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Exceptional Points of Degeneracy in Gyrator-Based Coupled Resonator Circuit

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***Abstract* – We propose a high-sensitive circuit scheme based on an exceptional point of degeneracy (EPD) using two LC resonators coupled with a gyrator. EPD is a point in a system's parameter space in which two or more eigenmodes coalesce in both their resonance frequency and eigenvectors into a single degenerate eigenmode by varying a parameter in the system. We present the conditions to obtain EPDs in this circuit and the typical bifurcation diagram that shows the extreme sensitivity of such a circuit operating at an EPD to system's perturbations. The very high sensitivity induced by an EPD can be used to explore a new generation of high-sensitive sensors.**

I. INTRODUCTION

High-sensitive circuits based on exceptional point of degeneracy (EPD) in multiple, coupled resonators have been investigated in various papers, for example, [1]–[4]. Although most of the works on EPDs are related to parity-time (PT) symmetry [2], [3], the occurrence of an EPD actually does not require a system to satisfy PT symmetry. Here we show that there is a new class of systems that exhibit EPD by using two resonators coupled via a gyrator. The circuit shows two real frequencies when perturbed.

Since the characterizing feature of an exceptional point is the strong full degeneracy of at least two eigenmodes, we emphasize the significance of referring to it as a "degeneracy" as in [5], including the D in EPD. In essence, an EPD is obtained when the system matrix is similar to a matrix that comprises a non-trivial Jordan block. In recent years frequency splitting phenomena at EPDs have been proposed for sensing applications using PT symmetry [2], [3], and also a time-varying circuit [4], [6]. Frequency splitting occurs at the degenerate resonance, and a tiny perturbation leads to a significant shift in the system resonance frequencies. This concept can be employed in modern sensing devices with a high-sensitivity feature.

This paper presents an investigation of EPDs in gyrator-based circuits. The ideal gyrator is a nonreciprocal passive device that neither stores nor dissipates energy. In the suggested circuit, two LC resonators are coupled to each other through a gyrator. The presented circuit in this paper is a dual version of the circuit shown in [7]. An important requirement of this new way to obtain EPDs is that two reactive components have negative values, which can be realized using operational amplifiers. We show the circuit's high sensitivity to a component variation and show that the Puiseux fractional power series expansion approximates the bifurcation of the frequency diagram around the EPD. The proposed circuit is more complex than a conventional single LC tank, but it is much more sensitive to perturbation because of the EPD. This circuit has the potential to constitute a new paradigm to realize ultra-high-sensitive capacitive or inductive sensors like pressure sensors, temperature sensors, etc. Examples where a change in capacitance is detected in an EPD-based sensing are in Refs. [8], [9].

II. EPD IN GYRATOR-BASED CIRCUIT

An ideal gyrator is a linear, lossless, passive two-port element that couples the current on one port to the voltage on the other and vice versa as $v_2 = R_g i_1$ and $v_1 = -R_g i_2$, where R_g is the gyration resistance. The arrow to the right in Fig. 1 represents the direction of gyration. A gyrator is a nonreciprocal element, so each practical gyrator circuit must contain at least one nonreciprocal component, like a transistor or an operational amplifier [10].

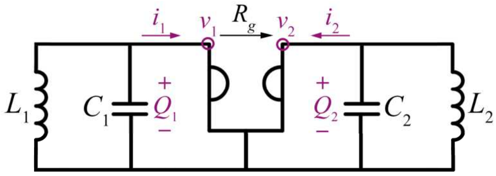

Fig. 1. A gyrator-based circuit to obtain EPD. The reactive elements on the right side have negative values.

We show how the gyrator coupling two resonant circuits, as shown in Fig. 1, leads to an EPD. By writing down the Kirchhoff current laws on both sides, we obtain the circuit equations. It is convenient to define the state vector as $\mathbf{\Psi} \equiv [Q_1, Q_2, \dot{Q_1}, \dot{Q_2}]$, which consists of a combination of stored charge ($Q_n = C_n V_n$) and its time derivative for each capacitor. Thus, we can write down the Liouvillian formalism for this circuit as

$$\frac{d\mathbf{\Psi}}{dt} = \underline{\mathbf{M}}\mathbf{\Psi}, \quad \underline{\mathbf{M}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-1}{C_1 L_1} & 0 & 0 & \frac{-1}{R_g C_2} \\ 0 & \frac{-1}{C_2 L_2} & \frac{1}{R_g C_1} & 0 \end{bmatrix}. \tag{1}$$

We are interested in determining the eigenfrequencies in the circuit. Assuming signals of the form $\exp(j\omega t)$, we write the eigenvalues problem. The eigenfrequencies of the circuit are found analytically as,

$$\omega_{1,4} = \pm\sqrt{a+b}, \qquad \omega_{2,3} = \pm\sqrt{a-b}, \tag{2}$$

$$a = \frac{1}{2}(\omega_{01}^2 + \omega_{02}^2 + \frac{1}{C_1 C_2 R_g^2}), \tag{3}$$

$$b^2 = \frac{1}{4}\Big((\omega_{01}^2 - \omega_{02}^2)^2 + (\frac{1}{C_1 C_2 R_g^2})^2 + \frac{2}{C_1 C_2 R_g^2}(\omega_{01}^2 + \omega_{02}^2)\Big), \tag{4}$$

with $\omega_{01}^2 = 1/(L_1 C_1)$ and $\omega_{02}^2 = 1/(L_2 C_2)$. In order to obtain an EPD, a necessary condition is that $b = 0$, so that $\omega_1 = \omega_2$. By satisfying this condition, we may have degenerate real or imaginary frequencies. Without loss of generality, we consider the case with real frequencies obtained when ω_{01}^2 and ω_{02}^2 are positive, both C_2 and L_2 are negative, and $a > 0$. We consider the following values for the components in the circuit that generate an EPD: $C_{1,e} = 4.45$ nF, $L_{1,e} = 33$ µH, $C_{2,e} = -33$ nF, $L_{2,e} = -33$ µH, and $R_{g,e} = 50\,\Omega$. With these values, $\omega_e = 1.5804$ Mrad/s. In Figs. 2(a) and (b), the complex resonance frequencies are shown varying the gyration resistance (only frequencies with positive real parts are shown). For a value $R_g = R_{g,e}$, the eigenfrequencies experience a transition from complex to purely real values, following the typical bifurcation diagram typical of an EPD perturbation [3]. The eigenvalues at EPDs are exceedingly sensitive to perturbations of the parameters of the circuits. We prove that the sensitivity of a system's observable to a specific change of an LC component value is boosted by the eigenvalues' degeneracy. We consider the circuit at its EPD condition with the aforementioned components and analyze the circuit's sensitivity near the EPD. We define the relative system perturbation of the capacitance $\delta = (C_1 - C_{1,\mathrm{e}})/C_{1,\mathrm{e}}$, where $C_{1,\mathrm{e}}$ is the unperturbed parameter that provides the EPD,

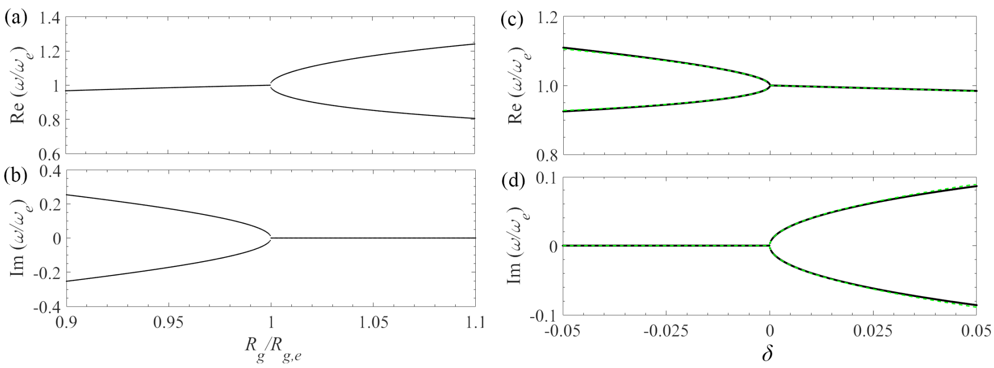

Fig. 2. Variation of the (a) real and (b) imaginary part of the eigenfrequencies to a gyration resistance variation. Change of the (c) real and (d) imaginary part of the eigenfrequencies to a capacitance perturbation. A solid black line shows the exact value, and the dashed green line shows the Puiseux approximation (5) truncated to its second order. The square root behavior near the EPD indicates very high sensitivity.

and C_1 is its perturbated value. In Fig. 2(c) and (d), we show the eigenfrequencies varying C_1 obtained by solving the eigenvalue problem corresponding to the perturbed matrix $\underline{\mathbf{M}}$. The diagram exhibits the typical bifurcation point of an EPD perturbation. Then, we calculate the corresponding eigenfrequencies using the Puiseux fractional power series expansion [11] truncated at the second order. As mentioned in [12], the most significant term in the Puiseux fractional power series expansion is the term that contains the first-order coefficient α_1 and the square root of relative system perturbation δ in

$$\omega_p(\delta) \approx \omega_e + (-1)^p \alpha_1 \sqrt{\delta} + \alpha_2 \delta \quad (5)$$

where ω_p $(p = 1, 2)$ are the two perturbed eigenfrequencies near the EPD. This plot shows the typical high sensitivity characteristics near an EPD. As a practical scenario, the sensing capacitor could be the one on the left resonator. Variation in the concentration of the material under test can change the capacitance value, which leads to frequency perturbation, as shown in the EPD-based schemes in [8].

VI. Conclusion

We have shown that two resonators coupled via an ideal gyrator circuit could yield an EPD and discussed the enabling condition. A perturbation of the capacitance leads to two real-valued frequency shifts from the EPD one, even if the circuit is not PT-symmetric. We can calculate the perturbation in the system by measuring the change in two resonance frequencies. Moreover, we discussed how eigenfrequencies are exceptionally sensitive to a perturbation of the system that may have very important implications in sensing technology and RF sensors. This new scheme of operation is promising for boosting the overall performance of ultra-high sensitive sensors.

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