω disappears from the expressions.

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We find that the same result could have been obtained starting with the amplitude corresponding to the diagram in figure 1 and using an *effective neutrino propagator in the neutrino medium* given by

$$S_F(p) = (\not{p}+m)\left\{\frac{i}{p^2-m^2+i\epsilon} \; - 2\,\pi\,\delta(p^2-m^2) \right. \\ \left. \times\left[\theta(p^0)\int d^3\vec{p}\,'|\omega_\nu(\vec{p},\vec{p}\,')|^2 f_\nu(\vec{p}\,') + \theta(-p^0)\int d^3\vec{p}\,'|\omega_\nu(\vec{p},\vec{p}\,')|^2 f_{\overline{\nu}}(\vec{p}\,')\right]\right\}\;, \tag{3.18}$$

which reproduces the thermal propagator eq. (2.1) in the $\sigma\to 0$ limit.

In fact, we can formally recover the results in the literature from eq. (3.17) by taking the limit of neutrino wavepackets to be plane waves

$$\lim_{\sigma\to 0}\,|\omega(\vec{k}',\vec{k})|^2 = \delta^3(\vec{k}'-\vec{k})\;. \tag{3.19}$$

In particular, if $f(\vec{k}')$ is a thermal distribution we recover the results obtained for the CνB [27–30]. Notice however, that as discussed above, the Pauli exclusion principle makes this limit unphysical for the non-thermal neutrino backgrounds from the Sun, nuclear reactors, or SN.

4 Results and conclusions

From the previous expressions, one can extract the corrections of the width of wavepackets in the 2-ν exchange potential. Although the integrals in eq. (3.17) cannot be performed analytically for most $f_\nu(\vec{p})$ distributions, the basic lessons can be extracted from the case of a monochromatic directional flux Φ_0

$$f_\nu(\vec{p}) = (2\pi)^3\delta^3(\vec{p}-\vec{p}_0)\Phi_0\,, \tag{4.1}$$

with $\vec{p}_0$ parallel to $\vec{r}$, which allows for an analytic treatment. This example can be used to estimate the modification of the potential due to solar, SN, and reactor neutrinos. Furthermore, *this is the scenario for which the largest background-induced enhancement is obtained in ref. [30]*. For this case, and neglecting the neutrino mass, we find for $E_\nu = |\vec{k}_0|$

$$V_\sigma^{\rm bkg}(r) = -\frac{G_F^2\, g_V^{f_1}\, g_V^{f_2}}{4\,\pi^{3/2}\,r^2}\Phi_0\frac{1}{(4\hat{E}^2+\hat{\sigma}^4)^2}\left\{-2e^{-\frac{\hat{E}_\nu^2}{\hat{\sigma}^2}}\hat{\sigma}(\hat{\sigma}^2-2\hat{E}^2)(4\hat{E}^2+\hat{\sigma}^4)\right. \\ +\sqrt{\pi}\left[2\hat{E}\left(8\hat{E}^4+2(1+\hat{E})^2\hat{\sigma}^4+\hat{\sigma}^6\right)\mathrm{Erf}\left[\frac{\hat{E}}{\hat{\sigma}}\right]\right. \\ +e^{-\hat{\sigma}^2}\left(8\hat{E}^4+\hat{\sigma}^6+2\hat{\sigma}^8+\hat{E}^2(6\hat{\sigma}^4-4\hat{\sigma}^2)\right)\left(\cos(2\hat{E})\mathrm{Im}(X)+\sin(2\hat{E})\mathrm{Re}(X)\right) \\ \left.\left.+e^{-\hat{\sigma}^2}\left(4\hat{E}\hat{\sigma}^2(2\hat{E}^2+\hat{\sigma}^2+\hat{\sigma}^4)\right)\left(\cos(2\hat{E})\mathrm{Im}(X)-\sin(2\hat{E})\mathrm{Re}(X)\right)\right]\right\}\,, \tag{4.2}$$

where for convenience we have introduced the dimensionless variables $\hat{E}\equiv E_\nu\,r$ and $\hat{\sigma}\equiv\sigma\,r$ (with $r=|\vec{r}|$), and $X\equiv\mathrm{Erf}\left[\frac{\hat{E}^2}{\hat{\sigma}^2}+i\,\hat{\sigma}\right]$.

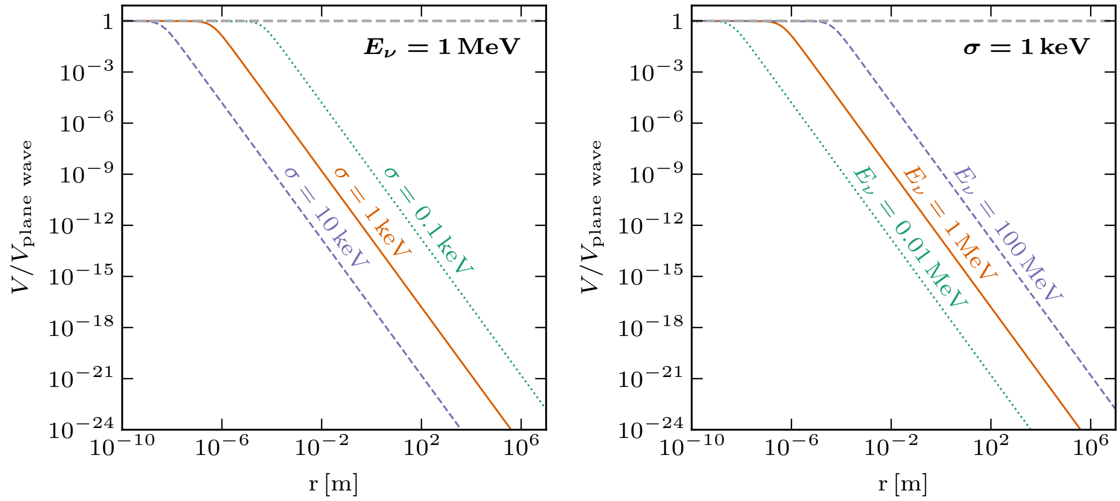

Figure 3. Suppression of the background-induced 2-ν-mediated potential between two fermions due to the inclusion of a finite width packet of the background states as a function of the distance between the fermions. The figures illustrate the dependence of the effect on the neutrino energy and the width of the wavepacket.

The comparison of the potential in eq. (4.2) with the expression obtained for $\sigma \to 0$

$$V^{\rm bkg}_{\sigma\to 0}(r) = -\frac{G_F^2\, g_V^{f_1}\, g_V^{f_2}}{4\pi\, r}\Phi_0 \left[E_\nu + \frac{\sin(2E_\nu r)}{2r}\right], \tag{4.3}$$

is shown in figure 3, where we plot $V^{\rm bkg}_{\sigma}(r)/V^{\rm bkg}_{\sigma\to 0}(r)$ as a function of r for energies around 1 MeV, characteristic of solar, SN, and reactor neutrinos; and different wavepacket widths. As we see from the figure, including the wavepacket width suppresses the effect of the background by orders of magnitude for distances probed by the most sensitive current experiments (which range from μm to $\sim 10^4$ km).

For widths $\sigma \ll E_\nu$ and distances $r \gg \frac{1}{E_\nu}$, eq. (4.2) is well approximated by

$$V^{\rm bkg}_{\sigma}(r) \simeq \frac{V^{\rm bkg}_{\sigma\to 0}(r)}{1 + r^2\sigma^4/(4E_\nu^2)}. \tag{4.4}$$

Although in figure 3 we use the full expression, this approximation is excellent. From the figure one reads that to have an unsuppressed potential due to a background of neutrinos of energies $\mathcal{O}$(MeV) over a distance of μm, the background neutrino states should have momentum widths $\sigma \ll 10\,$keV. For longer distances, $\sim 10^4\,$km, the width has to be even smaller, $\sigma \ll$ meV.

Estimations on the typical wavepacket sizes for neutrino backgrounds differ in the literature. For thermal systems with temperature T, collisions act as quantum-mechanical measurements that localize the particles, and $\sigma \sim T$ [34]. This would lead to $\sigma \sim$ keV for solar neutrinos (assuming that neutrinos inherit the wavepacket width of their parent nuclei), and $\sigma \sim$ MeV for SN neutrinos [34]. This implies that for a solar (SN) neutrino of 1 MeV, the potential is suppressed over distances larger than $\sim 10^{-6}$ m ($\sim 10^{-12}$ m). For reactor neutrinos, estimates are more uncertain, ranging from $\sim 20\,\text{MeV}$ to $\sim 1\,\text{eV}$ [34–38], with corresponding suppression of the potential from neutrinos of 1 MeV over distances larger than $\sim 10^{-14}$ m to $\sim$ m.

Overall, we find that the long-discussed 2-ν mediated force remains undetectable for fifth-force experiments and the discussed neutrino backgrounds. The effects only seem to be unsuppressed at extremely short distances (cf. figure 3), making it difficult to envision physical scenarios able to probe neutrino-induced long-range forces.

Note added. After our paper appeared on arXiv, ref. [30] updated their calculation to include spectral smearing effects. We emphasize that the spectral smearing they consider is physically different from the quantum-mechanical wavepacket size effect we discuss. In particular, after including spectral smearing they still find an enhancement in the 2-ν exchange potential for directional fluxes in directions very close to the incoming flux (in our notation, $\vec{p}_0$ parallel to $\vec{r}$). Our results show that the wavepacket width suppresses the potential irrespective of the direction. Therefore, contrary to their statement, we do not agree with their results.

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