

Exceptional Point of Degeneracy as a Desirable Point of Operation for Oscillator With Discrete Nonlinear Gain and Radiating Elements

Tarek Mealy, Alireza Nikzamir, Ahmed F. Abdelshafy, and Filippo Capolino

Abstract—An oscillator made of a periodic waveguide comprising of uniform lossless segments with discrete nonlinear gain and radiating resistive elements prefers to operate at exceptional point of degeneracy (EPD). The steady-state regime is an EPD with π phase shift between unit cells, for various choices of small signal gain of the nonlinear elements and number of unit cells. We demonstrated this fact by monitoring both current and voltage across each nonlinear gain element and finding its effective admittance at the oscillating frequency and checking the degeneracy of the eigenmodes at such point. The EPD studied here is very promising for many applications that incorporate discrete distributed coherent sources and radiation-loss elements. Operating in the vicinity of such special degeneracy conditions may lead to potential performance enhancement in the various microwave, THz and optical systems with distributed gain and radiation, paving the way for a new class of active integrated antenna arrays and radiating laser arrays.

Index Terms—Periodic structures, Oscillators, Exceptional points, Dispersion engineering, Transmission line.

I. INTRODUCTION

We focus on exceptional points of degeneracy (EPDs) in waveguides, hence eigenvalues represent modal wavenumbers and eigenvectors represent polarization states, that coalesce at an EPD.

In general, an EPD occurs in systems that are periodic in space [1]–[3] or in time [4], [5], or by having losses and gain in the system [3], [6] including systems satisfying parity-time (PT) symmetry [7]–[9].

The general conditions of EPD for a uniform waveguide periodically loaded with discrete gain and loss was studied in [10]. It was shown that because of those structures can be used as arrays of active antennas, all oscillating at a given frequency. Here, we show that the EPD with small gain is the desired point of the operation, and the system always tends to work at that point when at steady-state, i.e., after saturation, independently from the small-signal gain of the active elements.

II. DISTRIBUTED EPD OSCILLATOR

We consider an oscillator made of the finite-length loaded cavity comprised of 8 unit cells as shown in Fig. 1(a). We

This material is based upon work supported by the National Science Foundation under Grant Number ECCS-1711975, and by Northrop Grumman. (Corresponding author: Filippo Capolino)

The authors are with the Department of Electrical Engineering and Computer Science, University of California at Irvine, Irvine, CA 92697 USA (e-mail: f.capolino@uci.edu).

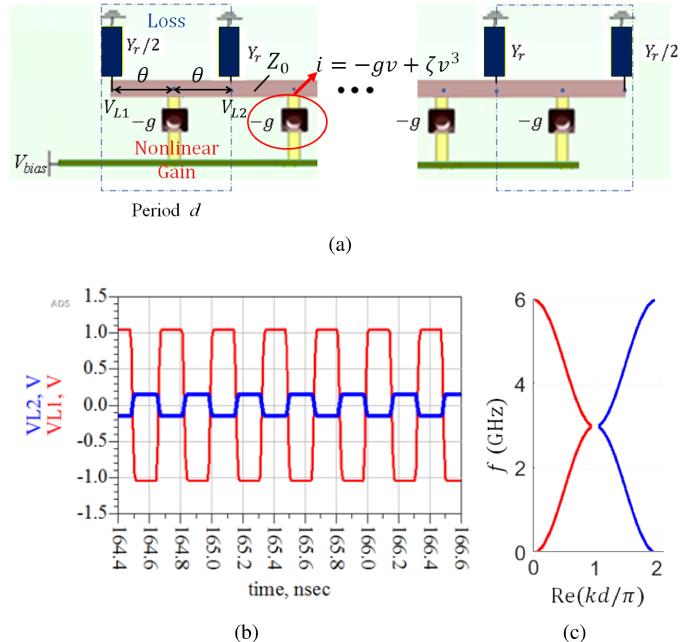


Fig. 1. Distributed EPD oscillator made of 8 cascaded unit cells (UCs) loaded with gain and radiating antennas (modeled as resistors). Active gain devices are placed in each UC, from the TL to the bias line (that acts as a ground for a.c. signals). (b) Voltage waveform at the first two UCs showing a phase shift of π . The oscillation occurs at 3 GHz, that corresponds to the EPD frequency. The shown results are for the case where $Y_r Z_0 = 2.5$ and $g Z_0 = 1.6$. The effective gain, after saturation, was found to be $g_{eff} Z_0 = 0.02$. (c) Dispersion diagram is calculated at steady-state, with $g_{eff} Z_0 = 0.02$. The figure shows that the steady-state operational point of the oscillator is an EPD at $kd = \pi$ which explains the waveform phase shift in (b).

performed time domain simulations using Cadence Virtuoso IC 616. The unit cell of the finite structure is chosen to have identical ideal TL segments with characteristic impedance $Z_0 = 50 \Omega$ and each has an electric length $\theta(3\text{GHz}) = \pi/2$. The loss elements are chosen as $Y_r Z_0 = 2.5$. The gain element is modeled via non-linear cubic i - v characteristic $i(t) = -gv(t) + \zeta v^3(t)$ of the active device [11], [12]. Here $-g$ is the small-signal slope of the i - v curve in the negative conductance region, and ζ is the third-order nonlinearity constant that models the saturation characteristic of the device. We set the turning point $v_b = \sqrt{g/(3\zeta)}$ of the i - v characteristics to be 1 volt, and accordingly, we set $\zeta = g/3$.

The effective value of the gain element is calculated using the Fourier transform of voltage and current signals at the gain element at steady state. The effective value of the gain element

evaluated after reaching saturation is not necessarily equal to the small-signal one (before reaching saturation), because of the cubic nonlinear elements. Next, the characteristics of the eigenmodes of the loaded TL are determined by using the effective gain value at steady state and not by the small-signal one.

A system state vector is defined as $\Psi(z) = [V(z), Z_0 I(z)]^T$, with T indicating the transpose action. We write the evolution equation of state vector as $\Psi(z + d) = \mathbf{T}_U(z)$, where the supported modes by the guiding structure are found by satisfying the Floquet's condition $\Psi(z + d) = e^{-jkd}\Psi(z)$, where d is the waveguide period, k is the Floquet-Bloch wavenumber, and a time convention of $e^{j\omega t}$ is implicitly assumed. The eigenmodes are found by solving the eigenvalue problem

$$[\mathbf{T}_U - \lambda \mathbf{I}] \Psi = \mathbf{0}, \quad (1)$$

where \mathbf{I} is the identity matrix, $\lambda = e^{-jkd}$ is an eigenvalue and Ψ is the associated eigenvector.

A "coalescence parameter" C is defined as a figure of merit to assess how close the steady-state operational point is to the EPD through observing the degree of coalescence of the system's eigenvectors. The coalescence parameter C is defined as

$$C = |\sin(\theta)|, \cos \theta = \frac{|\langle \Psi_1, \Psi_2 \rangle|}{\|\Psi_1\| \|\Psi_2\|}, \quad (2)$$

where the $\cos \theta$ is defined via the inner product $\langle \Psi_1, \Psi_2 \rangle$ of the two coalescing eigenvectors.

Multiple time domain simulations were performed using different number of unit cells and different small-gain values g . An oscillation frequency of 3 GHz was always observed. We show for example the resulting waveform in Fig. 1(b) for the case where we used 8 unit cells and $gZ_0 = 1.6$. For this case, after saturation, the effective gain of the active element in the middle of the structure was found to be $g_{eff}Z_0 = 0.02$ which is very far away from the small-signal one. Analogous scenarios with different number of unit cells and different gain values g were tested and interestingly the steady-state operational point was always found to be very close to EPD shown in Fig. 2 which corresponds to $f = 3$ GHz and $gZ_0 = 0$. We concluded that that the shown EPD is a desirable point where oscillations can be sustained using nonlinear gain element. Such property can be very useful to implement oscillators with very stable oscillation frequency accounting also for tolerances of the active elements.

In summary, we have demonstrated, via simulations, that an EPD is a preferable point of operation for oscillators made of cascaded unit cells with discrete nonlinear gain and radiation loss. The simulation results show that the system, after saturation, tends to work at this EPD independently of the number of unit cells and the small-signal gain value. Such property can be very useful to implement oscillators with very stable oscillation frequency despite the unavoidable presence of tolerance variations of the gain elements.

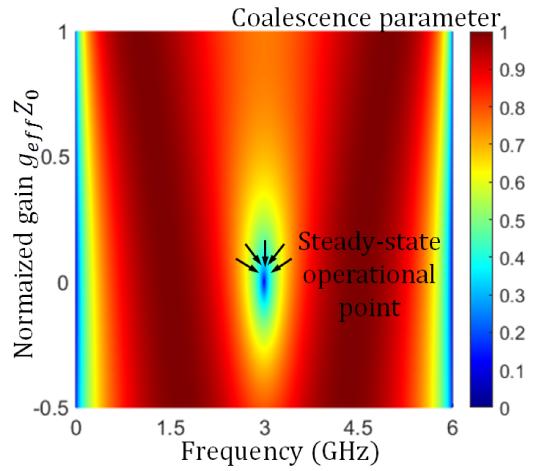


Fig. 2. Coalescence parameter calculated at different frequencies and effective gain value (after saturation) showing the occurrence of EPD at point where $f = 3$ GHz and $gZ_0 = 0$. Time-domain simulations showed that the system prefers to work very close to this point despite the value of the small-signal gain element value g .

REFERENCES

- [1] A. Figotin and I. Vitebskiy, "Gigantic transmission band-edge resonance in periodic stacks of anisotropic layers," *Physical Review E*, vol. 72, no. 3, p. 036619, Sep. 2005.
- [2] M. A. K. Othman, X. Pan, Y. Atmatzakis, and C. G. Christodoulou, "Experimental Demonstration of Degenerate Band Edge in Metallic Periodically-Loaded Circular Waveguide," *IEEE Microwave Theory and Techniques*, vol. 1611, no. 8, p. 9, 2017.
- [3] A. F. Abdelshafy, M. A. K. Othman, D. Oshmarin, A. Al-Mutawa, and F. Capolino, "Exceptional Points of Degeneracy in Periodically-Coupled Waveguides and the Interplay of Gain and Radiation Loss: Theoretical and Experimental Demonstration," *IEEE Transactions on Antennas and Propagation*, vol. 67, no. 09, 2019.
- [4] H. Kazemi, M. Y. Nada, T. Mealy, A. F. Abdelshafy, and F. Capolino, "Exceptional Points of Degeneracy Induced by Linear Time-Periodic Variation," *Physical Review Applied*, vol. 11, no. 1, p. 014007, Jan. 2019.
- [5] K. Rouhi, H. Kazemi, A. Figotin, and F. Capolino, "Exceptional Points of Degeneracy Directly Induced by Space-Time Modulation of a Single Transmission Line," *arXiv:2004.11423 [physics]*, Apr. 2020, arXiv: 2004.11423.
- [6] M. A. K. Othman and F. Capolino, "Theory of Exceptional Points of Degeneracy in Uniform Coupled-Waveguides and Balance of Loss and Gain," *IEEE Transactions on Antennas and Propagation*, vol. 65, no. 10, pp. 1–15, 2017.
- [7] W. D. Heiss, "The physics of exceptional points," *Journal of Physics A: Mathematical and Theoretical*, vol. 45, no. 44, p. 444016, 2012.
- [8] C. E. Rüter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev, and D. Kip, "Observation of parity-time symmetry in optics," *Nature Physics*, vol. 6, no. 3, pp. 192–195, 2010.
- [9] A. Guo, G. J. Salamo, D. Duchesne, R. Morandotti, M. Volatier-Ravat, V. Aimez, G. A. Siviloglou, and D. N. Christodoulides, "Observation of PT-symmetry breaking in complex optical potentials," *Physical Review Letters*, vol. 103, no. 9, p. 093902, 2009.
- [10] A. F. Abdelshafy, T. Mealy, E. Hafezi, A. Nikzamir, and F. Capolino, "Exceptional degeneracy in a waveguide periodically loaded with discrete gain and radiation loss elements," *Applied Physics Letters*, vol. 118, no. 22, p. 224102, 2021.
- [11] D. Oshmarin, F. Yazdi, M. A. Othman, J. Sloan, M. Radfar, M. M. Green, and F. Capolino, "New oscillator concept based on band edge degeneracy in lumped double-ladder circuits," *IET Circuits, Devices & Systems*, vol. 13, no. 7, pp. 950–957, 2019.
- [12] A. F. Abdelshafy, D. Oshmarin, M. A. K. Othman, M. M. Green, and F. Capolino, "Distributed degenerate band edge oscillator," *IEEE Transactions on Antennas and Propagation*, vol. 69, no. 3, pp. 1821–1824, 2021.