Parity-Time Glide-Symmetry and Third Order Exceptional Degeneracy in a Three-Way Microstrip Waveguide

Alireza Nikzamir, Robert Marosi, Tarek Mealy, Farshad Yazdi, and Filippo Capolino
Department of Electrical Engineering and Computer Science, University of California, Irvine, CA 92697, USA
email: f.capolino@uci.edu

***Abstract*— A microstrip-technology three-way waveguide has been conceived to display exceptional modal degeneracy in the presence of periodic gain and radiation losses satisfying parity-time (*PT*) glide-symmetry. A third order exceptional point of degeneracy (EPD) is obtained in the modal dispersion relation, where three Floquet-Bloch eigenmodes coalesce at a single frequency, in their eigenvalues and polarization states. In the proposed structure, *PT*-glide-symmetry is achieved using periodically spaced lumped (radiation) loss and gain elements, each shifted by half a period. This particular degeneracy can be utilized in the design of devices such as radiating arrays with distributed amplifiers, arrays of oscillators, and sensors.**

***Index Terms*—antennas, *PT*-symmetry, glide symmetry, exceptional points, periodic structures, waveguides.**

I. Introduction

We explore the properties of a periodic three-way microstrip waveguide with Parity Time (*PT*)-glide symmetry, which exhibits a third-order degeneracy of electromagnetic modes. The strong degeneracy of eigenmodes in guiding structures has been explored previously in [1]–[6], however, the investigation of exceptional points of degeneracy (EPD) in active devices (i.e., with gain) is a relatively recent topic to be studied, see for example [7]–[17]. The interest in active EPDs has been motivated by the work in *parity-time-* (*PT*-) symmetric systems, as in [18]–[29], where gain and loss have been seen as EPD-enabling.

The structure introduced in this paper, based on *PT*-glide-symmetry, is constructed by adding periodically spaced positive and negative lumped shunt admittances, each shifted by half a period with respect to each other, between the waveguide and the ground plane. These shunt admittances can be realized using active elements and passive elements, such as transistors and radiating antennas, respectively. The glide-symmetric three-way microstrip structure is illustrated in Fig 1. A one-dimensional periodic structure is said to be *glide-symmetric* if the geometry remains unchanged after a glide operation, which is a translation by half the geometric period, *d*, followed by a reflection in the so-called glide plane [30]–[33]. We define *PT-glide symmetry* as typical glide symmetry of the waveguide with lossless and gainless components, in addition to the switching of the sign of passivity/activity when reflecting the structure in the glide plane during the glide symmetry operation. In other words, it is a combination of PT symmetry and glide symmetry.

Stationary inflection points (SIPs), which are EPDs of 3rd order in *lossless and gainless* waveguides, have been

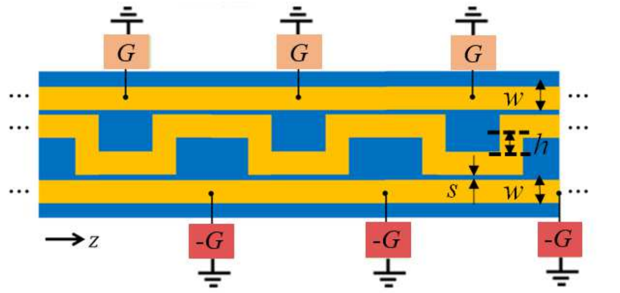

Fig. 1. 3-way periodic microstrip on a grounded substrate (blue) with periodic gain ($-G$) and loss (G, representing the radiation loss of arrayed antennas). The PT-glide symmetric structure is made of two transmission lines coupled through a third serpentine transmission line, loaded with periodic balanced gain and loss conductances.

previously demonstrated in [34]–[36], and the experimental demonstration of the SIP at microwaves in a reciprocal waveguide has been performed in [36]. Indeed, the microstrip geometry in this paper is based on the three-way microstrip structure of [36]. The presence of passive periodically spaced lumped elements in the structure causes the eigenmode dispersion to no longer exhibit third order modal degeneracy at its original design frequency. However, with the additional introduction of periodic gain elements, it is possible to recover a 3rd order EPD at a desired frequency, with real-valued wavenumber. While this EPD is not the same as an SIP due to the presence of active and passive elements, it is analogous in some respects.

Structures with *PT*-symmetry using balanced loss and gain have been previously explored in [11]–[13], [20], [21], [37] for EPDs of order 2 and 4. However, to the best of our knowledge, this is the first theoretical demonstration of EPDs of order 3 in a *PT*-glide-symmetric periodic structure.

In the vicinity of a 3rd order EPDs in the eigenmodes' dispersion diagram, the dispersion satisfies the condition $(\omega-\omega_e)\propto(k-k_e)^3$, where ω_e is the angular frequency at which three eigenmodes coalesce and k_e is the real-valued Floquet-Bloch wavenumber at the point of degeneracy, as demonstrated in Fig. 2 (b).

The concept of amplifiers based on the 3rd order EPDs shown here may be applied to the design of arrays of periodic antennas with coexisting distributed amplifiers. These kinds of EPDs are achieved in the presence of significant gain and loss, thus this kind of array can radiate high power. This is vastly different from the SIP concept in a lossless/gainless waveguide, where gain is introduced in a uniform way as in [17]; in that case, the SIP is degraded when high gain is presented in each unit cell, whereas the 3rd order EPD here is fully supported with large gain elements if appropriately designed, enabling applications of EPDs at very high power.

II. Unit Cell Design with Balanced Gain and Loss

We study the occurrence of the coalescence of three eigenmodes in a periodic structure made of three coupled waveguides. The periodic structure is shown in Fig. 1, where the two parallel transmission lines are coupled through a third adjacent serpentine-shaped transmission line in the middle. The uncoupled segments (where the serpentine transmission line is not close to a straight transmission line) are connected to periodic balanced lumped gain ($-G$) and loss (G) elements to make the structure *PT*-glide symmetric. Here, we assumed that both gain and loss are purely real-valued. For the sake of simplicity, we assume the transmission lines are identical in cross-section, with the same width $w = 5$ mm. The coupled transmission lines have a gap of $s = 0.5$ mm between them. The substrate has a relative dielectric constant is $\varepsilon_r = 2.2$ with no loss tangent ($\tan(\delta) = 0$), and a height of $h_s = 1.575$ mm. Each microstrip (when uncoupled) has a characteristic impedance $Z_0 = 50\ \Omega$. Then, by tuning the unit cell period d, lumped admittance G, and serpentine height h parameters, we obtain a 3rd order EPD as we minimize the coalescence parameter related to all three eigenmodes in the system, at SIP frequency $f_e = 2$ GHz. The details of the coalescence parameter are given in Section IV.

Small losses in the dielectric ($\tan(\delta) \neq 0$) and in the metal or any imperfections on the waveguide may cause the state of the system to move away from its original EPD. However, a new EPD in the lossy waveguide may be found by selecting different periodic loss and gain values. For example, one may fine tune the gain element to compensate for the small perturbation of loss in the waveguide and form a new EPD.

III. Transfer Matrix Formalism

In order to find the 3rd order EPD, the transfer matrix of the 6-port system is determined by first defining the state vector as

$$\mathbf{\Psi}(z) = [V_1,\ Z_0 I_1,\ V_2,\ Z_0 I_2,\ V_3,\ Z_0 I_3]^T, \qquad (1)$$

where T denotes the transpose operation. Voltages and currents are calculated at each position, z. The relation between the state vector calculated at two points in the structure, using the forward transfer matrix notation, is

$$\mathbf{\Psi}(z_2) = \underline{\mathbf{T}}(z_1, z_2)\mathbf{\Psi}(z_2), \qquad (2)$$

where $\underline{\mathbf{T}}$ is a 6×6 transfer matrix. The transfer matrix of a unit cell is defined such that $\mathbf{\Psi}(z + d) = \underline{\mathbf{T}}_U \mathbf{\Psi}(z)$. For an infinitely long structure, a periodic solution for the state vector exists in Bloch form, with an eigenvalue equation as

$$\underline{\mathbf{T}}_U \mathbf{\Psi}(z) = e^{-jkd}\mathbf{\Psi}(z), \qquad (3)$$

where k is the complex-valued Bloch wavenumber. The eigenvalues of the transfer matrix, and hence the Bloch wavenumbers, are calculated from $\det(\underline{\mathbf{T}}_U - \zeta \underline{\mathbf{I}}) = 0$, where the $\underline{\mathbf{I}}$ is 6×6 identity matrix and the six eigenvalues are $\zeta_i = e^{-jk_i d}$ with $i = 1,2,\ldots 6$. Each Bloch wavenumber has Floquet harmonics $k_i + n2\pi/d$, where n is an integer.

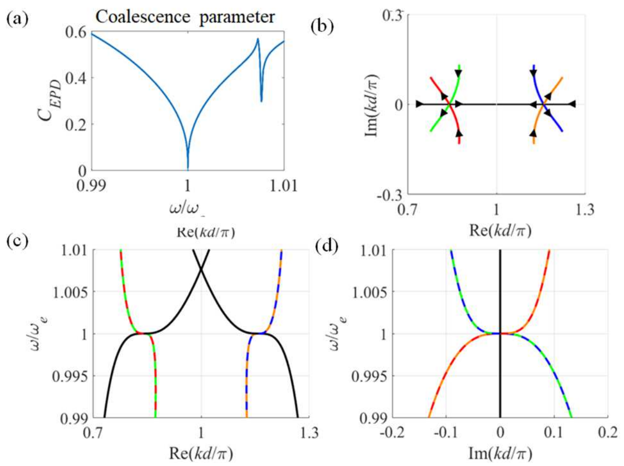

Fig. 2 (a) Coalescence parameter plotted versus normalized frequency to observe the degree of coalescence of three system's eigenvectors. (b) Complex wavenumber plotted in the complex k plane varying angular frequency, showing the EPD at two wavenumbers, k_e and $-k_e + 2\pi/d$. (c) and (d): real and imaginary parts of the complex Floquet–Bloch wavenumber k versus normalized angular frequency around the EPD angular frequency ω_e. The two solid black curves show the purely real wavenumber branches. Three modes coalesce at each of the two shown EPDs.

IV. Dispersion Relation and Coalescence Parameter of 3rd Order EPD

The proposed three-way microstrip in Fig. 1 is designed to have a 3rd order EPD at an operating frequency of 2 GHz. This is achieved by tuning the dimensional parameters like unit cell's period d, the height h of the serpentine sections, or electrical parameters such as the balanced gain $-G$ and loss G. It is convenient to introduce the coalescence parameter as a tool for verifying how close the system is to exhibiting an EPD. The coalescence parameters C_{EPD} for finding a 3rd order EPD is defined as

$$C_{EPD} = \frac{1}{3}\sum_{m=1,\, n=2,\, n>m}^{3} \left|\sin(\theta_{mn})\right| \qquad (4)$$

$$\cos(\theta_{mn}) = \frac{\mathrm{Re}|(\mathbf{\Psi}_m, \mathbf{\Psi}_n)|}{\|\mathbf{\Psi}_m\|\|\mathbf{\Psi}_n\|}, \qquad (5)$$

where θ_{mn} represents the angle between two eigenvectors and $(\mathbf{\Psi}_m, \mathbf{\Psi}_n)$ is the inner product of two eigenvectors. The coalescence parameter is a positive valued function, with smaller numbers indicating how well the eigenvectors of the structure coalesce. In the above equation, we consider the three eigenvectors associated to a degenerate eigenvalue, i.e., with wavenumber such that $0 < \mathrm{Re}(k_i) < \pi/d$. Using optimization routines in MATLAB to minimize the coalescence parameter, we found a 3rd order EPD with the conductance $G = 0.105\,\mathrm{S}$, serpentine height $h = 6.36\,\mathrm{mm}$, and period $d = 46.3\,\mathrm{mm}$. Aside from the introduced gain/loss lumped elements G, we have assumed the three-way microstrip waveguide structure to be lossless. Fig. 2 (a) shows the coalescence parameter C_{EPD} plotted versus normalized frequency around ω_e (corresponding to 2 GHz) to demonstrate the degree of coalescence of the system's eigenvectors. The imaginary versus the real part of the wavenumber is shown in Fig. 2 (b), which demonstrates the coalescence of the three modes at both k_e and $-k_e + 2\pi/d$, due to reciprocity. Also, we show the modal dispersion diagram of real and imaginary parts of the wavenumber versus frequency for modes in the infinite structure in Fig. 2 (c) and (d), respectively. The black-solid curve represents the branch with purely real wavenumber. The dashed curves with different colors are used to show two overlapping branches with complex wavenumbers.

V. Conclusion

We have shown the occurrence of a third order EPDs in a three-way waveguide with *PT*-glide-symmetry made of three microstrip coupled transmission lines structure. This involves simultaneous presence of gain and radiation losses (modeled as conductances). The dispersion diagram with the SIP on the left side of Fig. 2 shows that the propagating mode (black curve) contributing to the SIP is a forward wave. Thus, it does not involve backward waves that would otherwise cause undesirable oscillations, and therefore the proposed SIP may be beneficial to applications involving amplifiers. The finding in this paper paves the way for a new way of operation in high power radiating arrays of antennas with distributed gain. The goal of this paper is to show this kind of third order EPD for the first time at a conference; the investigation of possible advantages of this technique in realistic devices is left to future investigations.

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