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# Low-Frequency Magnetoelectric Effect in Bilayer Magnetostrictive-Piezoelectric Structures: Planar and Bending Deformations

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**Abstract.** The theory of the magnetoelectric effect in a bilayer magnetostrictive-piezoelectric structure magnetic-piezoelectric in the low-frequency region of the spectrum is presented. Based on the solution of the equations of the elastic theory and electrostatics, a simple expression for the magnetoelectric voltage coefficient in terms of the physical parameters of the material and the geometric characteristic of the structure is obtained. The contribution to the magnitude of the effect due to bending deformations and planar deformations is considered. It is shown that the contribution from planar deformations to the magnitude of the effect significantly exceeds the contribution from bending deformations. The calculation results for the structures nickel - PZT, permendur - PZT, Terfenol-D - PZT, Metglas - PZT are presented.

## INTRODUCTION

Layered magnetic-piezoelectric composite structures have better magnetoelectric (ME) characteristics in comparison with bulk composites [1-3]. One of the most important advantages of such structures is that in their manufacture it is possible to use ferromagnets with high magnetostriction, such as permendur, Terfenol-D, Metglas, etc., while in the manufacture of bulk composites used oxides of ferromagnets and their compounds such as ferrite-nickel, ferrite-cobalt spinels, etc. [2] with less magnetostriction. Ferromagnets, as a rule, are good conductors; therefore, their use in the manufacture of bulk composites leads to the occurrence of large leakage currents, which leads to a sharp increase in losses. In the layered structures, the ferromagnet layers are well insulated with a piezoelectric layer, as a result, the losses associated with leakage currents do not occur. Previously, the ME effect in bilayer structures was studied theoretically in the works [4-9]. The ME effect caused by planar vibrations of the structure was investigated in the works [4-7], and the ME effect caused by bending deformations was studied in the works [8,9]. Particular attention in these works was paid to the electromechanical resonance region, at which a peak increase in the effect occurs. The low-frequency region in these works has been paid very little attention, although due to the growing number of studies aimed at creating energy collectors - harvesters based on the ME effect, this area is extremely important [10]. The formulas for the magnitude of the ME effect obtained in [4-9] are extremely cumbersome and difficult to analyze. The authors did not give their passage to the limit for the low-frequency spectral region and did not sufficiently analyze the dependence of the magnitude of the effect on the parameters of the structure. In this work, the theory of the ME effect in the low-frequency region, based on the equations of the theory of elasticity and electrostatics, is presented; simple expressions are obtained for the magnetoelectric voltage coefficient in terms of the physical parameters of materials and the geometric characteristics of the structure. Contributions to the effect of planar and bending deformations and their dependence on the geometric parameters of the structure are analyzed.

## MODEL AND METHODS

As a model, we will consider a bilayer structure of a magnetic - piezoelectric, a schematic drawing of which is shown in Fig. 1. The origin of the coordinate system is compatible with the center of the sample, and the X-axis (1) is compatible with the interface between the piezoelectric layer and the magnetic layer.

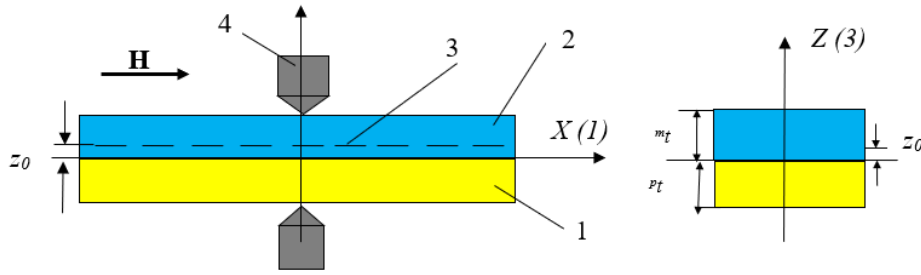


Fig. 1 Schematic drawing of the structure. 1 – piezoelectric layer, 2 – magnetic layer, 3 – neutral layer, 4 – electrodes (support).

We will assume that the sample length is much greater than its width  $W$  and thickness  $t = {}^m t + {}^p t$ . In this approximation, the constitutive equations for the piezoelectric and magnetostrictive phases will have the following form:

$${}^p S_1 = \frac{1}{{}^p Y} {}^p T_1 + {}^p d_{31} E_3, \quad (1)$$

$${}^m S_1 = \frac{1}{{}^m Y} {}^m T_1 + {}^m q_{11} H_1, \quad (2)$$

$${}^p D_3 = {}^p \varepsilon_{33} E_3 + {}^p d_{31} {}^p T_1, \quad (3)$$

where  ${}^p S_1$ ,  ${}^m S_1$  are strain tensor components of piezoelectric and magnetostrictive layers,  ${}^p Y$ ,  ${}^m Y$ , are their Young's moduli,  $E_3$ ,  ${}^p D_3$  are components of the vector of the electric field and electric induction,  ${}^p T_1$ ,  ${}^m T_1$ , are the stress tensor components of the piezoelectric and magnetostrictive phases,  ${}^p d_{31}$ ,  ${}^m q_{11}$  are piezoelectric and piezomagnetic coefficients,  ${}^p \varepsilon_{33}$  is the component of the permittivity.

When the sample is placed in a magnetic field in a magnet, due to magnetostriction, tensile deformations occur in the case of positive magnetostriction (permendur, D-Terfenol) or compression if the magnet has negative magnetostriction (nickel, ferrite-nickel spinel). By means of mechanical coupling through the interface, these deformations are transferred to the piezoelectric phase, because of which the sample can experience planar deformations such as tension or compression. Since these deformations are not axial, they lead to occur a bending moment and result occur bending deformations. We are supposed that layer thicknesses of the sample are thin, therefore, we can assume that for planar vibrations the layer strains are the same, i.e., the following equality hold:

$${}^m S_1 = {}^p S_1 = S_1 \quad (4)$$

The equilibrium condition of the sample, namely the equality to zero the  $X$  projection of the force, gives the following equation:

$${}^mT_1 {}^mt + {}^pT_1 {}^pt = 0 \quad (5)$$

Expressing the components of the stress tensor from Eq. (1) and Eq. (2) and substituting the obtained expressions into Eq. (5), we get the following expression:

$$({}^mY {}^mt + {}^pY {}^pt)S_1 - {}^mY {}^mt {}^mq_{11}H_1 - {}^pY {}^pt {}^pd_{31}E_3 = 0 \quad (6)$$

Hence, for planar deformations, we obtain an expression in the form:

$$S_1 = \frac{{}^mY {}^mt {}^mq_{11}H_1 + {}^pY {}^pt {}^pd_{31}E_3}{\bar{Y}t}, \quad (7)$$

where  $\bar{Y} = ({}^mY {}^mt + {}^pY {}^pt)/t$  is the average value of Young's modulus of the structure,  $t = {}^mt + {}^pt$  is total thickness of the sample.

Substituting the obtained expression into Eq. (3) and using the open-circuit condition, which in this case has the form  ${}^pD_3 = 0$  we get for the electric field induced in the piezoelectric due to planar deformations the following expression:

$$E_{3,plan} = - \frac{{}^pY {}^pd_{31} {}^mq_{11}}{\varepsilon_{33}(1 - k_p^2(1 - {}^pY {}^pt/\bar{Y}t))} \frac{{}^mY {}^mt}{\bar{Y}t} H_1 \quad (8)$$

Using the definition of MEVC in the form  $\alpha_E = E_3/H_1$ , we obtain the following expression for the contribution to it from planar deformations:

$$\alpha_{E,plan} = - \frac{{}^pY {}^pd_{31} {}^mq_{11}}{\varepsilon_{33}(1 - k_p^2(1 - {}^pY {}^pt/\bar{Y}t))} \frac{{}^mY {}^mt}{\bar{Y}t} \quad (9)$$

Expression (9) for MEVC coincides with the expression obtained in the work [7] during the passage to the limit when the frequency of the magnetic field tends to zero.

There is a parameter in expression (9)  $k_p^2 \ll 1$ ; therefore, this expression can be simplified by writing it in the form:

$$\alpha_{E,plan} = - \frac{{}^pY {}^pd_{31} {}^mq_{11}}{\varepsilon_{33}} \frac{{}^mY {}^mt}{\bar{Y}t} \quad (10)$$

Eq. (9) and Eq. (10) make it possible take to analyze the dependence of MEVC on the physical parameters of the magnet and piezoelectric and the ratio of their layer's thicknesses.

We will assume that the bond between the layers is ideal and, as a result, for the deformations of a piezoelectric and a magnet, the relation

$$S_1 = \frac{(z - z_0)}{\rho}, \quad (11)$$

where  $z_0$  is a coordinate of the neutral line,  $\rho$  is the radius of curvature of the neutral line, which is related to the bending moment by the relation [11]

$$\frac{1}{\rho} = \frac{M_y}{{}^m Y {}^m J_{z_0} + {}^p Y {}^p J_{z_0}}. \quad (12)$$

The following notations were introduced here  $M_y = \int_0^W dy \cdot \left( \int_{-{}^p t}^0 (z - z_0) {}^p T_1 dz + \int_0^{m_t} (z - z_0) {}^m T_1 dz \right)$  is a bending moment,  ${}^m J_{z_0}$ ,  ${}^p J_{z_0}$  are moments of inertia of sections about the neutral axis  $z_0$ , which, according to Steiner's theorem, are determined by the following expressions:

$${}^m J_{z_0} = \frac{1}{12} W ({}^m t)^3 + W {}^m t ({}^m t - z_0)^2 \quad (13)$$

$${}^p J_{z_0} = \frac{1}{12} W ({}^p t)^3 + W {}^p t ({}^p t + z_0)^2. \quad (14)$$

The neutral line position is determined from the equality to zero of the  $X$ -projection of the force, namely

$$\int_{-{}^p t}^0 {}^p T_1 dz + \int_0^{m_t} {}^m T_1 dz = 0 \quad (15)$$

Substituting into Eq. (15) the expressions for the components of the stress tensor and obtained from Eq. (1) and Eq. (2) and assuming the external influences to be weak, for the coordinate of the neutral line  $z_0$  we get the following expression:

$$z_0 = \frac{1}{2} \frac{{}^m Y {}^m t^2 - {}^p Y {}^p t^2}{\bar{Y} t}, \quad (16)$$

In the general case, the neutral layer can lie both in a piezoelectric and in a magnet. If the neutral layer is in a piezoelectric, then in this case one part of the piezoelectric that lies above the neutral layer undergoes tension (compression), the other part undergoes compression (tension). The resulting electric fields in different parts of the piezoelectric will have opposite directions, because of which the total electric field will decrease. If the neutral layer is in a magnet, then the bending moments arising under the action of the magnetic field in the parts located on opposite sides of the neutral layer will have opposite directions, as a result of which the total bending moment decreases. Optimal, from the point of view of obtaining the maximum ME response, is the case when the neutral layer is located at the interface between the magnet and piezoelectric, i.e., when  $z_0 = 0$ . This gives the optimal ratio between the thicknesses of the magnetic and piezoelectric, which, according to Eq. (16), will be determined by the equality

$${}^m Y {}^m t^2 = {}^p Y {}^p t^2. \quad (17)$$

The electric field arising in a piezoelectric because of bending deformations can be found from the open circuit condition, according to which we get

$$E_3(z) = -\frac{1}{{}^p \epsilon_{33}} {}^p d_{31} {}^p T_1 = -\frac{1}{{}^p \epsilon_{33}} {}^p d_{31} ({}^p Y {}^p S_1 - {}^p Y {}^p d_{31} E_3(z)). \quad (18)$$

Using the Eq. (After simple transformations, we get

$$E_3(z) = -\frac{{}^p d_{31} {}^p Y(z-z_0)}{{}^p \varepsilon_{33} D(1-k_p^2)} (0.5^m t - z_0)^m q_{11} {}^m Y^m t H_1. \quad (19)$$

where  $D = ({}^m Y^m J_{z0} + {}^p Y {}^p J_{z0})/W$  is cylindrical bending stiffness.

The average value of the electric field is determined from the relation

$$\langle E_3 \rangle = \frac{1}{p_t} \int_{-p_t}^0 E_3(z) dz. \quad (20)$$

Substituting Eq. (19) into Eq. (20) and integrating, we obtain

$$\langle E_3 \rangle = -\frac{{}^p d_{31} {}^p Y(0.5^p t + z_0)}{{}^p \varepsilon_{33} D(1-k_p^2)} (0.5^m t - z_0)^m q_{11} {}^m Y^m H_1 \quad (21)$$

Using the definition of MEVC  $\alpha_E = \langle E_3 \rangle / H_1$  we can obtain for it an expression due to bending deformations in the form

$$\alpha_{E,bend} = -\frac{{}^p d_{31} {}^m q_{11} {}^p Y {}^m Y^m}{{}^p \varepsilon_{33} D(1-k_p^2)} (0.5^p t + z_0)(0.5^m t - z_0). \quad (22)$$

Equation (22) allows calculating the MEVC arising from bending deformations, using the physical parameters of the composite materials and the geometric characteristics of the structure.

## RESULTS AND DISCUSSION

The MEVC of the structure is equal to the sum of the contributions arising from planar and bending deformations, i.e.

$$\alpha_{E,total} = \alpha_{E,plan} + \alpha_{E,bend}. \quad (23)$$

It should be noted that the contributions from planar and bending deformations enter the sum with opposite signs. In planar oscillations, deformations arising in a magnet under the action of a magnetic field cause deformations of the same sign in a piezoelectric. For example, in the case of a magnet with positive magnetostriction, tensile deformations occur in the magnetostrictive layer, which, being transmitted through the interface, cause tensile deformations in the piezoelectric. In the case of bending, tensile deformations in a magnet cause compression deformation in a piezoelectric, resulting in an electric field directed opposite to the electric field caused by planar deformations. Both contributions are proportional to the product of the piezoelectric tensor  $d$  by the piezomagnetic tensor  $q$  and Young's modulus of the piezoelectric  ${}^p Y$  and are inversely proportional to the permittivity  ${}^p \varepsilon_{33}$ ; they do not depend on the width and length of the sample, but in different ways depend on the ratio of the thicknesses of the piezoelectric and magnetic. Figures 2-5 show the MEVC dependences for the structures nickel - PZT, permendur - PZT, Terfenol-D - PZT, Metglas - PZT depending on the thickness of the magnet at a fixed thickness of the piezoelectric. The parameters used for the calculations are presented in Table 1.

Table 1 Parameters of materials of composite structures

Material	Young's modulus $Y$ , GPa	Density $\rho$ , kg/m <sup>3</sup>	Piezomodules $d_{31}$ , pC/N; $q_{11}$ , ppm/Oe	Permittivity $\epsilon$
PZT	66.7	8.2	-175	1750
Ni	215	8.9	-0.06	-
Pe	207	8.1	0.1	-
Terfenol-D	62.5	8.5	0.3	-
Metglas	110	8.2	0.3	-

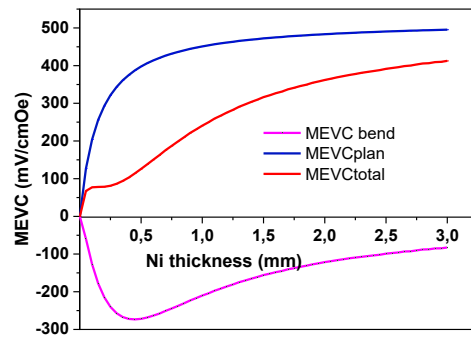


Fig. 2 Dependence of MEVC on the thickness of a magnet for the Ni - PZT structure with the piezoelectric thickness  $p_t=0.5$  mm.

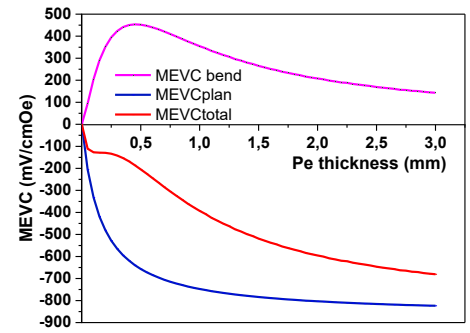


Fig. 3 Dependence of MEVC on the thickness of a magnet for the Pe - PZT structure with the piezoelectric thickness  $p_t=0.5$  mm

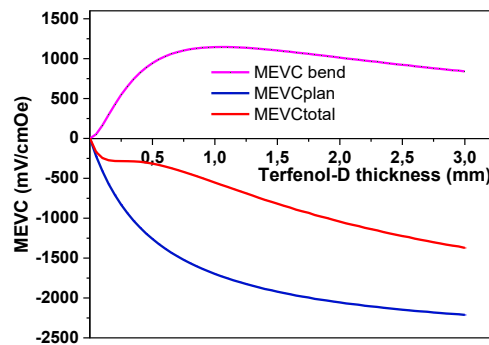


Fig. 4 Dependence of MEVC on the thickness of a magnet for the Terfenol-D - PZT structure with the piezoelectric thickness  $p_t=0.5$  mm.

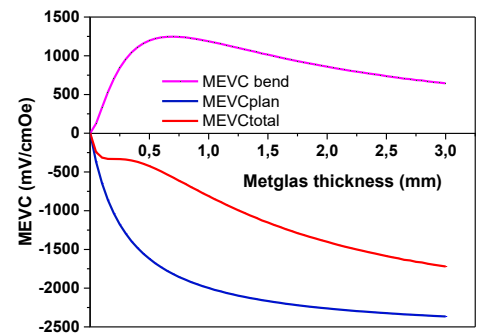


Fig. 5 Dependence of MEVC on the thickness of a magnet for the Metglas - PZT structure with the piezoelectric thickness  $p_t=0.5$  mm.

As can be seen from the figures, with an increase in the thickness of the magnet, the MEVC, caused by planar deformations, monotonically increases, and tends to saturation at values of the thickness of the magnet much greater than the thickness of the piezoelectric. The maximum value of MEVC at saturation due to planar deformations, according to Eq. (9), will be determined by the relation

$$(\alpha_{E,plan})_{sat} = -\frac{{}^pY^pd_{31}^mq_{11}}{\varepsilon_{33}(1-k_p^2)} \quad (24)$$

In contrast, the MEVC associated with bending deformations first increases, then reaches a maximum when the neutral line coincides with the magnetic - piezoelectric interface, and then decreases. As already noted, these contributions have different signs, and the contribution from planar deformations exceeds the contribution from bending deformations over the entire range of variation of the magnetic thickness. The total MEVC also increases with an increase in the thickness of the magnet, and in the range where the contribution from bending deformations reaches a maximum, the growth slows down, and a small plateau is observed. It should also be noted that since nickel has negative magnetostriction and permendur, Terfenol-D and Metglas are positive, the MEVC for the nickel - PZT structure and nickel – permendur, Terfenol-D – PZT and Metglas – PZT have opposite signs.

## CONCLUSION

In bilayer magnetostrictive-piezoelectric structures, the magnetoelectric effect is associated with planar deformations and bending deformations that occur when the structure is placed in a magnetic field. The contribution to MEVC from planar deformations increases monotonically with an increase in the thickness of the magnet and reaches saturation when the thickness of the magnet is much greater than the thickness of the piezoelectric. The value of the contribution from bending deformations first increases, then reaches a maximum when the relation is fulfilled, and then monotonically tends to zero. The contributions from planar and bending deformations have different signs, and the contribution from planar deformations is greater than the contribution from bending deformations over the entire range of variation of the magnetic thickness. The total MEVC eventually increases with the increasing thickness of the magnet and tends to saturation with the thickness of the magnet much greater than the thickness of the piezoelectrica

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