Highly Sensitive Exceptional Degeneracy in Coupled Transmission Lines With Balanced Gain and loss

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***Abstract*—We proposed a highly sensitive circuit scheme based on an exceptional point of degeneracy (EPD) using two finite-length coupled transmission lines terminated on balanced gain and loss. EPD is a point in a system's parameter space in which two or more eigenmodes coalesce in both their resonance frequency and eigenvectors into a single degenerate eigenmode by varying the system's parameter. We demonstrate that two PT-symmetric finite-length coupled transmission lines (CTLs), can generate an EPD at a desired frequency. We find the EPDs in this circuit and the bifurcation diagram that exhibits the ultra-sensitivity behavior to the system's perturbations. The very high sensitivity induced by an EPD can be used to conceive a new generation of high-sensitive sensors.**

I. Introduction

Recent advancements in the exceptional point of degeneracy (EPD) concepts have attracted a surge of interest due to their potential in sensing applications [1]–[4]. Most of the literatures on EPDs are related to parity-time (PT) symmetry [5], [6]. However, EPD is found in more general configurations which do not require a system to satisfy PT-symmetry [7], [8]. An EPD of order two is the full degeneracy of two eigenmodes. For instance, at an EPD, the system matrix is similar to a non-trivial Jordan block matrix. In recent years frequency splitting phenomena at EPDs have been proposed for sensing applications using PT symmetry [5], [6], and also a time-varying circuit in resonators [1] and a single transmission line [4]. Frequency splitting occurs at the EPD, significantly boosts its sensitivity performance to a tiny perturbation δ leads to a significant shift, square root behavior $\sqrt{\delta}$ in resonance frequencies of the system.

This paper presents a theoretical investigation of EPDs in two PT-symmetric finite-length *coupled* transmission lines (CTLs). The two CTLs are terminated with balanced loss (on the bottom right, Fig. 1) and gain (on the top left, Fig. 1). Moreover, we show the circuit high sensitivity (square root behavior) to a component perturbation and determine that the Puiseux fractional power series expansion [8] approximates the bifurcation of the frequency diagram around the exceptional point. This concept can be applied in current sensing devices with a high-sensitivity characteristic to sense chemical or physical changes.

II. EPD in CTLs

Two lossless coupled transmission lines with a finite length of $d = 40.15$ mm where terminated on the top left transmission line with a gain and terminated on the bottom right with loss, as shown in Fig. 1. The distributed inductance and capacitance of the transmission lines are

$$\underline{\mathbf{L}} = \begin{bmatrix} L_0 & L_m \\ L_m & L_0 \end{bmatrix}, \ \underline{\mathbf{C}} = \begin{bmatrix} C_0 + C_m & -C_m \\ -C_m & C_0 + C_m \end{bmatrix}, \qquad (1)$$

where $L_0 = 480$ nH, $C_0 = 579$ pF are the isolated, and $L_m = 367.4$ nH and $C_m = 102.7$ pF are coupling per-unit-length capacitance and inductance of TLs.

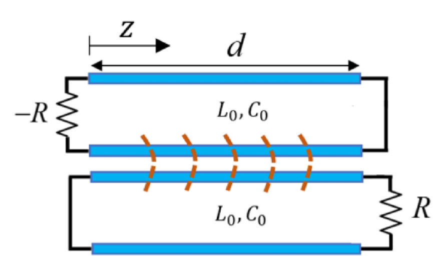

Fig. 1. Two finite-length CTLs terminated with gain and loss which CTLs are both electrically and magnetically coupled.

This structure has four different propagating modes with propagation constants $k_e = \pm\omega/u_e$ and $k_o = \pm\omega/u_o$ where $u_e = 1/\sqrt{(L_0 + L_m)C_0}$ and $u_o = 1/\sqrt{(L_0 - L_m)(C_0 + 2C_m)}$ are the phase velocities of the *even* and *odd* modes. It is convenient to define the voltage amplitude vector as $\mathbf{V} \equiv [V_e^+, V_e^-, V_o^+, V_o^-]$, which consists of the forward and backward of *even* and *odd* mode of voltages at $z = 0$. In order to derive the resonance frequencies for the two finite-length CTLs shown in Fig. 1, we enforce the boundary conditions at the four ports of the structure. Thus, we obtain the homogeneous linear system

$$\underline{\mathbf{A}}(\omega)\mathbf{V} = 0, \quad \underline{\mathbf{A}} = \begin{bmatrix} 1 - Y_e R & 1 + Y_e R & 1 - Y_o R & 1 + Y_o R \\ 1 & 1 & -1 & -1 \\ e^{-j\omega d/u_e} & e^{j\omega d/u_e} & e^{-j\omega d/u_o} & e^{j\omega d/u_o} \\ (1 - Y_e R)e^{-j\omega d/u_e} & (1 + Y_e R)e^{j\omega d/u_e} & -(1 - Y_o R)e^{-j\omega d/u_o} & -(1 + Y_o R)e^{j\omega d/u_o} \end{bmatrix}, \qquad (2)$$

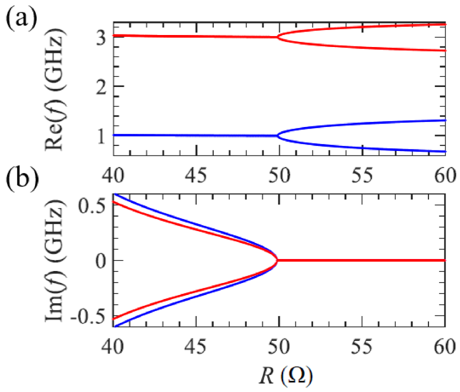

Fig. 2. Variation of the (a) real and (b) imaginary parts of the lowest eigenfrequencies to R variation.

where $Y_e = u_e C_0$ and $Y_o = u_o(C_0 + 2C_m)$ represent the characteristic admittances of the *even* and *odd* modes. We are interested in determining the eigenfrequencies of the circuit. They are calculated as the roots of the vanishing determinant $\det\left(\underline{\mathbf{A}}(\omega)\right) = 0$. In Fig. 2, the complex resonance frequencies are shown varying the resistance (only frequencies with positive real parts are shown). The EPD is referred to with the superscript "e". With these values, $f_e = 1\,\text{GHz}$. For a value $R = R_e = 49.88\,\Omega$ the eigenfrequencies experience a transition from complex to purely real values, following the common bifurcation diagram typical of an EPD perturbation.

As discussed earlier, the eigenfrequencies at EPDs are remarkably sensitive to perturbations of the circuits' parameters. Here we confirm that the sensitivity of a system to a specific change of R (both loss and gain) is boosted by the degeneracy. We define the relative system perturbation of both loss and gain resistance as $\delta_{\mathrm{R}} = (R - R_e)/R_e$, where R_e is the unperturbed parameter that provides the EPD, and R is its perturbated value. In Fig. 3, we show the eigenfrequencies varying δ_{R} obtained by solving the eigenvalue problem corresponding to the perturbed matrix $\underline{\mathbf{A}}$. The diagram exhibits the bifurcation point of an EPD perturbation. Then, we calculate the corresponding perturbed eigenfrequencies using the Puiseux fractional power series expansion truncated at its first-order [10],

$$f_p(\delta_{\mathrm{R}}) \approx f_e + (-1)^p \alpha_1 \sqrt{\delta_{\mathrm{R}}} \qquad (3)$$

where f_p $(p = 1,2)$ are the two perturbed eigenfrequencies near the EPD. Eigenfrequencies are proportional to the square root of the system's perturbation δ_{R}. The plot in Fig. 3 shows the typical high-sensitivity square root characteristics near an EPD.

In conclusion, we have shown the occurrence of EPDs in a pair of finite-length CTLs resonators, terminated with balanced gain and loss. We have demonstrated the system's eigenfrequencies are highly sensitive to perturbation. Furthermore, to implement such a circuit, negative conductance could be designed by simple circuit structures such as a cross-coupled transistor pair or opamp-based circuits. Using the implementation with nonlinear gain, we could achieve the EPD and aforementioned sensitive behavior to sense the local physical or chemical changes.

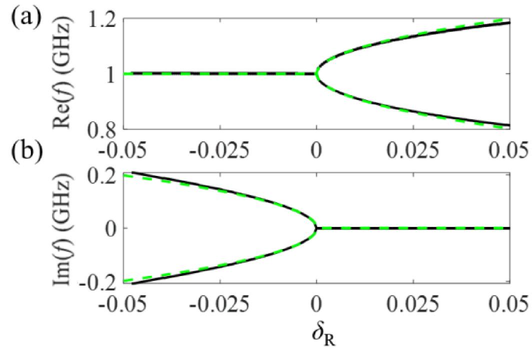

Fig. 3. Variation of the (a) real and (b) imaginary parts of the eigenfrequencies to a resistance perturbation δ_{R}. A solid black line shows the exact value, and the dashed green line shows square root behavior near the EPD using the first-order Puiseux series approximation.

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