

Exceptional Points of Degeneracy in Waveguides with or without Loss and Gain

A. Nikzamir, N. Furman, A. Herrero, T. Mealy, and F. Capolino

Department of Electrical Engineering and Computer Science, University of California, Irvine, CA 92697 USA
anikzami@uci.edu, furmann@uci.edu, aherrero@uci.edu, tmealy@uci.edu, and f.capolino@uci.edu

Abstract – There are two kinds of exceptional points of degeneracy (EPD) in waveguides: those in the absence of loss and gain, related to slow light, and those where the waveguide has distributed gain and/or loss. Here, we discuss both kinds and highlight their differences. We show EPDs of order 2, 3, 4 and 6 in waveguides supporting two or three modes in each direction, and how the coalescence parameter is a good tool to measure the degree of degeneracy by measuring the angle between the eigenvectors (polarization states). In highlighting the differences between the two kinds of EPDs, we also show different sets of applications, which include sensors, delay lines, distributed amplifiers, antennas, and oscillators.

I. INTRODUCTION

An exceptional point of degeneracy (EPD) is a point at which two or more eigenvectors of the system coalesce, besides the eigenvalues [1], [2]. The relationship between angular frequency and wavenumber in proximity of an EPD of order m is $(\omega - \omega_e) \propto (k - k_e)^m$, where subscript e denotes EPD. Examples of waveguides with EPD without loss and gain are in [3], [4], whereas examples with loss and gain are in [5], [6]. Parity time (PT) symmetry is an enabling condition [7]–[10], but it is not necessary to obtain an EPD. Systems working at EPD are highly sensitive to perturbations, which is a useful property for sensing applications. Other applications are high-quality factor cavities, delay lines, lasers with low lasing threshold [11], lasers with high power, distributed amplifiers, high power electron beam devices, etc. Geometries supporting EPDs in the two categories, without loss/gain and with loss/gain, are shown in Fig. 1(a,b) and (c,d), respectively. The EPDs are shown in the corresponding dispersion diagrams in Fig. 2. Finally, we discuss the different applications and the advantages of using an EPD, with some examples summarized in Fig. 3.

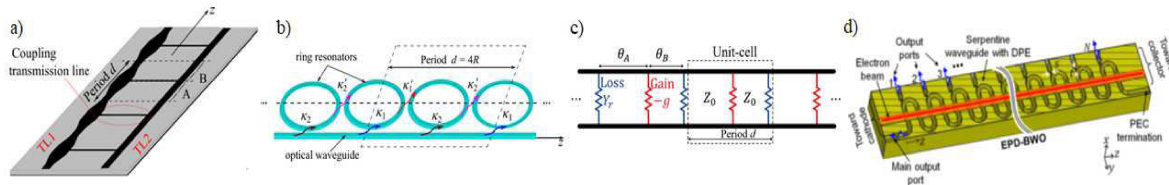


Fig 1. Periodic structures without, (a) (b), and with, (c) (d), distributed gain and loss that exhibit different orders of EPDs: (a) microstrip waveguide [3]; (b) coupled resonator optical waveguide (CROW) [4]; (c) periodic waveguide loaded with loss and gain [5]; (d) backward-wave oscillators made of folded waveguide with distributed power extraction [6].

II. WAVEGUIDE GEOMETRIES AND DIFFERENT APPLICATIONS SUPPORTING EPD

The four geometries in Fig. 1 support a fourth order EPD, known as the degenerate band edge (DBE) (a)(b), a third order EPD, called stationary inflection point (SIP) (b), a second order EPD with loss and gain (c) (d). Fig. 1(a) shows a microstrip geometry that exhibits DBE characteristics even with radiative and fabrication losses [3]. The DBE of Fig. 1(a) is shown in the dispersion relation given in Fig. 2(a) [3]. This dispersion relation was found using the scattering matrix obtained using measurements from a vector network analyzer. When a gain element in the form of a negative conductance (lumped or distributed) is added to the DBE structure, the system oscillates at the DBE resonance frequency as shown in Fig. 3(a) [12]. Fig. 1(b) depicts a coupled resonator optical waveguide (CROW) consisting of a chain of coupled ring resonators of radius R , side-coupled to a rectangular straight waveguide [4]. The radius of the rings is $R = 10 \mu\text{m}$ and the other parameters are in [4]. Fig. 2(b) shows the Floquet-Bloch wavenumber dispersion diagram of the gainless and lossless CROW [4]. It shows a DBE at

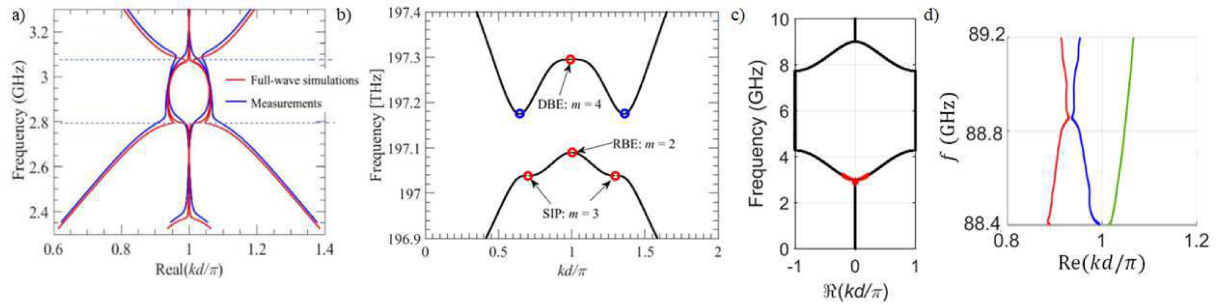


Fig 2. Dispersion diagrams for the four waveguides in Fig. 1, showing various kinds of EPDs, without loss and gain (a [3], b [4]) and with loss and gain (c [5], d [6]).

frequency $f \cong 197.3$ THz, an SIP at $f \cong 197.0$ THz, and an RBE (second-order EPD) at $f \cong 197.1$ THz. Fig. 3(b) highlights the very peculiar lasing threshold scaling of the DBE laser in terms of number of unit cells N , as N^{-5} , the RBE-based laser instead shows a scaling as N^{-3} , whereas the uniform FPC cavity exhibits the conventional scaling versus waveguide length of N^{-1} .

Fig. 1(c) exhibits a structure representing a periodic waveguide loaded with periodic loss and gain elements, which supports a second order EPD [5]. Fig. 2(c) shows the dispersion relation of the complex-valued wavenumber with a second order EPD at $f = 3$ GHz. Fig. 3(c) shows a stable oscillator at the EPD frequency when the waveguide is periodically loaded with loss and nonlinear gain. Here, losses may represent radiation.

We show another important application of waveguides uniformly loaded with gain and (radiation) loss. Standard backward oscillators (BWOs) operating at high frequencies (i.e., mm waves) typically generate output power not exceeding tens of watts with very poor power conversion efficiency in the order of 1%. Fig. 1(d) shows the backward-wave oscillator (BWO) that uses a folded waveguide, where the power is extracted in a distributed fashion to enable the occurrence of the exceptional synchronization regime when working at an EPD [6]. The distributed power is extracted by introducing a small slot in each folded waveguide period that couples a portion of the power traveling in the folded waveguide to the outgoing rectangular waveguides, and then it can be radiated away. Fig. 2(d) shows the dispersion of the complex-valued wavenumbers in the fundamental Brillouin zone, of three hot modes showing the wavenumber degeneracy when the used current is about 2.055 A. The distributed power extraction (i.e., distributed loss, from a waveguide point of view) along a folded waveguide in Fig. 1(d) is useful to satisfy the necessary conditions to have an EPD at the synchronization point (Fig. 2(d)). The use of EPD enables a high starting current for oscillation that indicates a high level of power extraction from the e-beam kinetic energy. Fig. 3(d) shows that EPD-BWO, i.e., a BWO operating at an EPD regime, is capable of generating output power exceeding 3 kW with a conversion efficiency of exceeding 20% [6].

We have demonstrated also waveguides made of three coupled microstrips (not shown here for brevity) that operate near a third order EPD with gain and loss, giving rise to high power radiation applications. Other waveguide systems allow for stable oscillators at the DBE with low oscillation threshold or potential high radiating power efficiency in array oscillator designs [3]. In optical systems, the DBE in a waveguide has been studied for laser applications demonstrating low threshold and very stable frequency of oscillation. DBE lasers are of particular interest because of order-of-magnitude reductions in the pumping rate compared to cavities not operated in the DBE regime, and of ultra stable lasing frequency. Other applications in sensing technology and delay lines have been demonstrated with the geometries presented here and other waveguides operating at an EPD.

In conclusion, we will show how we classify waveguides with EPD in two categories and how we determine the EPDs using either simulations or measurement techniques. EPD are recognized by the vanishing of the coalescence parameter [3] that quantifies the separation of the eigenvectors, and it is an excellent tool to measure their degeneracy. Despite the two kinds of waveguides discussed here have analogous classifications, their various possible applications remain distinct, especially for oscillator/laser systems, that can be designed at any frequency, at high or low power operations. Waveguides with EPDs without loss and gain are usually used for delay lines, sensors, low threshold oscillators with low phase noise, distributed amplifiers, etc. The introduction of gain and loss in such systems is possible, but they do not retain the EPD if gain and loss are substantially large. Instead, waveguides where an EPD is obtained with the presence of loss and gain are used for high power amplifiers and

oscillators because the EPD is designed in the presence of arbitrarily high gain and loss. In some applications, the distributed loss represents the radiation by a periodic array of antennas.

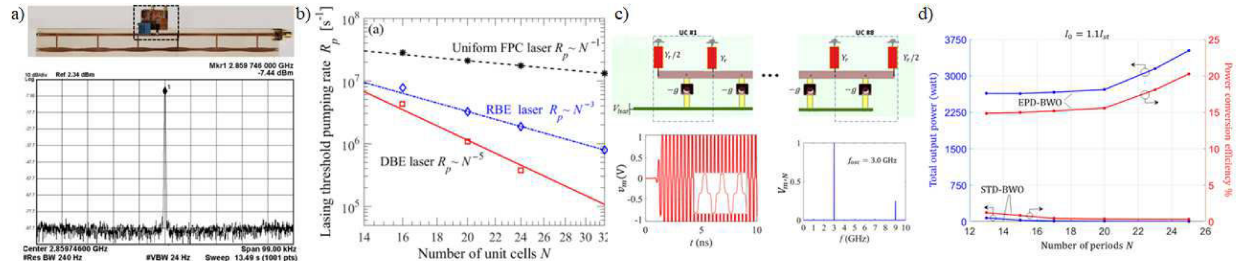


Fig 3. (a) Finite length implementation of DBE geometry with gain element and single frequency oscillations demonstrated [12]. (b) Scaling of the lasing threshold pumping rate of three regimes of laser operation based on three types of cavities varying as a function of the cavities' length (number of unit cells N). The DBE laser exhibits a low threshold and a new peculiar scaling law [11]. (c) Distributed oscillator with periodically radiative losses and nonlinear gain [5]. (d) Comparison between the output power and power conversion efficiency of a standard BWO and a BWO operating at an EPD that use folded waveguide and a linear electron beam. The EPD regime with distributed power extraction is capable of generating more power than the standard BWO regime [6].

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