

Lasing with multimode guided waves with exceptional degeneracy

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AFOSR
AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

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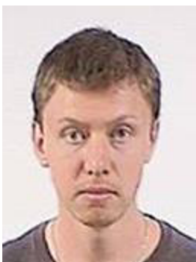
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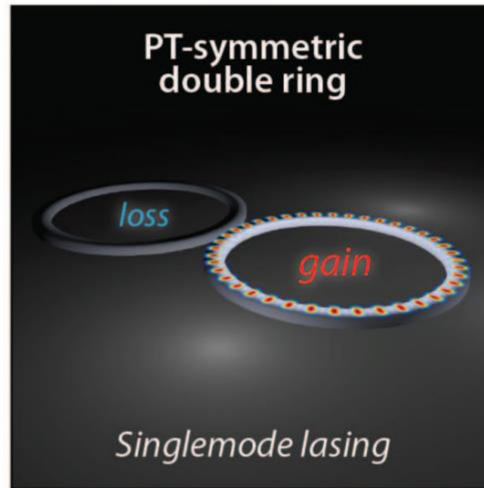


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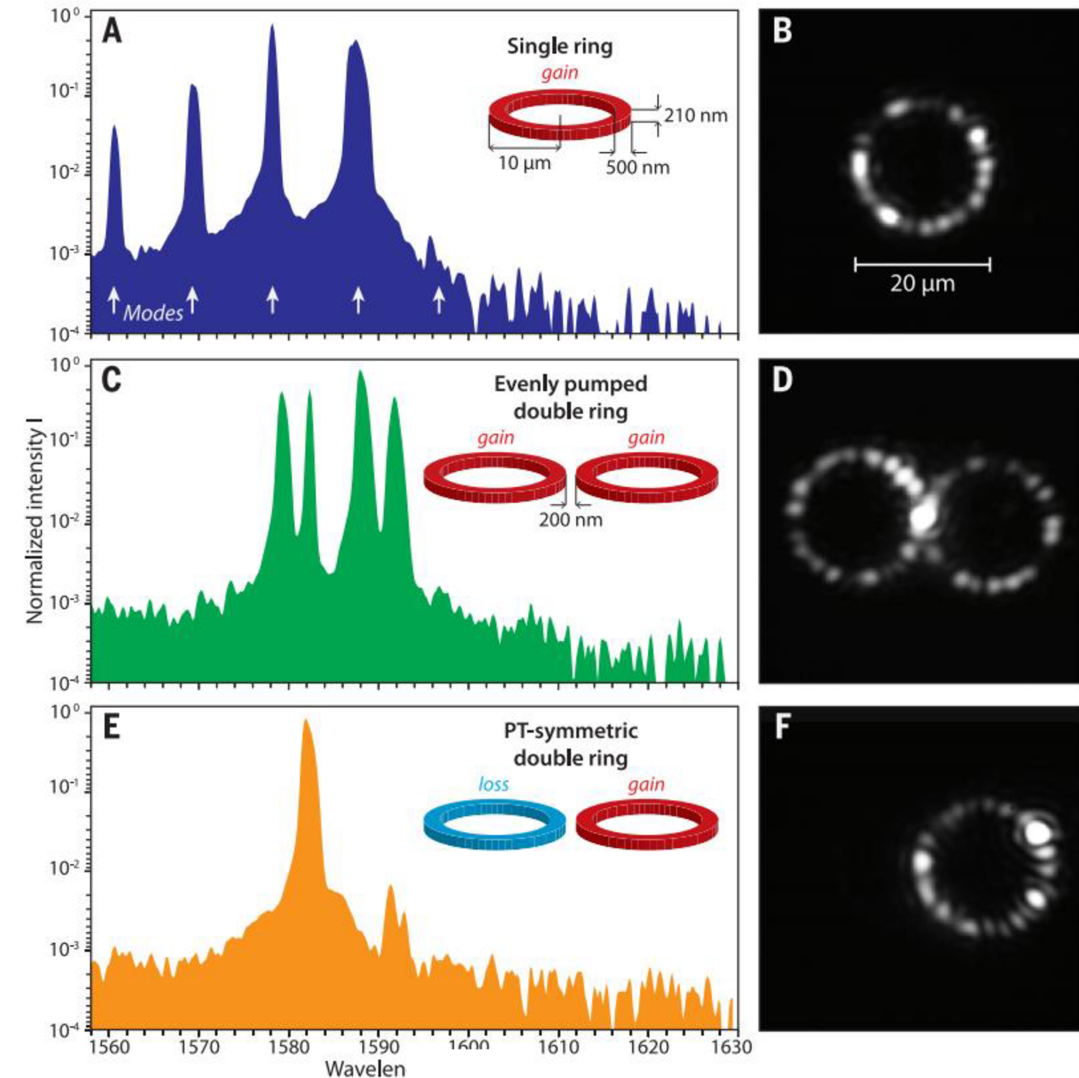
Gain and loss balanced laser



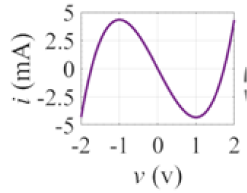
Example based on PT-symmetry

- 2nd order EPD in coupled ring resonators with balanced loss and gain
- EPD can be used to enhance some properties of a desired lasing mode

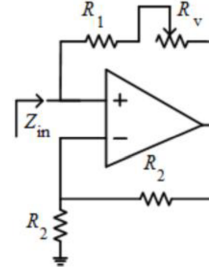
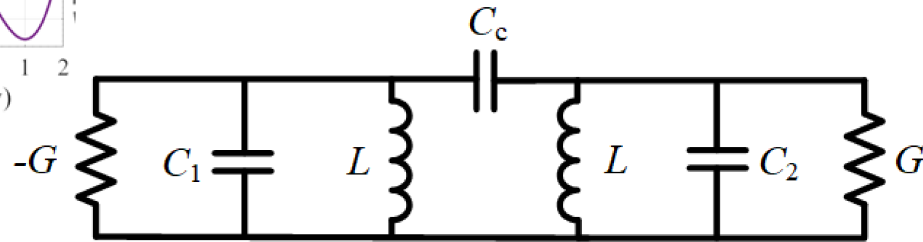
Hodaei, Miri, Heinrich, Christodoulides, Khajavikhan, *Science* **346**, no. 6212 (2014)



Experimental proof – RF, high sensitivity



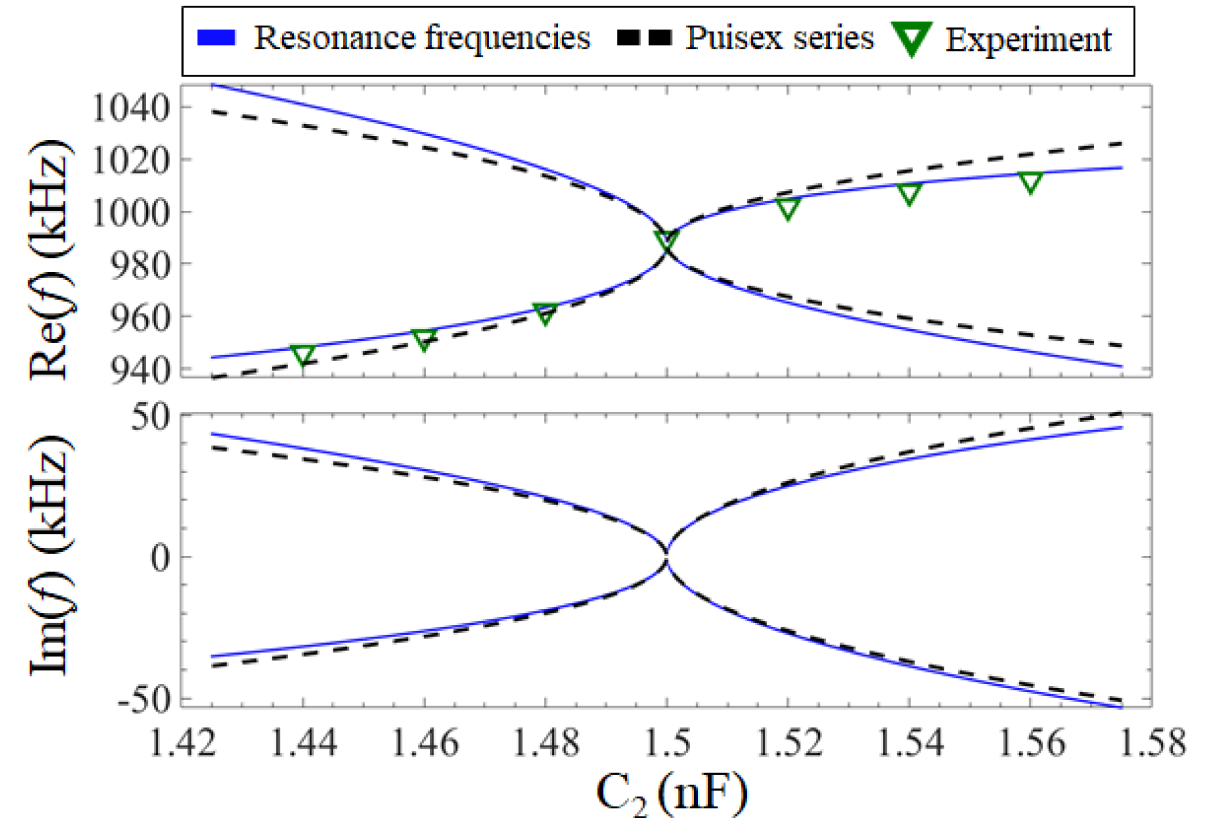
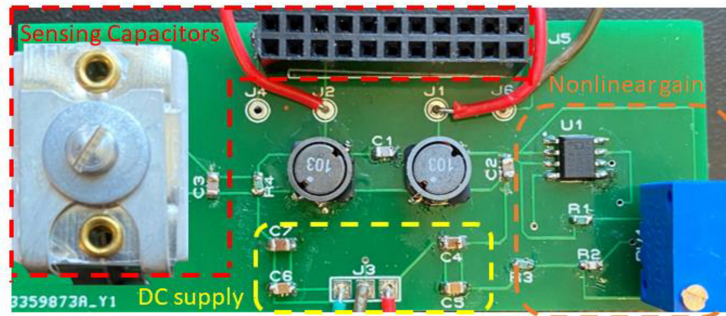
Nonlinear gain $-G$



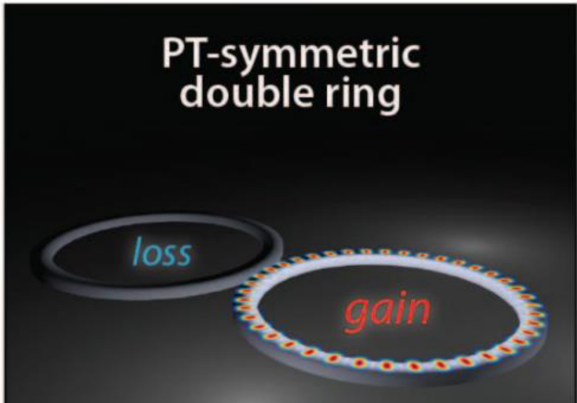
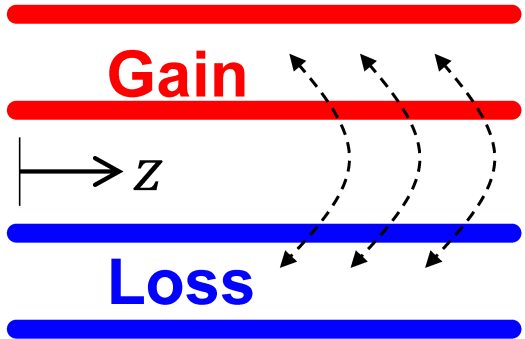
Square root sensitivity to the capacitance change C_2

$$\omega_p(\Delta C_2) \approx \omega_e + \alpha_1(-1)^p \sqrt{\Delta C_2}$$

$L = 10 \mu\text{H}$
 $C_1 = C_2 = C_c = 1.5 \text{ nF}$
 $G = 9 \text{ mS}$

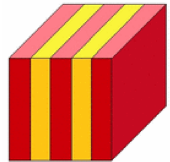
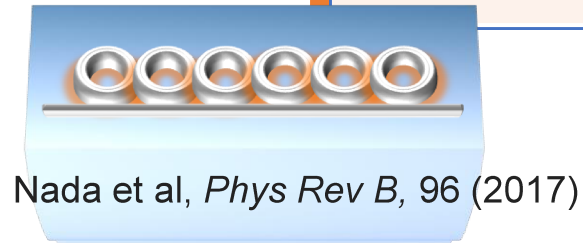


Nikzamir, Capolino, "Highly Sensitive Coupled Oscillator Based on an Exceptional Point of Degeneracy and Nonlinearity," arXiv:2206.04031

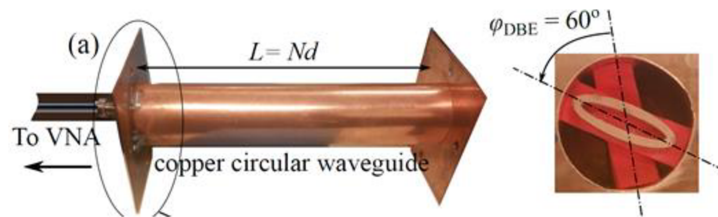
	“Temporal” EPD	“Spatial” EPD
Structure	PT-symmetric double ring  2 coupled resonators Hodaei, Miri, Heinrich, Christodoulides, Khajavikhan, Science 346, no. 6212 (2014)	 2 coupled waveguides Klaيمان, et al, Phys Rev Letters, vol. 101, no. 8, 2008. Othman, Capolino, IEEE Trans. Antennas Propag., vol. 65, no. 10, 2017
Mode function	$\phi(t) \propto e^{-i\omega t}$	$\phi(z, t) \propto e^{-i\omega t} e^{ikz}$
Degeneracy	$\omega_1 = \omega_2$ Coalescence in resonance frequency	$k_1 = k_2 \quad @ \quad \omega = \omega_e$ Coalescence in wavenumber

EPD in Waveguides

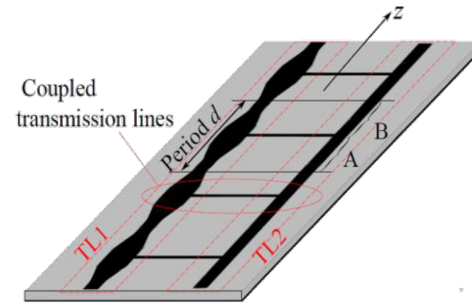
Lossless



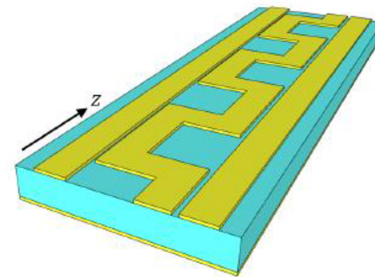
Figotin, Vitebskiy, *Phys. Rev. E* **68**(3), (2003)



Othman et al, *IEEE MTT*, 65, 6, 2017

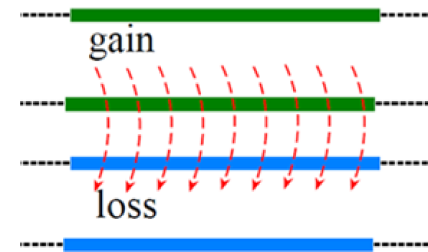


Abdelshafy et al, *IEEE TAP*, 67, no. 9, 2019

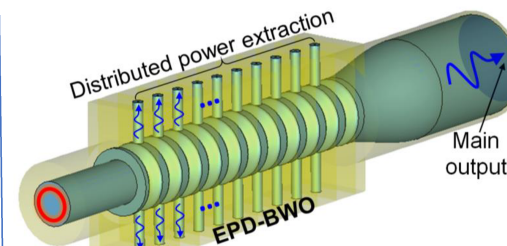
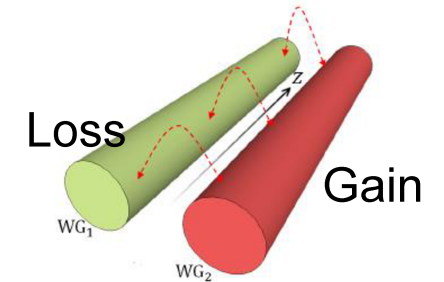


Nada, Mealy, Capolino, *IEEE MWCL*, 2020

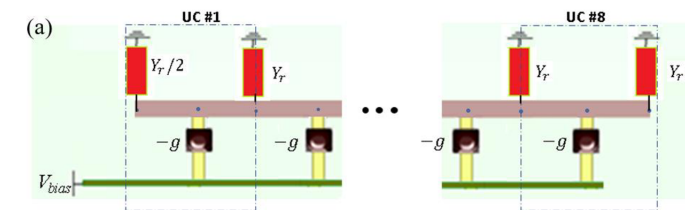
With gain/loss



Othman, Capolino, *IEEE TAP*, 65, 10, 2017



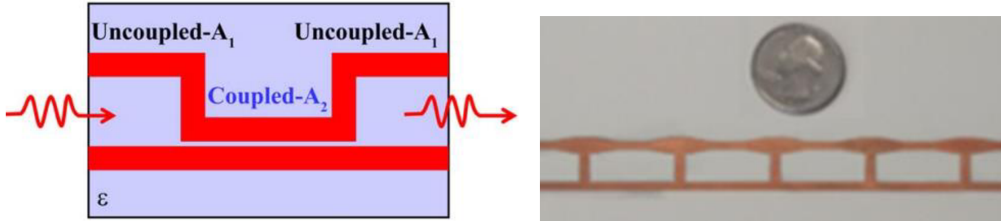
Mealy, Abdleshafy, Capolino, *PR Appl*, 2021



Abdelshafy et al, *Appl. Phys. Lett.*, 118, 224102, 2021

Some passive waveguides supporting EPDs

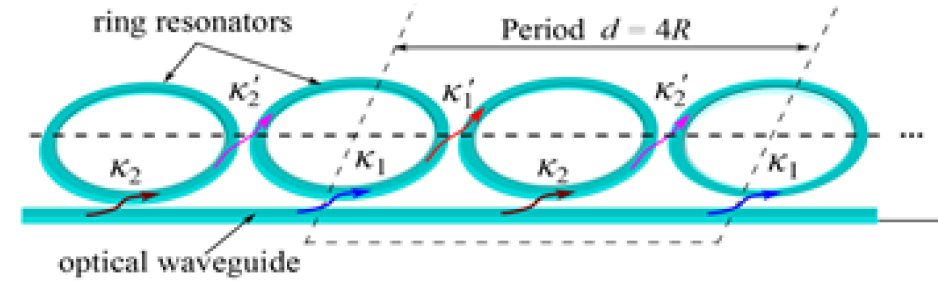
➤ Microstrip coupled TL



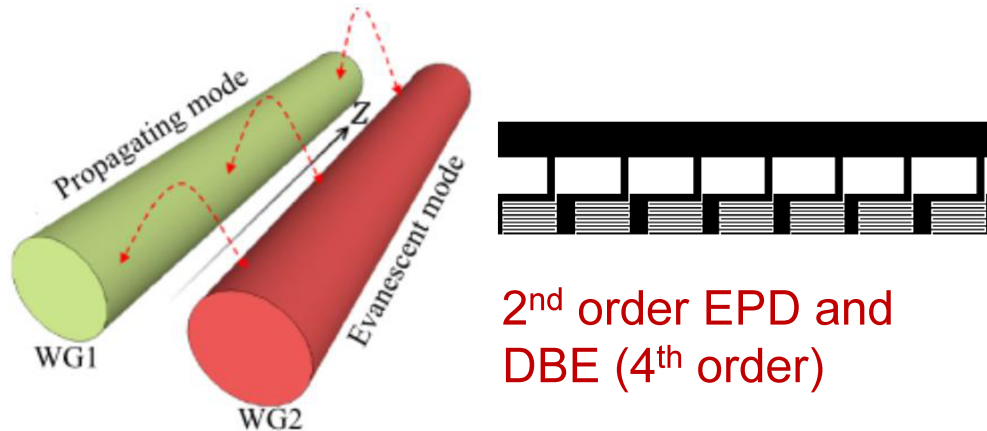
Mumcu, Sertel, Volakis *IEEE TAP* 57.6, 2009

Abdelshafy, Othman, Oshmarin, Al-Mutawa, Capolino, *IEEE TAP*, 67, 11, 2019

➤ Optical waveguide coupled with Ring Resonators



Nada, Othman, Capolino, *Phys. Rev. B*, vol. 96, no. 18, 2017

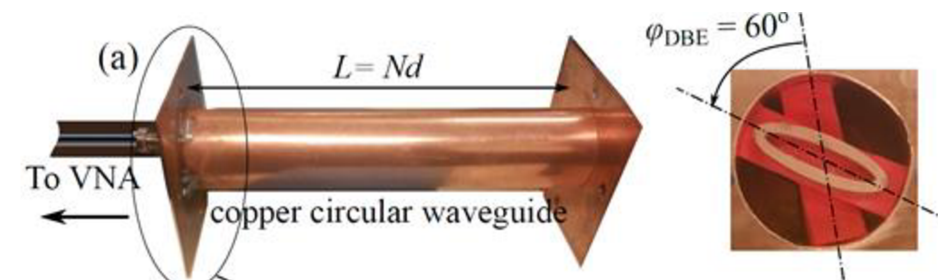


2nd order EPD and
DBE (4th order)

**Coupling between propagating and
evanescent modes**

Mealy, Capolino, *IEEE TMTT*, 68, 8, 2020

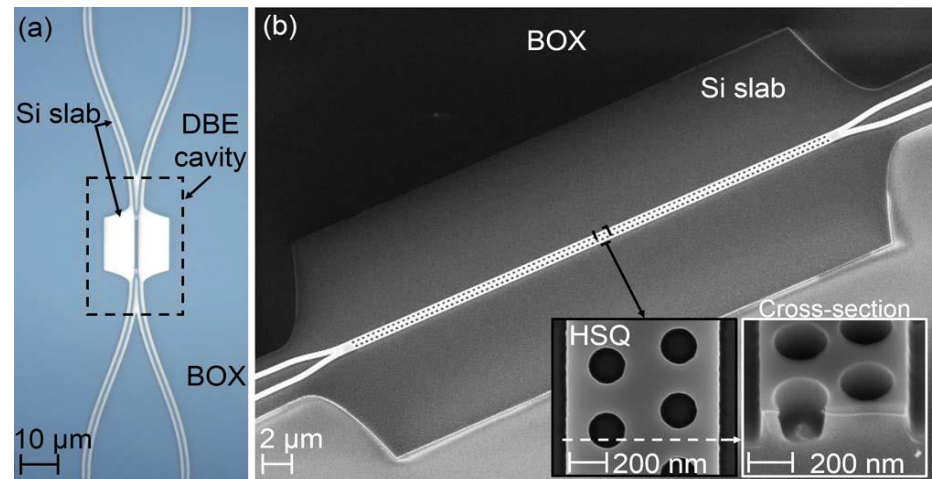
➤ Waveguides with periodic loading



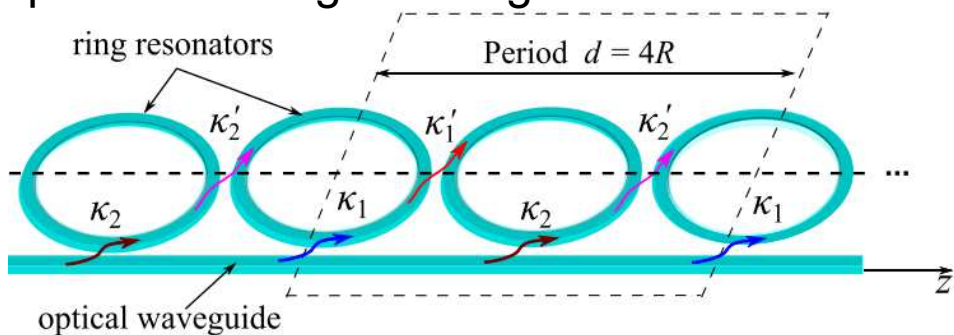
Othman, Pan, Atmatzakis, Christodoulou, Capolino, *IEEE TMTT.*, vol. 65, no. 6 , 2017

DBE (4th order) in optical periodic coupled waveguides (no loss, no gain)

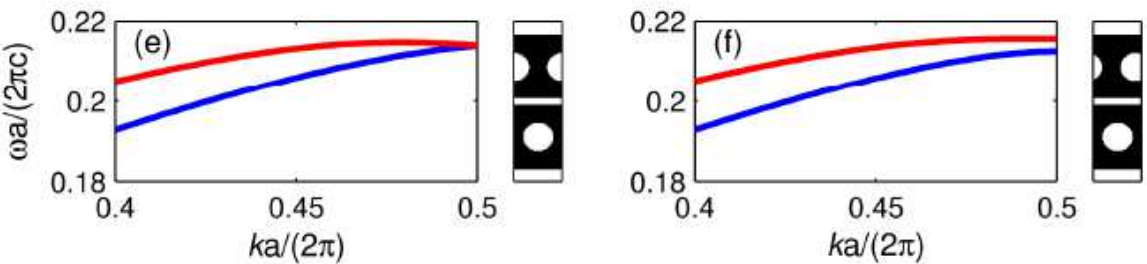
Silicon based photonic crystal waveguide



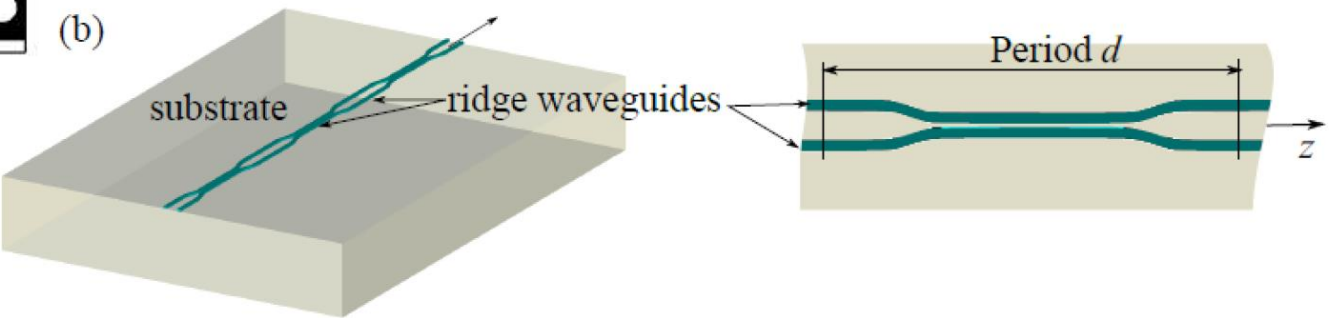
Coupled ring resonators waveguides (CROW) coupled to a straight waveguide



Nada, Othman, Capolino, *Phys. Rev B*. vol. 96, no.18, 2017
Nada, Othman, Boyraz, Capolino, *JLT*, 2017

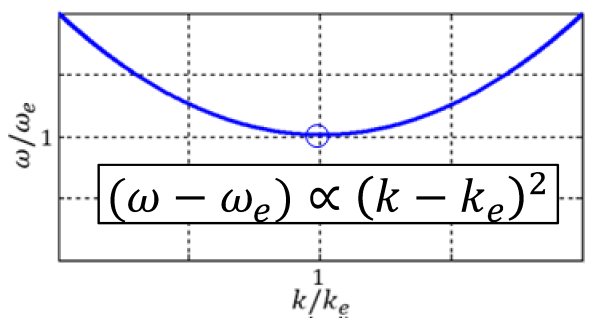
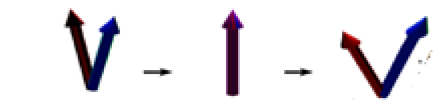
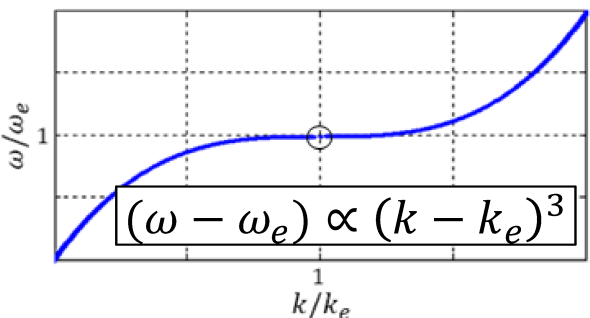
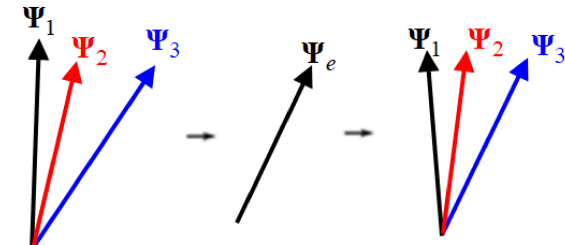
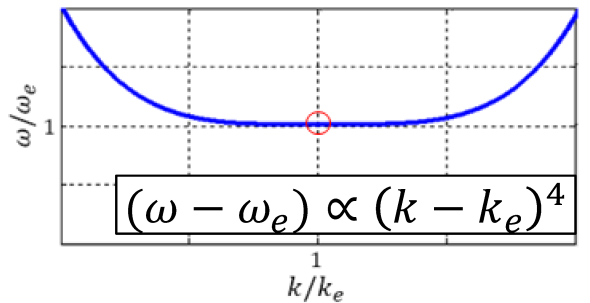
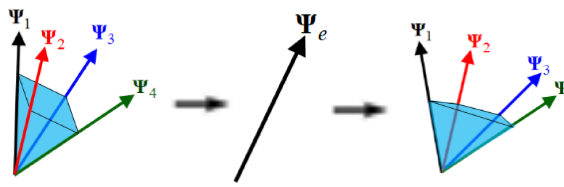


Wood, Burr, Reano, *Opt. Lett.* 40,11, 2015



Veysi, Othman, Figotin, Capolino, *Phys. Rev. B*. Vol. 97, no. 19, 2018

EPD in lossless waveguides - linear algebra perspective

EPD order	Purely-real wavenumber	Eigenvector evolution	System matrix @ EPD	Eigenmode distribution
2 (RBE)	 $(\omega - \omega_e) \propto (k - k_e)^2$	 Varying system parameter	$\mathbf{M} \sim \begin{bmatrix} k_e & 1 \\ 0 & k_e \end{bmatrix}$ 2x2 Jordan block	$\Psi(z) \propto z e^{ik_e z}$
3 (SIP)	 $(\omega - \omega_e) \propto (k - k_e)^3$	 Varying system parameter	$\mathbf{M} \sim \begin{bmatrix} k_e & 1 & 0 \\ 0 & k_e & 1 \\ 0 & 0 & k_e \end{bmatrix}$ 3x3 Jordan block	$\Psi(z) \propto z^2 e^{ik_e z}$
4 (DBE)	 $(\omega - \omega_e) \propto (k - k_e)^4$	 Varying system parameter	$\mathbf{M} \sim \begin{bmatrix} k_e & 1 & 0 & 0 \\ 0 & k_e & 1 & 0 \\ 0 & 0 & k_e & 1 \\ 0 & 0 & 0 & k_e \end{bmatrix}$ 4x4 Jordan block	$\Psi(z) \propto z^3 e^{ik_e z}$

EPD conditions in two uniform lossless waveguides

- Coupled mode theory equations

$$\frac{d}{dz} \begin{bmatrix} A(z) \\ B(z) \end{bmatrix} = i \begin{bmatrix} \beta_a & \kappa_{ab} \\ \kappa_{ba} & \beta_b \end{bmatrix} \begin{bmatrix} A(z) \\ B(z) \end{bmatrix}$$

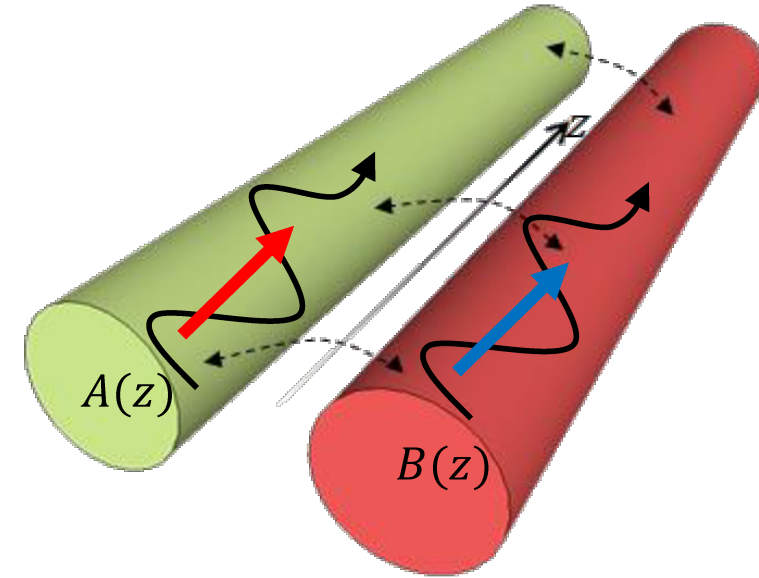
- EPD condition $k_1 = k_2 \rightarrow \boxed{(\beta_a - \beta_b)^2 = -4\kappa_{ab}\kappa_{ba}}$

- Possible Scenarios

1. Codirectional coupling ($\kappa_{ab} = \kappa_{ba}^*$):

$$\boxed{(\beta_a - \beta_b)^2 = -4|\kappa|^2}$$

Requires gain and loss



T.Mealy, F. Capolino, "Exceptional Points of Degeneracy with Indirect Bandgap Induced By Mixing Forward and Backward Propagating Waves," arXiv:2205.06844, 2022

EPD conditions in two uniform waveguides

Based on coupled mode theory

- Coupled mode theory equations

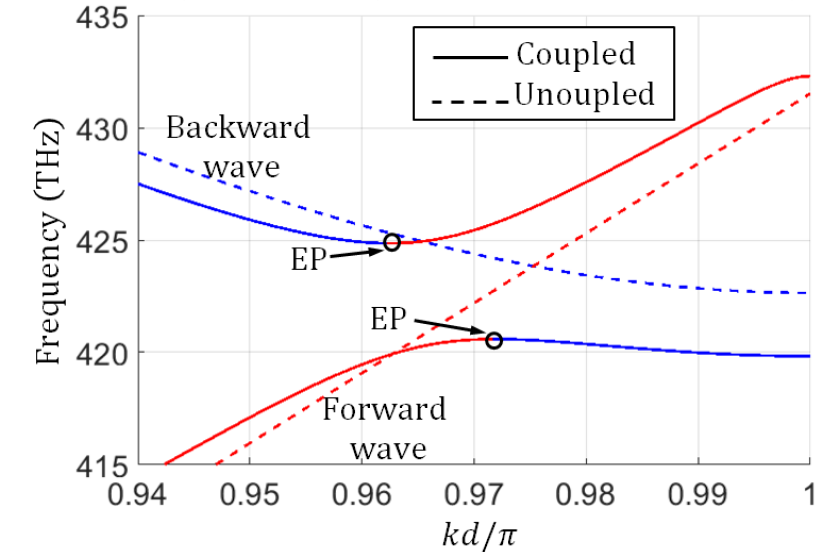
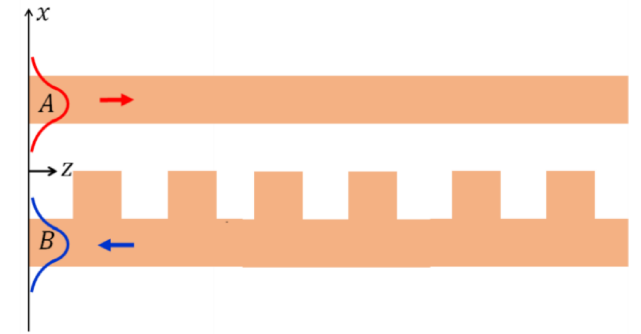
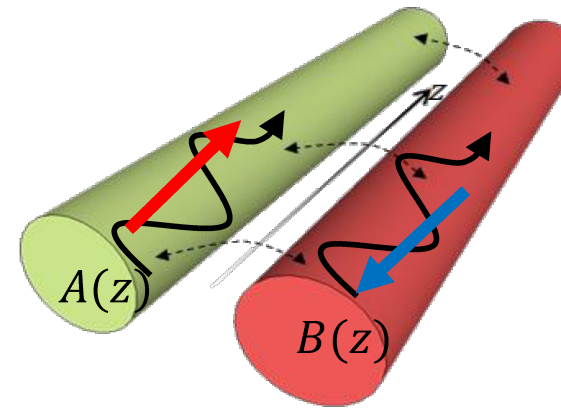
$$\frac{d}{dz} \begin{bmatrix} A(z) \\ B(z) \end{bmatrix} = i \begin{bmatrix} \beta_a & \kappa_{ab} \\ \kappa_{ba} & \beta_b \end{bmatrix} \begin{bmatrix} A(z) \\ B(z) \end{bmatrix}$$

- EPD condition $k_1 = k_2 \rightarrow (\beta_a - \beta_b)^2 = -4\kappa_{ab}\kappa_{ba}$

- Possible Scenarios

2. Contradirectional coupling ($\kappa_{ab} = -\kappa_{ba}^*$): $(\beta_a - \beta_b)^2 = 4|\kappa|^2$

Does not require gain and loss



Coupling between a forward wave and a backward wave yields an EPD

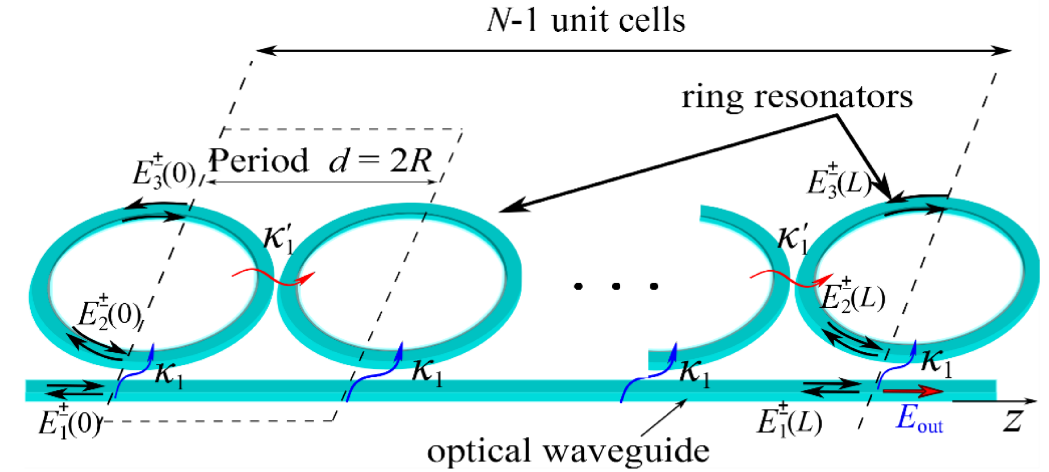
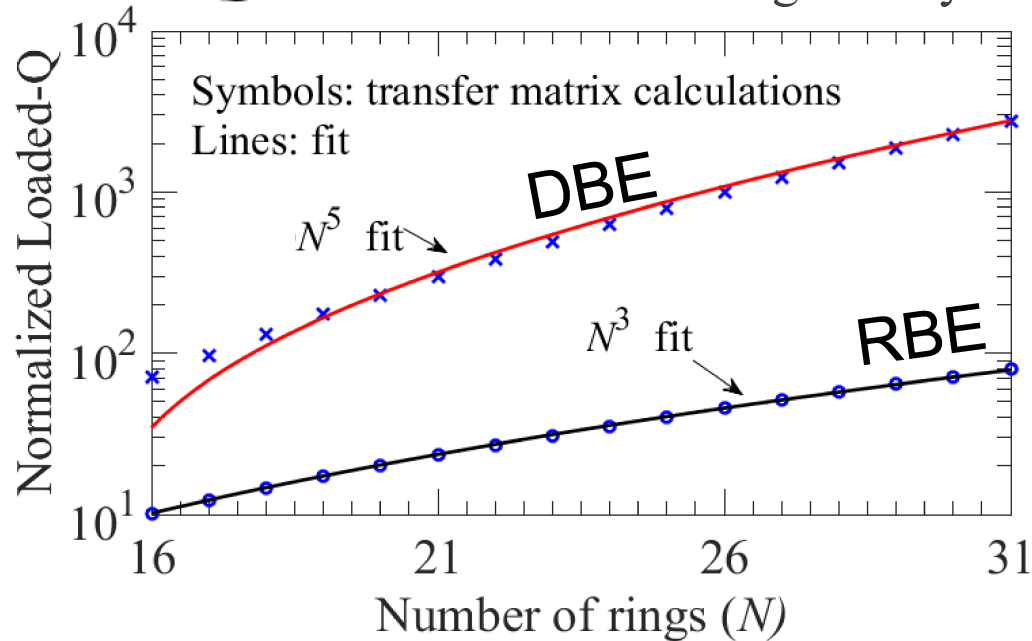
T.Mealy, F. Capolino, "Exceptional Points of Degeneracy with Indirect Bandgap Induced By Mixing Forward and Backward Propagating Waves," arXiv:2205.06844, 2022

Scaling of quality factor near a DBE degeneracy

Anomalous scaling of Q factor for a finite lossless CROW

$$Q_{\text{loaded}} \approx \omega_{r,d} \tau_g / 2$$

$$Q \propto N^{m+1}, m: \text{Order of degeneracy}$$



Operating at the **DBE** provides a new scaling for the quality factor

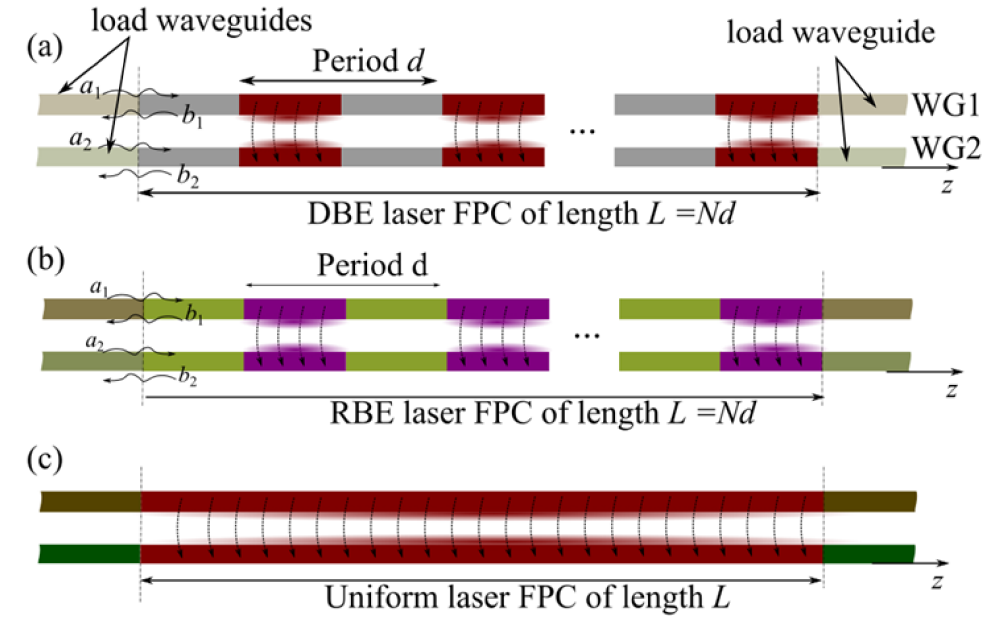
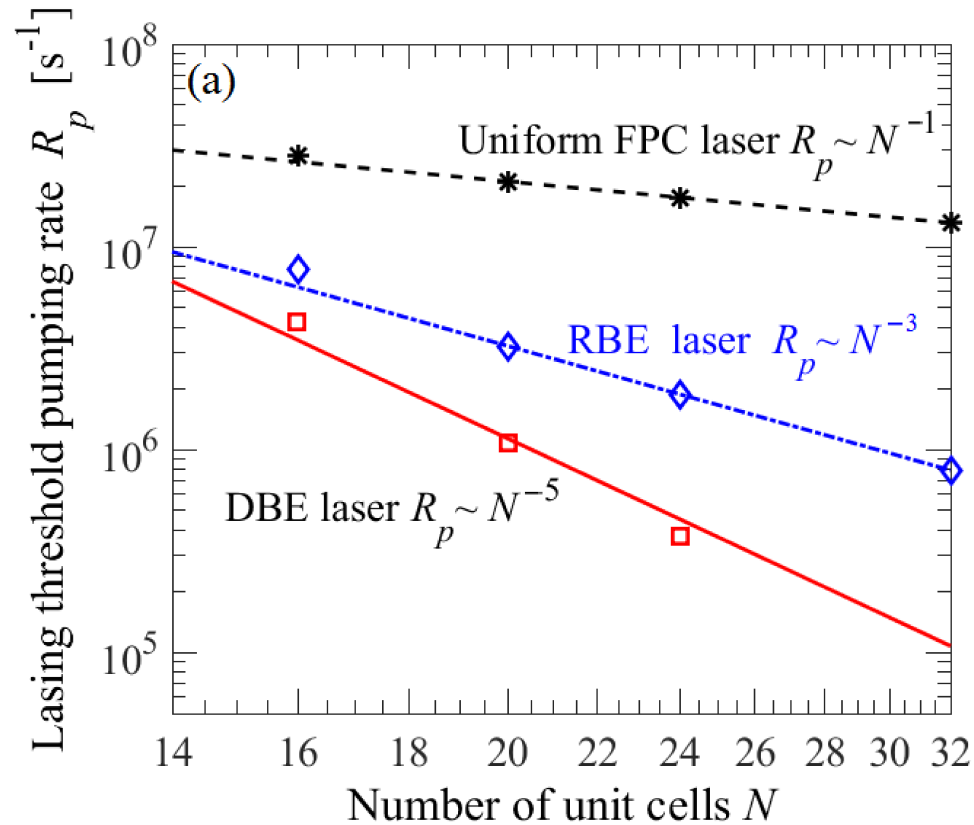
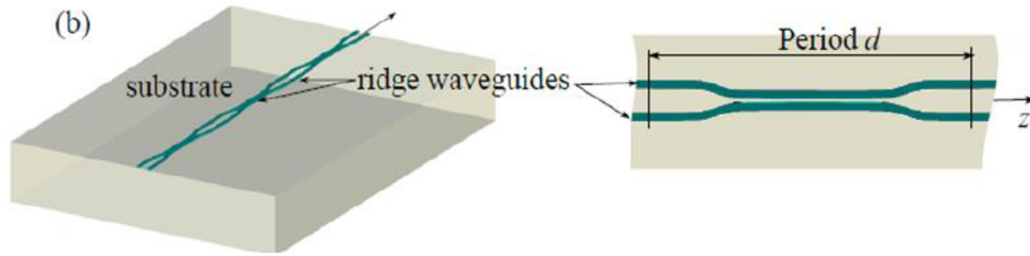
- DBE is an EPD of order 4
- RBE is an EPD of order 2

This anomalous scaling of the cavity Q factor is obtained **without using external mirrors.**

Nada, Othman, Capolino, *Phys Rev B* 96.18 (2017): 184304

Nada, Othman, Boyraz, and Capolino, *J. Light. Technol.*, 2018

Applications: DBE laser

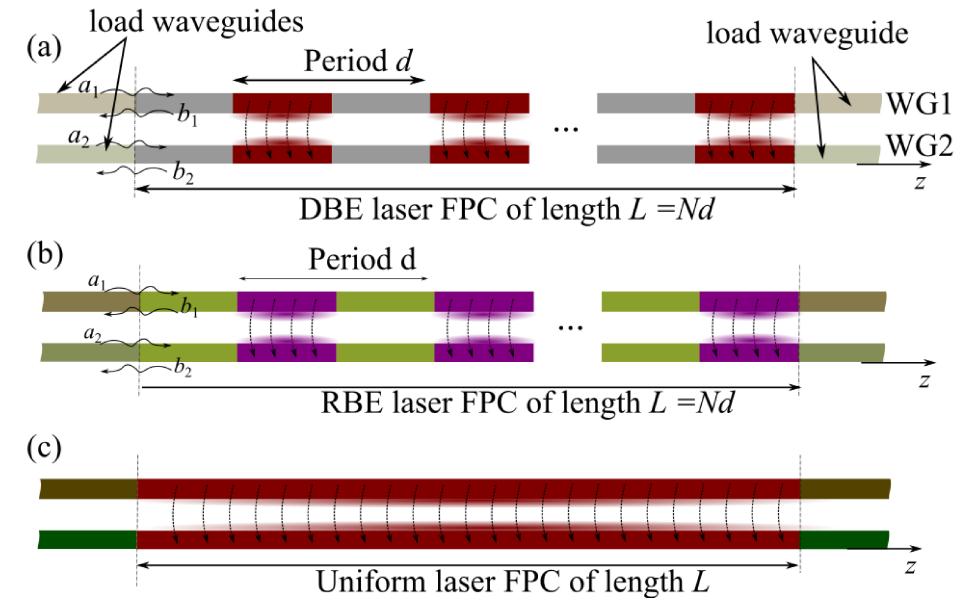
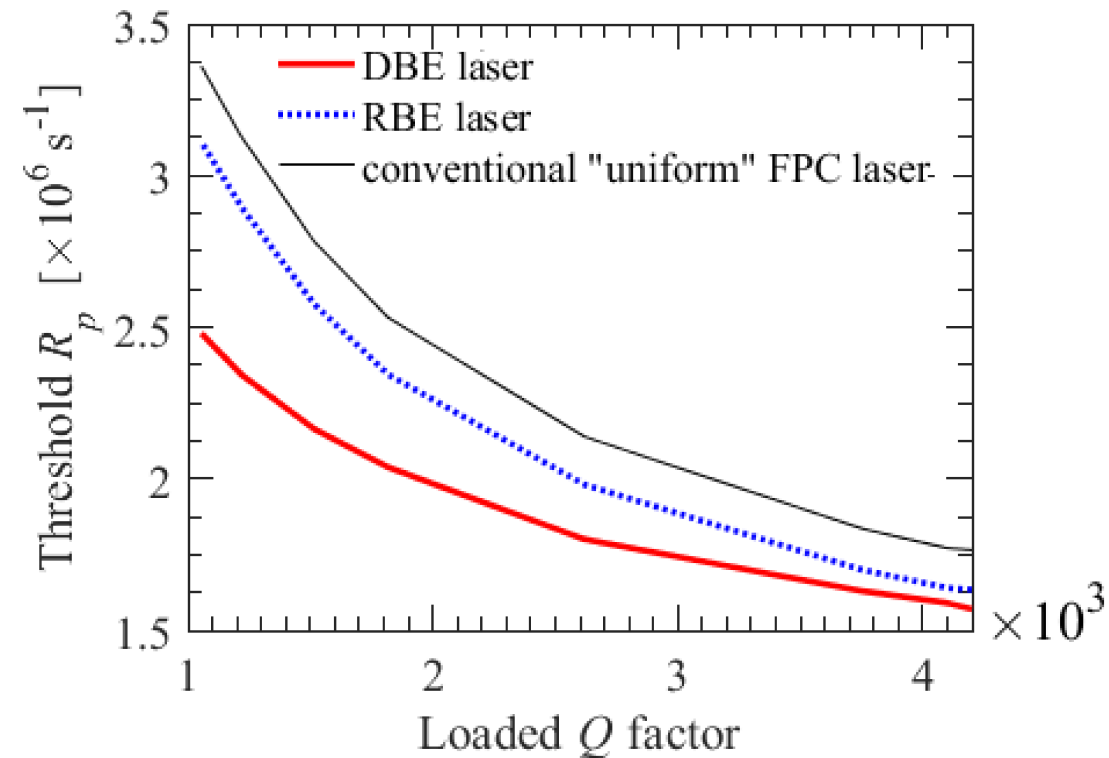


- DBE laser threshold, calculated using Time-domain coupled mode theory
- DBE laser has the lowest threshold
- Scales as N^{-5}

Comparisons with conventional lasers

DBE, RBE, and uniform laser cavities (having the same length, resonant frequencies, and loaded Q-factors)

Lasing threshold pumping rate comparison for the **same loaded Q factor**



DBE laser have the lowest threshold for the same loaded Q

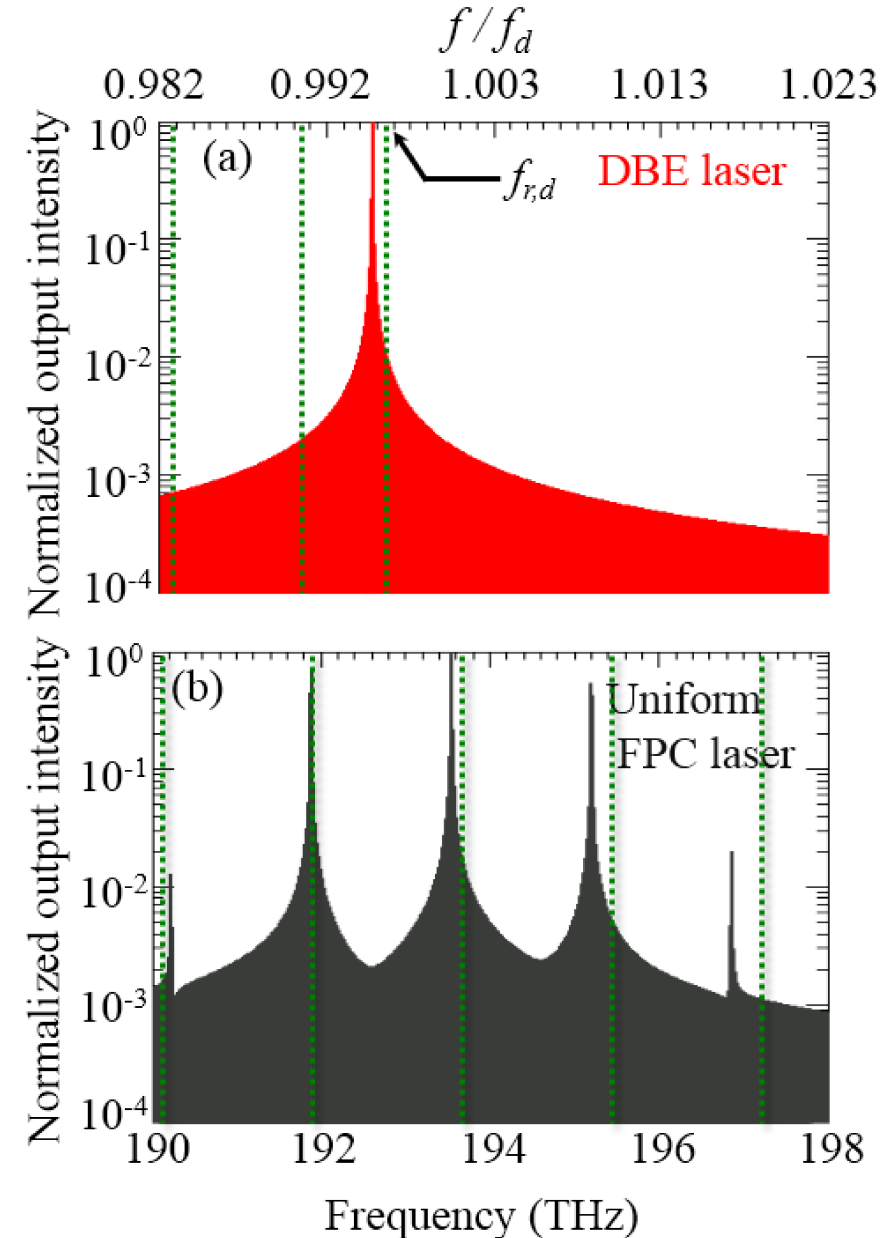
Veysi, Othman, Figotin, Capolino, "DBE Laser," *Phys Rev B*, 97 (2018)

Applications: DBE laser, mode selectivity

- Comparison between **DBE laser** and **uniform FPC laser**

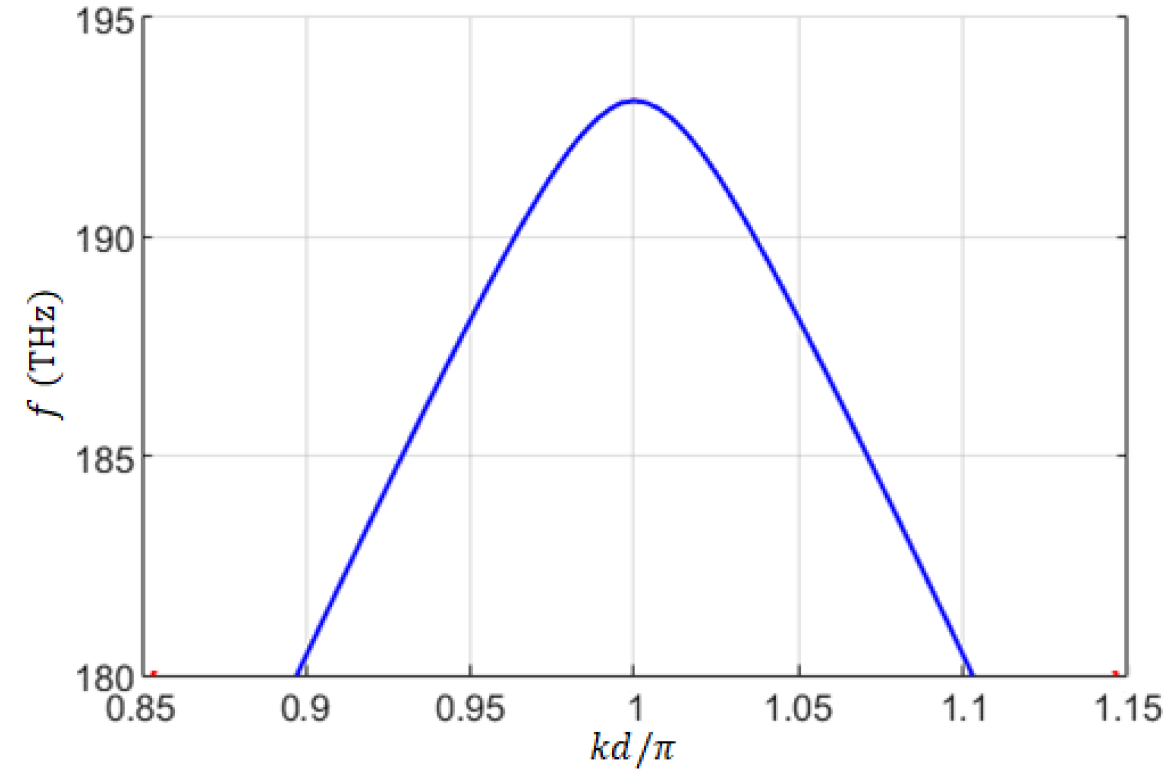
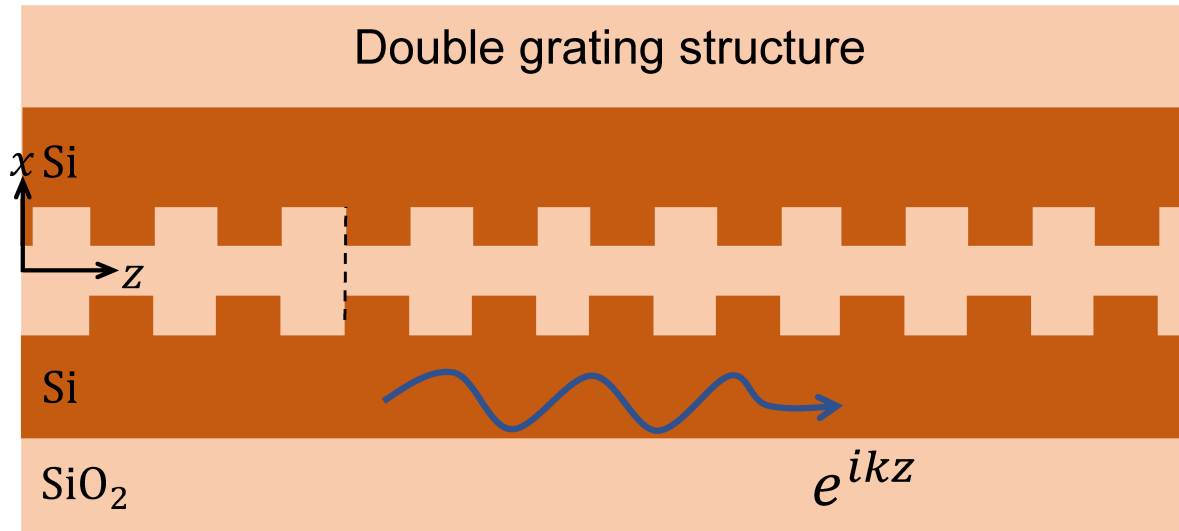
- Same Q-factor
- Both pumped at rate 10^6 s^{-1}

- The **DBE laser** has a **single oscillation mode** (single frequency)
- **Uniform optical cavities** have multimode operation



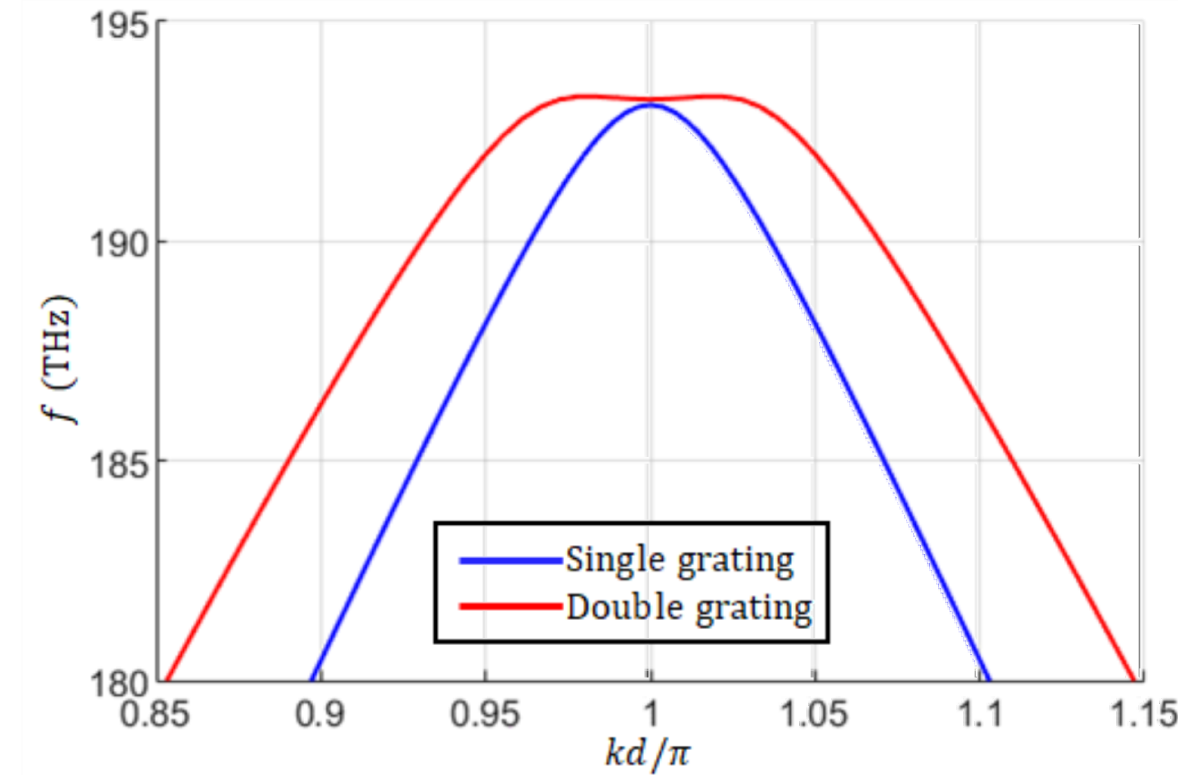
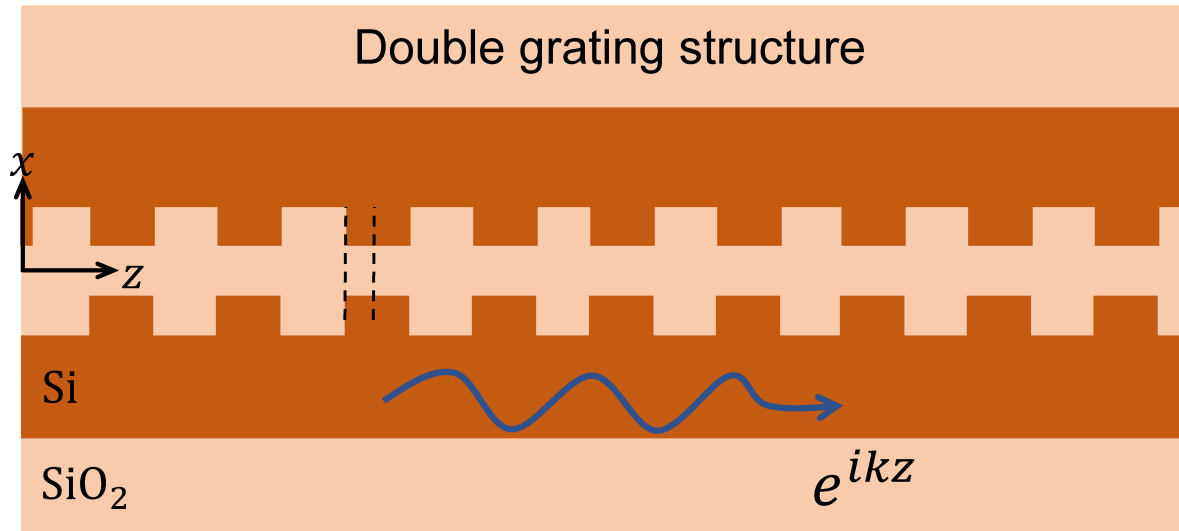
Veysi, Othman, Figotin, Capolino, "DBE Laser,"
Phys. Rev. B, vol. 97, no. 19 (2018)

Role of symmetry on the existence of DBE (Degenerate Bragg condition)



T. Mealy, F. Capolino, "Degenerate Distributed Feedback Photonic Structure with Double Grating Exhibiting Degenerate Band Edge," arXiv:2205.05639, 2022

Role of symmetry on the existence of DBE (Degenerate Bragg condition)

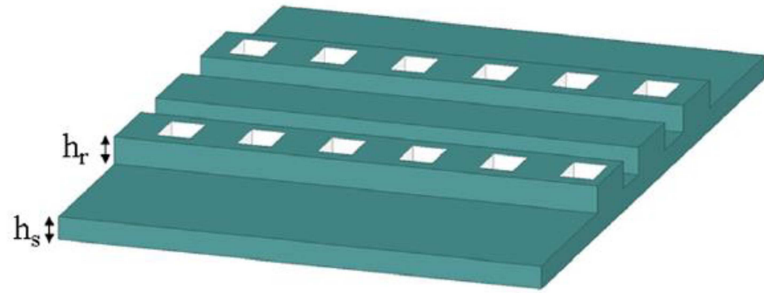


- Based on mirror symmetry breaking, a degenerate version of the Bragg condition based on DBE is proposed
- The proposed structure is a good candidate to conceive a degenerate distributed feedback laser operating at the DBE

Mealy, Capolino, "Degenerate Distributed Feedback Photonic Structure with Double Grating Exhibiting Degenerate Band Edge," arXiv:2205.05639, 2022

Optics passive waveguides supporting SIP (3rd order EPD)

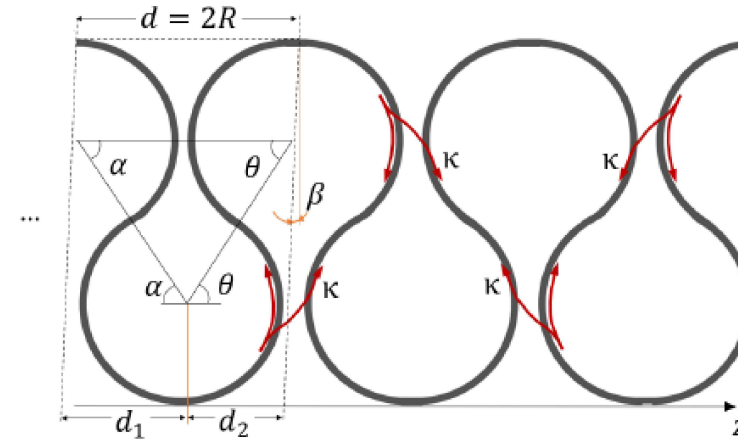
➤ Ridge waveguide



SIP (3rd order)
DBE (4th order)

Paul et al, JOSA B, 38, 5, 2021

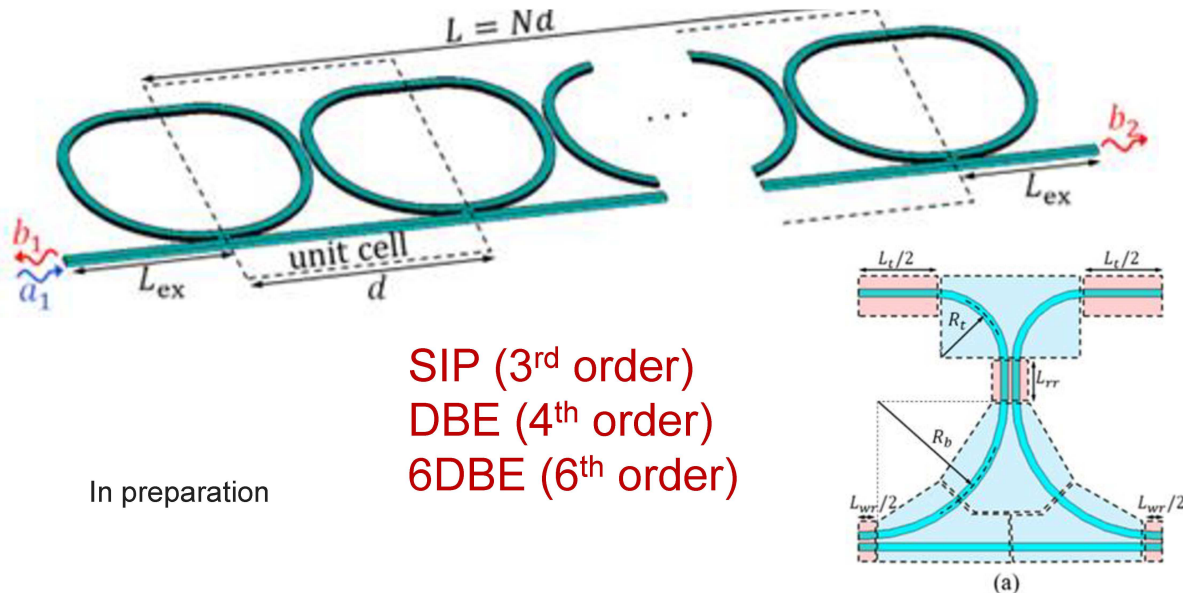
➤ Serpentine waveguide



SIP (3rd order)
DBE (4th order)
6DBE (6th order)

Herrero-Parareda, et al, Adv. Phot. Research, 2100377, 2022

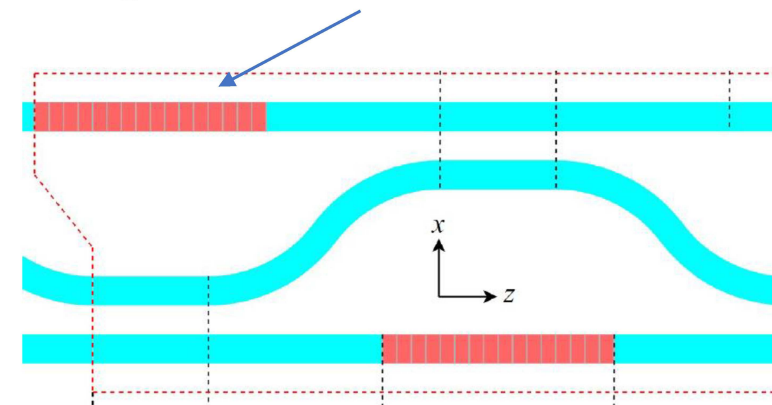
➤ Modified CROW



SIP (3rd order)
DBE (4th order)
6DBE (6th order)

In preparation

➤ Waveguide with DBR

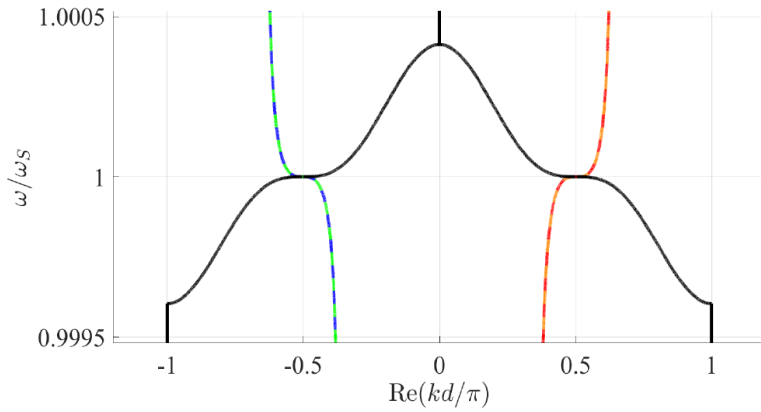


SIP (3rd order)

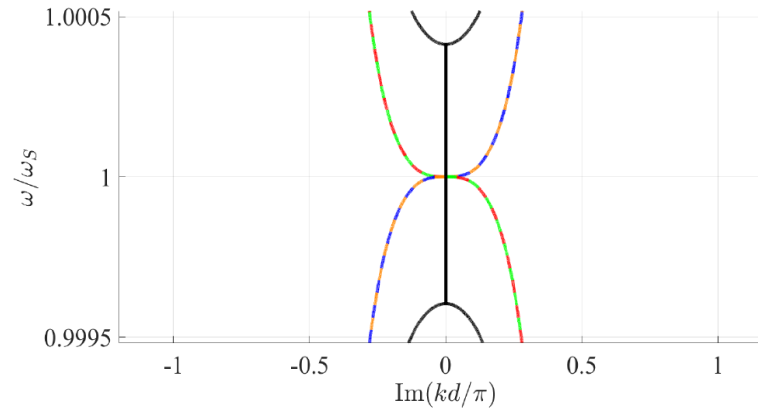
In preparation

SIP (frozen mode) in passive waveguide

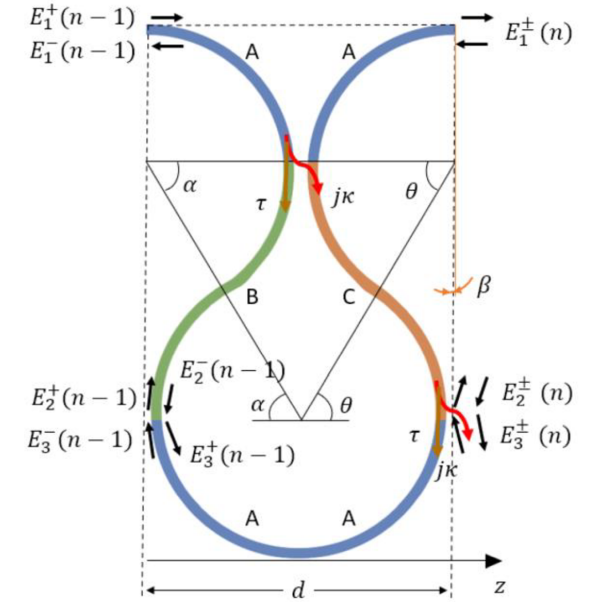
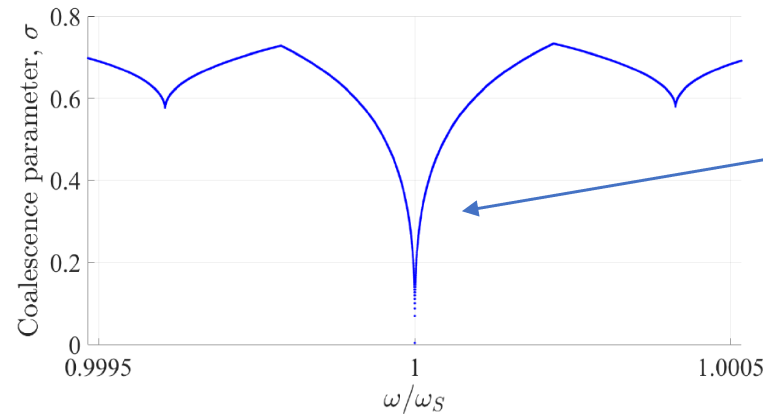
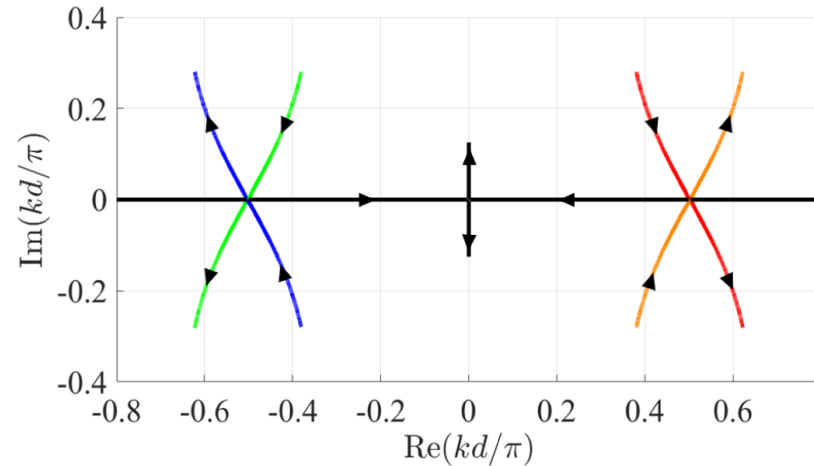
- Degeneracy of 3 wavenumbers and 3 polarization states



(a)



(b)



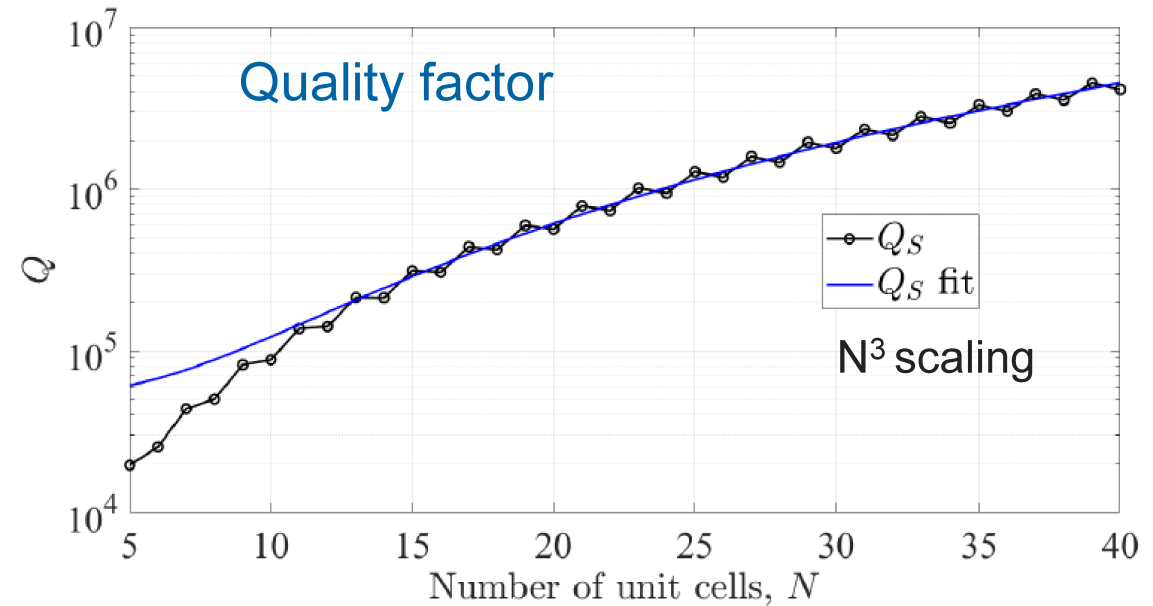
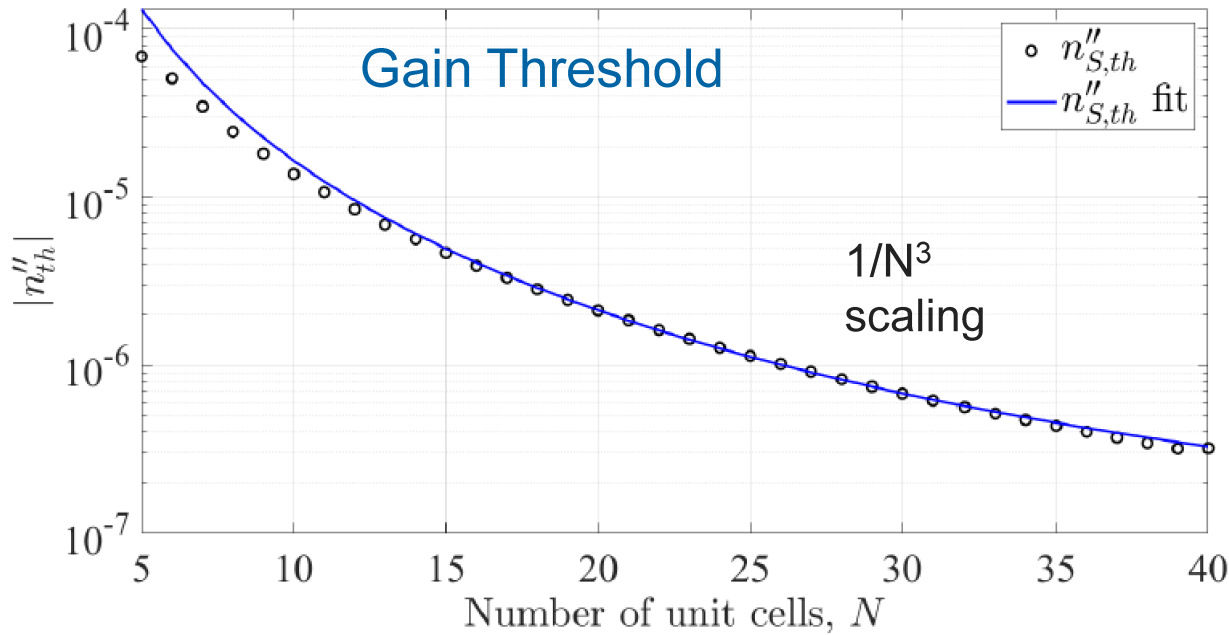
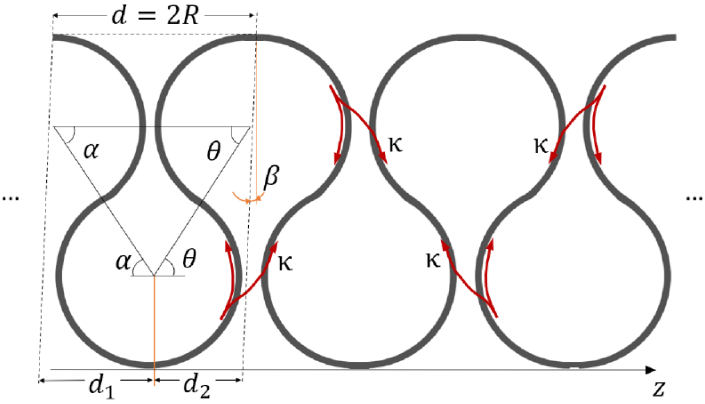
Coalescence parameter

$$\sigma' = \sqrt{\sum_{m=1, n=2, n>m}^3 |\theta_{mn}|^2}$$

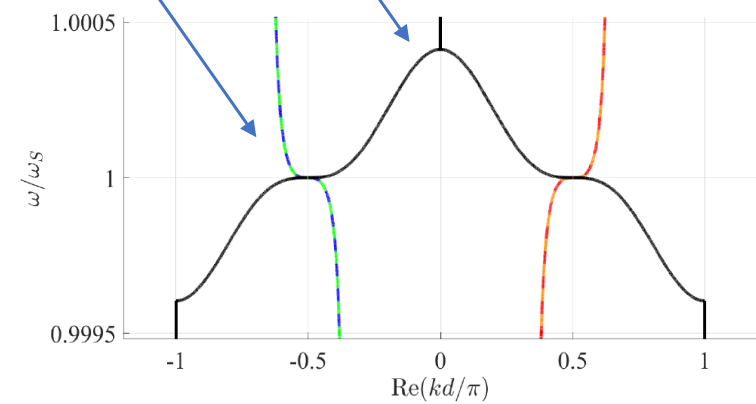
$$\cos(\theta_{mn}) = \frac{|\langle \psi_m | \psi_n \rangle|}{\|\psi_m\| \|\psi_n\|}$$

Herrero-Parareda, et al, "Frozen mode in an asymmetric serpentine optical waveguide," *Adv. Phot. Research*, 2100377, 2022

SIP (frozen mode) laser



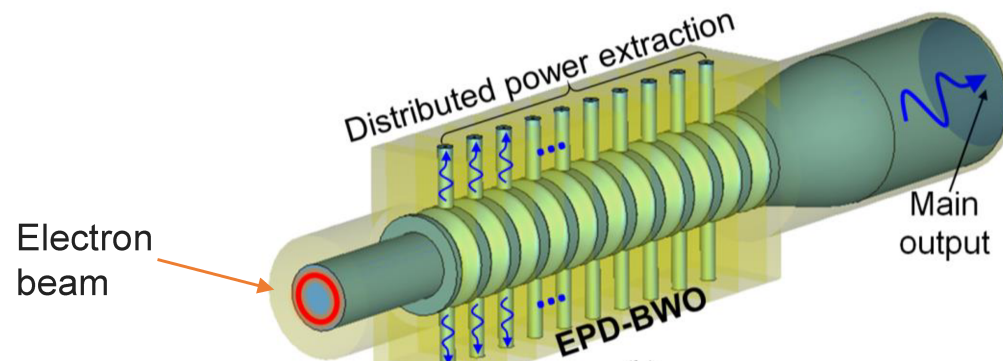
Challenge: is the SIP or the RBE that is providing lower threshold?



Herrero-Parareda, et al, "Frozen mode in an asymmetric serpentine optical waveguide," *Adv. Phot. Research*, 2100377, 2022

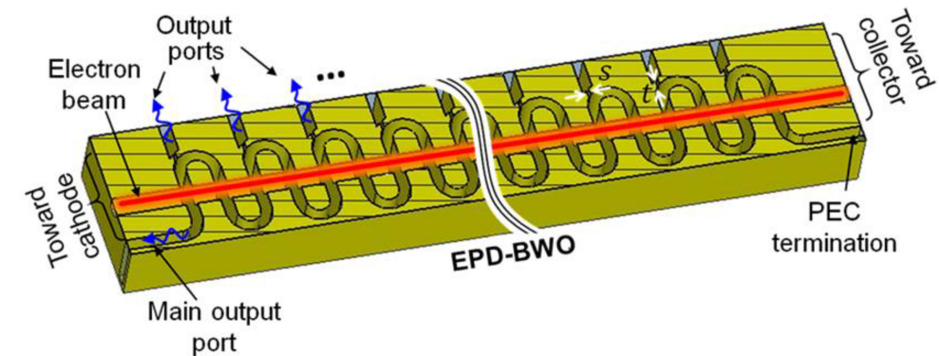
Active waveguides supporting EPDs (with gain and “loss”)

➤ RF backward wave oscillator



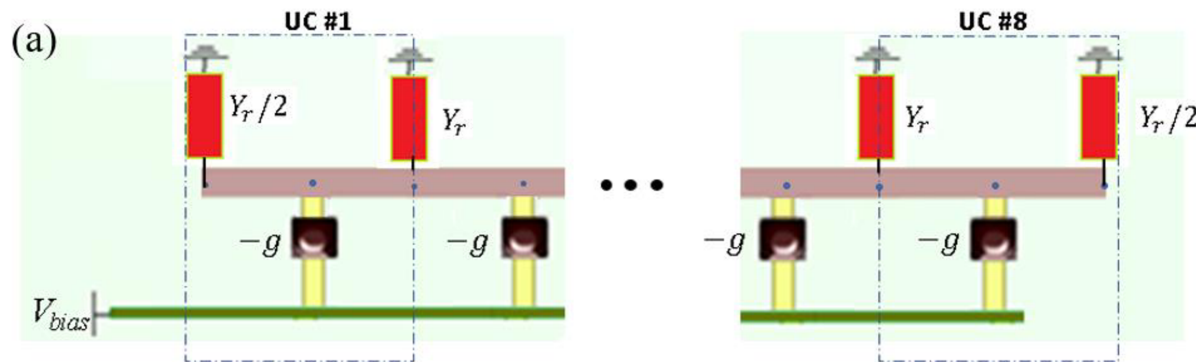
Mealy, Abdelshafy, Capolino, *PR Appl*, 2021

➤ mm-waves/THz backward wave oscillator



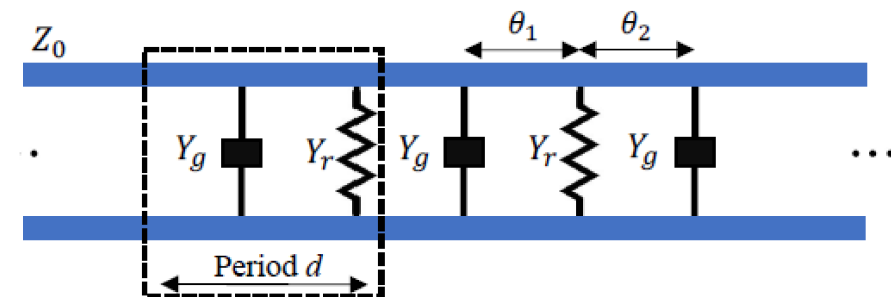
Mealy, Abdelshafy, Capolino, *IEEE Trans Electr Dev*, 2021

➤ Radiating active array (oscillating)

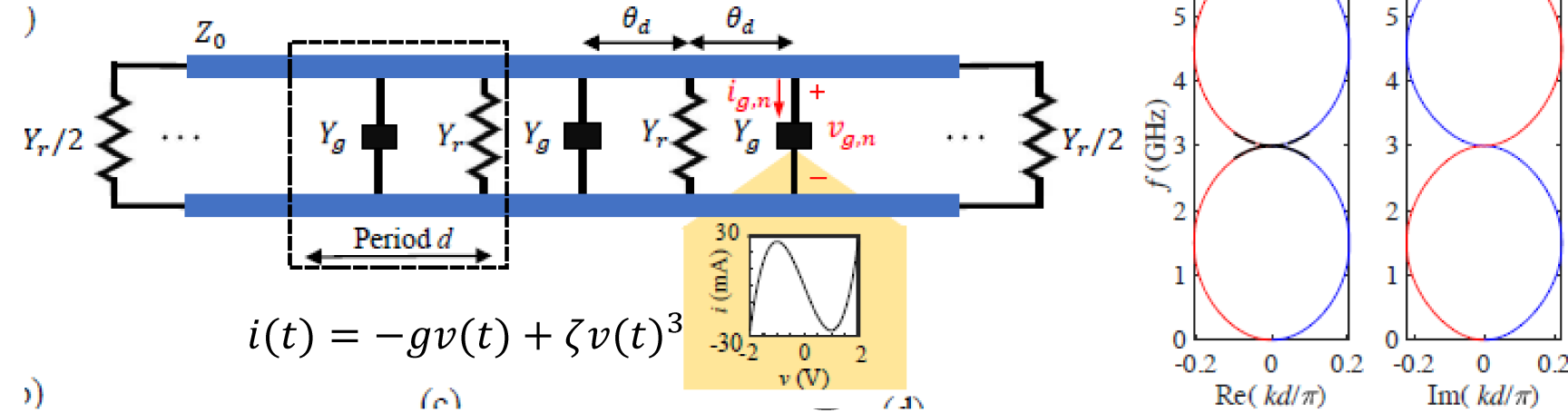


Abdelshafy et al, *Appl. Phys. Lett.*, 118, 224102, 2021

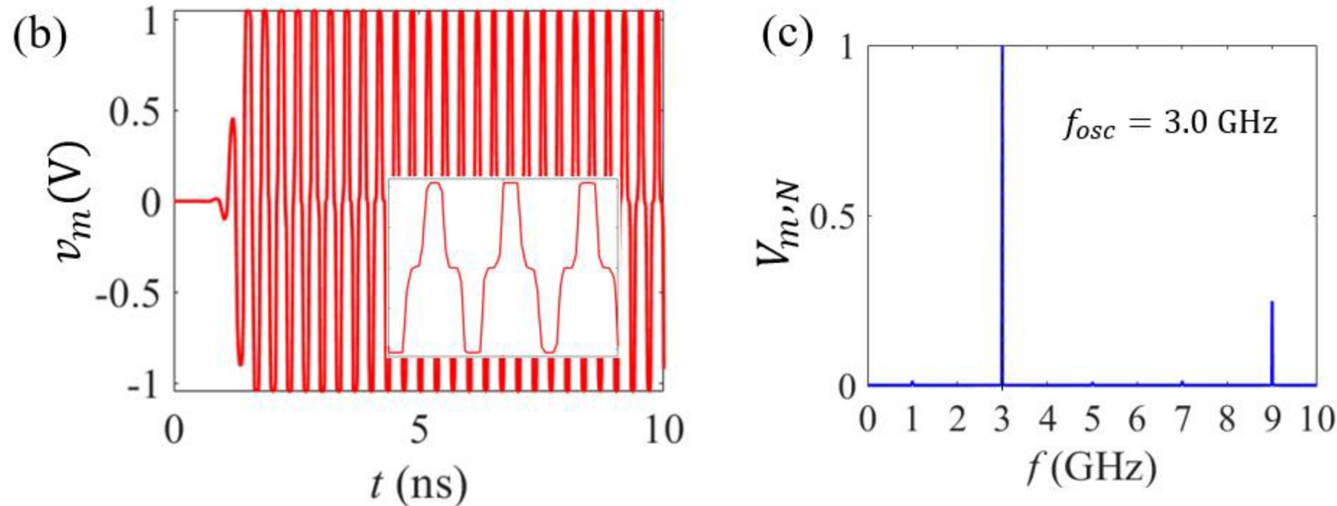
➤ Radiating active array (oscillating)



EPD- Radiation array oscillator



8 cascaded unit cells loaded with nonlinear gain and loss (representing radiating antennas)



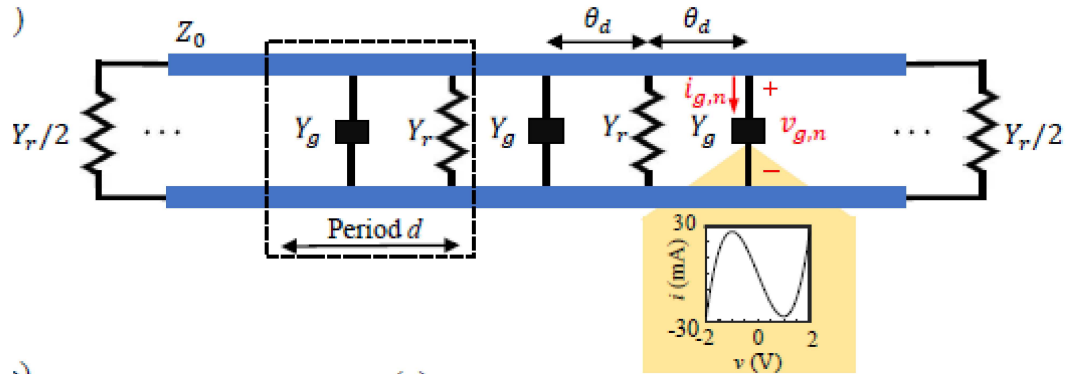
(Left) Voltage waveform $v_m(t)$ monitored at the Y_r load in the middle of the structure where steady-state oscillation is observed

(Right) Normalized voltage spectrum $V_{m;N}(f)$ shows that oscillations occur at around 3 GHz, that corresponds to the EPD frequency.

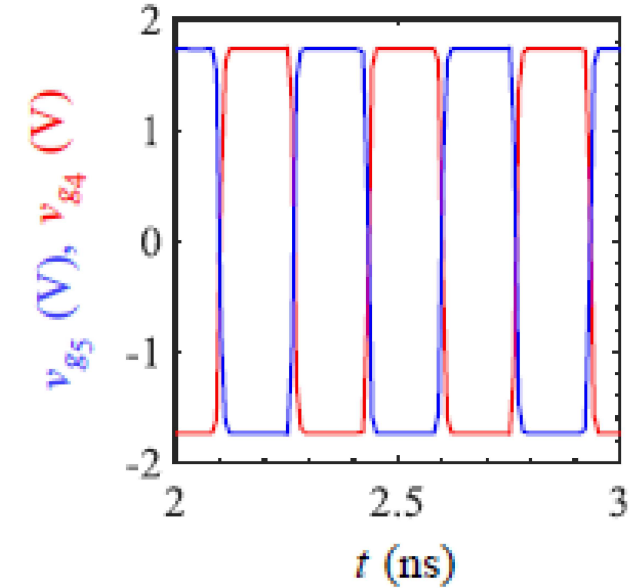
- Oscillation frequency is at 3 GHz
- EPD at 3GHz

Abdelshafy et al, "Exceptional Degeneracy in a Waveguide Periodically Loaded with Gain and Radiation Loss," *Appl. Phys. Lett.*, **118**, 224102, 2021

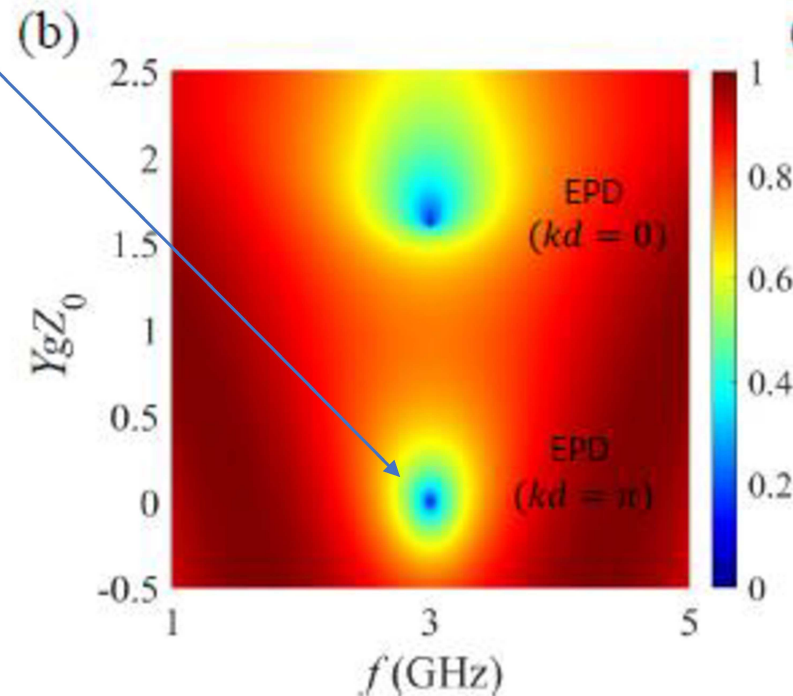
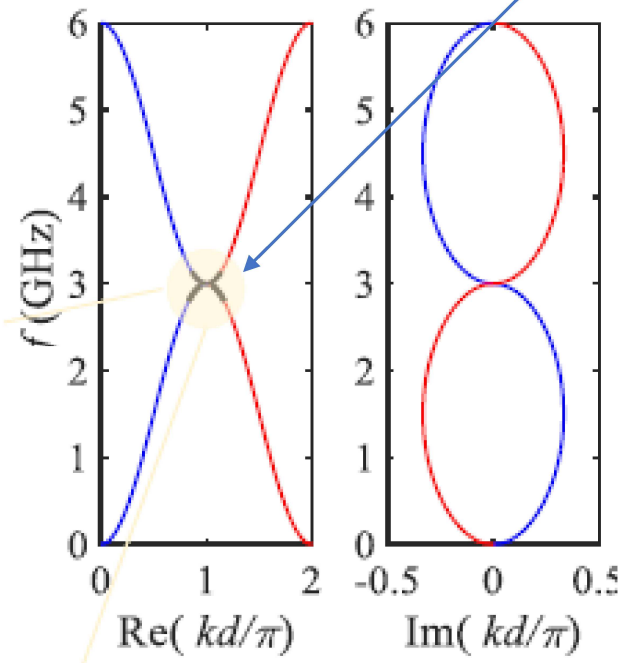
EPD- Radiation array oscillator



However, the phase shift between cell to cell is π



- There is another EPD at 3GHz, associated to a phase shift of π , which is reached by the smaller saturated gain value.



EPDs offer interesting physics aspects that can be used for various applications, like oscillators/lasers

1) In waveguides (lossless-gainless):

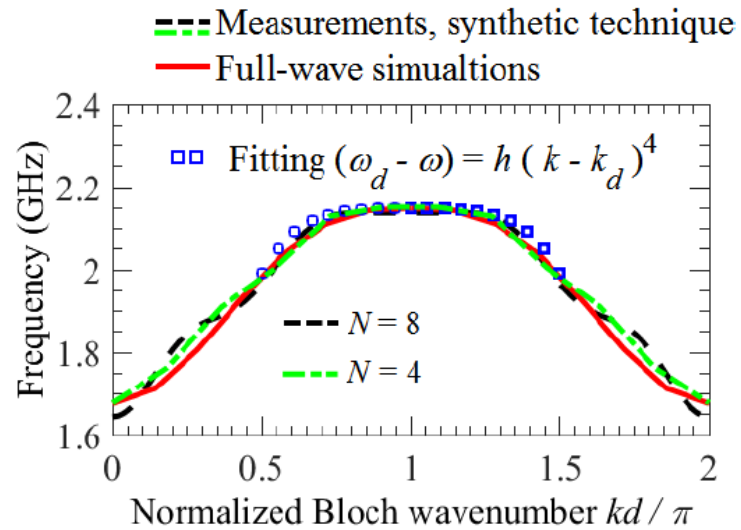
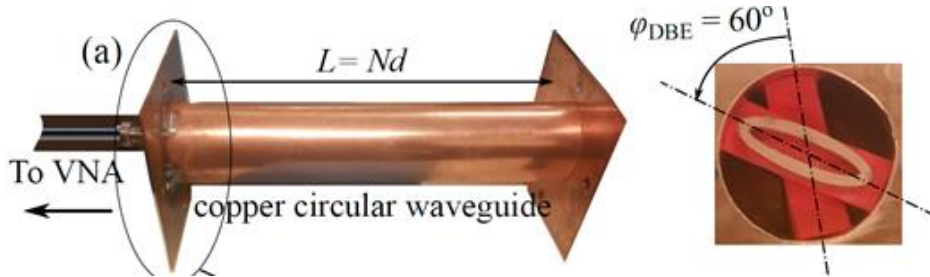
- High LDOS
- RBE, SIP, DBE: very stable/robust lasers/oscillators
- DBE: pulse generation
- SIP: delay lines
- RBE, SIP, DBE: sensors (k is highly sensitive), looking at group delay or radiation angle (spectrometers)
- Switches (on/off), Modulators

2) In waveguides (with loss and gain):

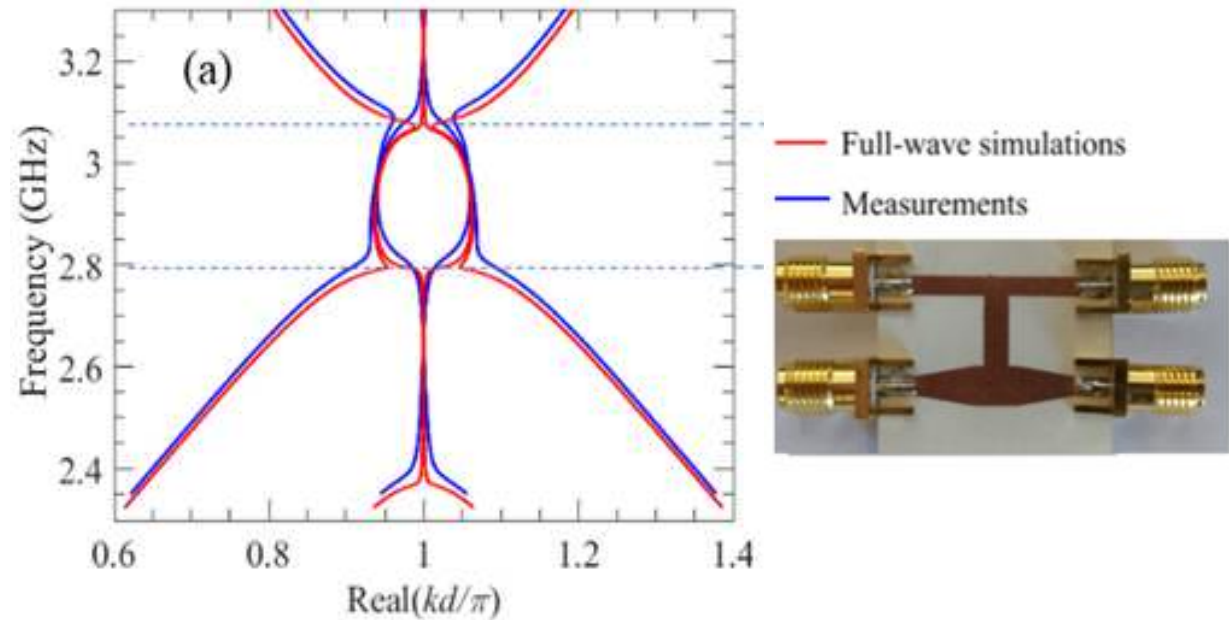
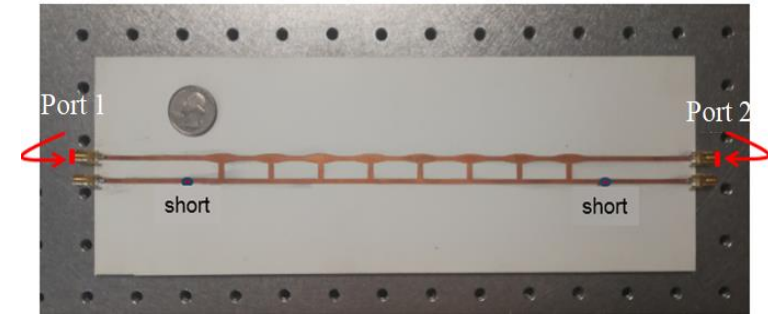
- High power distributed oscillators/radiators
- Synchronization of distributed oscillators

Experimental Demonstrations

Experimental demonstrations: DBE (no loss, no gain), EPD of order 4

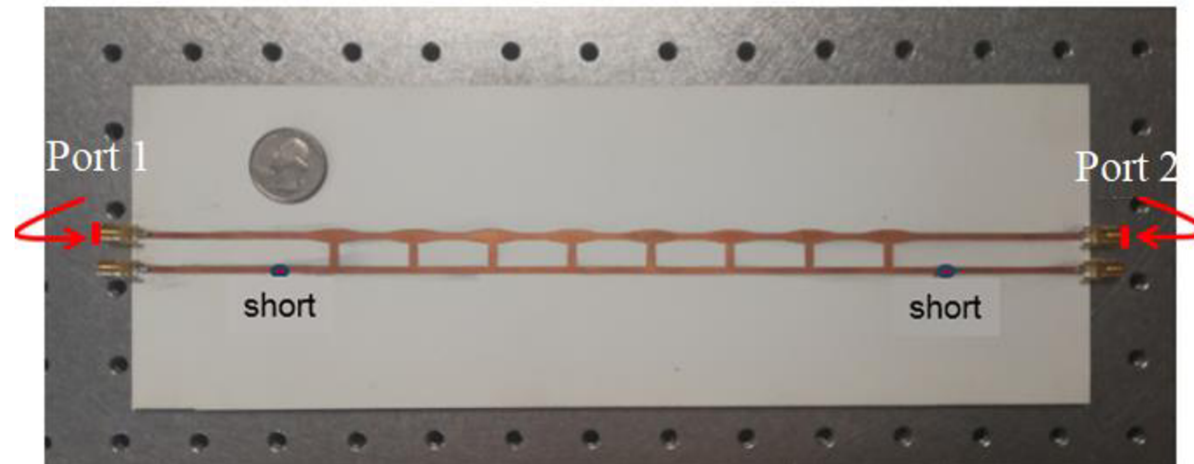


Othman, Pan, Atmatzakis, Christodoulou, Capolino, *IEEE Trans. Microw. Theory Techn.*, vol. 65, no. 6, 2017

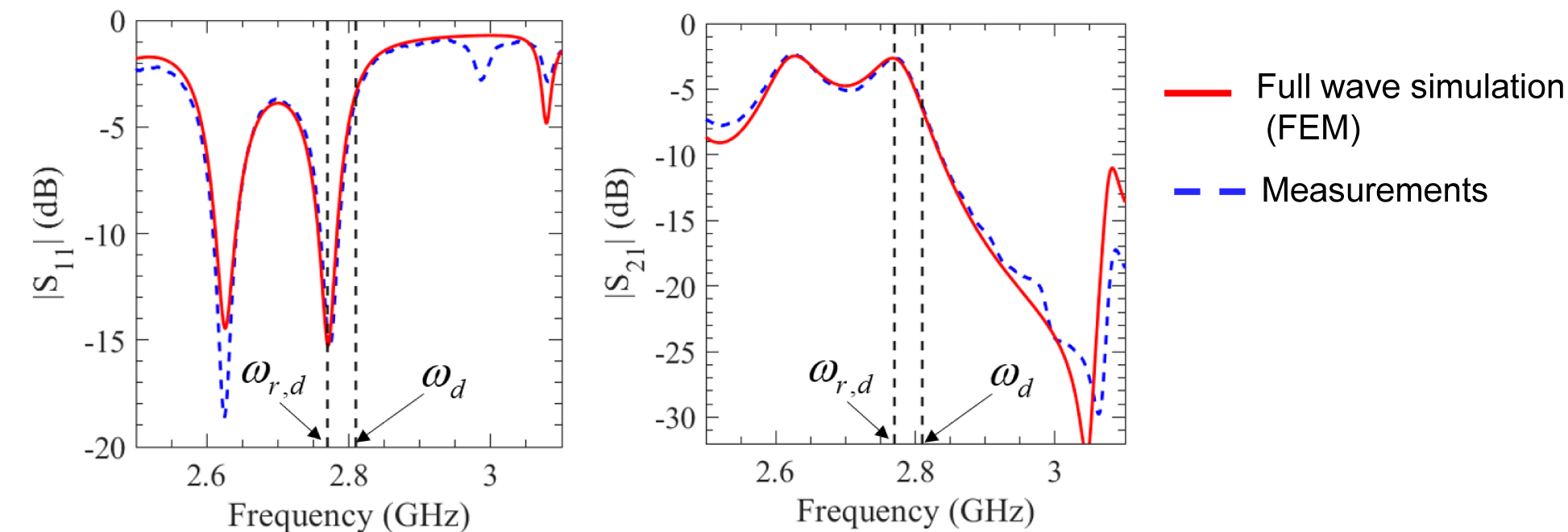


Abdelshafy, et al., *IEEE Trans. Antenna Propag.*, vol. 67, no. 9, 2019

➤ Finite structure:



➤ Simulations and measurements of the scattering parameters S_{11} and S_{21} for an 8-unit-cell array

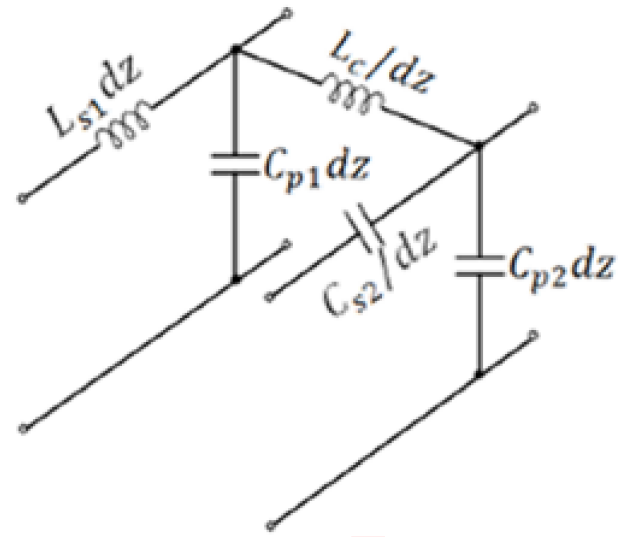


Resonance DBE frequency

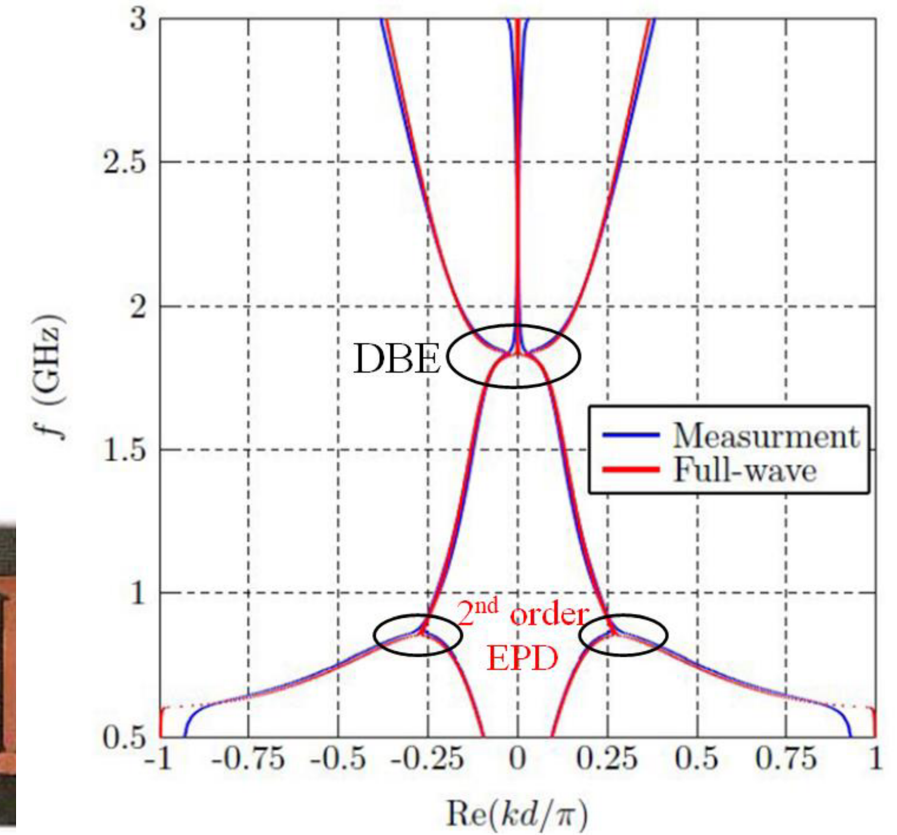
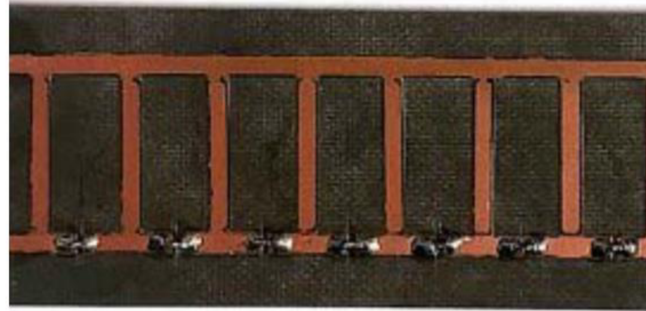
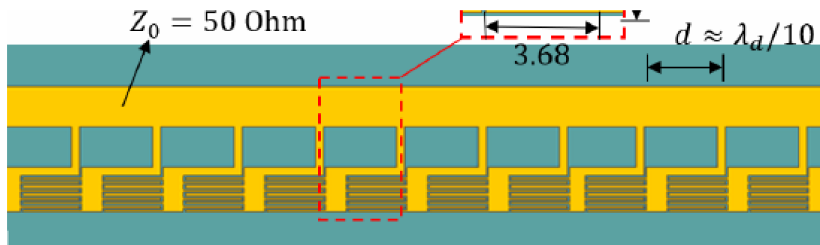
$$(\omega_{r,d} - \omega_d) \propto \left(\frac{\pi}{Nd} \right)^4$$

Figotin, Vitebskiy, "Slow-wave resonance in periodic stacks of anisotropic layers", PRA 2007

DBE (EPD of order 4)



Microstrip implementation



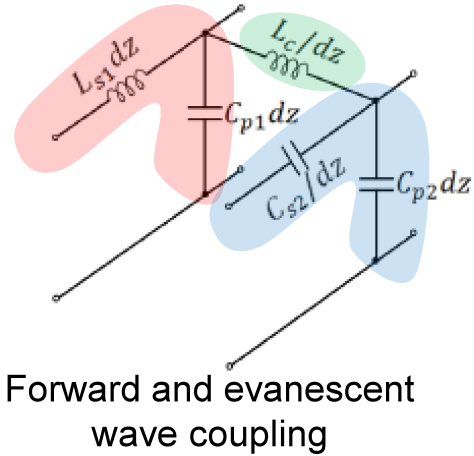
4th order DBE($k=0$) and 2nd order RBE are obtained in uniform coupled transmission lines where one line supports forward propagating modes and the other supports evanescent modes

Tarek Mealy and Filippo Capolino, "General conditions to realize exceptional points of degeneracy in two uniform coupled transmission lines," *IEEE Transactions on Microwave Theory and Techniques*, vol. 68, no. 8, pp. 3342–3354, 2020

EPD conditions in two uniform waveguides

Experimental verification

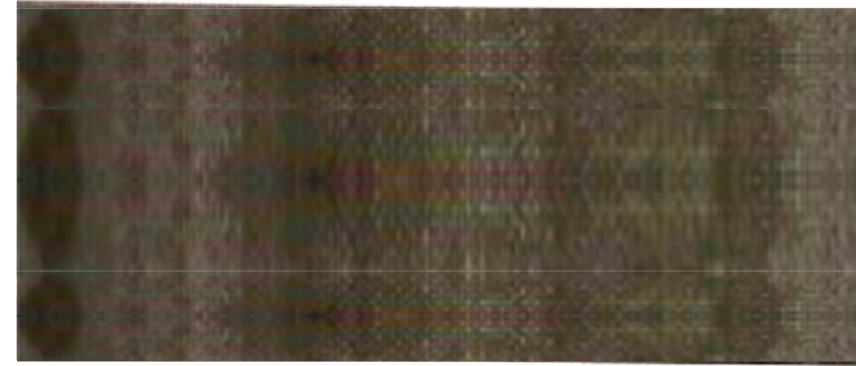
- Sub-wavelength microstrip realization



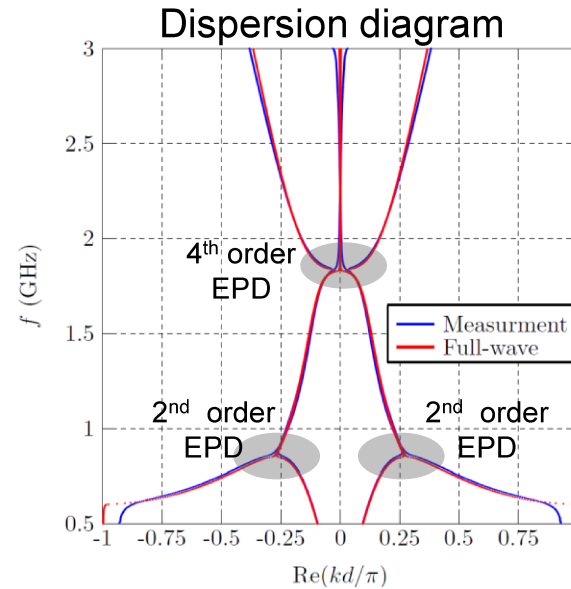
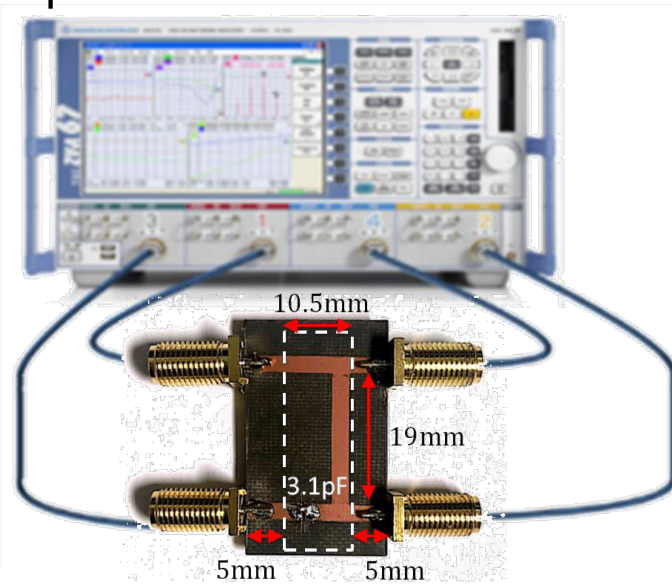
Physical realization



Microstrip Implementation

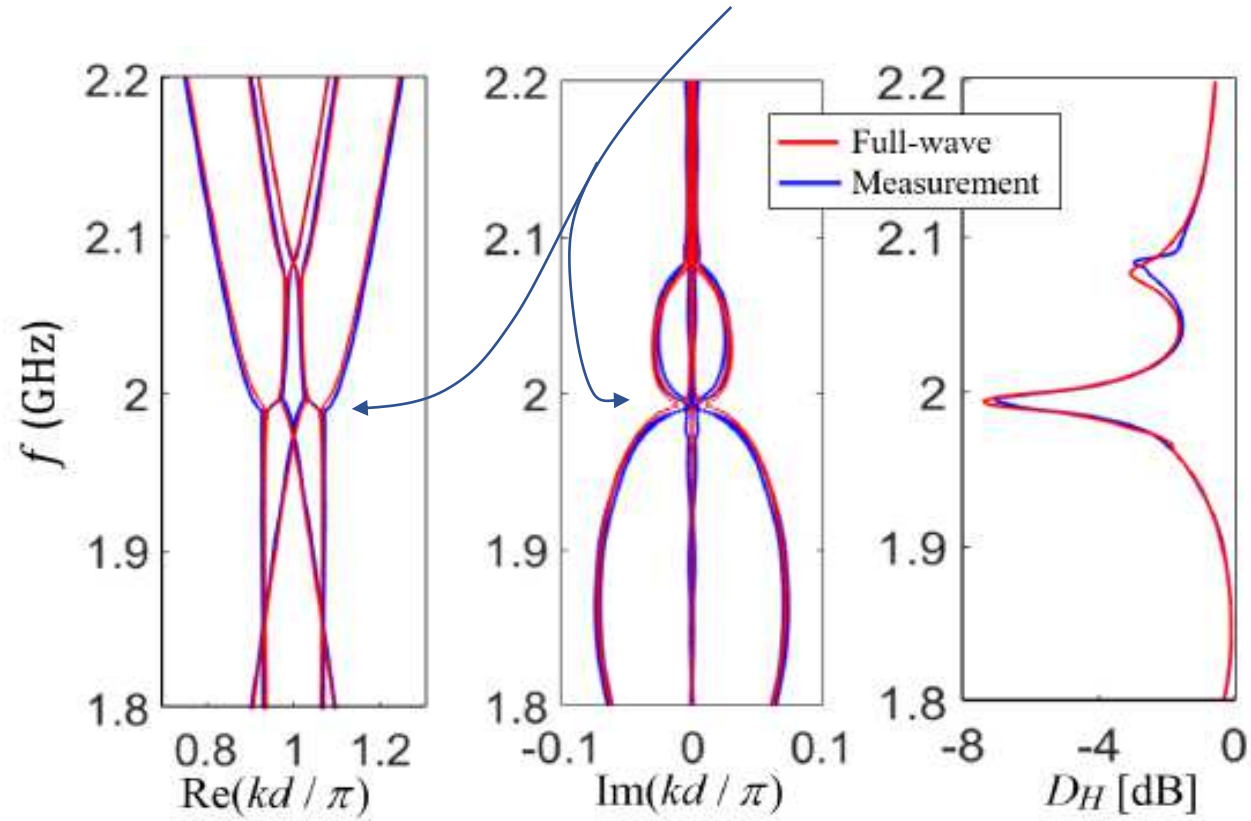


- Experiment result

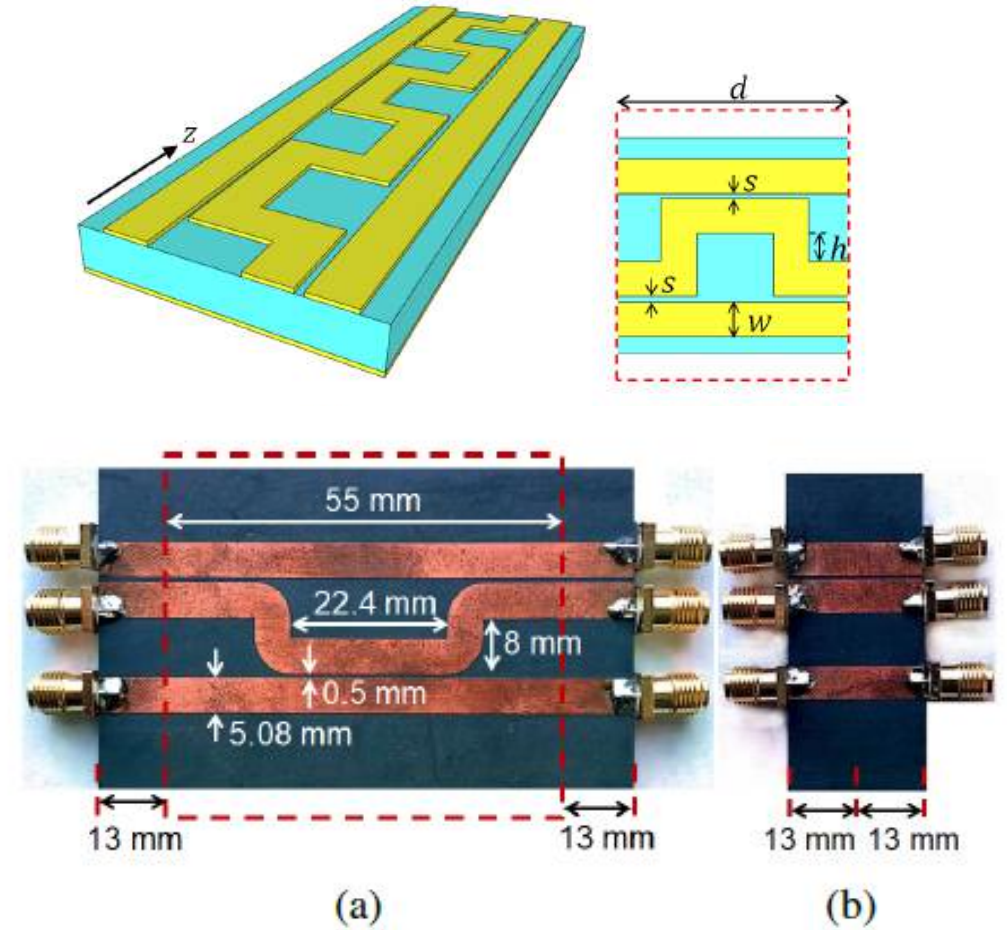


Measurement shows the existence of 4th order EPD and is in good agreement with full-wave simulation

Stationary inflection point (SIP), 3rd order EPD



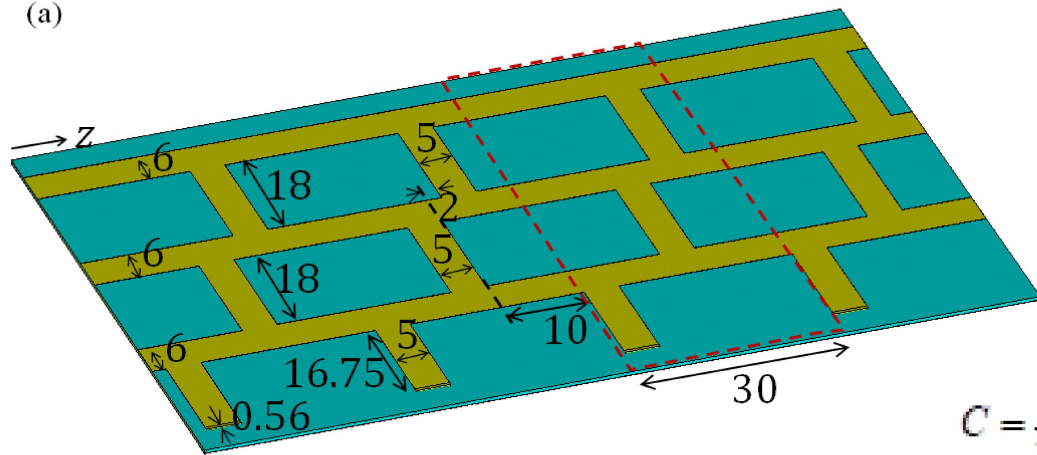
“Coalescing” parameter:
$$D_H = \frac{1}{3} \sum_{\substack{m=1, n=2 \\ n>m}}^3 |\sin(\theta_{mn})|$$



Nada, Mealy, Capolino, IEEE MWCL, 2021

6DBE- 6th order EPD

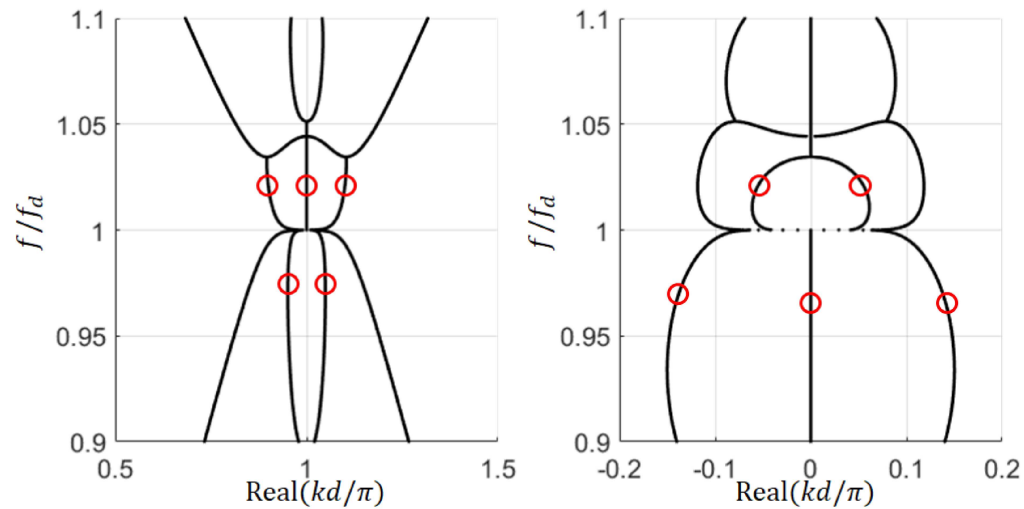
(a)



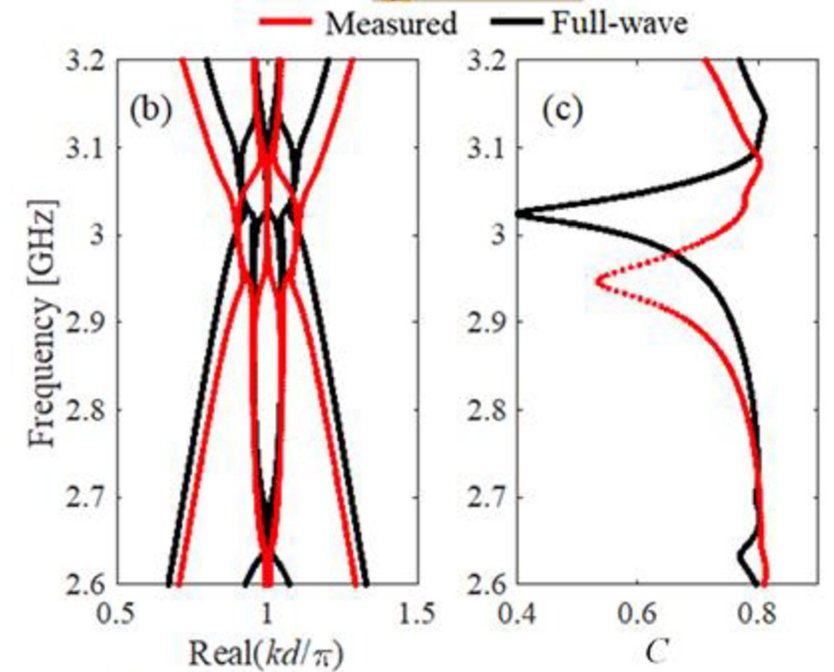
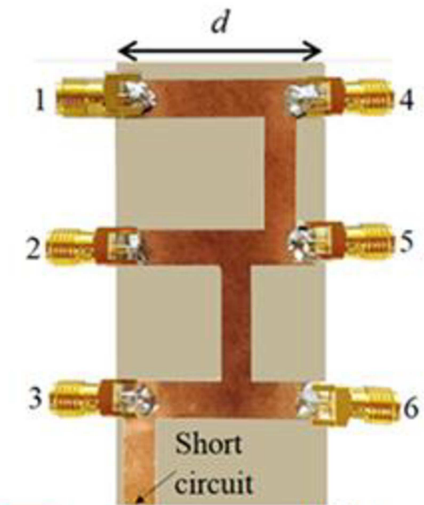
“Coalescence”
parameter:

$$C = \frac{1}{15} \sum_{\substack{m=1, N-1 \\ n=1}}^6 |\sin(\theta_{mn})|, \quad \cos(\theta_{mn}) = \frac{(\Psi_m, \Psi_n)}{\|\Psi_m\| \|\Psi_n\|}$$

(b)

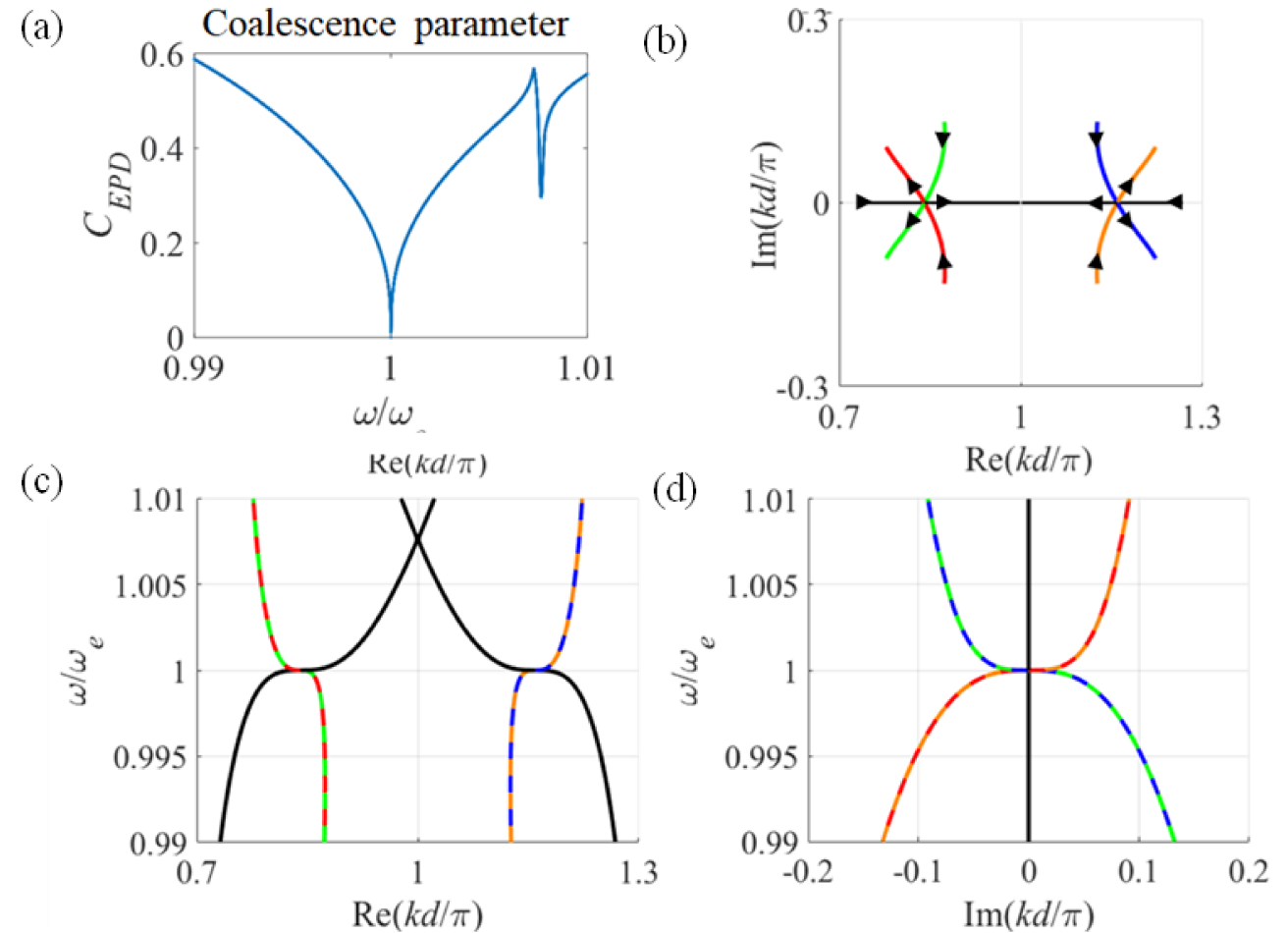
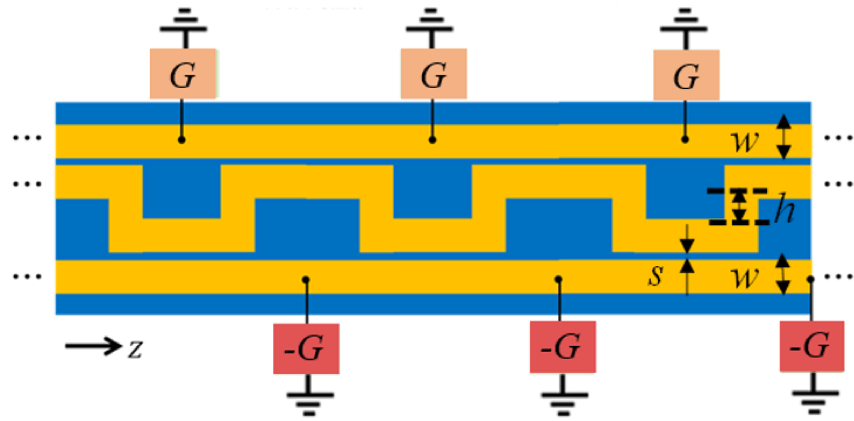


(a)



Yazdi, Oshmarin, Mealy, Almutawa, Nikzamid, Capolino,
arXiv:1906.03331, 2019

EPD in GT symmetric waveguide

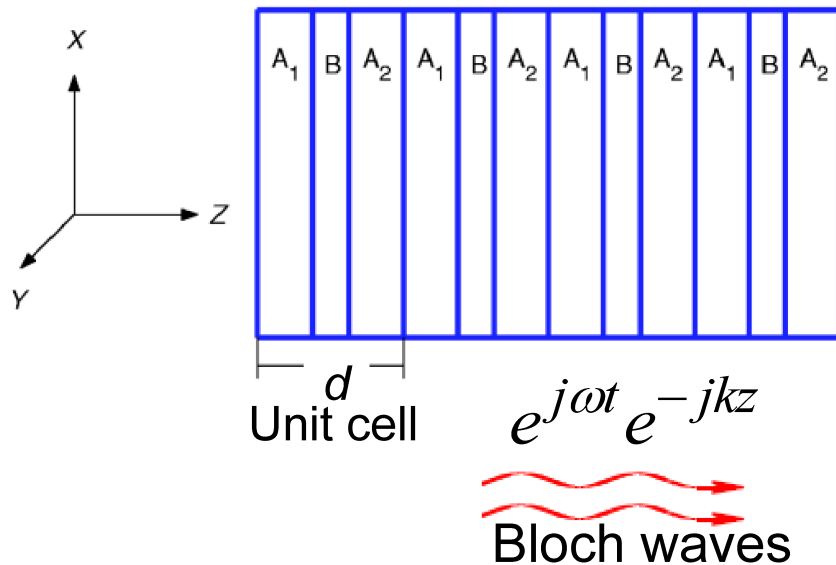


EPD is found in Glide-time symmetric waveguide

Degenerate band edge (DBE)

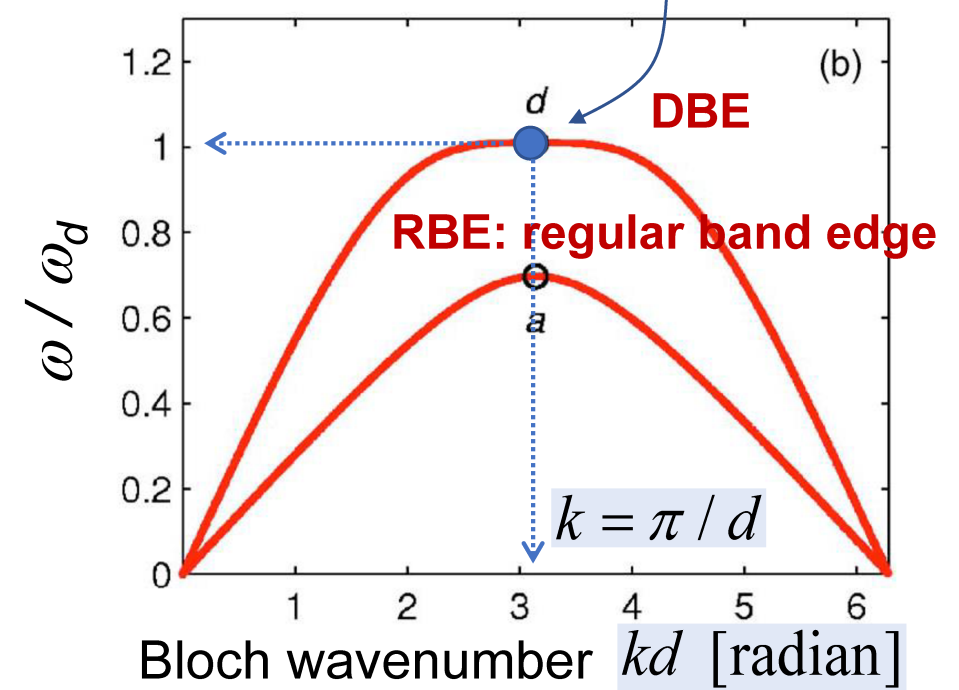
DBE: a 4th order EPD

Stack of lossless anisotropic layers



Dispersion diagram

$$(\omega_d - \omega) \propto (k - k_d)^4$$



Figotin, Vitebskiy, *Phys. Rev. E* 68(3) 2003

Figotin, Vitebskiy, *Phys. Rev. B* 72(3) 2005

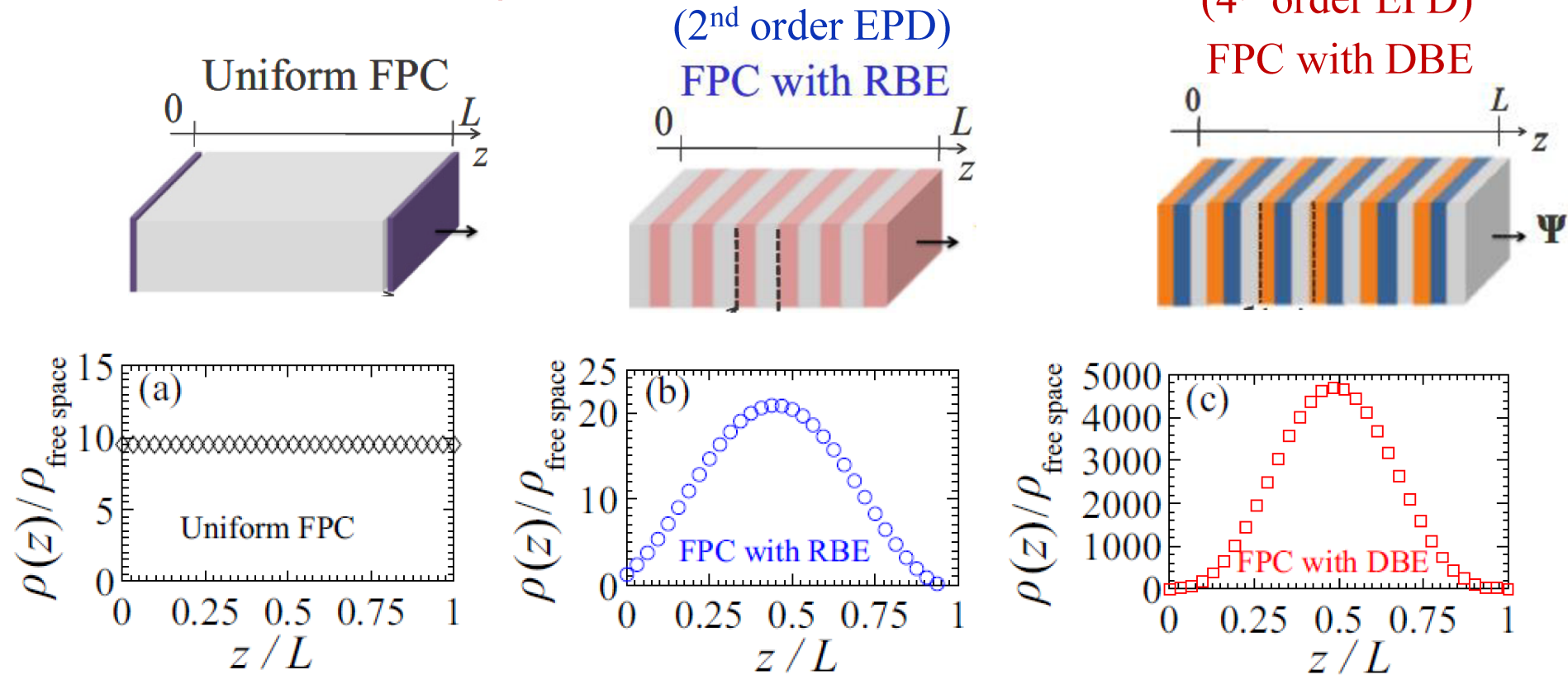
Othman and Capolino, *IEEE Microw. Wirel. Compon. Lett.*, vol. 25, no. 11, 2015

Othman, Yazdi, Figotin, Capolino, *Phys. Rev. B*, vol. 93, no. 2, p. 024301, 2016

Tamma, Figotin, Capolino, *IEEE Trans. Microw. Theory Tech.*, vol. 64, no. 3, pp. 742–755, 2016

Othman, Veysi, Figotin, Capolino, *IEEE Trans Plasma Sci*, vol 44, no. 6, 918-929, 2016

DBE and local density of states



- There is no need of mirrors when RBE or DBE are used
- LDOS is nonuniform in the cavity, peaking at the center
- LDOS reaches high values for the DBE case